Quantum Information Processing by NMR: A Status Report

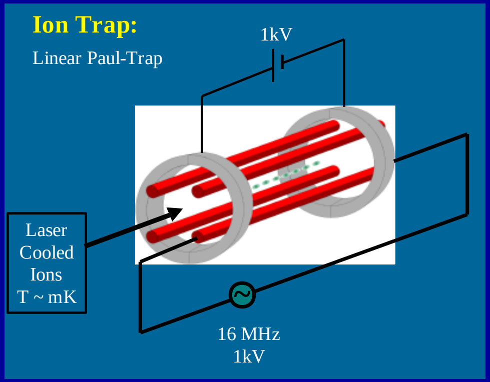
Anil Kumar

Department of Physics and NMR Research Centre
Indian Institute of Science, Bangalore-560012

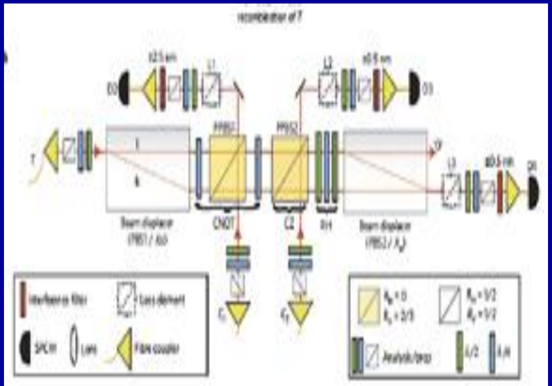
Quantum Information Processing and Applications
Harish-Chandra Research Institute, Allahabad. 02 - 08 December 2018

Experimental Techniques for Quantum Computation:

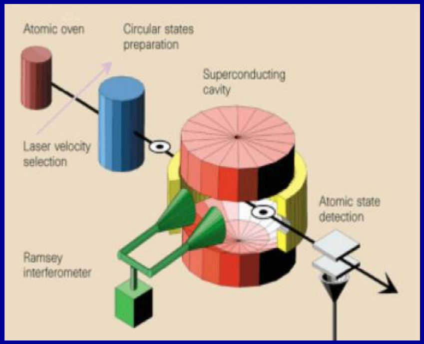
1. Trapped Ions



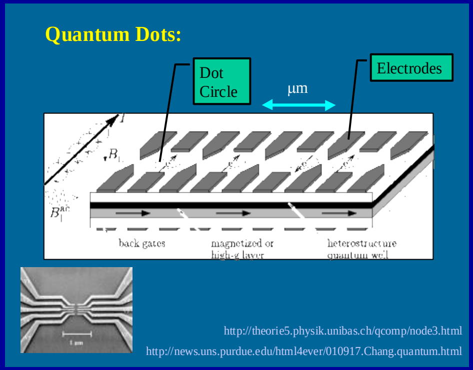
2. Polarized Photons Lasers



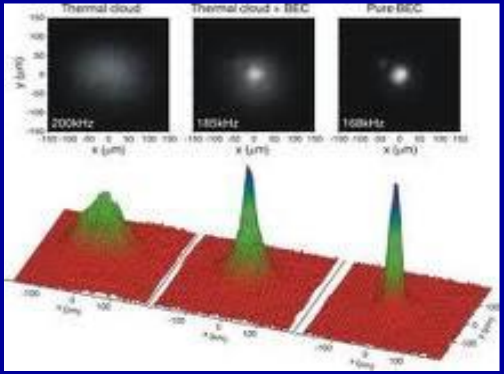
3. Cavity Quantum Electrodynamics (QED)



4. Quantum Dots



5. Cold Atoms



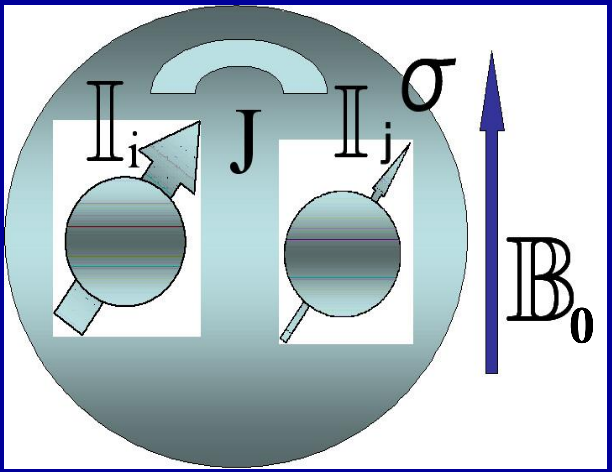
6. NMR

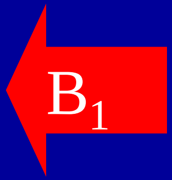
7. **Josephson junction/SQUID based qubits**

8. Fullerence based ESR quantum computer

Nuclear Magnetic Resonance (NMR)

1. Nuclear spins have small magnetic moments and behave as tiny quantum magnets.





2. When placed in a magnetic field (B_0), spin ½ nuclei orient either along the field (|0⟩ state) or opposite to the field (|1⟩ state) .

3. A transverse radio-frequency field (B_1) tuned at the Larmor frequency of spins can cause transition from |0⟩ to |1⟩ (by a 180^0 pulse = NOT Gate). Or put them in coherent superposition (by a 90^0 pulse = Hadamard Gate).

Single qubit gates.

4. Spins are coupled to other spins by indirect spin-spin (J) coupling, and controlled (C-NOT) operations can be performed using J-coupling.

Multi-qubit gates

NUCLEAR SPINS ARE QUBITS

NMR Research Centre, IISc

Field/ Frequency stability $= 1{:}10^{9}$

1 PPB

Why NMR?

> A major requirement of a quantum computer is that the coherence should last long.

> Nuclear spins in liquids retain coherence ~ 100's millisec and their longitudinal state for several seconds.

> A system of N coupled spins (each spin 1/2) form an N qubit Quantum Computer.

> Unitary Transform can be applied using R.F. Pulses and J-evolution and various logical operations and quantum algorithms can be implemented.

Addressability in NMR

NMR sample has ~ 10^{18} spins.

↓

Do we have 10^{18} qubits?

No - because, all the spins can't be individually addressed.

↓ **Progress so far**

Spins having different Larmor frequencies can be addressed in the frequency domain resulting-in as many "qubits" as Larmor frequencies, each having ~10^{18} spins. (ensemble computing).

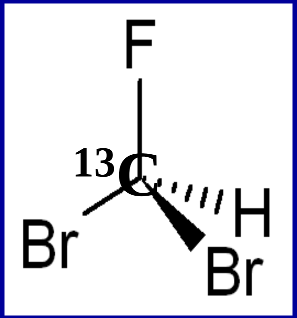
↓

One needs un-equal couplings between the spins, yielding resolved transitions in a multiplet, in order to encode information as qubits.

NMR Qubits

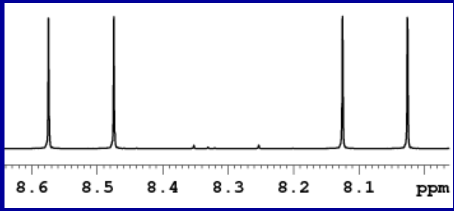
An example of a Hetero-nuclear three qubit system.

$^{13}CHFBr_2$

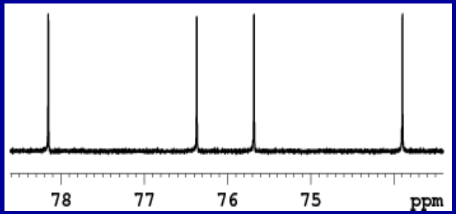


J_{CH} = 225 Hz
J_{CF} = -311 Hz
J_{HF} = 50 Hz

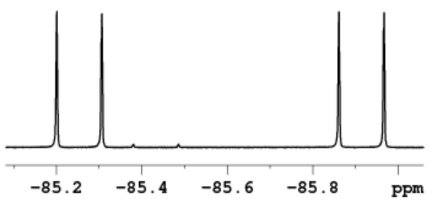
^{1}H = 500 MHz



^{13}C = 125 MHz



^{19}F = 470 MHz

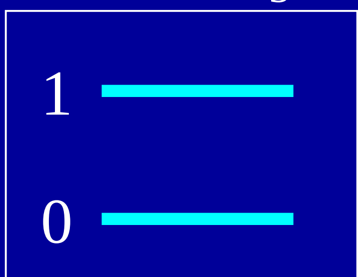


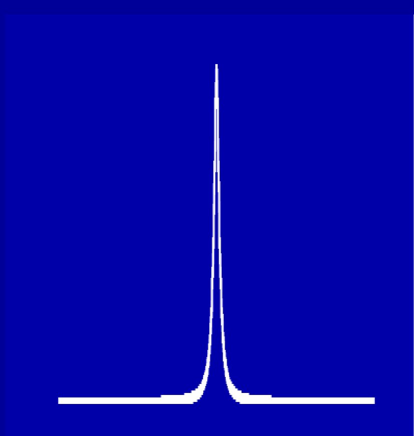
Br (spin 3/2) is a quadrupolar nucleus, is decoupled from the rest of the spin system and can be ignored.

Homo-nuclear spins having different Chemical shifts (Larmor frequencies) also form multi-qubit systems

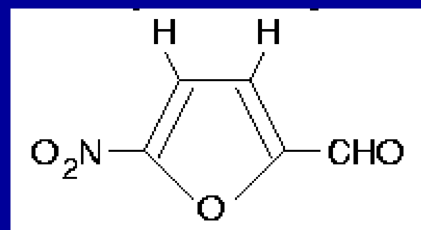
1 Qubit

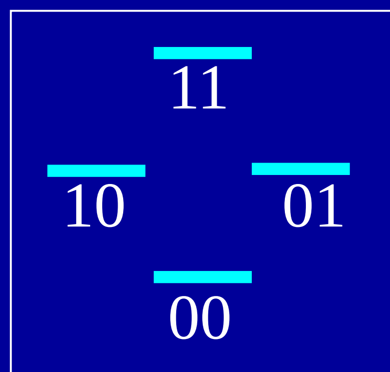
$CHCl_3$

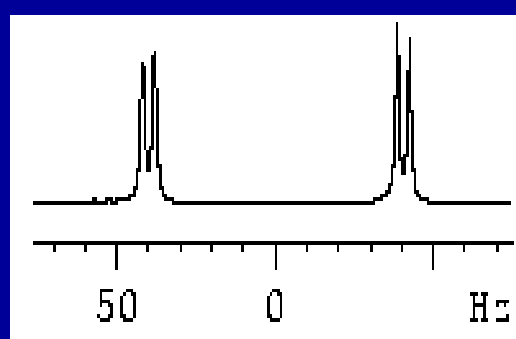




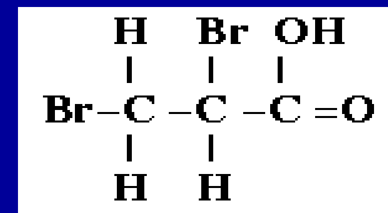
2 Qubits

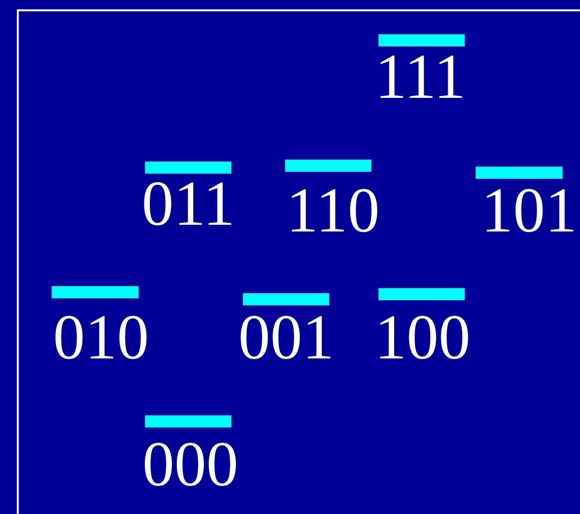


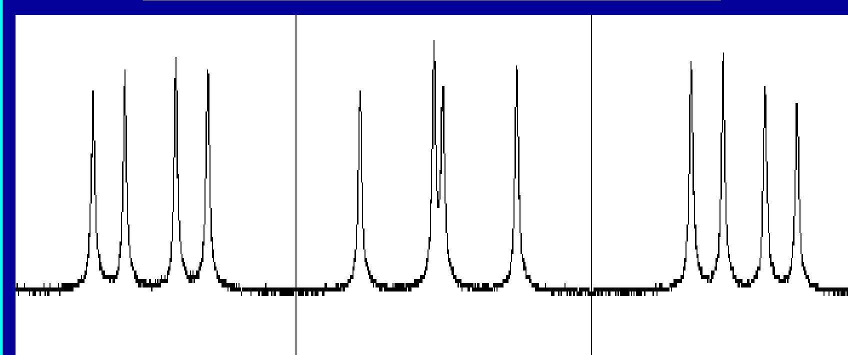




3 Qubits







Pure States:

$$Tr(\rho) = Tr(\rho^2) = 1$$

For a diagonal density matrix, this condition requires that all energy levels except one have zero populations.

Such a state is difficult to prepare in NMR

Pseudo-Pure States

Under High Temperature Approximation

$\boldsymbol{\rho = 1/N (\alpha 1 + \Delta\rho)}$ Here $\boldsymbol{\alpha = 10^5}$ **and** $\boldsymbol{U\, 1\, U^{-1} = 1}$

We create a state in which all levels except one have EQUAL populations. Such a state mimics a pure state.

Pseudo-Pure State

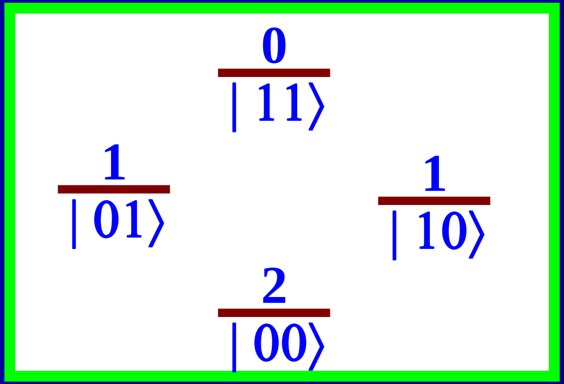
In a two-qubit Homo-nuclear system:

(Under High Field Approximation)

(i) Equilibrium:

$$\rho = 10^5 + \Delta\rho = \{2, 1, 1, 0\}$$

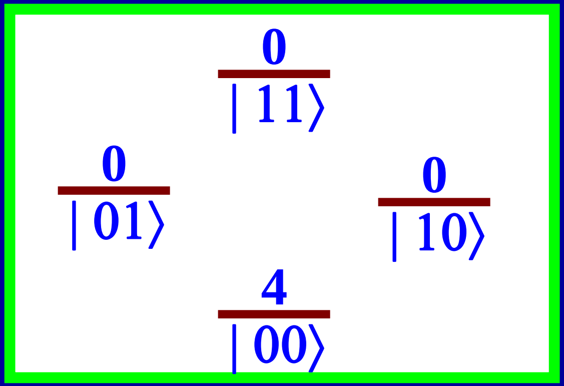
$$\Delta\rho \sim I_{z1}+I_{z2} = \{1, 0, 0, -1\}$$



(ii) Pseudo-Pure

$$\Delta\rho = \{4, 0, 0, 0\}$$

$$\Delta\rho \sim I_{z1}+I_{z2} + 2\, I_{z1}I_{z2}$$
$$= \{3/2, -1/2, -1/2, -1/2\}$$



Preparation of Pseudo-Pure States

- Spatial Averaging — Cory, Price, Havel, PNAS, 94, 1634 (1997)
- Logical Labeling — N. Gershenfeld et al, Science, 275, 350 (1997)
  Kavita, Arvind, Anil Kumar, Phy. Rev. **A 61**, 042306 (2000)
- Temporal Averaging — E. Knill et al., Phy. Rev. **A57**, 3348 (1998)
- Pairs of Pure States (POPS) — B.M. Fung, Phys. Rev. **A 63**, 022304 (2001)
- Spatially Averaged Logical Labeling Technique (SALLT)

  T. S. Mahesh and Anil Kumar, Phys. Rev. **A 64**, 012307 (2001)
- Using long lived Singlet States

  S.S. Roy and T.S. Mahesh, Phys. Rev. **A 82**, 052302 (2010).

Spatial Averaging:

Cory, Price, Havel, PNAS, 94, 1634 (1997)

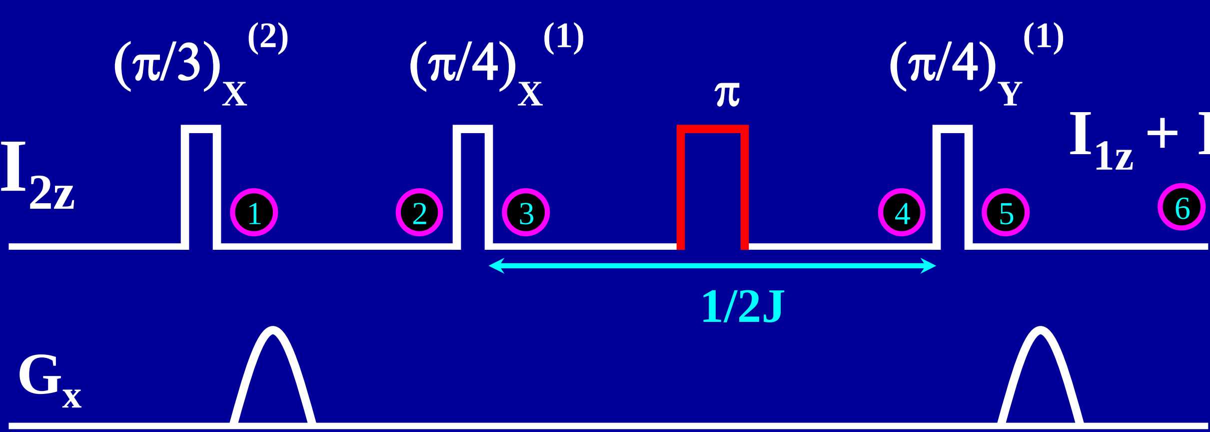
Most commonly used method

$$I_{1z} = 1/2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad I_{2z} = 1/2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad 2I_{1z}I_{2z} = 1/2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$I_{1z} + I_{2z} + 2I_{1z}I_{2z} = 1/2 \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \longleftarrow$$

Pseudo-pure state

$Eq. = I_{1z} + I_{2z}$



$I_{1z} + I_{2z} + 2I_{1z}I_{2z}$

Gradient Pulses make this a non-unitary operation

Achievements of NMR - QIP

√ 1. **Preparation of Pseudo-Pure States**

√ **2. Quantum Logic Gates**

√ **3. Deutsch-Jozsa Algorithm**

√ **4. Grover's Algorithm**

√ **5. Hogg's algorithm**

√ **6. Berstein-Vazirani parity algorithm**

√ **7. Quantum Games**

√ **8. Creation of EPR and GHZ states**

√ **9. Entanglement transfer**

√ **10. Quantum State Tomography**

√ **11. Geometric Phase in QC**

√ **12. Adiabatic Algorithms**

√ **13. Bell-State discrimination**

14. Error correction

15. Teleportation

16. Quantum Simulation √

17. Quantum Cloning

18. Shor's Algorithm

√ **19. No-Hiding Theorem**

√ Also performed in our Lab.

Maximum number of qubits achieved in our lab: 8

In other labs.: 12 qubits;

Negrevergne, Mahesh, Cory, Laflamme et al., Phys. Rev. Letters, 96, 170501 (2006).

Our own contributions are distributed into 8 Ph.D. theses and nearly 40 Publications.

A few of these are briefly highlighted in the following.

Some Selected Developments From Our Laboratory

(i) **Multipartite quantum correlations reveal frustration in quantum Ising spin systems: Experimental demonstration.**
K. Rama Koteswara Rao, Hemant Katiyar, T. S. Mahesh, Aditi Sen(De), Ujjwal Sen and Anil Kumar; Phys. Rev. A 88, 022312 (2013).

(ii) An NMR simulation of Mirror inversion propagator of an XY spin Chain.
K. R. Koteswara Rao, T.S. Mahesh and Anil Kumar, Phys. Rev. A 90, 012306 (2014).

(iii) Quantum simulation of 3-spin Heisenberg XY Hamiltonian in presence of DM interaction- entanglement preservation using initialization operator.
V.S. Manu and Anil Kumar, Phys. Rev. A 89, 052331 (2014).

Quantum simulation of frustrated Ising spins by NMR

K. Rama Koteswara Rao[1], Hemant Katiyar[3], T.S. Mahesh[3], Aditi Sen (De)[2], Ujjwal Sen[2] and Anil Kumar[1]:

Phys. Rev A 88 , 022312 (2013).

[1] **Indian Institute of Science, Bangalore**
[2] **Harish-Chandra Research Institute, Allahabad**
[3] **Indian Institute of Science Education and Research, Pune**

A spin system is frustrated when the minimum of the system energy does not correspond to the minimum of all local interactions. Frustration in electronic spin systems leads to exotic materials such as spin glasses and spin ice materials.

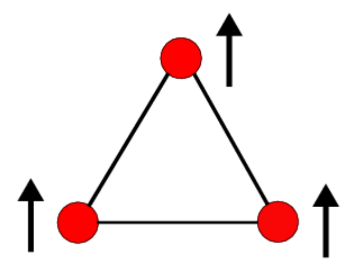
3-spin transverse Ising system

$$\mathcal{H} = h(\sigma_{1x} + \sigma_{2x} + \sigma_{3x}) + J(\sigma_{1z}\sigma_{2z} + \sigma_{2z}\sigma_{3z} + \sigma_{1z}\sigma_{3z})$$

$$J \gg h$$

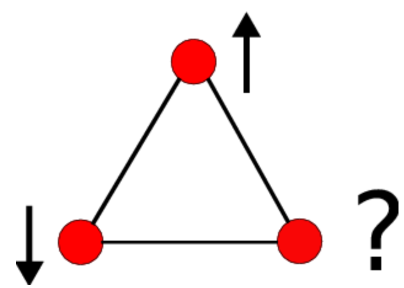
If J is negative $\longrightarrow$ Ferromagnetic

The system is non-frustrated



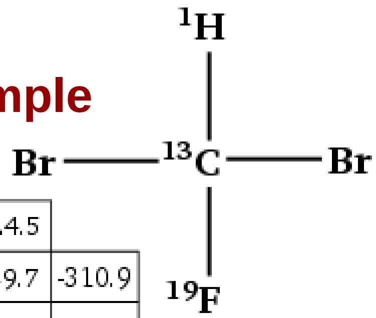
If J is positive $\longrightarrow$ Anti-ferromagnetic

The system is frustrated



Experiment 1: Using a hetero-nuclear spin system

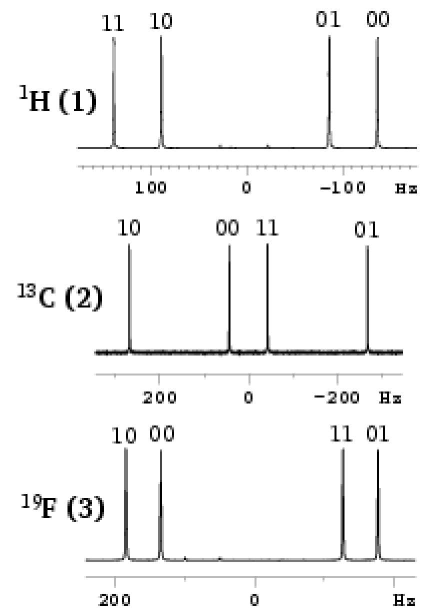
Sample



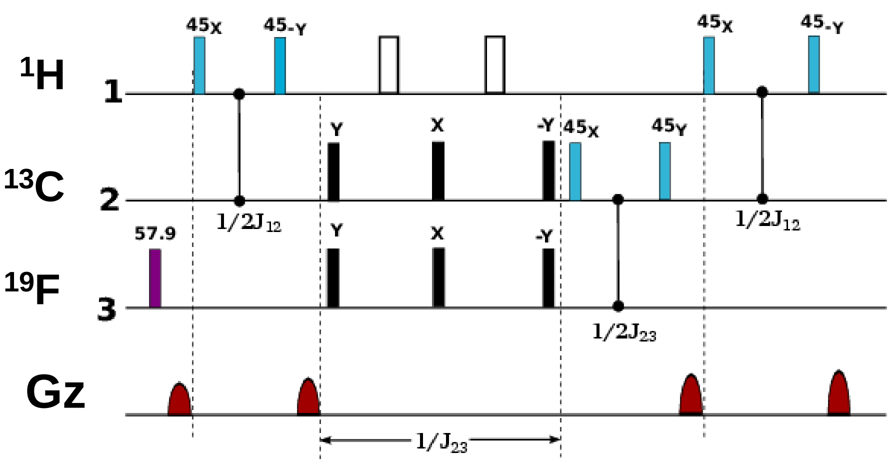
C	224.5	
F	49.7	-310.9
	H	C

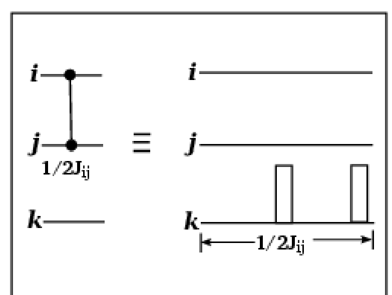
$$\mathcal{H}_{\text{int}} = -\sum_{i=1}^{3} \pi \nu_i \, \sigma_z^i + \sum_{i<j,=1}^{3} \frac{\pi}{2} J_{ij} \sigma_z^i \sigma_z^j$$

Equilibrium spectra

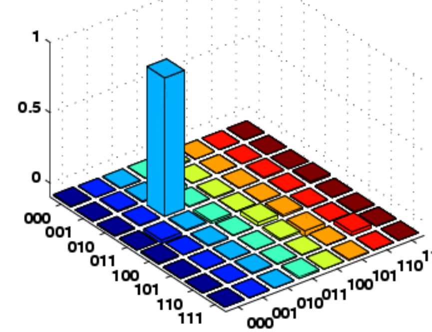


Pulse sequence to prepare PPS (using only the nearest neighbour couplings)





Tomography of 010 PPS



The system is initially prepared in the ground state of

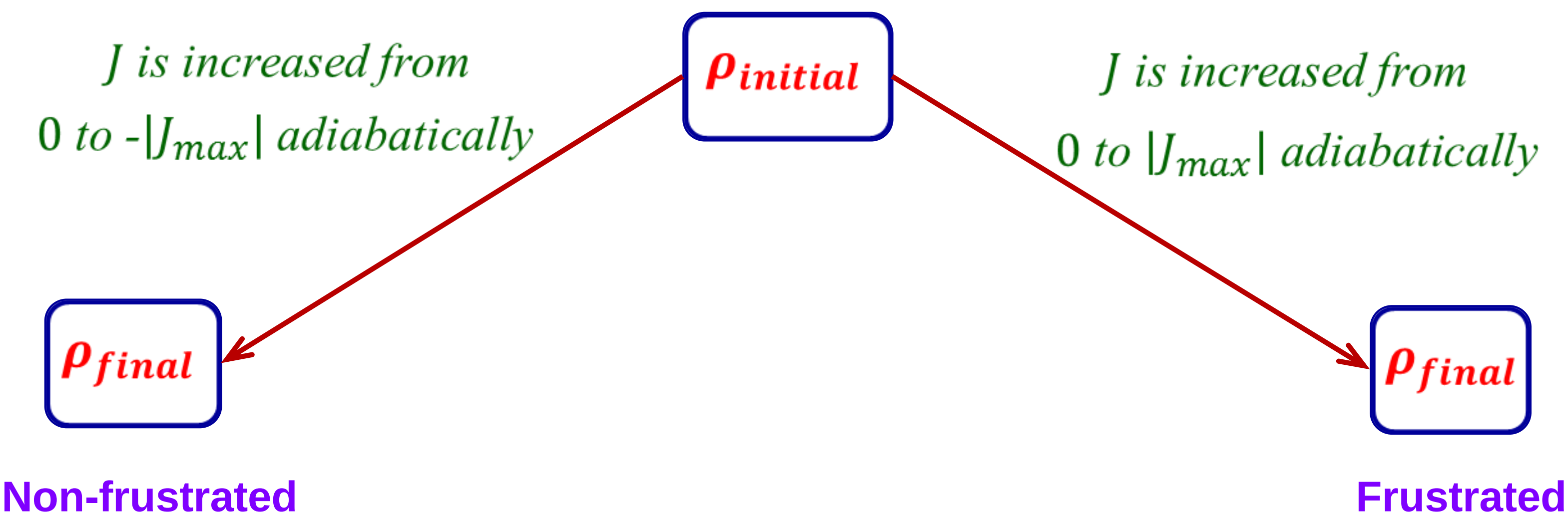
$$\mathcal{H}(t=0) = h(\sigma_{1x} + \sigma_{2x} + \sigma_{3x})$$

$$\rho_{initial} = |{-}{-}{-}\rangle\langle{-}{-}{-}|;$$

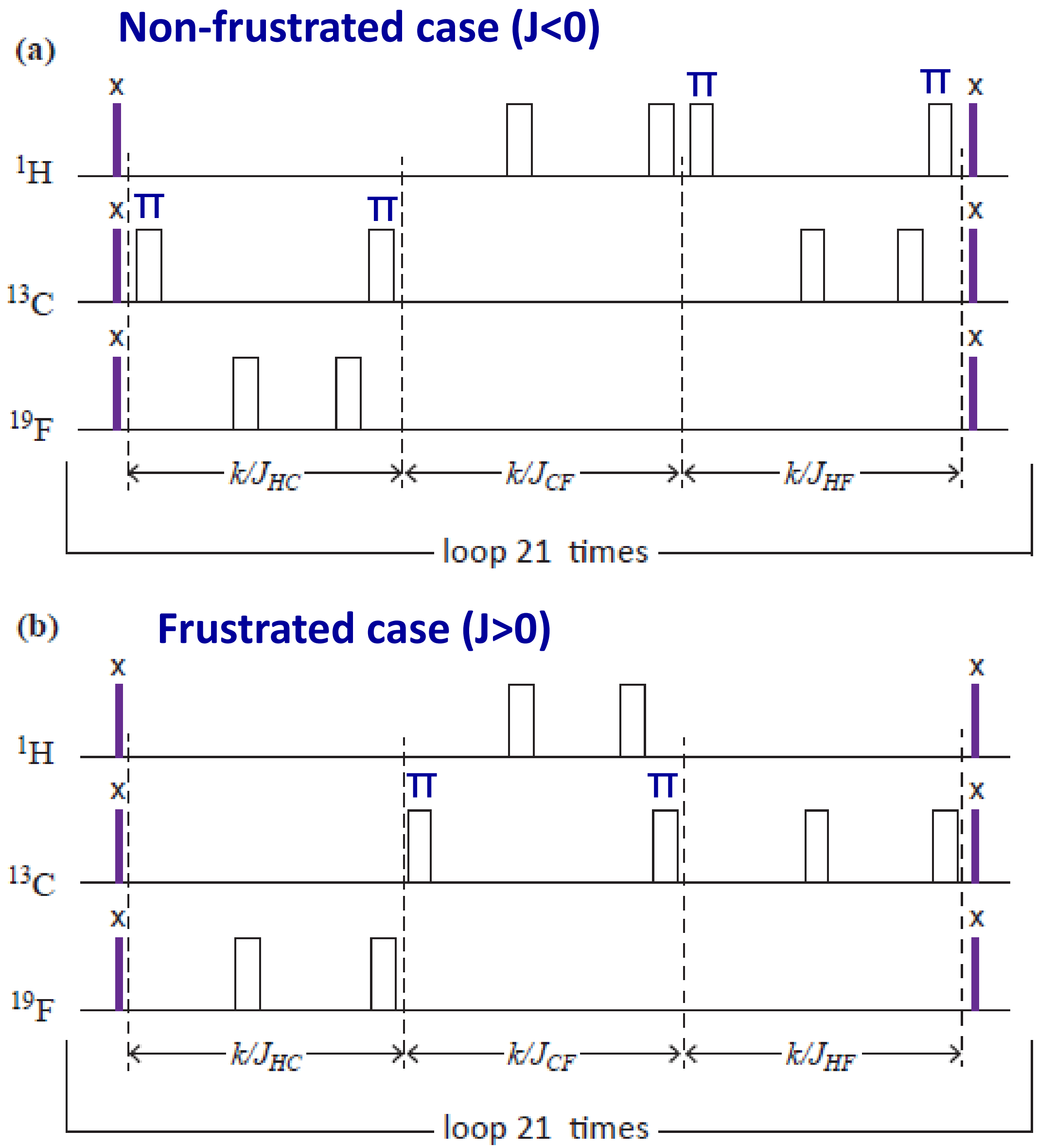
$$|{-}\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$\rho_{initial}$ is prepared by first creating a 3-qubit |000> PPS , using spatial averaging, followed by a Hadamard gate on each qubit.

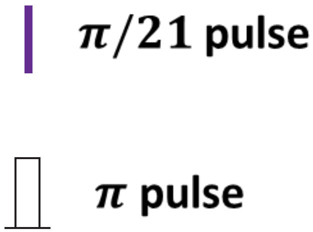
$$\mathcal{H}(t) = h(\sigma_{1x} + \sigma_{2x} + \sigma_{3x}) + J(t)(\sigma_{1z}\sigma_{2z} + \sigma_{2z}\sigma_{3z} + \sigma_{1z}\sigma_{3z})$$



Pulse Sequences for Frustration Study

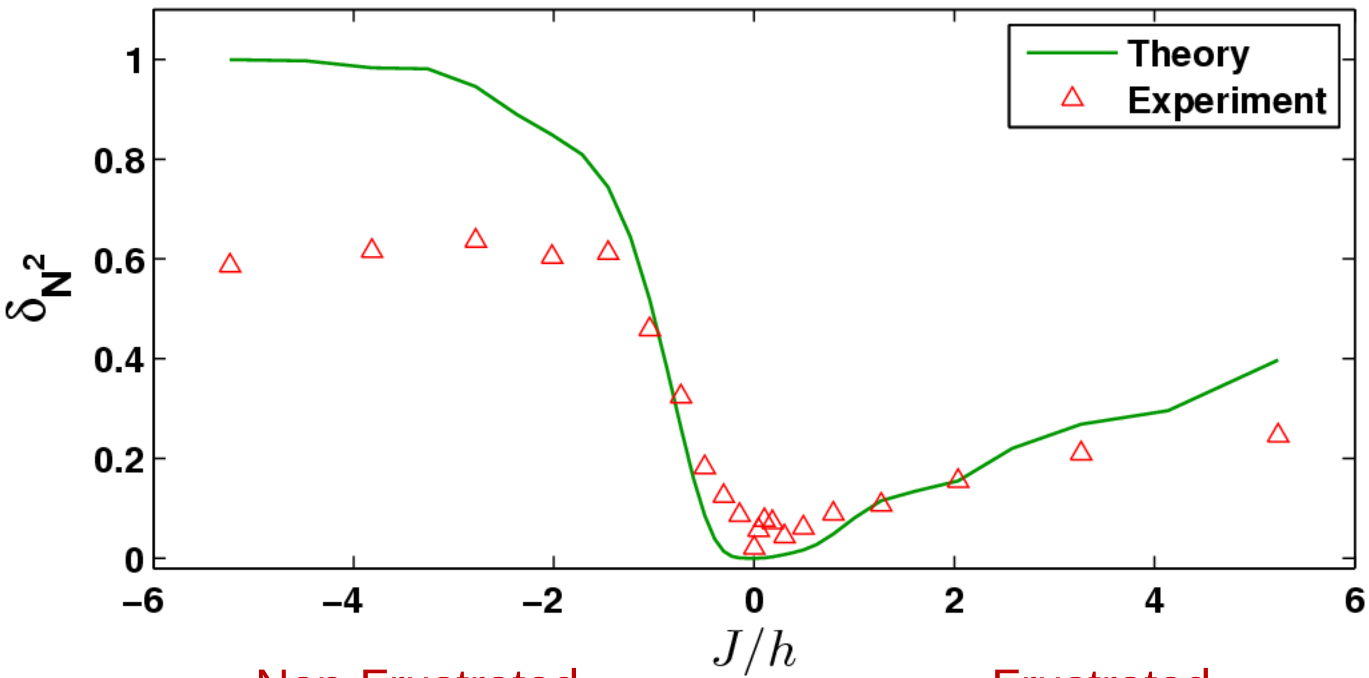


$J_{CF} < 0$, J_{HC}, $J_{HF} > 0$.
π pulses on C and H effectively make them < 0



$k = \frac{m}{2M}$,
$\frac{m}{M}$ is varied from 0 to 1 in 21 steps

Results



Non-Frustrated Frustrated

While the trend was correct, the experimental results did not match well with Theory, especially in the Non-Frustrated region. The RF In-homogeneity and evolution during RF pulses were suspected to be the reasons.

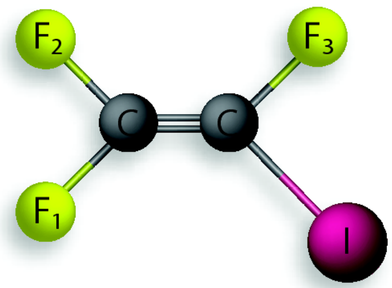
We therefore used numerical optimization techniques which could take into account these features.

Experiment 2

$$\mathcal{H}_{NMR} = -\sum_{i=1}^{3} \pi\nu_i \sigma_z^i + \sum_{i<j,=1}^{3} \frac{\pi}{2} J_{ij} \sigma_z^i \sigma_z^j$$

$$\mathcal{H}_{rf} = x \sum_{i=1}^{3} \frac{1}{2}\sigma_x^i + y \sum_{i=1}^{3} \frac{1}{2}\sigma_y^i$$

Chemical Structure of trifluoroiodoethylene and Hamiltonian parameters



A three qubit system

	F_1	F_2	F_3
F_1	11860.8	69.9	47.4
F_2		0	-128.3
F_3			-17379.1

Diagonal elements are the chemical shifts (ν_i) and off-diagonal elements are the scalar coupling constants (J_{ij})

$|000\rangle$ Pseudo-Pure State (PPS) is prepared from the equilibrium by using the spatial averaging method.

The initial state $|---\rangle$ is prepared from the $|000\rangle$ PPS by applying a $\frac{\pi}{2}$ rotation with respect to $-y$ axis on all the three spins.

This rotation was realized by a numerically optimized amplitude and phase modulated radio frequency (RF) pulse using GRadient Ascent Pulse Engineering (GRAPE) technique[1].

The experiments have been carried out at a temperature of 290 K on Bruker AV 500 MHz liquid state NMR spectrometers.

[1]N. Khaneja and S. J. Glaser et al., J. Magn. Reson. 172, 296 (2005).

- All the unitary operators corresponding to the adiabatic evolution are also implemented by using GRAPE pulses.

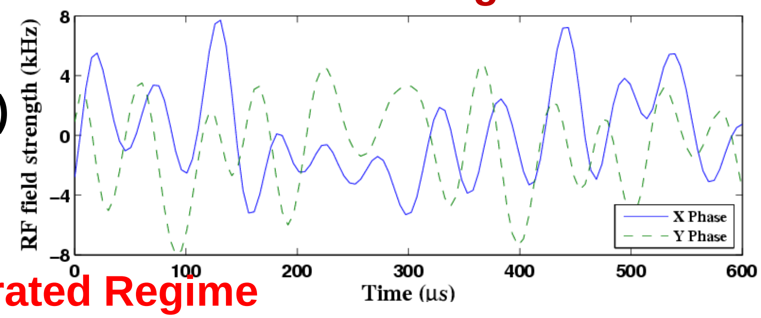
- The length of these pulses ranges between 2ms (for first data point) to 30 ms (the last (21) data point).

- Robust against RF field in-homogeneity.

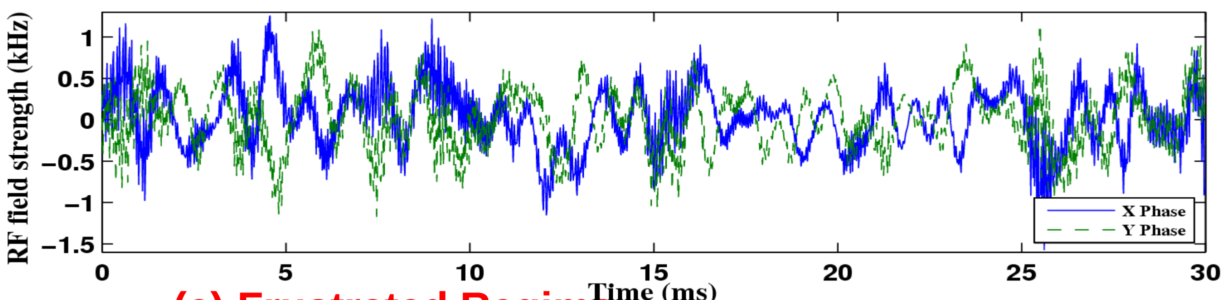
- The average Hilbert-Schmidt fidelity is greater than 0.995

π/2 rotation using GRAPE

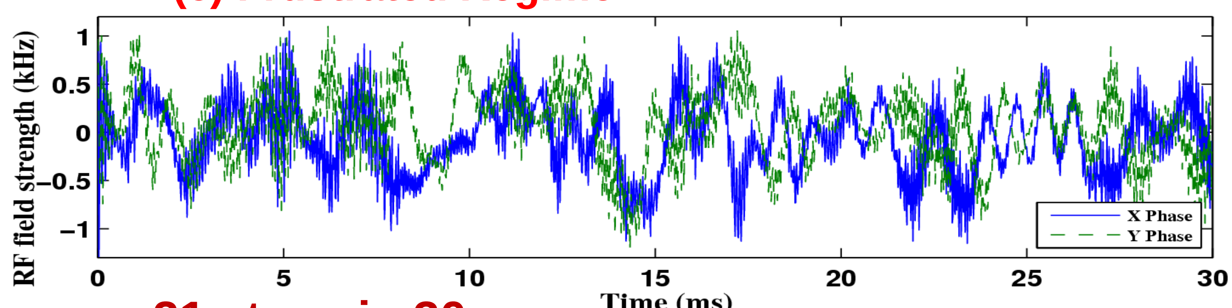
(a)



(b) Non-Frustrated Regime



(c) Frustrated Regime



21 steps in 30 ms

Quadrature components of the control fields of the GRAPE pulses corresponding to (a) $\frac{\pi}{2}-y$ rotation of all the spins and combined unitary operators which implements the last step in (b) non-frustrated and (c) frustrated regions.

Multipartite quantum correlations

Entanglement Score using deviation Density matrix

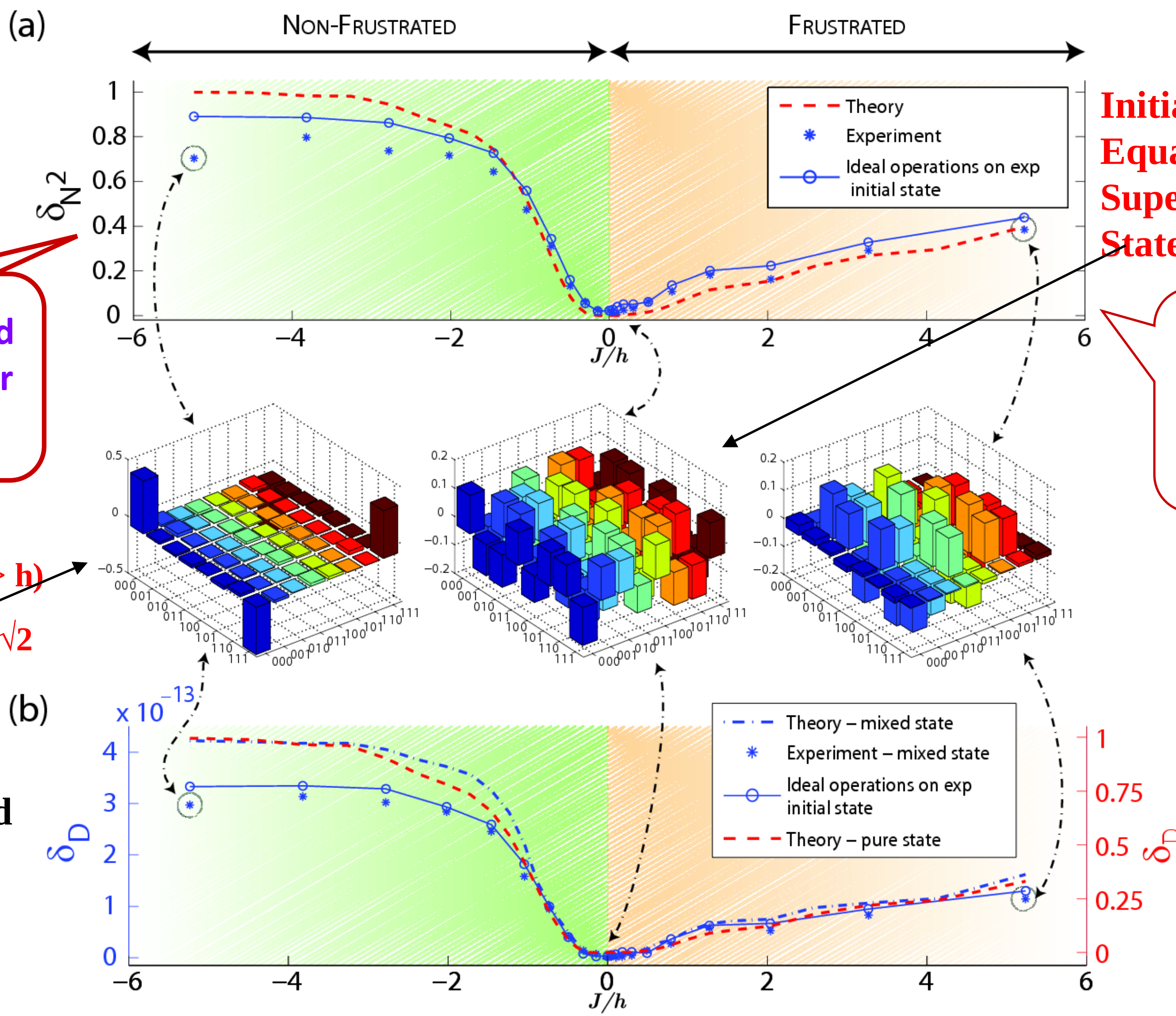
Non-frustrated regime: Higher correlations

Ground State GHZ State (J >> h)

(|000> - |111>)/√2

Fidelity = .984

Quantum Discord Score using full density matrix



Initial State: Equal Coherent Superposition State. Fidelity = .99

Frustrated regime: Lower correlations

Koteswara Rao et al. Phys. Rev A 88, 022312 (2013).

Conclusion

- The ground state of the 3-spin transverse Ising spin system has been simulated experimentally in both the frustrated and non-frustrated regimes using Nuclear Magnetic Resonance.

- To analyze the experimental ground state of this spin system, we used two different multipartite quantum correlation measures which are defined through the monogamy considerations of (i) negativity and of (ii) quantum discord. These two measures have similar behavior in both the regimes although the corresponding bipartite quantum correlations are defined through widely different approaches.

- The frustrated regime exhibits higher multipartite quantum correlations compared to the non-frustrated regime and the experimental data agrees with the theoretically predicted ones.

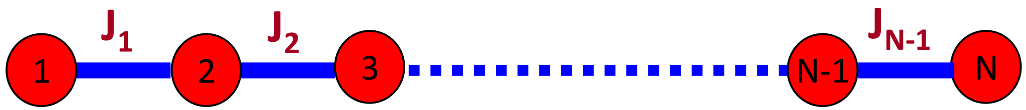
(ii) An NMR simulation of Mirror inversion propagator of an XY spin Chain.

K. R. Koteswara Rao, T.S. Mahesh and Anil Kumar, Phys. Rev. A 90, 012306 (2014).

In the last decade, there have been many interesting proposals in using spin chains to efficiently transfer quantum information between different parts of a quantum information processor.

Albanese et al have shown that mirror inversion of quantum states with respect to the center of an XY spin chain can be achieved by modulating its coupling strengths along the length of the chain. The advantage of this protocol is that non-trivial entangled states of multiple qubits can be transferred from one end of the chain to the other end.

Mirror Inversion of quantum states in an XY spin chain*



$$\exp(-it_0\mathcal{H}_{XY})\,|\psi_1\rangle|\psi_2\rangle\ldots|\psi_N\rangle = e^{i\varphi}|\psi_N\rangle|\psi_{N-1}\rangle\ldots|\psi_1\rangle$$

$$\mathcal{H}_{XY} = \sum_{i=1}^{N-1}\frac{J_i}{2}\left(\sigma_x^i\sigma_x^{i+1} + \sigma_y^i\sigma_y^{i+1}\right) \qquad \mathrm{J}_i = [i(N-i)]^{1/2}$$

- The above XY spin chain Hamiltonian generates the mirror image of any input state up to a phase difference.
- Entangled states of multiple qubits can be transferred from one end of the chain to the other end

*Albanese et al., Phys. Rev. Lett. 93, 230502 (2004)
*P Karbach, and J Stolze et al., Phys. Rev. A 72, 030301(R) (2005)

NMR Hamiltonian of a weakly coupled spin system

$$\mathcal{H}_{\text{int}} = -\pi \sum_i \nu_i \, \sigma_i^z + \frac{\pi}{2} \sum_{i<j} J_{ij} \, \sigma_i^z \sigma_j^z$$

Control Hamiltonian

$$\mathcal{H}_{\text{rf}}(t) = x(t) \sum_m \sigma_m^x + y(t) \sum_m \sigma_m^y$$

Simulation

$$U_{XY}\left(\tfrac{\pi}{2}\right) = \exp(-i\tfrac{\pi}{2}\mathcal{H}_{XY}) \;\approx\; U_{\text{sim}} = \mathcal{T} \exp\left[-i \int_0^T dt \;\; (\mathcal{H}_{\text{int}} + \mathcal{H}_{\text{rf}}(t))\right]$$

where $\mathcal{T}$ is the Dyson time-ordering operator

In practice

$$\boldsymbol{U}_{\textbf{sim}} = \boldsymbol{U}_{\mathbf{1}} \boldsymbol{U}_{\mathbf{2}} \cdots \boldsymbol{U}_{\boldsymbol{m}}$$

Here, $U_j = \exp\left[-i \, \Delta t_j \; (\mathcal{H}_{\text{int}} + \mathcal{H}_{\text{rf}}(t_j))\right]$

where $\mathcal{H}_{\text{rf}}(t_j)$ is constant in each step, and $\sum_{j=1}^{m} \Delta t_j = T$.

Simulation

$$U_{\text{sim}} = U_1 U_2 \cdots U_m$$

1) GRAPE algorithm

- Restricted to small number of spins
- Takes a lot of time to find a pulse sequence for arbitrary U

2) An algorithm by *A Ajoy et al. Phys. Rev. A 85, 030303(R) (2012)*

Product-decomposes any arbitrary U into a chosen operator basis.

i.e., $U = \prod_k \exp(-i\theta_k D_k)$, where $D_k \in \mathcal{B}$

- Here, we use a combination of these two algorithms to simulate the unitary evolution of the XY spin chain
- Specifically, we first product-decompose $U_{XY}\left(\frac{\pi}{2}\right)$ into the Pauli operator basis, and then the resulting unitaries are implemented with GRAPE technique

Ashok Ajoy, KRK Rao, Anil Kumar and P Rungta, Phys. Rev A (R), 85, 030303 (2012)

Product-decomposition of $U_{XY}\left(\frac{\pi}{2}\right)$ into the Pauli operator basis

4-spin chain

$$U_{XY}\left(\tfrac{\pi}{2}\right) = \exp\left(i\tfrac{\pi}{4}\,\sigma_1^x\sigma_2^z\sigma_3^z\sigma_4^x\right)\exp\left(i\tfrac{\pi}{4}\,\sigma_1^y\sigma_2^z\sigma_3^z\sigma_4^y\right)\exp\left(i\tfrac{\pi}{4}\,\sigma_2^x\sigma_3^x\right)\exp\left(i\tfrac{\pi}{4}\,\sigma_2^y\sigma_3^y\right)$$

5-spin chain

$$U_{XY}\left(\tfrac{\pi}{2}\right) = \exp\left(i\tfrac{\pi}{4}\,\sigma_1^x\sigma_2^z\sigma_3^z\sigma_4^z\sigma_5^y\right)\exp\left(i\tfrac{\pi}{4}\,\sigma_1^y\sigma_2^z\sigma_3^z\sigma_4^z\sigma_5^x\right)\exp\left(i\tfrac{\pi}{4}\,\sigma_2^x\sigma_3^z\sigma_4^y\right)$$
$$\times\ \exp\left(i\tfrac{\pi}{4}\,\sigma_2^y\sigma_3^z\sigma_4^x\right)\exp\left(i\tfrac{\pi}{2}\,\sigma_1^x\sigma_2^y\sigma_4^y\sigma_5^x\right)$$

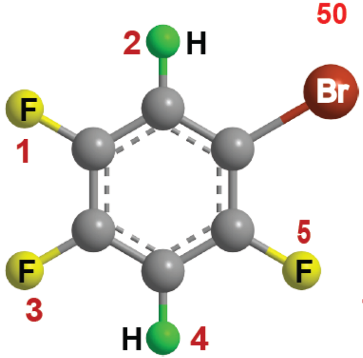
In the experiments, each of these decomposed operators are simulated using GRAPE technique

The number of operators in the decomposition increases only linearly with the number of spins (N).

Experiment

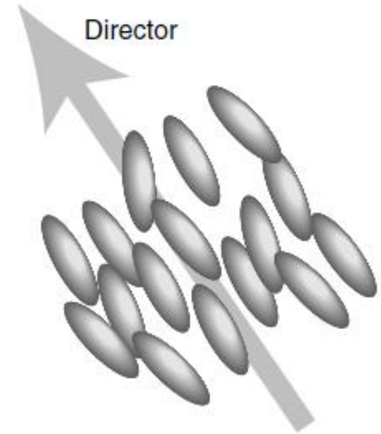
Molecular structure and Hamiltonian parameters

1	2	3	4	5	
-6743	1532	-26	55	116	1
	50	106	-7.6	54	2
		-3680	1270	277	3
			29	1556	4
				6029	5



5-spin system

The diagonal elements are ν_i and the off-diagonal elements are $(J_{ij} + 2D_{ij})$ in Hz



The dipolar couplings of the spin system get scaled down by the order parameter (~ 0.1) of the liquid-crystal medium.

The sample 1-bromo-2,4,5-trifluorobenzene is partially oriented in a liquid-crystal medium MBBA

The Hamiltonian of the spin system in the doubly rotating frame:

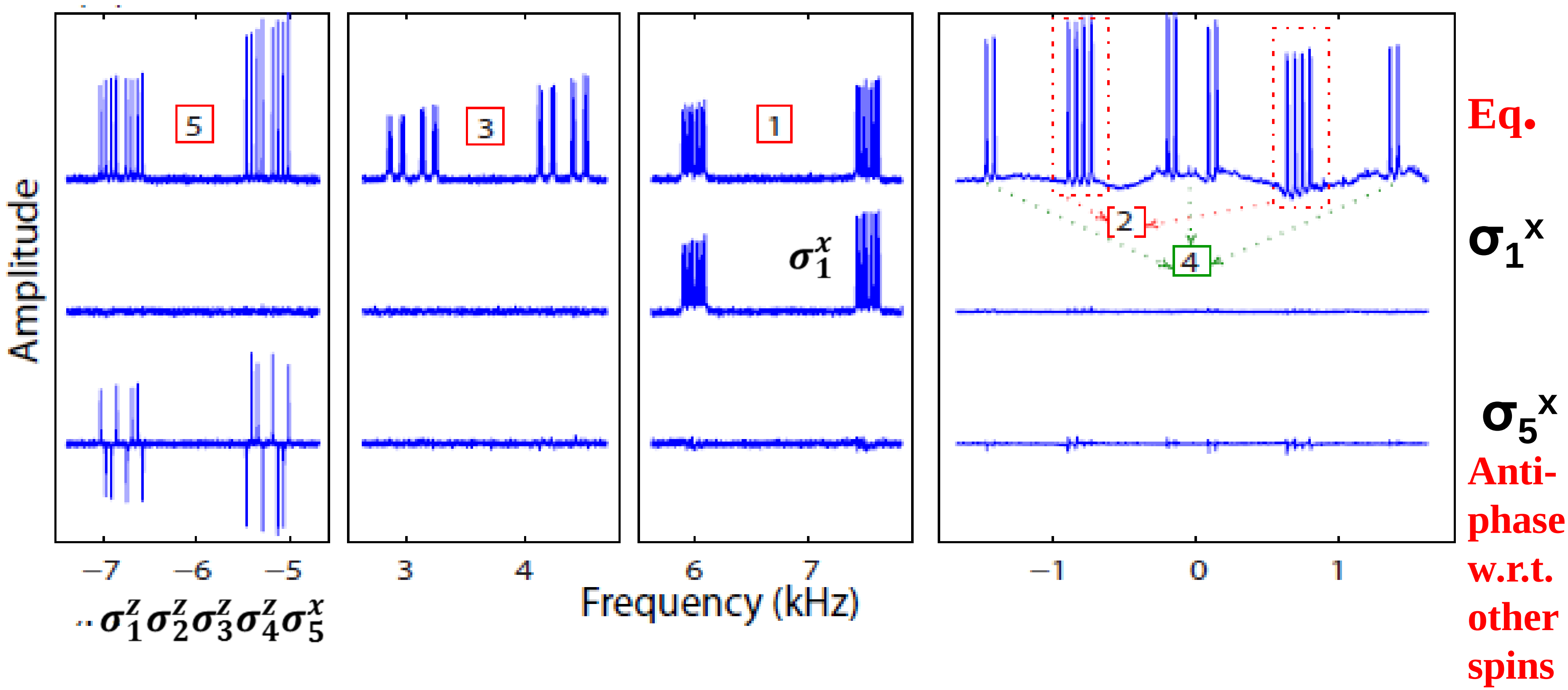
$$\mathcal{H}_{\text{int}} = -\pi \sum_i \nu_i \, \sigma_i^z + \frac{\pi}{2} \sum_{i<j} (J_{ij} + 2D_{ij}) \;\sigma_i^z \sigma_j^z$$

Coherence Transfer:
Mirror Inversion of a 5-spin initial state

$$\sigma_1^x \xrightarrow{U_{XY}(\pi/2)} \sigma_1^z\sigma_2^z\sigma_3^z\sigma_4^z\sigma_5^x$$

σ_5^x
Anti-phase w.r.t. other spins

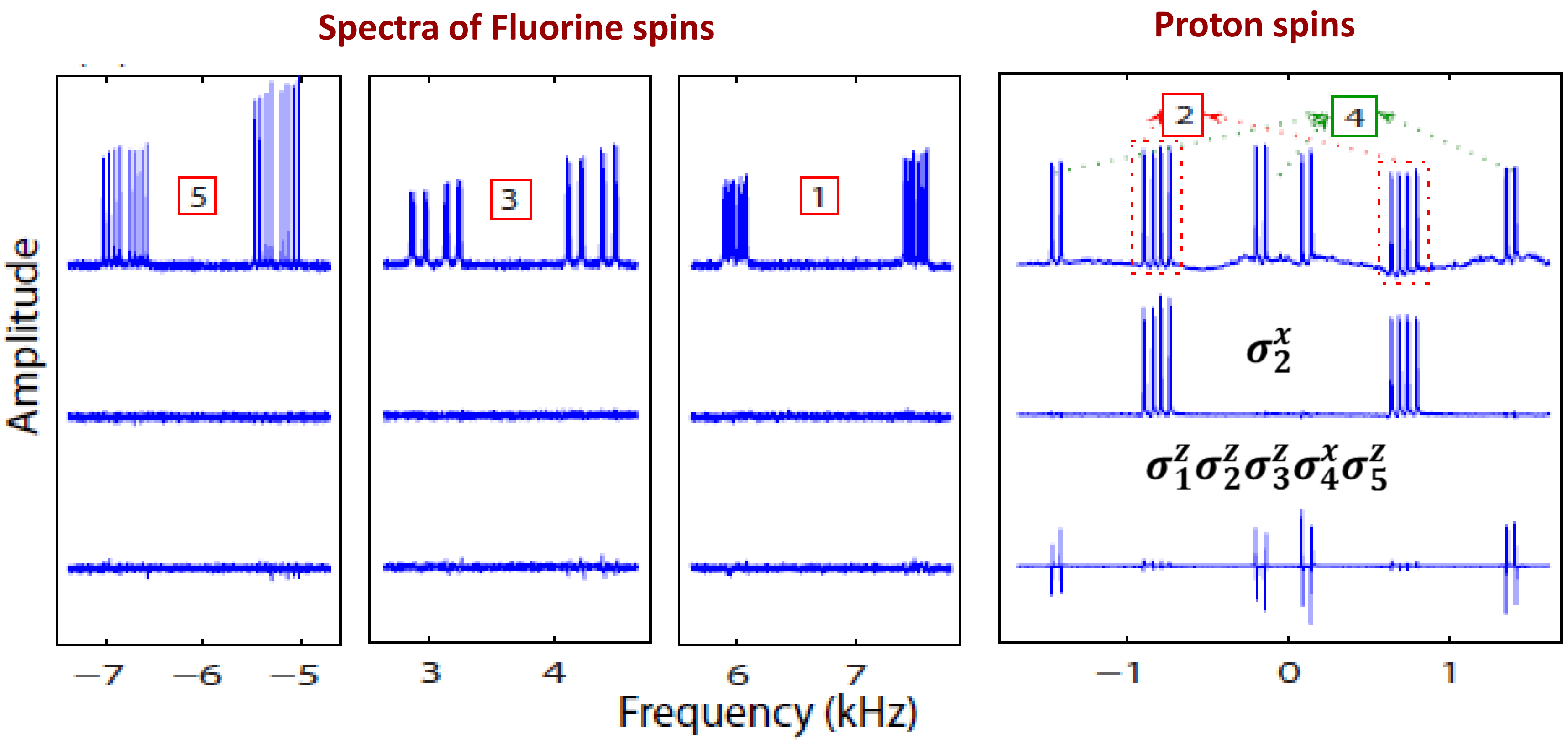
Spectra of Fluorine spins **Proton spins**



K R K Rao, T S Mahesh, and A Kumar, Phys. Rev. A , 90, 012306 (2014).

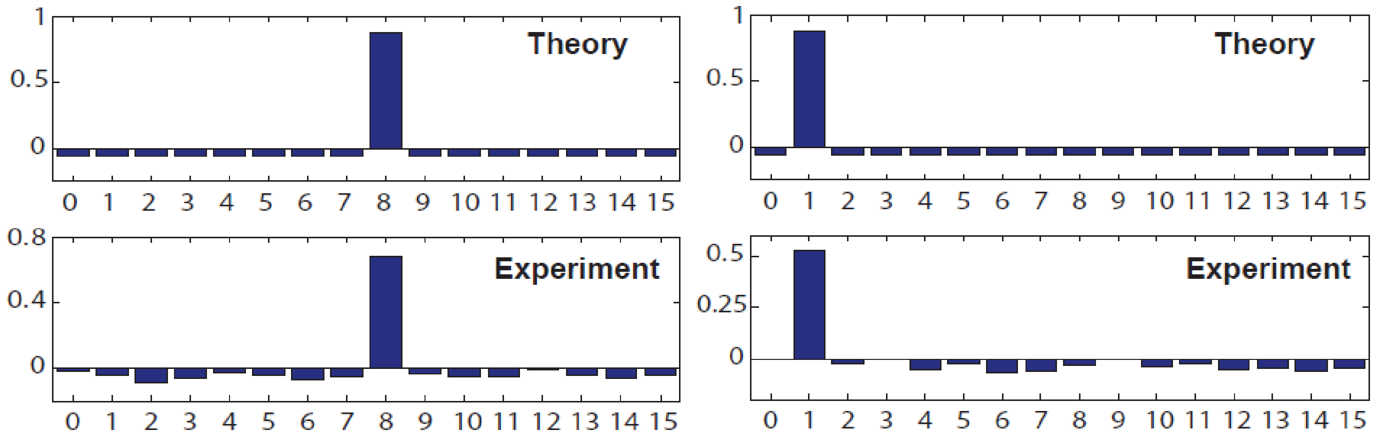
Coherence Transfer:
Spin 2 (in- phase) magnetization transferred to spin 4 (anti-phase w.r.t. other spins)

$$\sigma_2^x \xrightarrow{U_{XY}(\pi/2)} \sigma_1^z\sigma_2^z\sigma_3^z\sigma_4^x\sigma_5^z$$



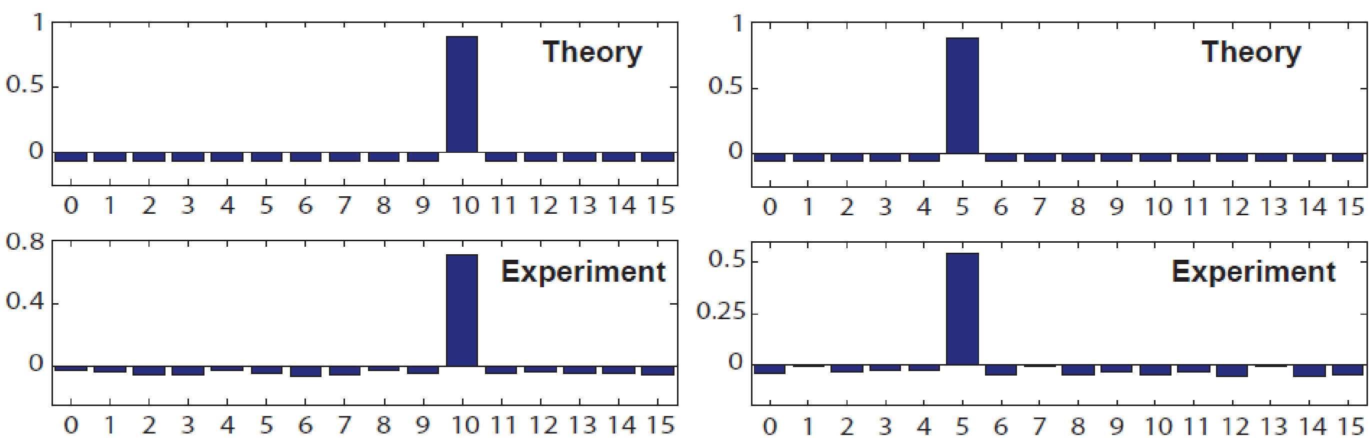
Quantum State Transfer:
Mirror Inversion of a 4-spin pseudo-pure initial states

$$|1000\rangle\langle 1000| \xrightarrow{U_{XY}(\pi/2)} |0001\rangle\langle 0001|$$



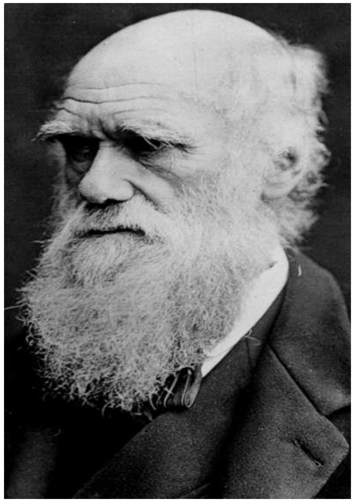
Diagonal part of the deviation density matrices (traceless)
The *x*-axis represents the standard computational basis in decimal form

$$|1010\rangle\langle 1010| \xrightarrow{U_{XY}(\pi/2)} |0101\rangle\langle 0101|$$

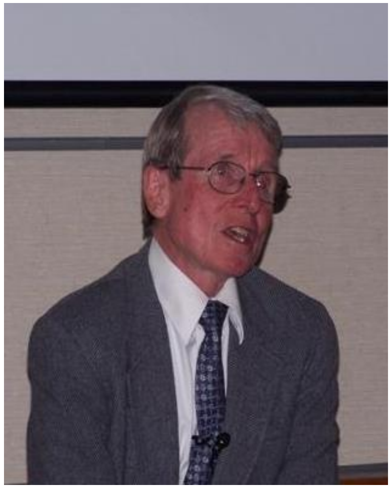


3. Use of Genetic Algorithm for Quantum Simulation of Dzyaloshinsky-Moriya (DM) interaction in presence of Heisenberg XY interaction. Entanglement preservation.

The Genetic Algorithm



Charles Darwin 1866
1809-1882



John Holland

Directed search algorithms based on the mechanics of biological evolution

Developed by John Holland, University of Michigan (1970's)

Genetic Algorithm

"Genetic Algorithms are good at taking large, potentially huge, search spaces and navigating them, looking for optimal combinations of things, solutions one might not otherwise find in a lifetime"

Here we apply Genetic Algorithm to Quantum Information Processing

In the first part (a) we have used GA for

Quantum Logic Gates (operator optimization)
and
Quantum State preparation (state-to-state optimization)

V.S. Manu et al. Phys. Rev. **A 86**, 022324 (2012)

Representation Scheme

Representation scheme is the method used for encoding the solution of the problem to individual genetic evolution. Designing a good genetic representation is a hard problem in evolutionary computation. Defining proper representation scheme is the first step in GA Optimization.

In our representation scheme we have selected the gene as a combination of

(i) an array of pulses, which are applied to each channel with amplitude (θ) and phase (φ),

(ii) An arbitrary delay (d).

It can be shown that the repeated application of above gene forms the most general pulse sequence in NMR

The Individual, which represents a valid solution can be represented as a matrix of size (n+1)x2m. Here ‘m’ is the number of genes in each individual and ‘n’ is the number of channels (or spins/qubits).

$$\begin{pmatrix} \theta_{11} & \varphi_{11} & & & \theta_{m1} & \varphi_{m1} \\ \theta_{12} & \varphi_{12} & \cdot & \cdot & \theta_{m1} & \varphi_{m1} \\ \vdots & & \cdot & \cdot & & \vdots \\ & & & & & \\ \theta_{1n} & \varphi_{1n} & \cdot & \cdot & \theta_{mn} & \varphi_{mn} \\ d_1 & 0 & & & d_m & 0 \end{pmatrix}$$

So the problem is to find an optimized matrix, in which the optimality condition is imposed by a “Fitness Function”

Fitness function

In operator optimization

GA tries to reach a preferred target Unitary Operator (U_{tar}) from an initial random guess pulse sequence operator (U_{pul}).

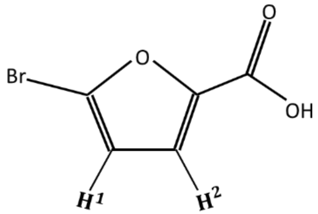
Maximizing the Fitness function

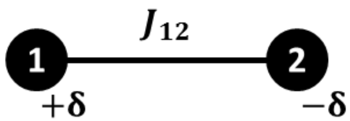
$$F_{pul} = \text{Trace}\,(U_{pul} \times U_{tar})$$

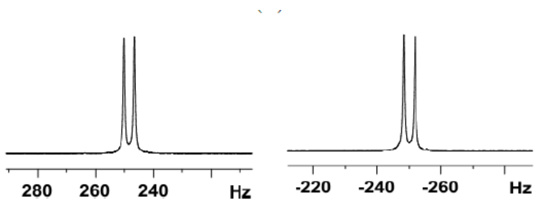
In State-to-State optimization

$$F_{pul} = \text{Trace}\,\{ U_{pul}\,(\rho_{in})\, U_{pul}^{(-1)}\, \rho_{tar}^{\dagger} \}$$

Two-qubit Homonuclear case







δ = 500 Hz, J= 3.56 Hz

$$H = 2\pi\,\delta\,(I_{1z} - 1_{2z}) + 2\pi\,J_{12}\,(I_{1z}I_{2z})$$

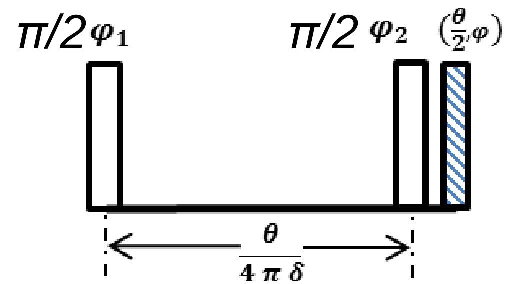
Hamiltonian used

$$H = H_{cs} = 2\,\pi\,\delta\,(I_z^1 - I_z^2)$$

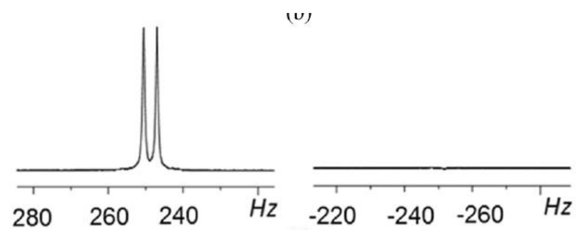
Simulated using J = 0

Non-Selective *(Hard) Pulses applied in the centre*

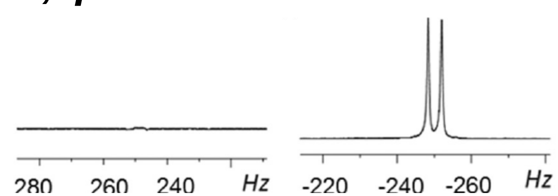
Single qubit rotation



$\varphi_1 = 2\pi$, $\varphi_2 = \pi$,
$\Theta = \pi/2$, $\varphi = \pi/2$

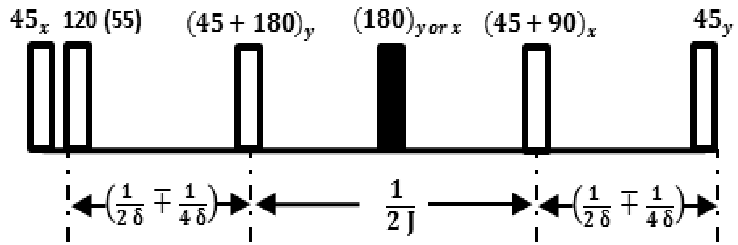


$\varphi_1 = \pi$, $\varphi_2 = 0$
$\Theta = \pi/2$, $\varphi = \pi/2$

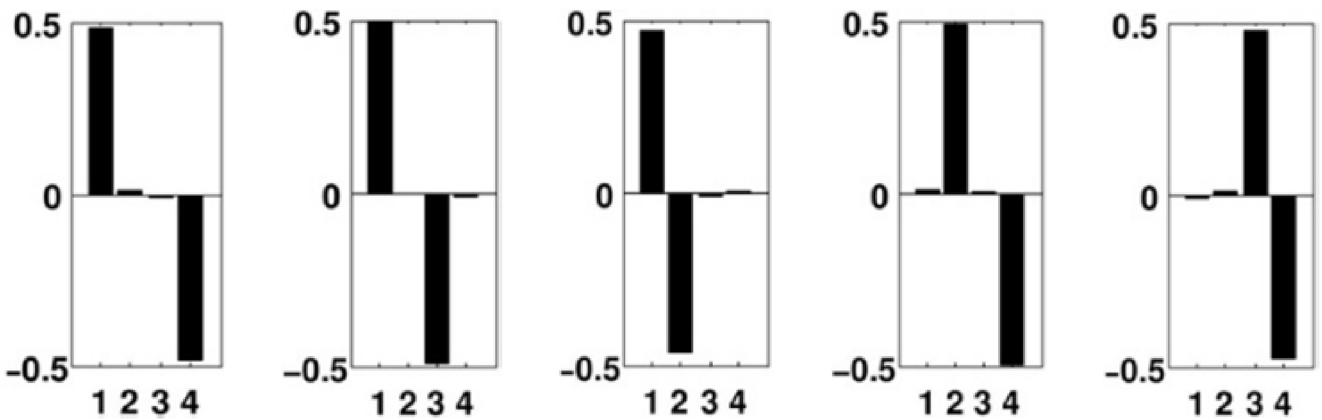


Controlled- NOT:

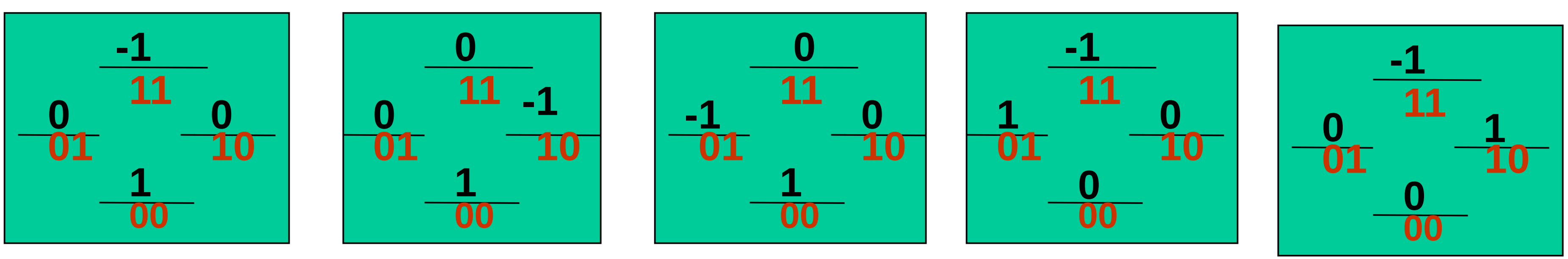
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



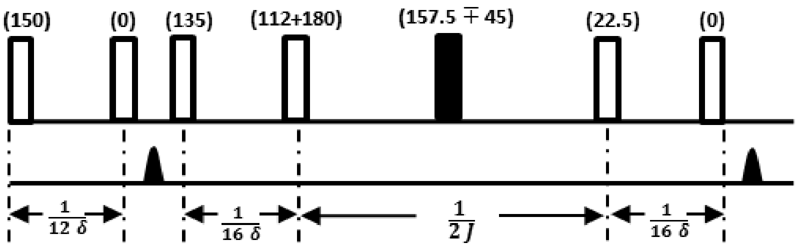
The phase of π pulse (*y or x*) decides the operation type (C-NOT or C-$\overline{NOT}$) and $\mp$ in first and third delays decides the Control Qubit.



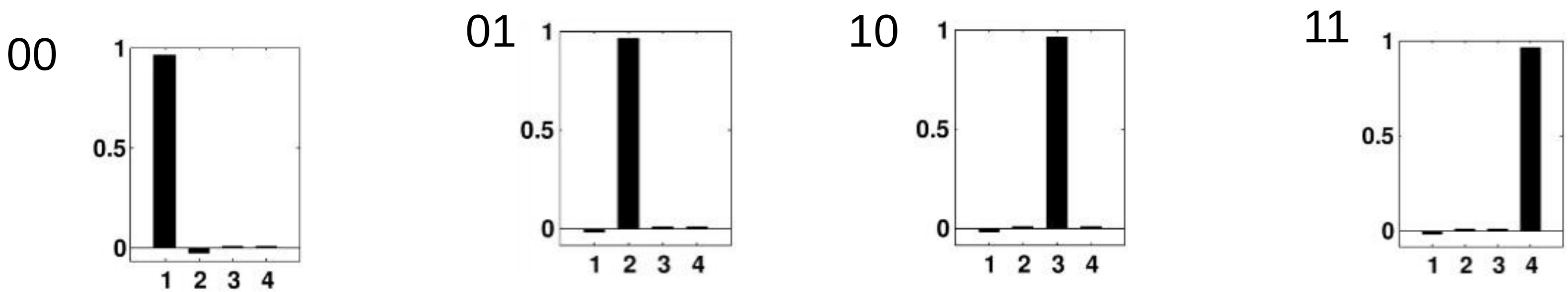
Equilibrium C-NOT(1,2) C-NOT(2,1) C-$\overline{\text{NOT}}$ (1,2) C-$\overline{\text{NOT}}$ (2,1)



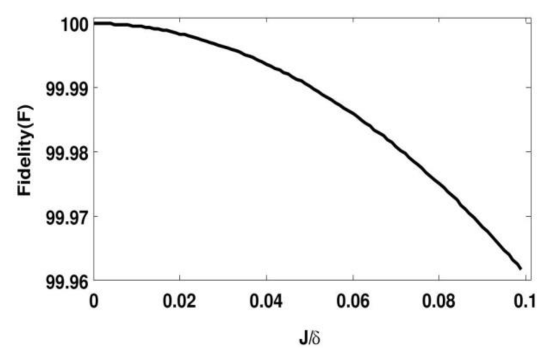
Pseudo Pure State (PPS) creation



All unfilled rectangles represent 90^0 pulse
The filled rectangle is 180^0 pulse.
Phases are given on the top of each pulse.

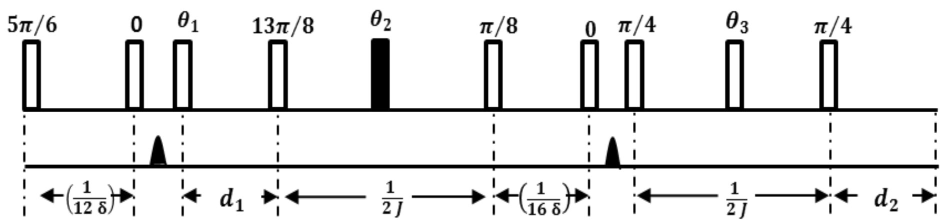


Fidelity w.r.t. to J/δ



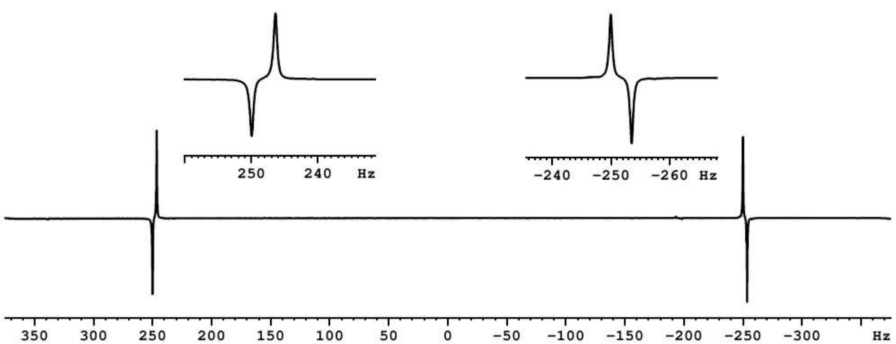
Bell state creation: From Equilibrium (No need of PPS)

Bell states are maximally entangled two qubit states.



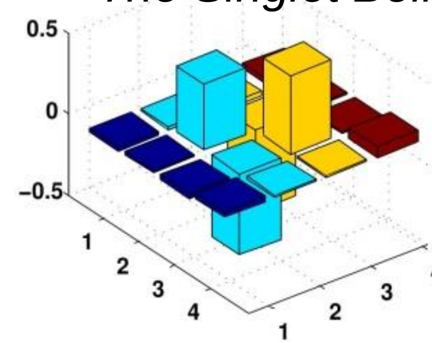
	θ_1	θ_2	θ_3	d_1	d_2
$\frac{1}{\sqrt{2}}(\lvert 01\rangle + \lvert 10\rangle)$	0	$5\pi/8$	$3\pi/4$	$9/48\delta$	$1/8\delta$
$\frac{1}{\sqrt{2}}(\lvert 01\rangle - \lvert 10\rangle)$	0	$5\pi/8$	$\pi/4$	$9/48\delta$	$1/8\delta$
$\frac{1}{\sqrt{2}}(\lvert 00\rangle + \lvert 11\rangle)$	$3\pi/4$	$9\pi/8$	$3\pi/4$	$1/16\delta$	0
$\frac{1}{\sqrt{2}}(\lvert 00\rangle - \lvert 11\rangle)$	$3\pi/4$	$9\pi/8$	$\pi/4$	$1/16\delta$	0

All blank pulses are 90^0 pulses. Filled pulse is a 180^0 pulse.
Phases and delays Optimized for best fidelity.



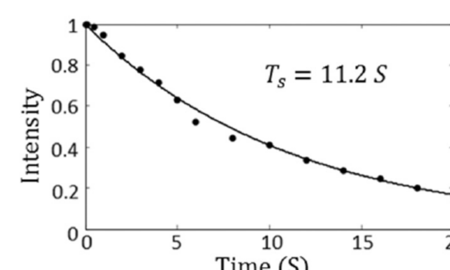
Experimental Fidelity > 99.5 %

The Singlet Bell State



Shortest Pulse Sequence for creation of Bell States directly from Equilibrium

T_1=8.7 s
T_s=11.2 s



V.S. Manu et al. Phys. Rev. A 86, 022324 (2012)

(b) Quantum Simulation of Dzyaloshinsky-Moriya (DM) interaction (H_{DM}) in presence of Heisenberg XY interaction (H_{XY}) for study of Entanglement Dynamics

DM Interaction[1,2]

- Anisotropic antisymmetric exchange interaction arising from spin-orbit coupling.
- Proposed by Dzyaloshinski to explain the weak ferromagnetism of antiferromagnetic crystals (Fe_2O_3, $MnCO_3$).

$$H_{DM} = \frac{D}{2}(\sigma_{1+}\sigma_{2-} - \sigma_{1-}\sigma_{2+})$$

Quantum simulation of a Hamiltonian H requires unitary operator decomposition (UOD) of its evolution operator, ($U = e^{-iHt}$) in terms of experimentally preferable unitaries.

Using **Genetic Algorithm** optimization, we numerically evaluate the most generic UOD for DM interaction in the presence of Heisenberg XY interaction.

1. I. Dzyaloshinsky, J. Phys & Chem of Solids, 4, 241 (1958).
2. T. Moriya, Phys. Rev. Letters, 4, 228 (1960).

The Hamiltonian

$$H(J,D) = \underbrace{J(\sigma_{1x}\sigma_{2x} + \sigma_{1y}\sigma_{2y})}_{\text{Heisenberg XY interaction}} + \underbrace{D(\sigma_{1x}\sigma_{2y} - \sigma_{1y}\sigma_{2x})}_{\text{DM interaction}}$$

Evolution Operator: $U(D,J,t) = exp(-iH(J,D) \times t)$

$$\gamma = D/J \qquad \tau = J \times t$$

$$U(\gamma,\tau) = exp(-i[(\sigma_{1x}\sigma_{2x} + \sigma_{1y}\sigma_{2y}) + \gamma(\sigma_{1x}\sigma_{2y} - \sigma_{1y}\sigma_{2x})]\,\tau)$$

The UOD is performed for Υ = 0 - ∞;

Υ = 0 -> pure H_{XY} interaction and Υ = ∞ -> pure H_{DM} interaction.

Decomposing the U in terms of Single Qubit Rotations (SQR) and ZZ- evolutions.

SQR by Hard pulse $\quad R^n(\theta,\phi) = exp(-i\theta/2 \times [Cos\phi\ \sigma_{nx} + Sin\phi\ \sigma_{ny}])$

ZZ evolutions by Delays $\quad U_{zz}(\theta) = exp\left(-i\ \frac{\theta}{2}\ \sigma_z^1\sigma_z^2\right)$

B. Decomposition for ϒ = 1 - ∞

ϒ' = 1/ ϒ When ϒ > 1 -> ϒ' < 1

$$U'(\gamma', \tau') = exp(-i[\gamma'(\sigma_{1x}\sigma_{2x} + \sigma_{1y}\sigma_{2y}) + (\sigma_{1x}\sigma_{2y} - \sigma_{1y}\sigma_{2x})] \ \tau'),$$

$$U'(\gamma', \tau') = R^1(\tfrac{\pi}{2}, \tfrac{\pi}{2})R^2(\tfrac{\pi}{2}, \theta_3)U_{zz}(\tfrac{\pi}{4})R^1(\theta_2 + \theta_3, 0)$$
$$R^2(\theta_1, \theta_4)U_{zz}(\tfrac{\pi}{4})R^1(\tfrac{\pi}{2}, \tfrac{\pi}{2})R^2(\tfrac{\pi}{2}, \theta_3),$$

2

$$\theta = [0.09812 \ exp(-2.42\gamma) + 0.4023 \ exp(0.5524\gamma)]\tau,$$
$$\theta_1 = -\theta + 3.142,$$
$$\theta_2 = \theta - [1.242 \ exp(-0.9617\gamma) + 0.3546 \ exp(-0.1145\gamma)],$$
$$\theta_3 = 1.259 \ exp(-0.957\gamma) + 3.479 \ exp(-0.0087\gamma),$$
$$\theta_4 = 1.256 \ exp(-0.959\gamma) + 1.912 \ exp(-0.0166\gamma),$$

So we have a complete decomposition for ϒ = 1 - ∞, which means all arbitrary amounts of DM and XY interaction can be simulated.

Using above decomposition, we studied entanglement preservation in a two-qubit system.

[3]Phys. Rev. A, 69 012313 (2009)

Entanglement Preservation

Hou et al. [1] demonstrated a mechanism for entanglement preservation using H(J,D). They showed that preservation of initial entanglement is performed by free evolution interrupted with a certain operator O, which makes the state to go back to its initial state.

$$OU(\gamma,\tau)OU(\gamma,\tau) \equiv I \qquad O = I_1 \otimes \sigma_{2z}$$

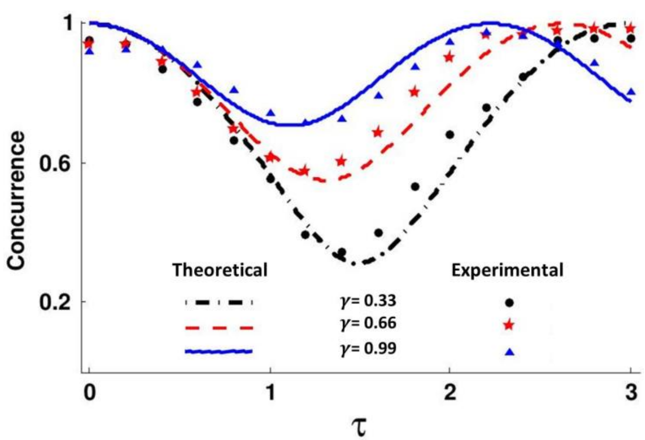
concurrence

$$C = \max\left(0, 2\max\{\sqrt{\mu_i}\} - \sum_i \sqrt{\mu_i}\right)$$

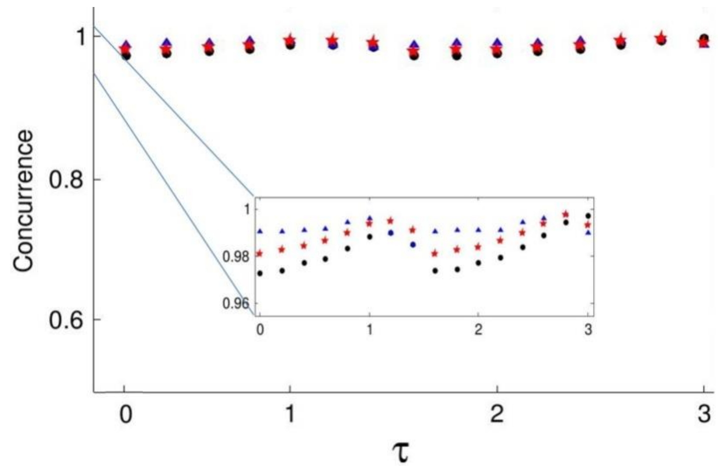
μ_i are eigen values of the operator $\rho S \rho^* S$, where $S = \sigma_{1y} \otimes \sigma_{2y}$

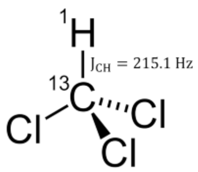
We experimentally demonstrated this with an initial Bell state : $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$

Without Operator *O*



With Operator *O*





Similar to Quantum Zeno Effect

Entanglement (concurrence) oscillates during Evolution.

Entanglement (concurrence) is preserved during Evolution. This confirms the Entanglement preservation method of Hou et al.[1]

Manu et al. Phys. Rev. A 89, 052331 (2014).

[1]Hou et al. *Annals of Physics, 327 292 (2012)*

Pause

Do I still have some time?

IBM recently (last Year) released a 5-qubit (and a 10-qubit) Superconductivity based Quantum Computer (Quantum Experience) and placed it on the cloud for use of one-and-all (free of cost).

My 2017 Summer students* used the 5-qubit computer and verified Three of our NMR experiments which we had done earlier, namely

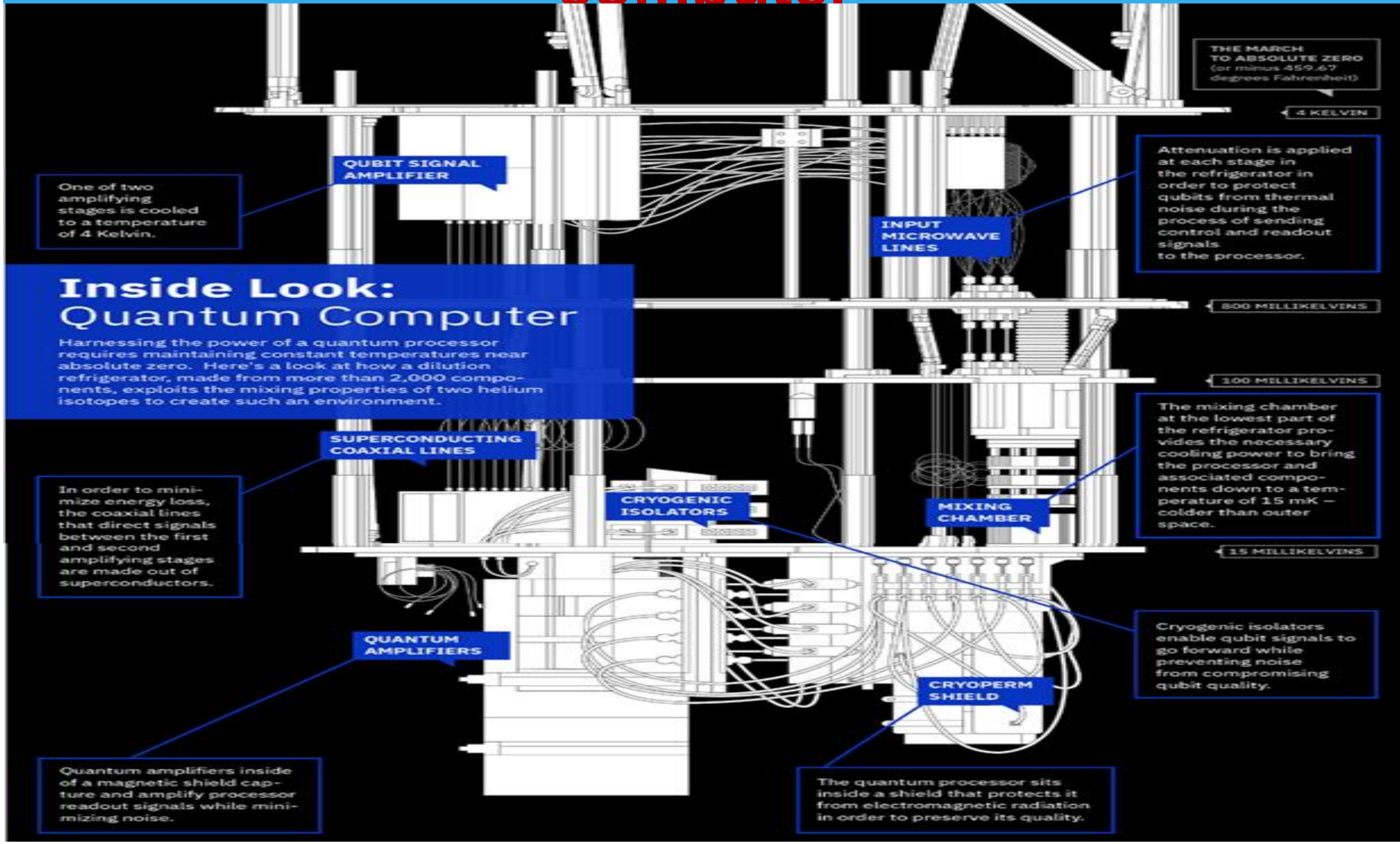
1. **Non-destructive discrimination of Bell States.**
   Jharana Rani Samal, Manu Gupta, P.K. Panigrahi and Anil Kumar, J.Phys. B, 43, 095508 (2010)

2. **Non-destructive discrimination of arbitrary set of orthogonal quantum States by phase estimation.**
   V.S. Manu and Anil Kumar (75 years of Entanglement, Foundations and Information Theoretic Applications, Koltata Jan., 2011, AIP conf. Proceedings; 1384, 229-240 (2011).

3. **Experimental Test of Quantum of No-Hiding theorem.**
   Jharana Rani Samal, Arun K. Pati and Anil Kumar,
   Phys. Rev. Letters, 106, 080401 (25 Feb., 2011)

1. **Ayan Majumdar, IISER-Mohali**
2. **Santanu Mohapatra, IIT Khrgpur**
3. **Porvika Bala, NIT, Trichy**

Hardware of the IBM quantum computer



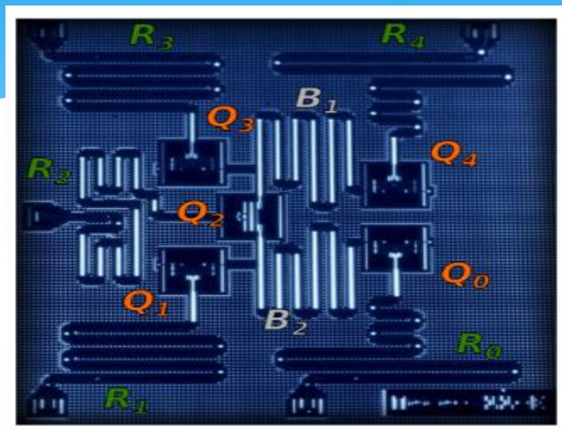
- http://research.ibm.com/ibm-q/learn/what-is-quantum-computing/

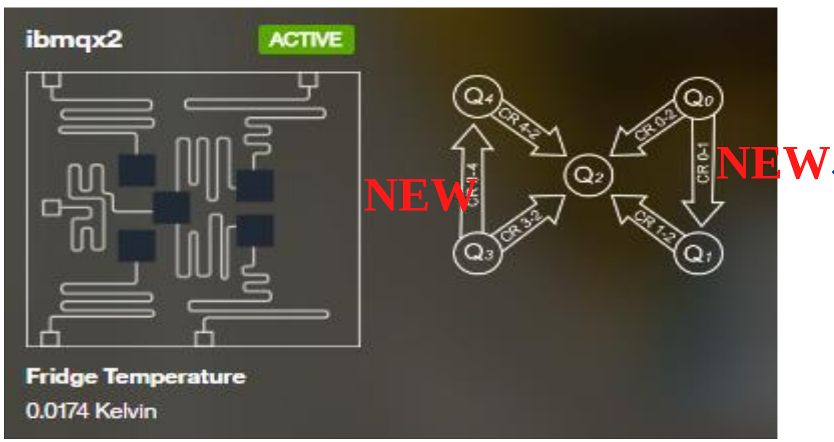
In order to minimize energy loss, the coaxial lines that direct signals between the first and second amplifying stages are made out of superconductors.

- ❖ **Superconducting coaxial lines**
  - ❑ https://www.youtube.com/watch?v=S52rxZG-zi0

New version

- ❑ This device went online January 24th, 2017

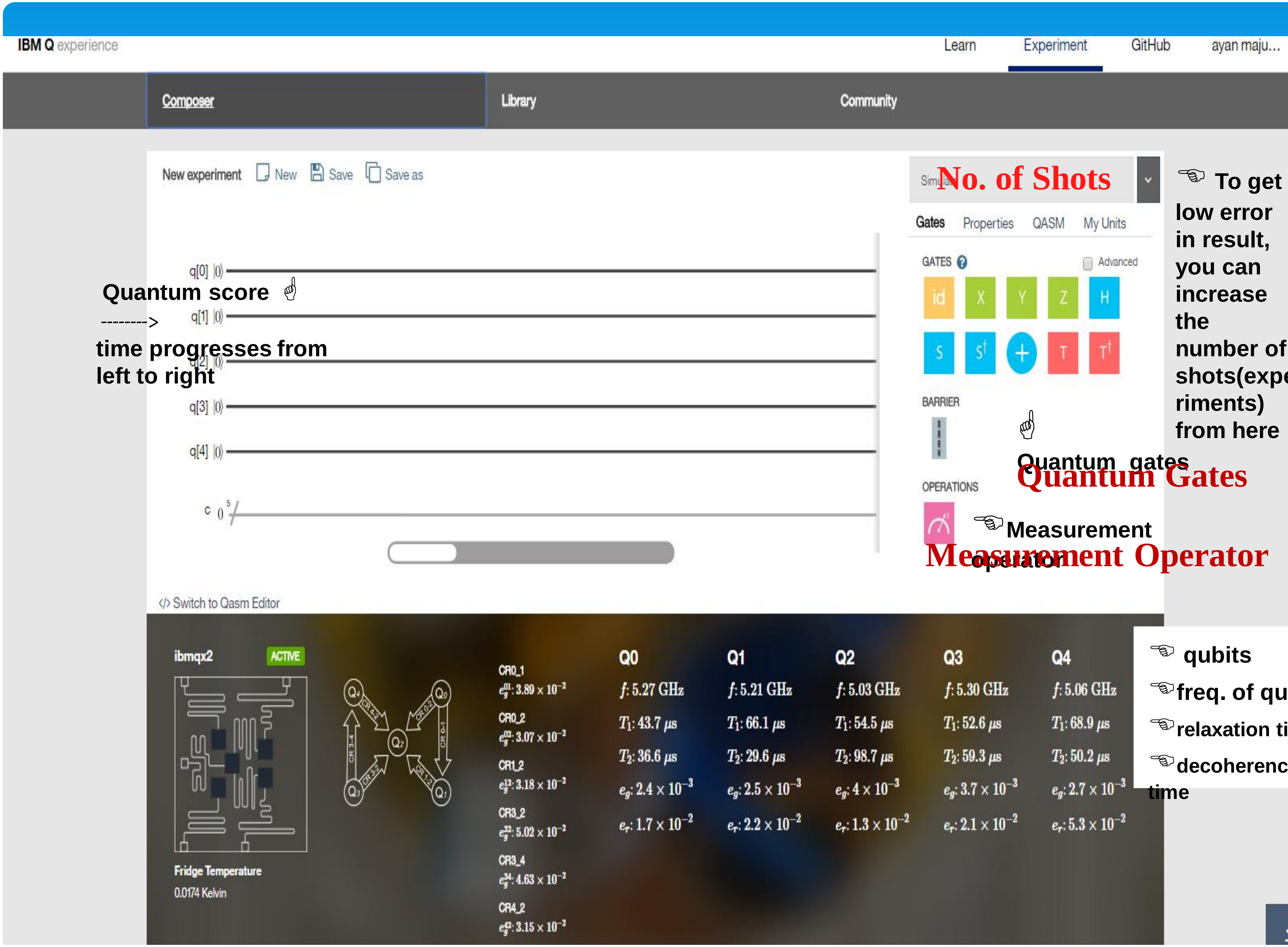




IBM Quantum Experience ibmqx2 device

- Coupling map = {0: [1, 2], 1: [2], 3: [2, 4], 4: [2]} where, a: [b] means a CNOT with qubit a as control and b as target can be implemented.

- The connectivity is provided by two coplanar waveguide (CPW) resonators with resonances around 6.0 GHz (coupling Q2, Q3 and Q4) and 6.5 GHz (coupling Q0, Q1 and Q2). Each qubit has a dedicated CPW for control and readout. This picture shows the chip layout.



IBM Q experience
Learn
Experiment
GitHub
ayan maju...
Composer
Library
Community
New experiment
New
Save
Save as
No. of Shots
To get low error in result, you can increase the number of shots(experiments) from here
Gates
Properties
QASM
My Units
GATES
Advanced
id
X
Y
Z
H
S
S†
T
T†
BARRIER
Quantum gates
Quantum Gates
OPERATIONS
Measurement operator
Measurement Operator
q[0] |0⟩
q[1] |0⟩
q[2] |0⟩
q[3] |0⟩
q[4] |0⟩
c 0 5
Quantum score
-------->
time progresses from left to right
Switch to Qasm Editor
ibmqx2
ACTIVE
Fridge Temperature
0.0174 Kelvin
CR0_1 e_g^{01}: 3.89 x 10^{-2}
CR0_2 e_g^{02}: 3.07 x 10^{-2}
CR1_2 e_g^{12}: 3.18 x 10^{-2}
CR3_2 e_g^{32}: 5.02 x 10^{-2}
CR3_4 e_g^{34}: 4.63 x 10^{-2}
CR4_2 e_g^{42}: 3.15 x 10^{-2}
Q0 f: 5.27 GHz T_1: 43.7 μs T_2: 36.6 μs e_g: 2.4 x 10^{-3} e_r: 1.7 x 10^{-2}
Q1 f: 5.21 GHz T_1: 66.1 μs T_2: 29.6 μs e_g: 2.5 x 10^{-3} e_r: 2.2 x 10^{-2}
Q2 f: 5.03 GHz T_1: 54.5 μs T_2: 98.7 μs e_g: 4 x 10^{-3} e_r: 1.3 x 10^{-2}
Q3 f: 5.30 GHz T_1: 52.6 μs T_2: 59.3 μs e_g: 3.7 x 10^{-3} e_r: 2.1 x 10^{-2}
Q4 f: 5.06 GHz T_1: 68.9 μs T_2: 50.2 μs e_g: 2.7 x 10^{-3} e_r: 5.3 x 10^{-2}
qubits
freq. of qubit
relaxation time
decoherence time

Your Quantum Results

Bell State

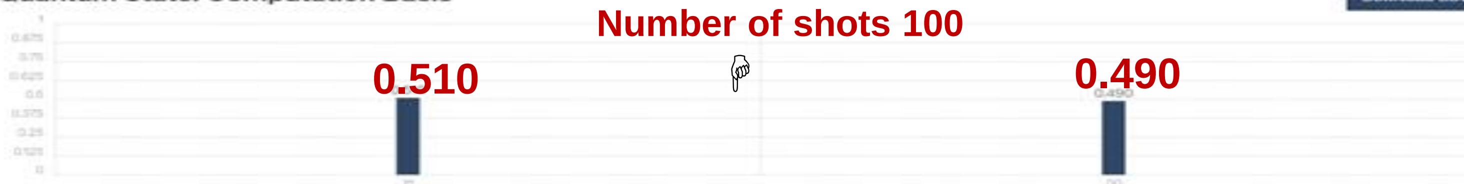
Bell state

Quantum State: Quantum Sphere

Quantum State: Computation Basis

Download CSV

Number of shots 100



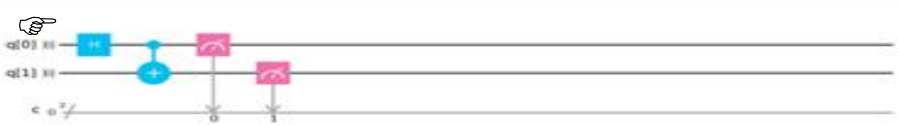
Probability of finding the system in state 11

Probability of finding the system in state 00

Quantum Circuit

the circuit for Bell state

$$\frac{1}{\sqrt{2}}(|00> +|11>)$$



OPENQASM 2.0

```
include "qelib1.inc";

qreg q[2];
creg c[2];

h q[0];
cx q[0],q[1];
measure q[0] -> c[0];
```

Open in Composer

Edit in QASM Editor

Executed on: Jun 8, 2017 4:56:08 PM

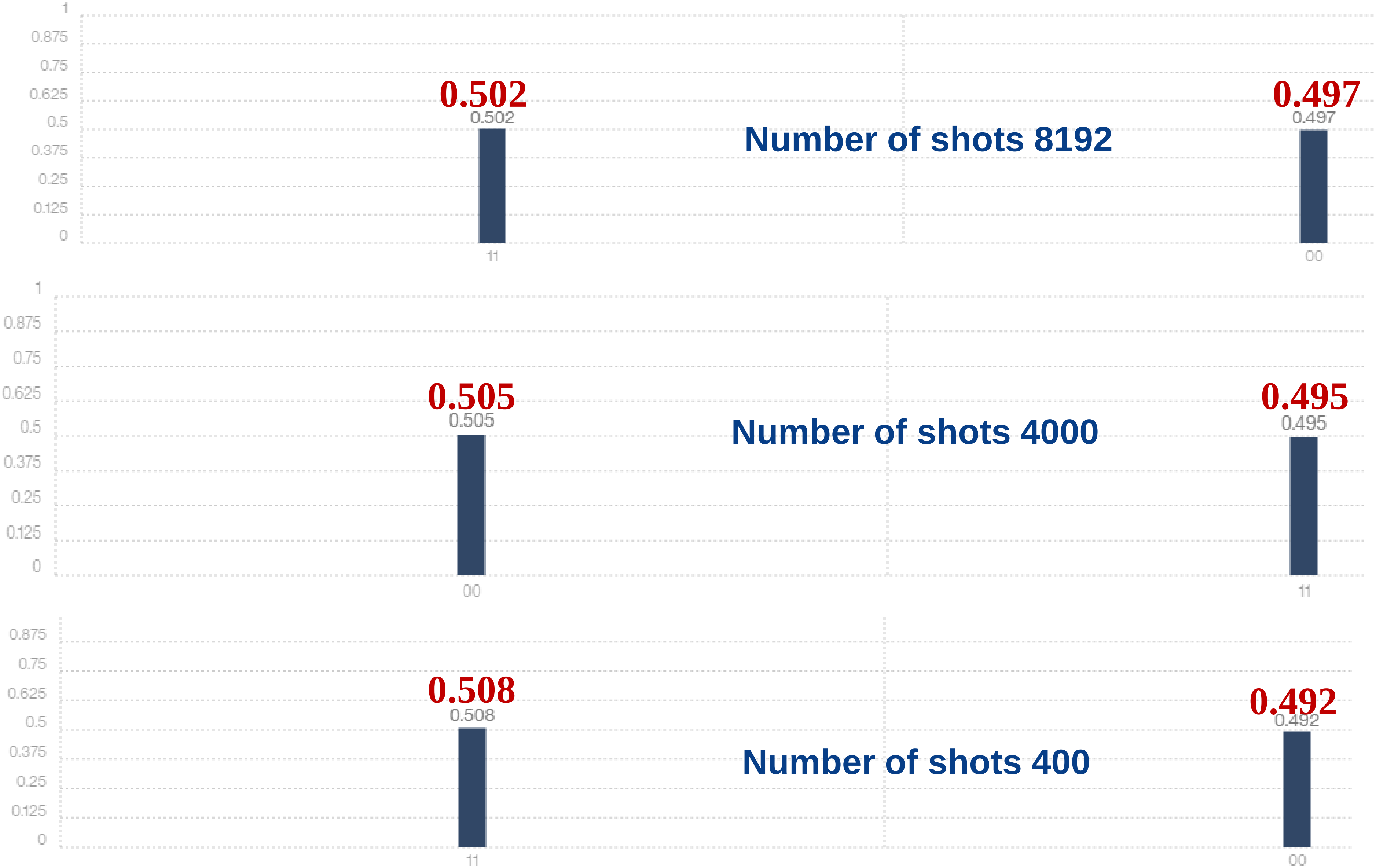
Results date: Jun 8, 2017 4:56:09 PM

Number of shots: 100

Seed: 232143396

Download All Data

Fidelity improves as the number of shots is increased.



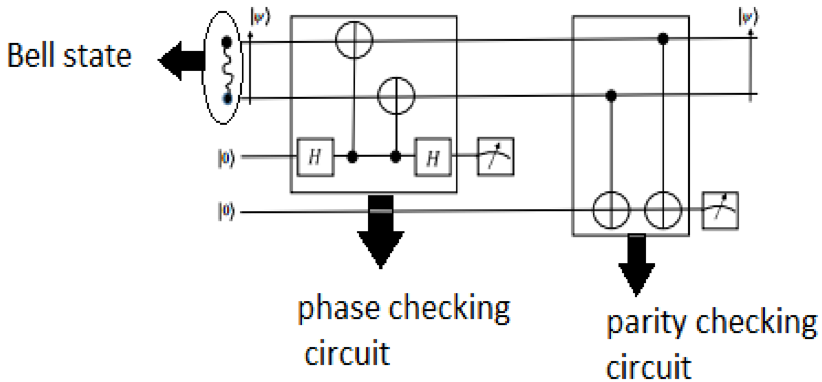
Nondestructive discrimination of Bell states using phase & parity checking circuit

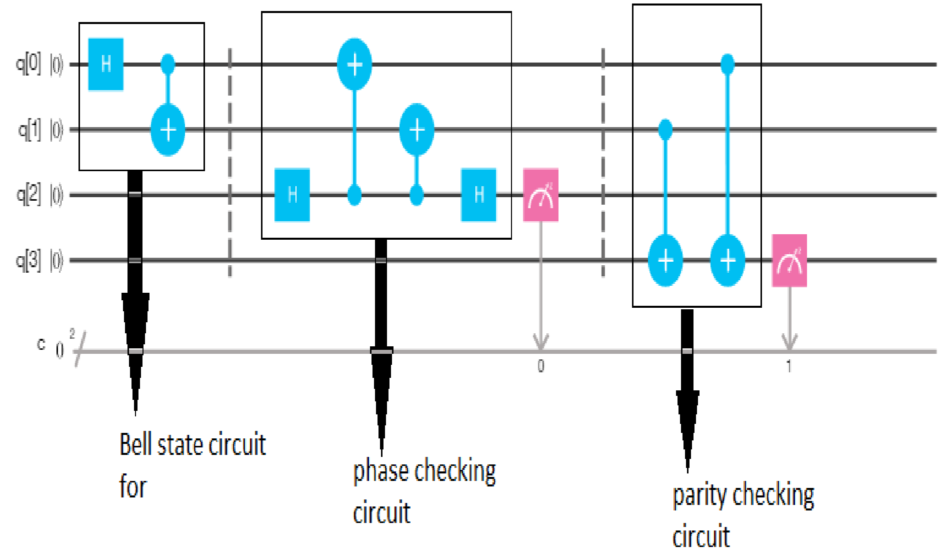
- **This experiment already verified by NMR**

NMR Fidelity 4.0%

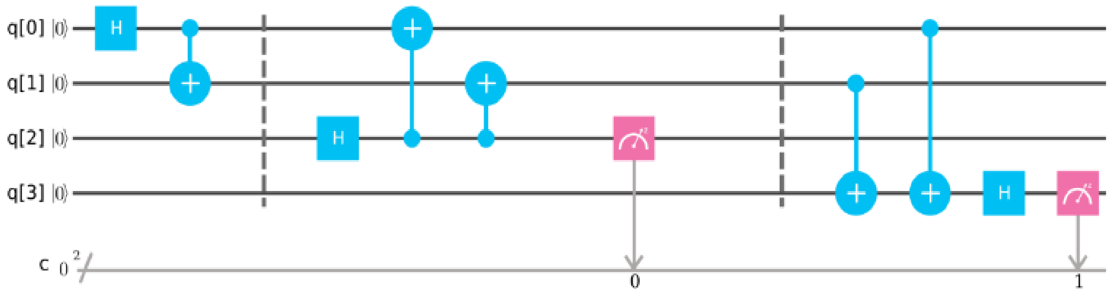
- **Recently this experiment was also implemented in IBM quantum experience by**
- **Mitali Sisodia, Abhishek Shukla, Anirban Pathak, arXiv:1705.00670 [quant-ph])**

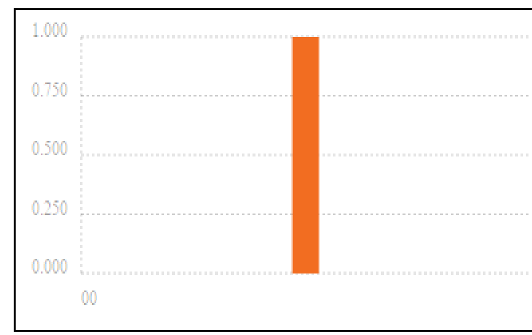
Ayan used the New version of the ibmqx2 device, had fewer gates, and got high Fidelity (0.9%)





- **Measurement in x-basis**





states	Ancilla-1	Ancilla-2
$\vert B_1\rangle$	$\vert 0\rangle$	$\vert 0\rangle$
$\vert B_2\rangle$	$\vert 1\rangle$	$\vert 0\rangle$
$\vert B_3\rangle$	$\vert 0\rangle$	$\vert 1\rangle$
$\vert B_4\rangle$	$\vert 1\rangle$	$\vert 1\rangle$

❖ **High Fidelity = 0.9%**

Instrument	Average absolute deviation	Maximum absolute deviation
New version of **ibmqx2 device**	0.0281%	0.9%

Nondestructive discrimination of arbitrary set of orthogonal quantum states

- This protocol already verified by a NMR
- Manu V S & Anil Kumar, AIP Conf. Proc. 1384,229-240(2011).

Also Verified here by using the New version of the ibmqx2 device (Ayan)

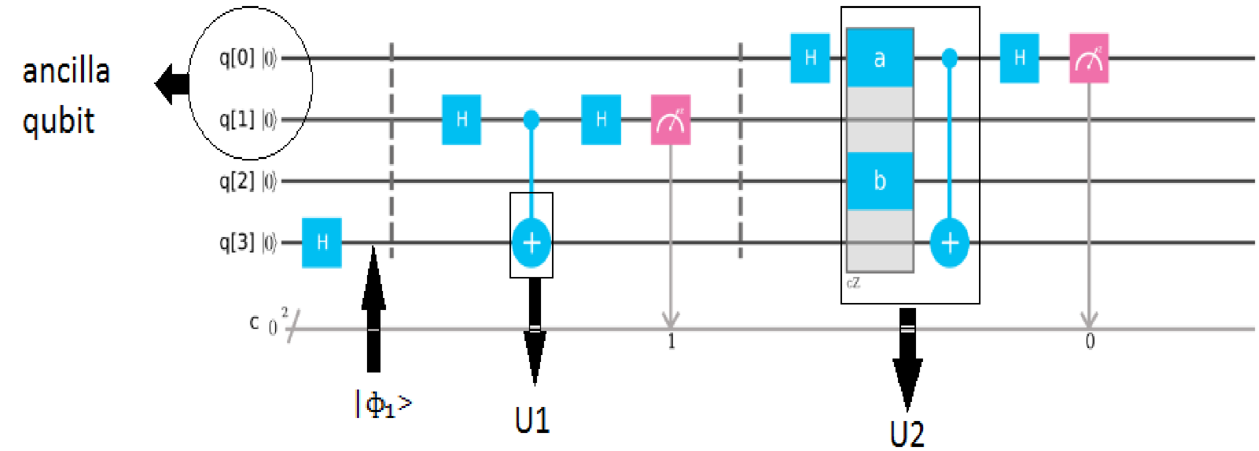
- **Possible orthogonal states are,**

$|\phi_1\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$

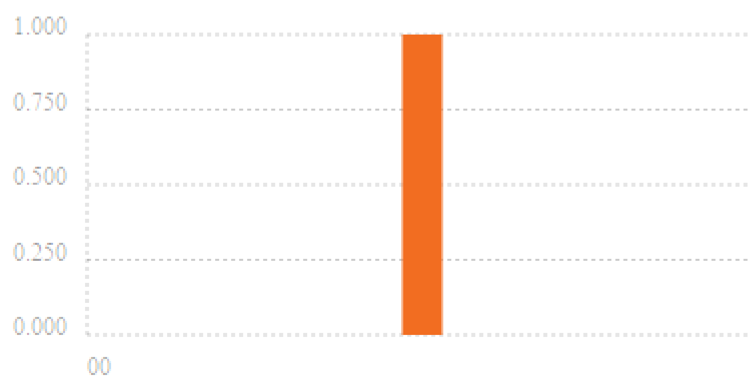
$|\phi_2\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |11\rangle)$

$|\phi_3\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |11\rangle)$

$|\phi_4\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$



states	Ancilla-1	Ancilla-2
$\|\phi_1\rangle$	$\|0\rangle$	$\|0\rangle$
$\|\phi_2\rangle$	$\|0\rangle$	$\|1\rangle$
$\|\phi_3\rangle$	$\|1\rangle$	$\|0\rangle$
$\|\phi_4\rangle$	$\|1\rangle$	$\|1\rangle$



- **Fidelity: NMR = 7.2%, ibmqx2 = 2.0 %**

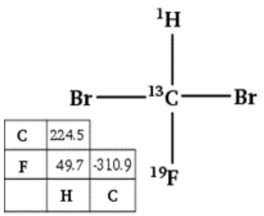
Instrument	Average absolute deviation	Maximum absolute deviation
NMR	4.0%	7.2%
IBM quantum experience	0.3750%	2%

Experimental Test of Quantum of No-Hiding theorem by NMR .

Jharana Rani Samal, Arun K. Pati and Anil Kumar,
Phys. Rev. Letters, 106, 080401 (25 Feb., 2011)

- **Has now been implemented by new version of ibmqx2 by Santanu Mohapatra in my lab.**

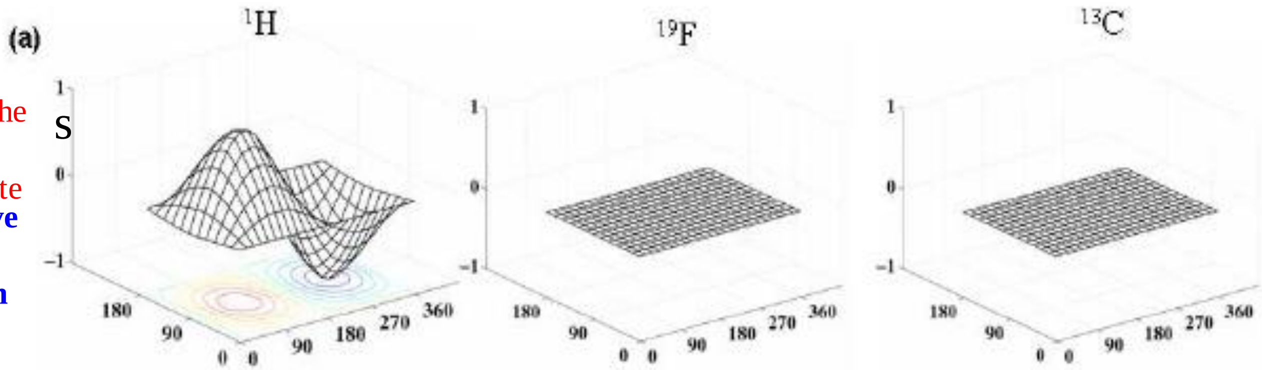
Experimental Result for the No-Hiding Theorem.



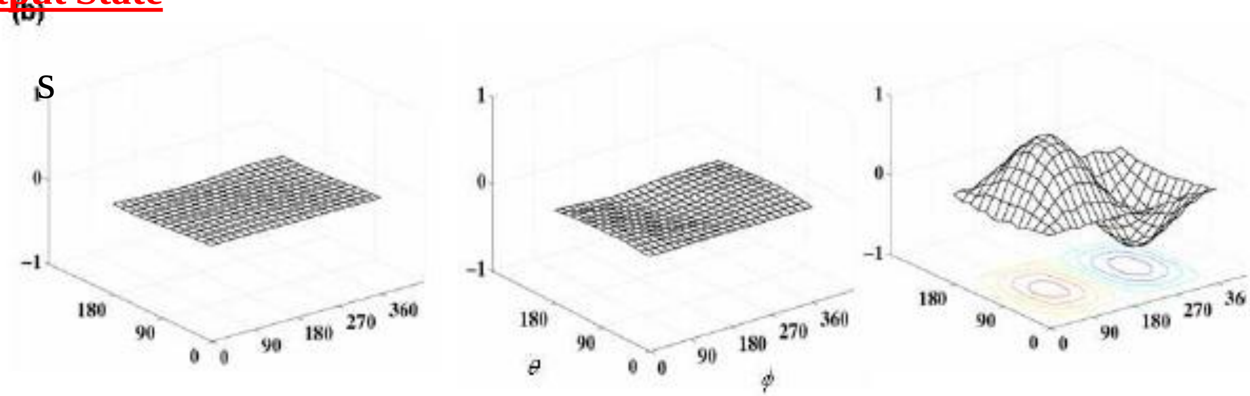
<u>Input State</u>

Each point in this figure is a point on the Bloch sphere and represents a pure state

325 experiments have been performed by varying θ and φ in steps of 15°



<u>Output State</u>



The state ψ is completely transferred from first qubit to the third qubit

S = Integral of real part of the signal for each spin

Jharana Rani Samal, Arun K. Pati and Anil Kumar, Phys. Rev. Letters, <u>106</u>, 080401 (25 Feb., 2011).

IMPLEMENTING IT IN IBM QUANTUM COMPUTER

- In order to implement the above pulse sequences in this quantum computer, We need to convert these into quantum gates. We already know that

$$U=[\pi/2]^{3}_{-z}\,[\pi/2]^{1}_{-x}\,U_{12}\,[\pi/2]^{1}_{x}\,U_{13}\,[\pi/2]^{1}_{-x}\,[\pi/2]^{1}_{-z}$$

So, the sequence of quantum gates for the randomization op

$$S^{+}_{3}\left(H_{1}\,S^{+}_{1}\,H_{1}\,X_{1}\right)[H_{2}\,X_{2}\,H_{1}\,H_{1}\,S^{+}_{1}\,H_{1}\,X_{1}\,H_{1}\,X_{1}\,CNOT_{12}\,H_{2}$$

$$H_{1}\,(S_{1}\,H_{1}\,X_{1}\,H_{3}\,X_{3}\,H_{1}\,H_{1}\,S^{+}_{1}\,H_{1}\,X_{1}\,H_{1}\,X_{1}\,CNOT_{13}\,H_{3}\,S^{+}_{3}\,H_{3}\,X_{3}\,H_{3}$$

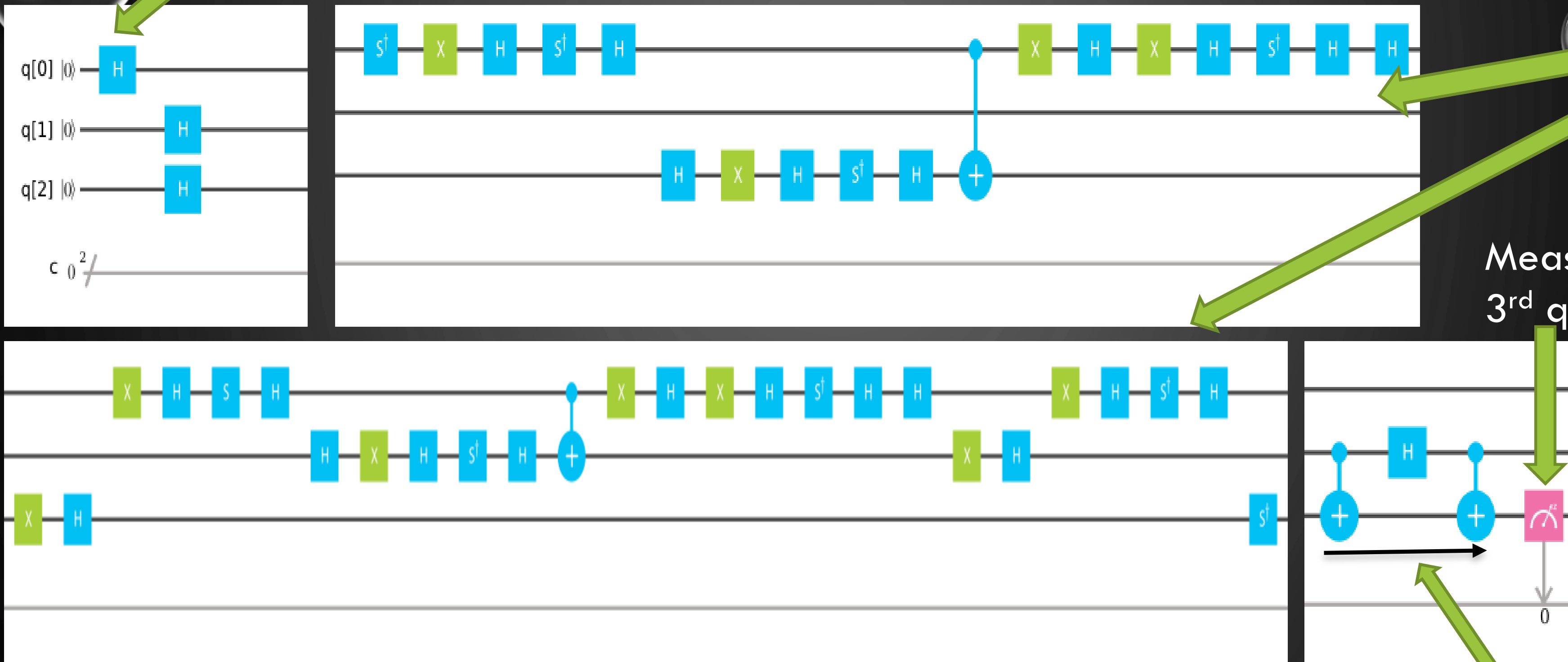
Formulas used:

1. $[\pi/2]_{-y}$=H, $[\pi/2]_{y}$=H X
2. $[\pi/2]_{z}$=S, $[\pi/2]_{-z}$=S^{+}
3. $\varphi_{x}=[\pi/2]_{-y}\,\varphi_{z}\,[\pi/2]_{y}$
4. $\varphi_{-x}=[\pi/2]_{-y}\,\varphi_{-z}\,[\pi/2]_{y}$
5. $CNOT_{12}=[\pi/2]^{1}_{-y}\,[\pi/2]^{1}_{x}\,[\pi/2]^{1}_{y}\,[\pi/2]^{2}_{-y}$

For θ=π/2 and ø=0, |ψ>=(|0>+|1>)/√2

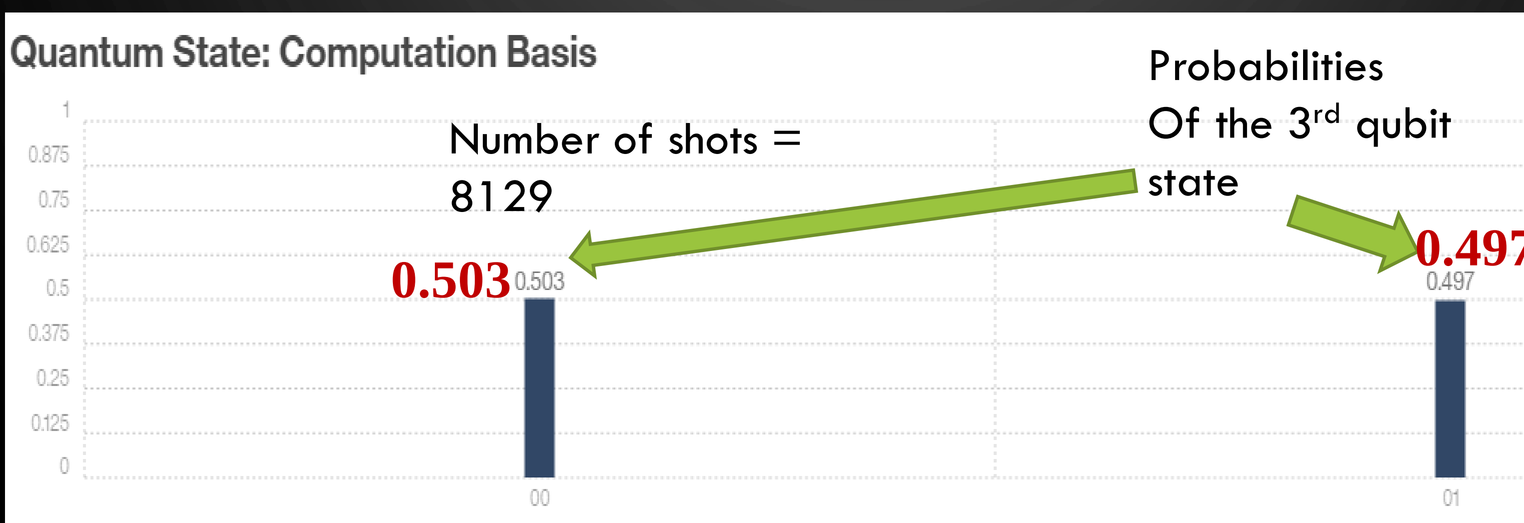
Initial state

U

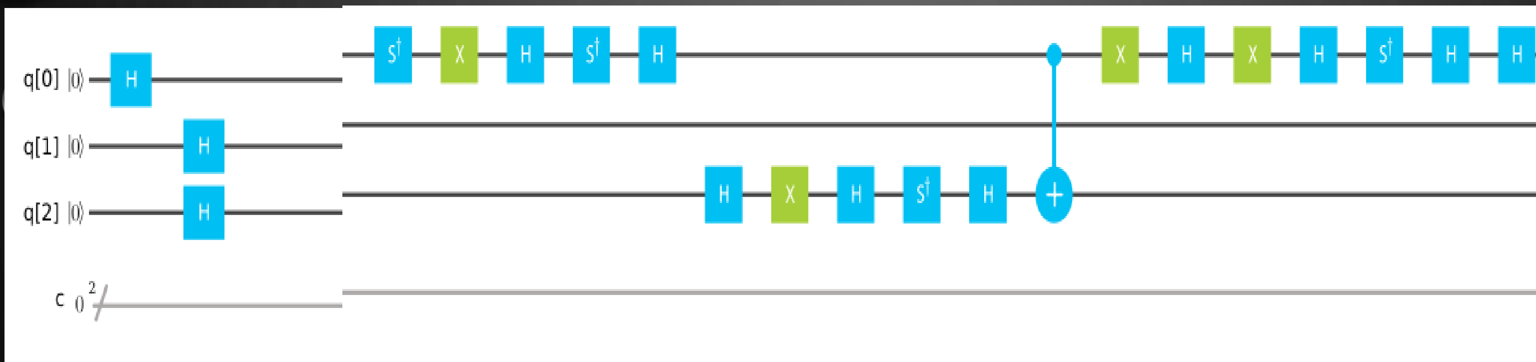


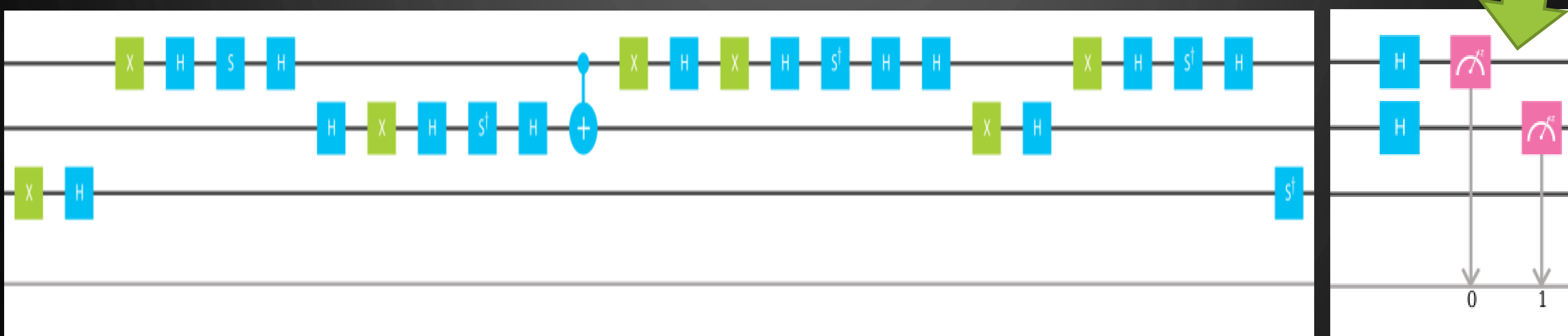
Measuring 3rd qubit

Local unitary Operation for Extraction of s

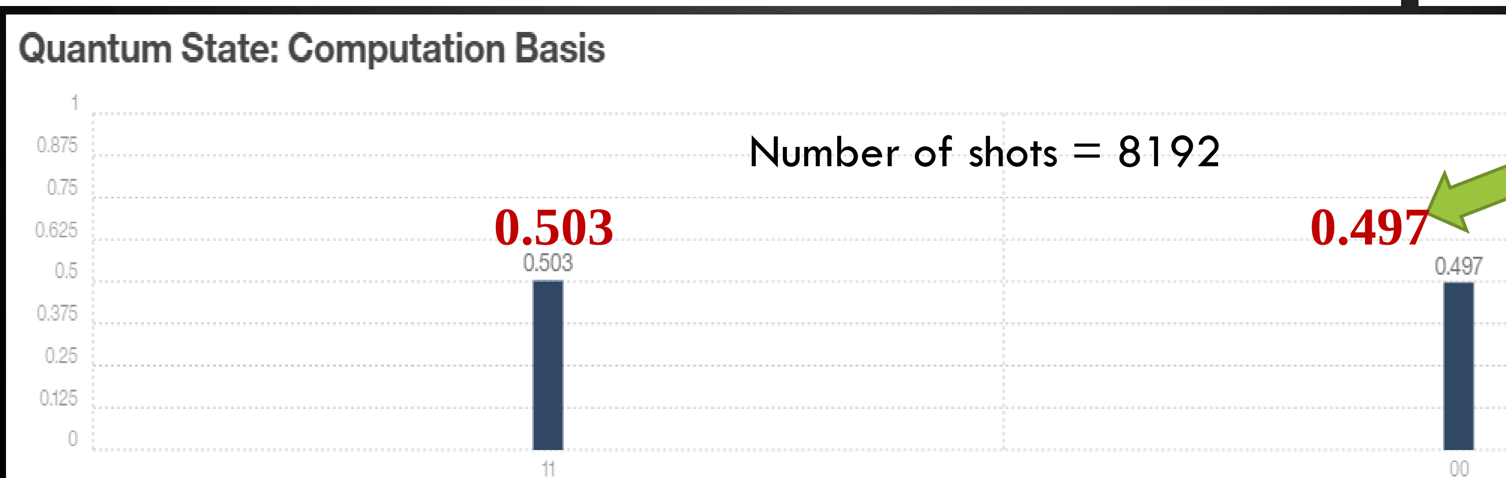


Extracting the Bell state





Measuring 1st
And 2nd qubit
X basis



Bell state

- **Thanks to the IBM for developing such a wonderful experimental setup and making it available to one and all**

Summary

NMR is continuing to provide a test bed for many quantum Phenomenon and Quantum Algorithms.

Acknowledgements

Former QC- IISc-Associates/Students

Prof. Arvind - IISER Mohali
Prof. Kavita Dorai - IISER Mohali
Prof. T.S. Mahesh - IISER Pune
Dr. Neeraj Sinha - CBMR Lucknow
Dr. K.V.R.M.Murali - IBM, Bangalore
Dr. Ranabir Das - NCIF/NIH USA
Dr. Rangeet Bhattacharyya - IISER Kolkata
Dr.Arindam Ghosh - NISER Bhubaneswar
Dr. Avik Mitra - Philips Bangalore
Dr. T. Gopinath - Univ. Minnesota
Dr. Pranaw Rungta -
Dr. Tathagat Tulsi – IIT Bombay

This lecture is dedicated
to the memory of
Ms. Jharana Rani Samal*
(*Deceased, Nov., 12, 2009)

Recent QC IISc - Students

Dr. R. Koteswara Rao - Dortmund
Dr. V.S. Manu - Univ. Minnesota

Other IISc Collaborators

Prof. Apoorva Patel
Prof. K.V. Ramanathan
Prof. N. Suryaprakash

Other Collaborators

Prof. Malcolm H. Levitt - UK
Prof. P.Panigrahi IISER Kolkata
Prof. Arun K. Pati HRI-Allahabad
Prof. Aditi Sen HRI-Allahabad
Prof. Ujjwal Sen HRI-Allahabad
Mr. Ashok Ajoy BITS-Goa-MIT-UCB

Funding: DST/DAE/DBT

Thanks: NMR Research Centres at IISc, Bangalore for spectrometer time

Thank You

Non-destructive discrimination of Bell States

Bell States are Maximally Entangled 2-qubit states.
There are 4 Bell States namely

$|\Phi^{+}\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$

$|\Phi^{-}\rangle = (|00\rangle - |11\rangle)/\sqrt{2}$

$|\psi^{+}\rangle = (|01\rangle + |10\rangle)\ \sqrt{2}$

$|\psi^{-}\rangle = (|01\rangle - |10\rangle)\sqrt{2}$

Protocol for Non-destructive Discrimination of Bell States

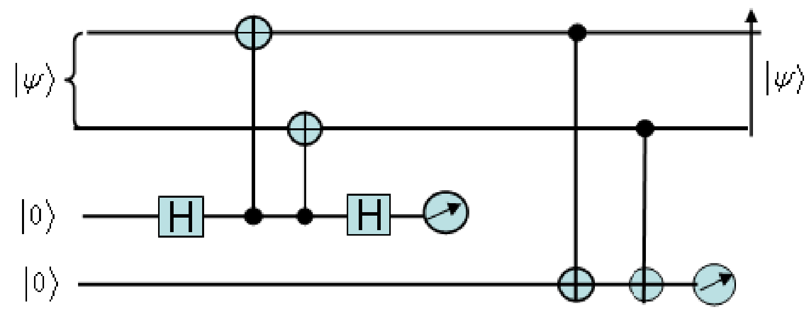
Manu Gupta and P. Panigrahi (quant-ph/0504183v);
Int. J. of Quantum Information 5, 627 (2007)

Theory

Jharana Rani Samal*, Manu Gupta, P. Panigrahi and Anil Kumar,
J. Phys. B, 43, 095508 (2010).

Experimental verification by NMR

(a)



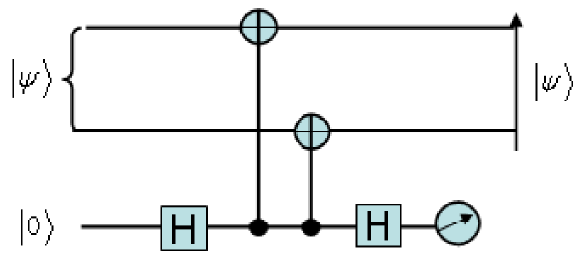
Panigrahi Circuit

Needs two Ancilla Qubits

Jharana Circuits

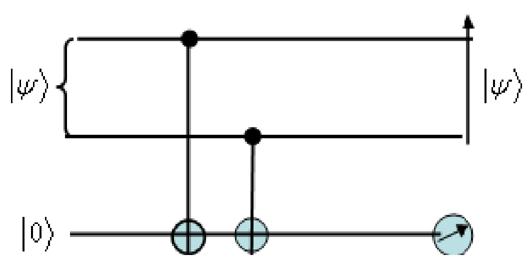
Needs one Ancilla but two measurements

(b)



Phase Measurement

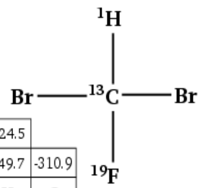
(c)



Parity Measurement

Bell State	1st Measurement	2nd Measurement
$\lvert\phi^{+}\rangle$	$\lvert 0\rangle$	$\lvert 0\rangle$
$\lvert\phi^{-}\rangle$	$\lvert 1\rangle$	$\lvert 0\rangle$
$\lvert\psi^{+}\rangle$	$\lvert 0\rangle$	$\lvert 1\rangle$
$\lvert\psi^{-}\rangle$	$\lvert 1\rangle$	$\lvert 1\rangle$

NMR Pulse Sequence for Discrimination of Bell States using one Ancilla Qubit



C	224.5	
F	49.7	-310.9
	H	C

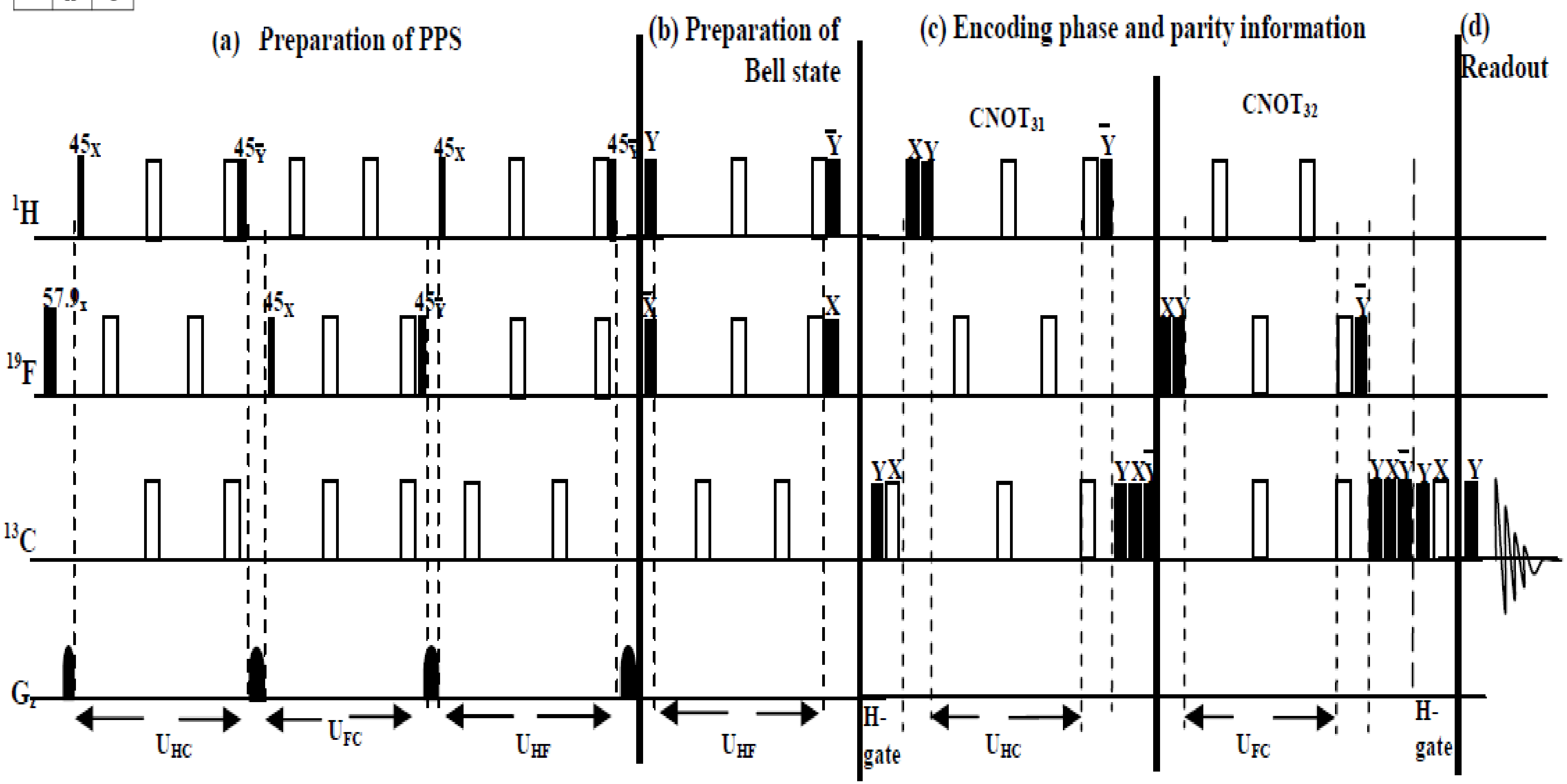


Fig. 2

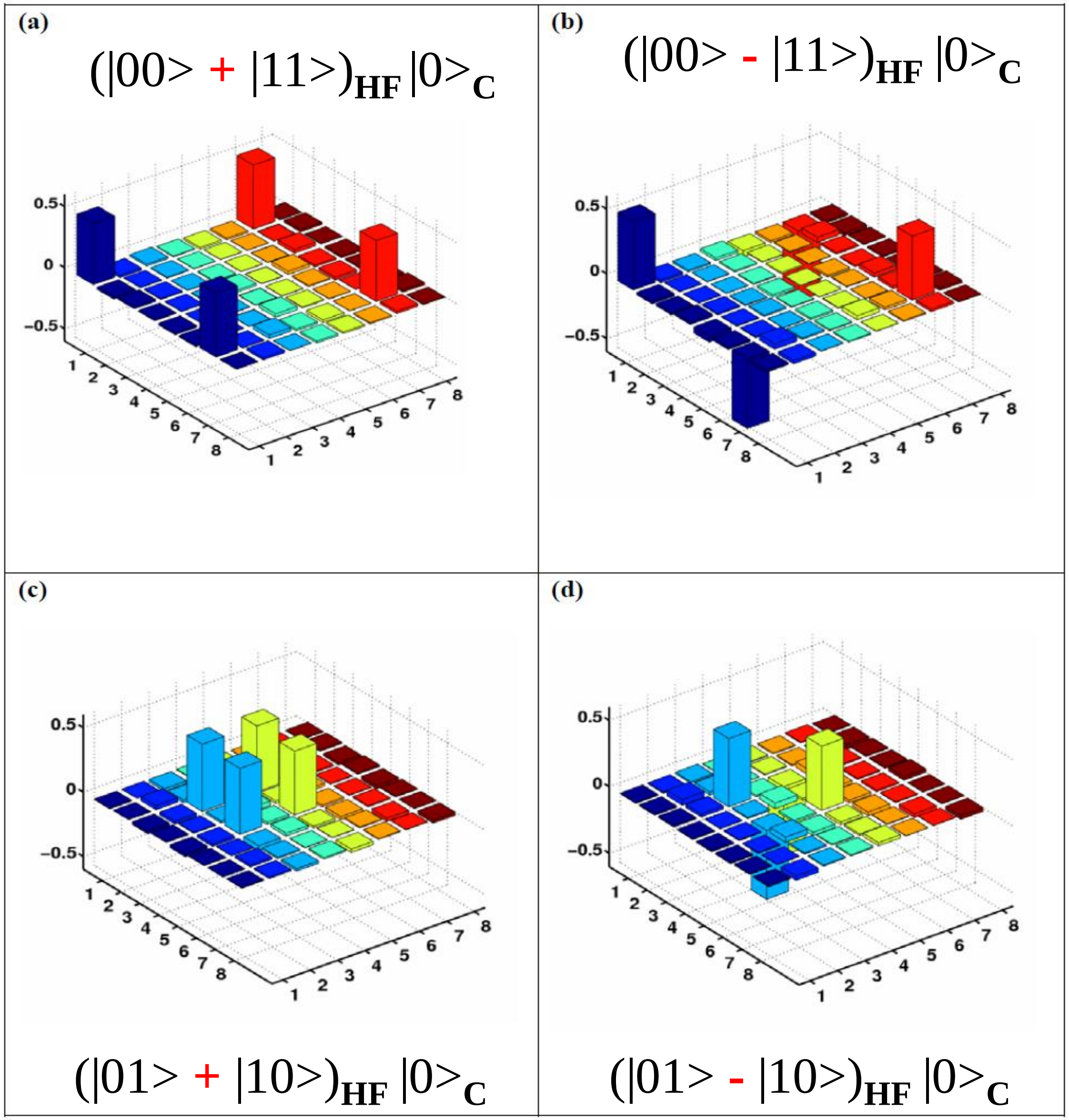
For Parity measurement the Hadamard gates are removed and the CNOT Gates are reversed

Jharana et al, J.Phys. B., 43, 095508 (2010)

Created Bell States

$|\Phi^+\rangle$

$|\Phi^-\rangle$



$|\psi^+\rangle$

$|\psi^-\rangle$

1 = |000>; 7 = |110>; 3 = |010>; 5 = |100>

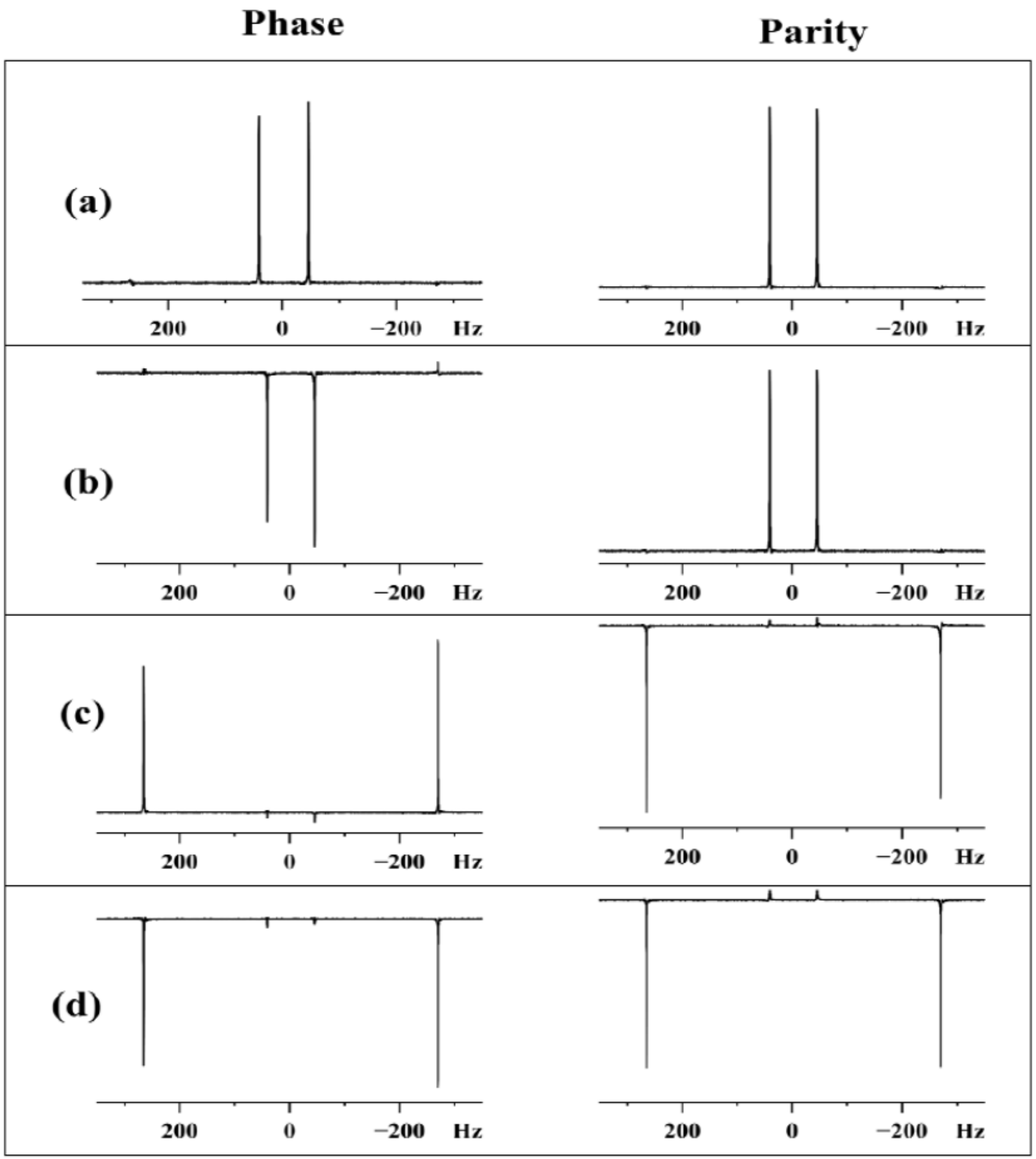
Population Spectra of ^{13}C

$|\Phi^{+}\rangle$

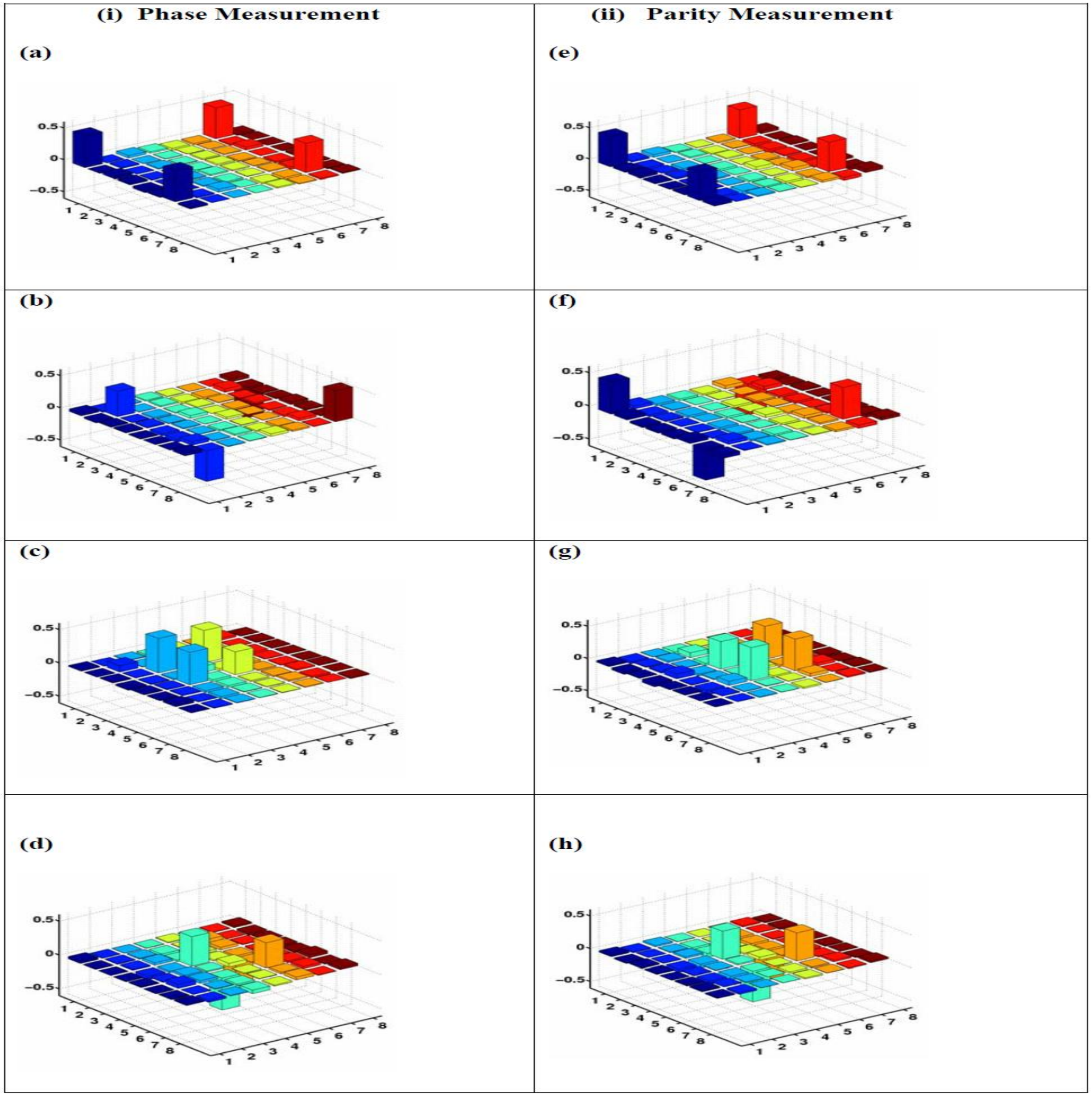
$|\Phi^{-}\rangle$

$|\psi^{+}\rangle$

$|\psi^{-}\rangle$



Tomograph of the real part of the Density matrix confirming the Phase and Parity measurement.



Jharna et al J.Phys.B 43, 095508 (2010)

Non-Destructive Discrimination of Arbitrary set of Orthogonal Quantum states by NMR using Quantum Phase Estimation.

For this algorithm, the states need not have definite PARITY (and can even be in a coherent superposition state).

This algorithm is thus more general than the just described Bell-State Discrimination.

V.S. Manu and Anil Kumar (75 years of Entanglement, Foundations and Information Theoretic Applications, Koltata Jan., 2011, AIP conf. Proceedings; 1384, 229-240 (2011).

For a given eigen-vector |φ> of a Unitary Operator U, Phase Estimation Circuit, can be used for finding the eigen-value of |φ>.

Conversely, with defined eigen-values, the Phase Estimation can be used for discriminating eigenvectors.

By logically defining the operators with preferred eigen-values, the discrimination, as shown here, can be done with certainty.

Quantum Phase Estimation

- **Suppose a unitary operation *U* has a eigen vector |*u*> with eigen value $e^{-i\varphi}$.**
- **The goal of the Phase Estimation Algorithm is to estimate *φ*.**

As the state is the eigen-state, the evolution under the Hamiltonian during phase estimation will preserve the state.

Finding the *n* Operators U_j

Let M_j be the diagonal matrix formed by eigen-value array $\{e^i\}_j$ of U_j.

And

V is the matrix formed by the column vectors $\{|\varphi_k>\}$,

$$U_j = V^{-1} \times M_j \times V$$

Forming Eigen-value arrays

1. Eigen-value arrays $\{ e^i \}$ should contain equal number of +1 and -1

2. 1st eigen value array can have any order of +1 and -1.

3. 2nd onwards should also contain equal number of +1 and -1, but should not be equal to earlier arrays or their complements.

Two Qubit Case

Consider a set

$$\left\{S\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)\right\}=\left\{\begin{matrix}\frac{1}{\sqrt{2}}(|00\rangle+|01\rangle),\frac{1}{\sqrt{2}}(|10\rangle+|11\rangle),\\ \frac{1}{\sqrt{2}}(|10\rangle-|11\rangle),\frac{1}{\sqrt{2}}(|00\rangle-|01\rangle)\end{matrix}\right\}$$

A complete set of orthogonal States, which are not Bell states.
They have the 1st qubit in state |0> or 1> and the 2nd qubit in a superposed State (0> ± 1>)

U_1 and U_2 can be shown as,

$$U_1=\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix},\quad U_2=\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&-1&0\end{pmatrix}\qquad \text{.........}\ (3)$$

Experimental implementation of this case is performed here by NMR

For the operators U_1 and U_2 described in Eqn. (3)

$$Controlled - U_1 = e^{-iH_1} \qquad Controlled - U_2 = e^{-iH_2}$$

In terms of NMR Product Operators The Hamiltonians are given by

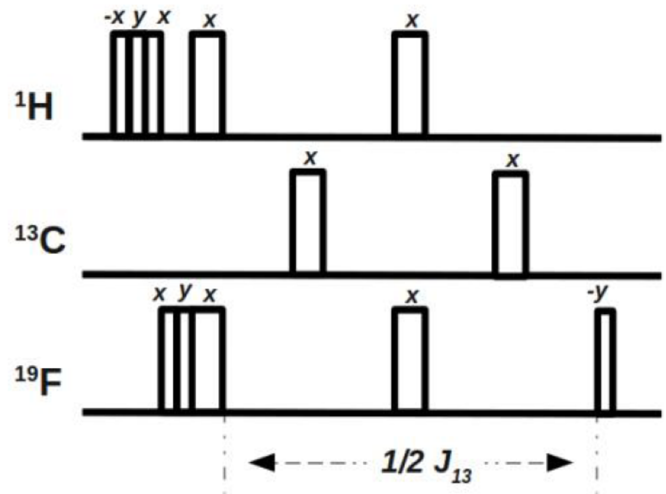
$$H_1 = \left(\frac{\pi}{4}I - \frac{\pi}{2}I_z^1 - \frac{\pi}{2}I_z^3 + \pi\, I_z^1 I_x^3\right)$$

$$H_2 = \left(\frac{\pi}{4}I - \frac{\pi}{2}I_z^1 - \pi\, I_z^2 I_x^3 + 2\pi\, I_z^1 I_z^2 I_x^3\right).$$

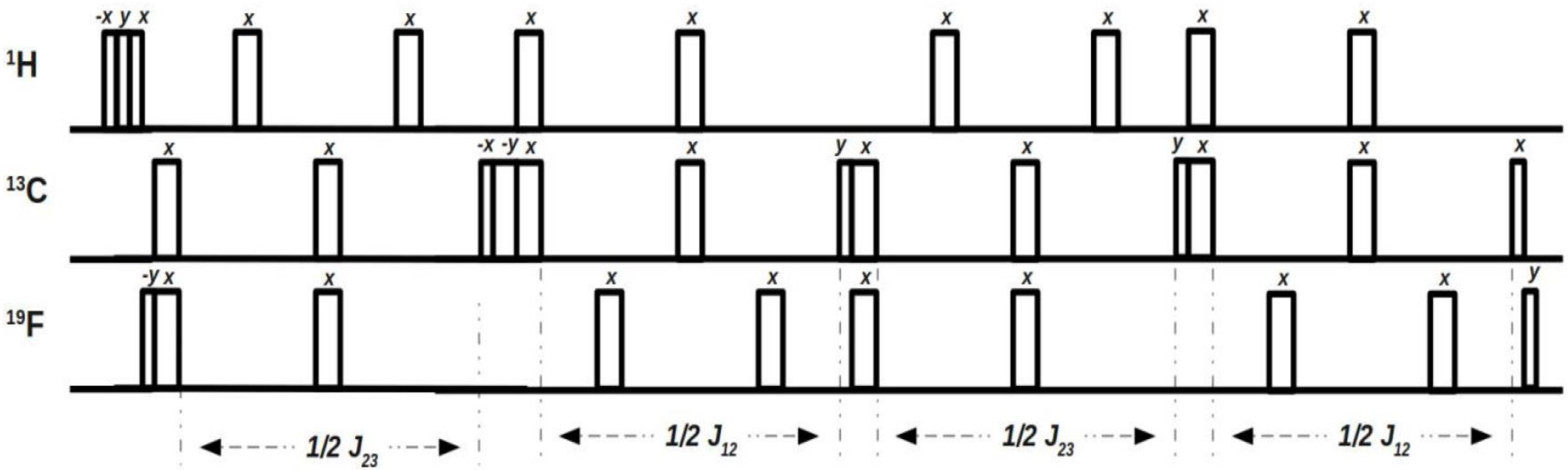
Since various terms in H_1 and H_2 commute each other, we can write,

$$Controlled - U_1 = e^{i\frac{\pi}{4}I} \times e^{-i\frac{\pi}{2}I_z^1} \times e^{-i\frac{\pi}{2}I_x^3} \times e^{i\pi I_z^1 I_x^3}.$$

$$Controlled - U_2 = e^{i\frac{\pi}{4}I} \times e^{-i\frac{\pi}{2}I_z^1} \times e^{i\pi I_z^1 I_x^3} \times e^{i2\pi I_z^1 I_z^2 I_x^3}$$



(a) *Controlled*-U_1

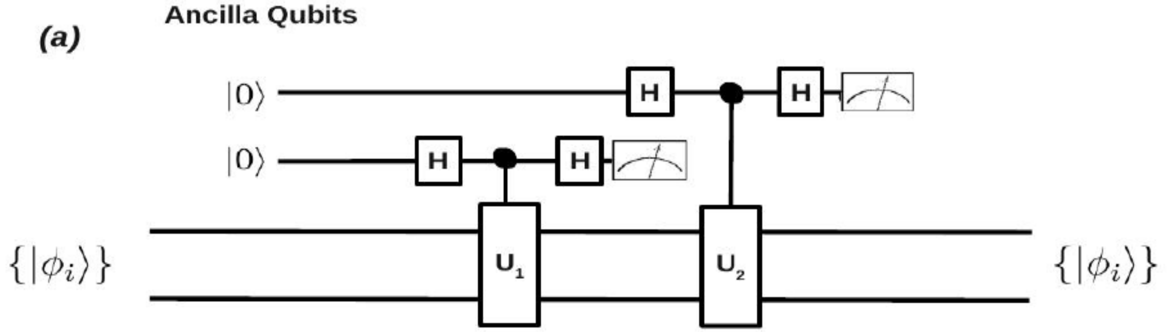


(b) *Controlled*-U_2

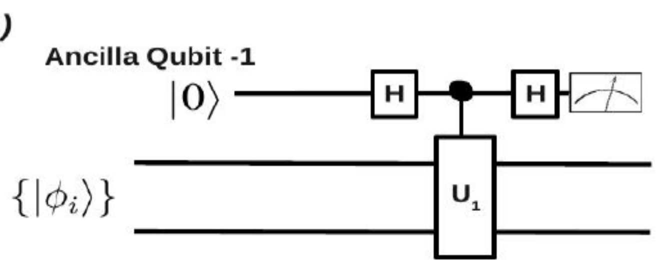
Thin pulses are π/2 and broad pulses are π pulses. Phase of pulses on top

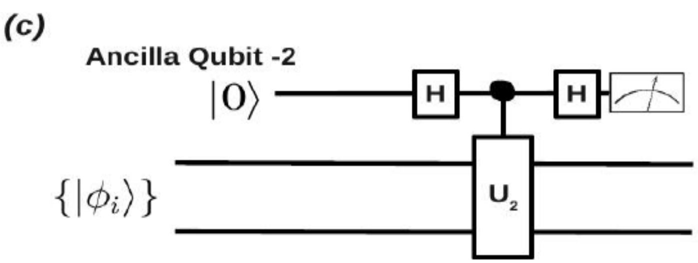
Non-destructive Discrimination of two-qubit orthonormal states.

Original Circuit Needing 2-ancilla qubits

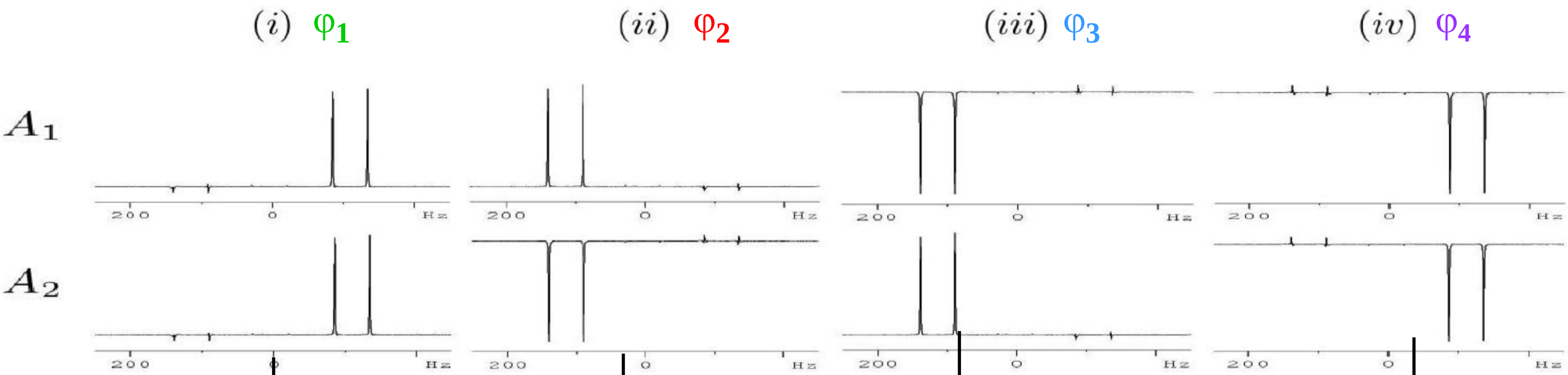


Split Circuit needing 1-ancilla qubit





Results for Ancilla measurements



A_1 → +ve signal → |0> state.
A_2 → +ve signal → |0> state.

→ $(1/\sqrt{2})\,(|00\rangle + |01\rangle)$

A_1 → +ve signal → |0> state.
A_2 → -ve signal → |1> state.

→ $(1/\sqrt{2})\,(|10\rangle + |11\rangle)$

A_1 → -ve signal → |1> state.
A_2 → +ve signal → |0> state.

→ $(1/\sqrt{2})\,(|10\rangle - |11\rangle)$

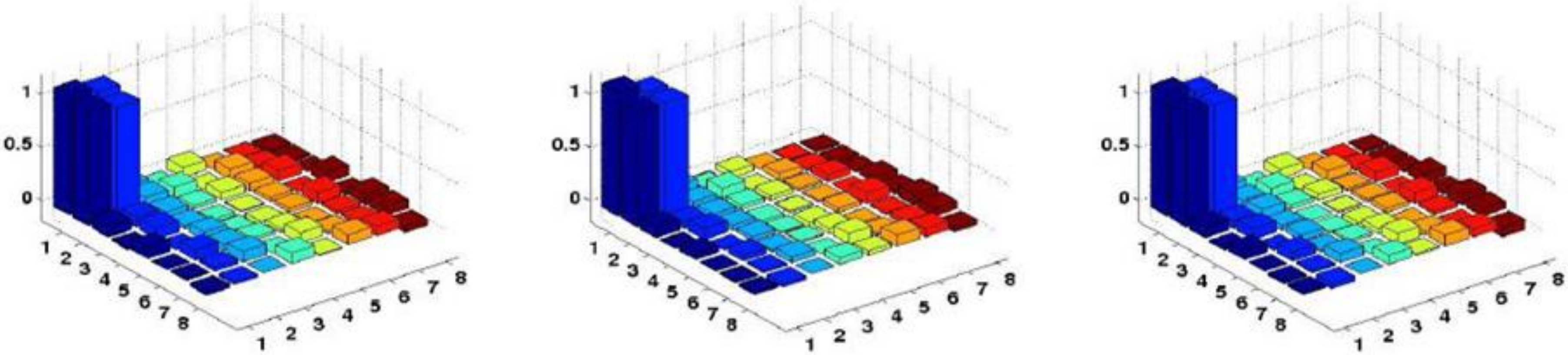
A_1 → -ve signal → |1> state.
A_2 → -ve signal → |1> state.

→ $(1/\sqrt{2})\,(|00\rangle - |01\rangle)$

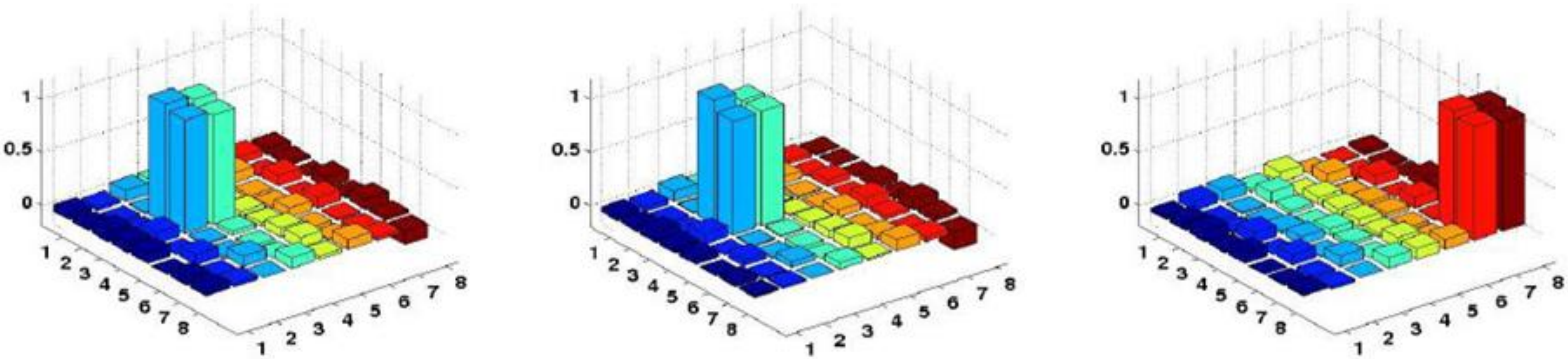
Complete density matrix tomography has done to

1. Show the state is preserved
2. Compute fidelity of the experiment.

Initial State	After First Experiment	After Second Experiment
$(i)\quad \vert 0\rangle(\frac{1}{\sqrt{2}}(\vert 00\rangle + \vert 01\rangle))$	$\vert 0\rangle(\frac{1}{\sqrt{2}}(\vert 00\rangle + \vert 01\rangle))$	$\vert 0\rangle(\frac{1}{\sqrt{2}}(\vert 00\rangle + \vert 01\rangle))$



$(ii)\quad \vert 0\rangle(\frac{1}{\sqrt{2}}(\vert 10\rangle + \vert 11\rangle))$	$\vert 0\rangle(\frac{1}{\sqrt{2}}(\vert 10\rangle + \vert 11\rangle))$	$\vert 1\rangle(\frac{1}{\sqrt{2}}(\vert 10\rangle + \vert 11\rangle))$



Conclusions of the State Discrimination

- **A general scalable method for quantum state discrimination using quantum phase estimation algorithm is discussed, and experimentally implemented for a two qubit case by NMR.**

- **As the direct measurements are performed only on the ancilla, the discriminated states are preserved.**

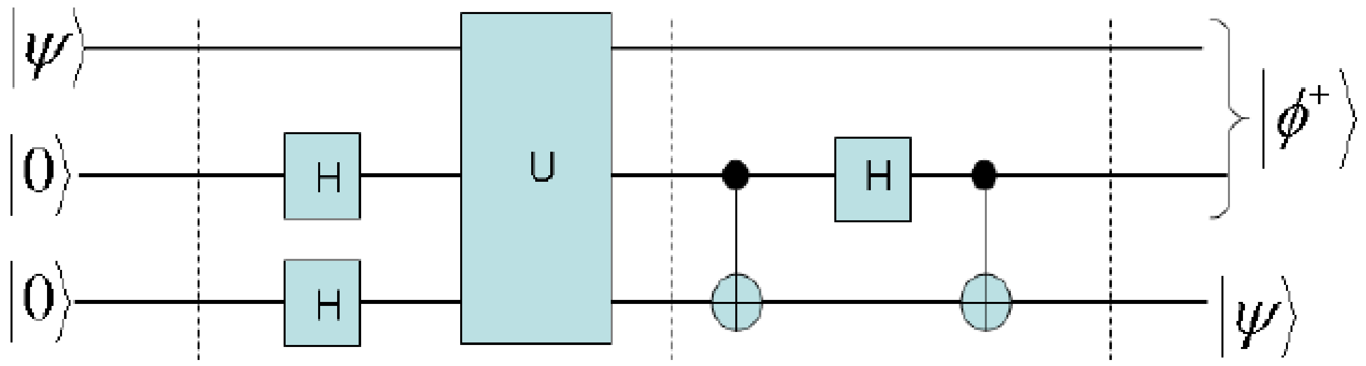
V.S. Manu and Anil Kumar (75 years of Entanglement, Foundations and Information Theoretic Applications, Koltata Jan., 2011, AIP conf. Proceedings; 1384, 229-240 (2011).

No-Hiding Theorem

S.L. Braunstein & A.K. Pati, Phys.Rev.Lett. 98, 080502 (2007).

Any physical process that bleaches out the original information is called "Hiding". If we start with a pure state, this bleaching process will yield a "mixed state" and hence the bleaching process in Non-Unitary". However, in an enlarged Hilbert space, this process can be represented as a "unitary". The No-Hiding Theorem demonstrates that the initial pure state, after the bleaching process, resides in the ancilla qubits from which, under local unitary operations, is completely transformed to one of the ancilla qubits.

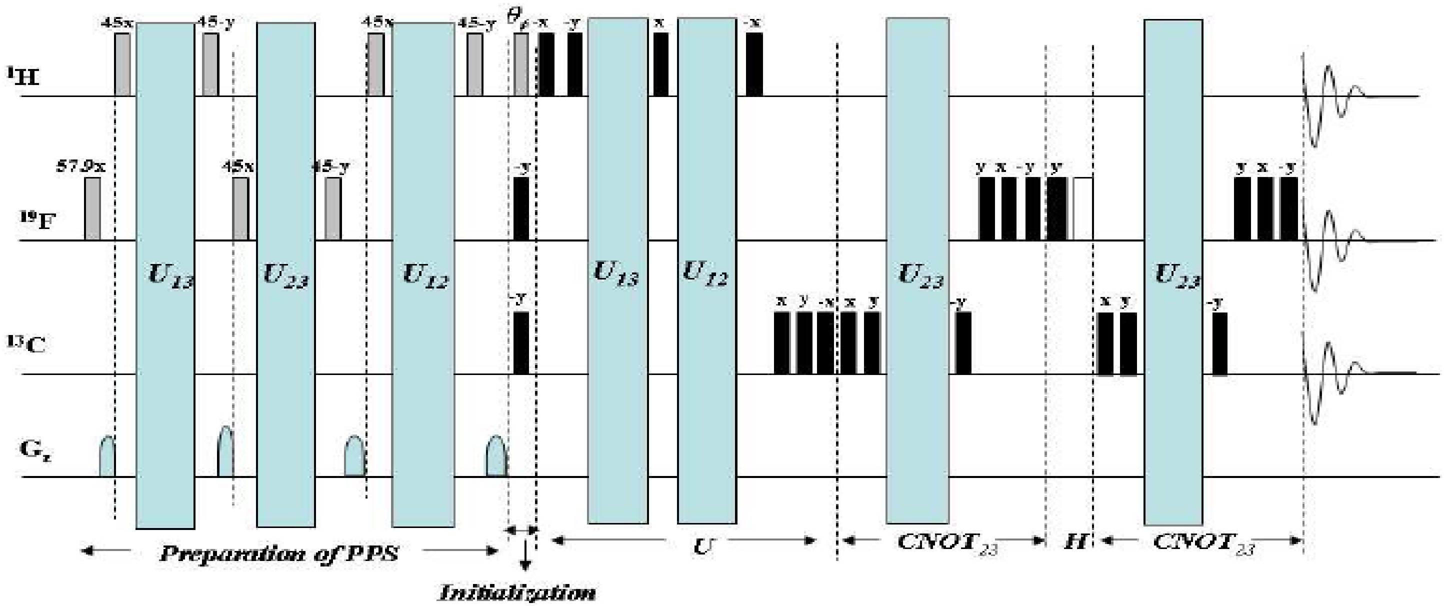
Quantum Circuit for Test of No-Hiding Theorem using State Randomization (operator U). H represents Hadamard Gate and dot and circle represent CNOT gates.



After randomization the state |ψ> is transferred to the second Ancilla qubit proving the No-Hiding Theorem.

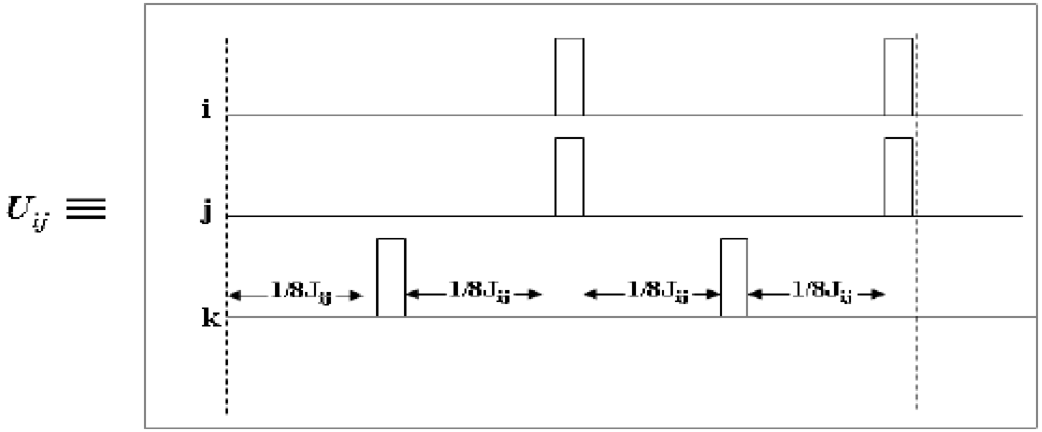
(S.L. Braunstein, A.K. Pati, PRL 98, 080502 (2007).

NMR Pulse sequence for the Proof of No-Hiding Theorem



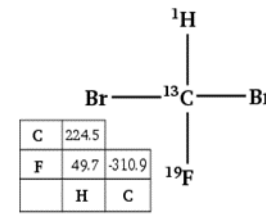
(a)

The initial State ψ is prepared for different values of θ and φ



Jharana et al

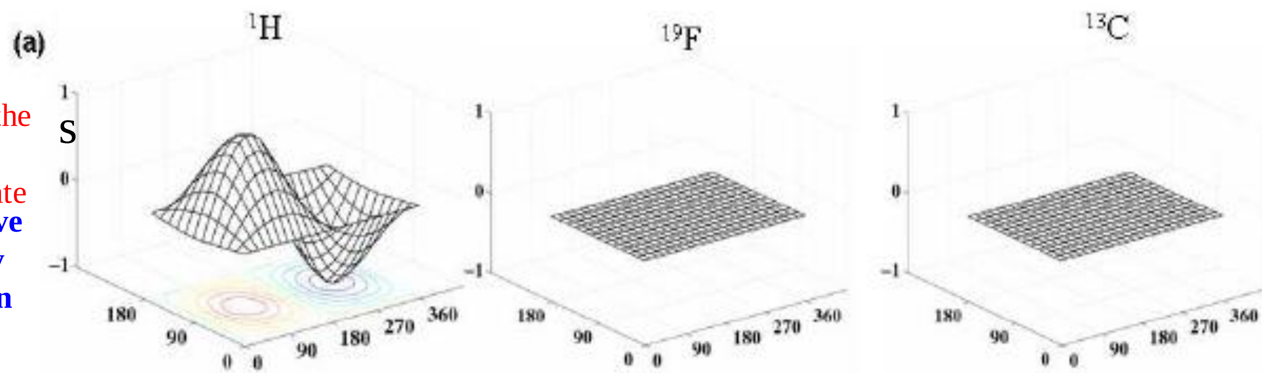
Experimental Result for the No-Hiding Theorem.



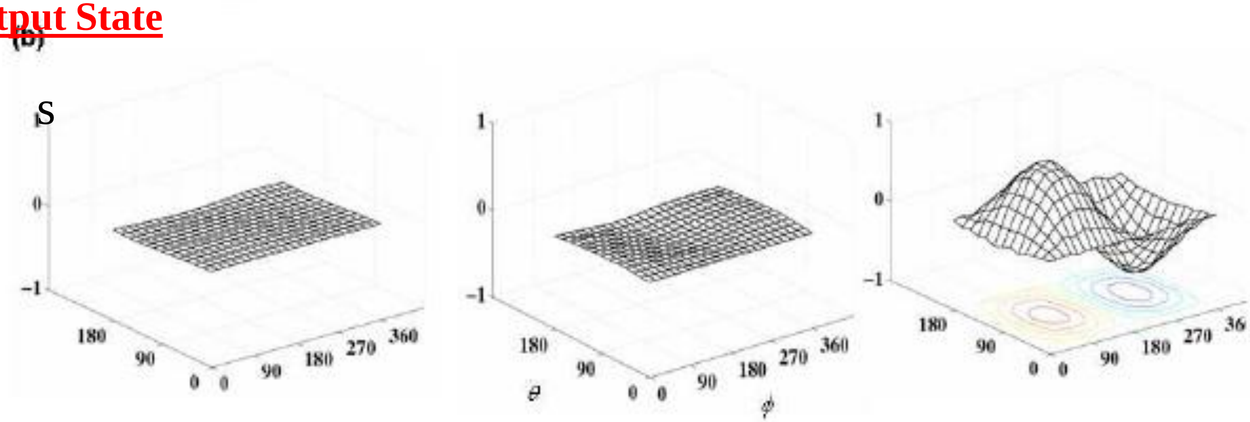
Input State

Each point in this figure is a point on the Bloch sphere and represents a pure state

325 experiments have been performed by varying θ and φ in steps of 15°



Output State



The state ψ is completely transferred from first qubit to the third qubit

S = Integral of real part of the signal for each spin

Jharana Rani Samal, Arun K. Pati and Anil Kumar, Phys. Rev. Letters, 106, 080401 (25 Feb., 2011).