Operational perspective of quantum simulations using quantum walks

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Outline

- Quantum simulations : Operational approach and why is it important ?

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- Disorder QW and localization of quantum state
  - Dynamic localization
  - Anderson localization

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- Discrete-time quantum walks (DTQW)
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  - Dynamic localization
  - Anderson localization
- Accelerate DTQW
  - one particle case
  - two particle localization and entanglement generation

Quantum simulation

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when dealing with large system or an high energy quantum phenomenon

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• Application : study of condensed matter physics, high-energy physics, atomic physics, quantum chemistry and cosmology

• Can be implemented in quantum computer and also with simpler analog devices that require less control and easier to construct
For example :neutral atoms, ion, polar molecules, electron in semiconductor, superconducting circuits, nuclear spins and photons

Quantum Simulation : Operational approach (algorithmic) approach using a qubit

Qubit : Basic Element

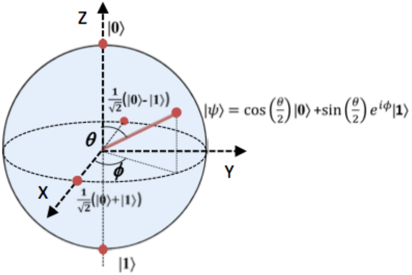
- Qubit - Ideal two-state quantum system : $|\uparrow\rangle = \begin{bmatrix}1\\0\end{bmatrix}$ and $|\downarrow\rangle = \begin{bmatrix}0\\1\end{bmatrix}$

  logical /computational states

  - photons (V and H polarization, transmission mode - path encoding)
  - electrons or other spin-$\frac{1}{2}$ systems (spin up and down)
  - systems defined by two energy levels of atoms or ions

Allows superposition : $|\Psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$

$\alpha, \beta \in \mathbb{C}$; $|\alpha|^2 + |\beta|^2 = 1$; $|\Psi\rangle = e^{i\eta}|\Psi\rangle$



Discrete-time quantum walk

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- Walk is defined on the Hilbert space $\mathcal{H} = \mathcal{H}_c \otimes \mathcal{H}_p$

  $\mathcal{H}_c$ (particle) is spanned by $|\uparrow\rangle$ and $|\downarrow\rangle$
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  - Coin operation : $C(\theta) = \begin{bmatrix} \cos(\theta) & -i\sin(\theta) \\ -i\sin(\theta) & \cos(\theta) \end{bmatrix}$

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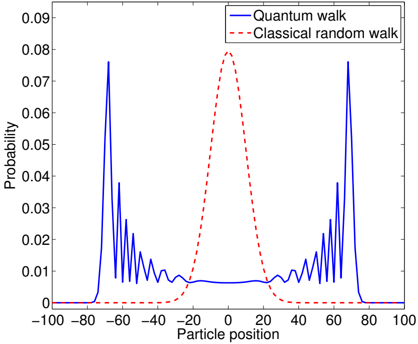
  - Coin operation : $C(\theta) = \begin{bmatrix} \cos(\theta) & -i\sin(\theta) \\ -i\sin(\theta) & \cos(\theta) \end{bmatrix}$
  - Conditional unitary shift operation S:

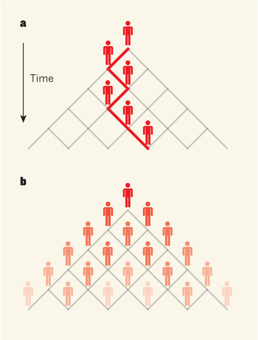
$$S = \sum_{j\in\mathbb{Z}} \Big[|\uparrow\rangle\langle\uparrow| \otimes |j-1\rangle\langle j| + |\downarrow\rangle\langle\downarrow| \otimes |j+1\rangle\langle j| \Big]$$

state $|\uparrow\rangle$ moves to the left and state $|\downarrow\rangle$ moves to the right

Quantum walk

- Each step of QW : $W = S(C(\theta) \otimes \mathbb{1})$





100 step of CRW and QW $[S(C(\pi/4) \otimes \mathbb{1})]^{100}$on a particle with initial state $\frac{1}{\sqrt{2}}(|\uparrow\rangle + i|\downarrow\rangle)$

- G. V. Riazanov (1958), R. Feynman (1986)
- K.R. Parthasarathy, Journal of applied probability 25, 151-166 (1988)
- Y. Aharonov, L. Davidovich and N. Zugury, Phys. Rev. A, 48, 1687 (1993)
- Use of word Quantum ~~random~~ walk

Quantum walks

- Superposition and interference : Evolution (dynamics) exploits superposition and interference aspects of quantum mechanics entangling the Hilbert space involved in the dynamics (particle and position space)
  Explores multiple possible paths simultaneously with amplitude corresponding to different paths interfering

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- Experimentally implemented and control over the dynamics demonstrated
  NMR system, ion traps, photons in optical waveguide, neutral atoms on optical lattice.......

Dirac equation

$$\left(i\hbar\frac{\partial}{\partial t} - \hat{\mathbf{H}}_{\mathbf{D}}\right)\Psi = \left(i\hbar\frac{\partial}{\partial t} + i\hbar c\hat{\alpha}\cdot\frac{\partial}{\partial x} - \hat{\beta}mc^2\right)\Psi = 0$$

Continuum limit and simulation of Dirac equation

Dirac equation

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From DTQW

$$\psi^{\uparrow}_{x,t+1} = \cos(\theta)\psi^{\uparrow}_{x+1,t} - i\sin(\theta)\psi^{\downarrow}_{x-1,t}$$
$$\psi^{\downarrow}_{x,t+1} = \cos(\theta)\psi^{\downarrow}_{x-1,t} - i\sin(\theta)\psi^{\uparrow}_{x+1,t}$$

when $\theta = 0$, the expression in continuum limit takes the form

$$\left[i\hbar\frac{\partial}{\partial t} - i\hbar\sigma_3\frac{\partial}{\partial x}\right]\Psi(x,t) = 0$$

Massless Dirac equation

David Mayer (1996) ; Fredrick Strauch (2006) ; CMC (2010)

DE with mass term : Split-step / 2-period QW

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$$C(\theta_1) = \begin{pmatrix} \cos(\theta_1) & -i\sin(\theta_1) \\ -i\sin(\theta_1) & cos(\theta_1) \end{pmatrix},$$
$$C(\theta_2) = \begin{pmatrix} \cos(\theta_2) & -i\sin(\theta_2) \\ -i\sin(\theta_2) & \cos(\theta_2) \end{pmatrix}$$

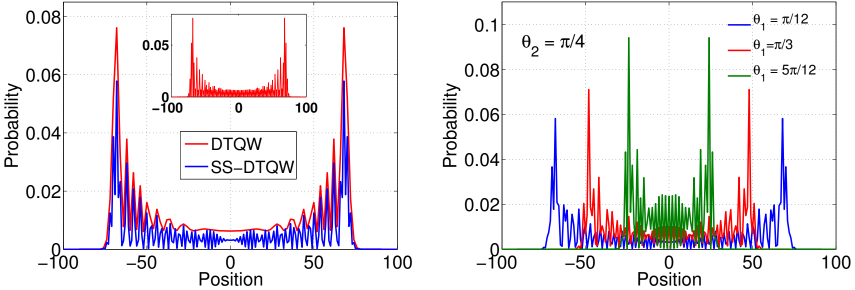
and a two half-shift operators,

$$S_- = \begin{pmatrix} T_- & 0 \\ 0 & I \end{pmatrix}, \quad S_+ = \begin{pmatrix} I & 0 \\ 0 & T_+ \end{pmatrix} \quad S = \begin{pmatrix} T_- & 0 \\ 0 & T_+ \end{pmatrix}$$

$$T_- = |j-1\rangle\langle j| \quad ; \quad T_+ = |j+1\rangle\langle j|$$

$$U_{SQW} = S_+\Big(C(\theta_2)\otimes I\Big)S_-\Big(C(\theta_1)\otimes I\Big) \equiv S\Big(C(\theta_2)\otimes I\Big)S\Big(C(\theta_1)\otimes I\Big)$$

DCA and SS-QW



SSQW ($\theta_1 = 0, \theta_2 = \pi/4$) = DCA $\alpha = \beta = \frac{1}{\sqrt{2}}$ Substituting $\theta_1 = \phi_1 = \delta_1 = \delta_2 = 0$ we get,

$$U_{SQW} = \begin{pmatrix} \cos(\theta_2)T_- & -i\sin(\theta_2)I \\ -i\sin(\theta_2)I & \cos(\theta_2)T_+ \end{pmatrix}$$

which is in the same form as U_{DA} where $\beta = \sin(\theta_2) \equiv \frac{mca}{\hbar}$ and $\alpha = \cos(\theta_2)$.

SS-QW and DE

$$\left[\frac{\partial}{\partial t} - \cos(\theta_2)\begin{bmatrix}\cos(\theta_1) & -i\sin(\theta_1)\\ i\sin(\theta_1) & -\cos(\theta_1)\end{bmatrix}\frac{\partial}{\partial x} - \begin{bmatrix}\cos(\theta_1+\theta_2)-1 & -i\sin(\theta_1+\theta_2)\\ -i\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2)-1\end{bmatrix}\right]\begin{bmatrix}\psi^{\downarrow}_{x,t}\\ \psi^{\uparrow}_{x,t}\end{bmatrix} =$$

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1. Setting $\theta_1 = 0$ and θ_2 to a small value (mass of sub-atomic particles) :

$$i\hbar\left[\frac{\partial}{\partial t} - \left(1-\frac{\theta_2^2}{2}\right)\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\frac{\partial}{\partial x} + i\theta_2\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\right]\begin{bmatrix}\psi^{\downarrow}_{x,t}\\ \psi^{\uparrow}_{x,t}\end{bmatrix} \approx 0$$

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2. Setting $\cos(\theta_1+\theta_2) = 1$:

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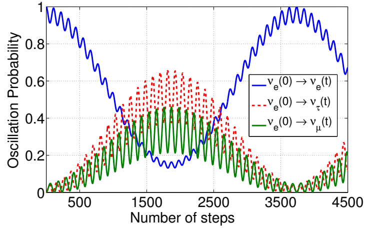
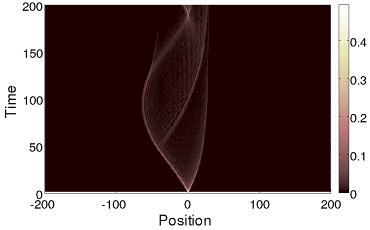
3. Setting θ_1 to be extremely small and $\cos(\theta_1+\theta_2) = 1$:

$$i\hbar\left[\frac{\partial}{\partial t} - \cos(\theta_2)\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\frac{\partial}{\partial x}\right]\begin{bmatrix}\psi^{\downarrow}_{x,t}\\ \psi^{\uparrow}_{x,t}\end{bmatrix} \approx 0$$

Scientific Reports 6, 25779 (2016); Phys. Rev. A 97, 012116 (2018)

Examples of simulations





(1) Neutrino flavour oscillation Eur. Phys. J. C (Particles and Fields), 77: 85 (2017)
(2) Dirac Hamiltonian in curved space with U(1) gauge potential arXiv:1712.03911

Disorder and localization

Disorder induced by randomized unitary evolution

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- Temporal disorder

$$S\Big(B(\theta_t)\otimes\mathbb{1}\Big)\cdots S\Big(B(\theta_j)\otimes\mathbb{1}\Big)\cdots S\Big(B(\theta_0)\otimes\mathbb{1}\Big)|\Psi_{in}\rangle$$

randomly chosen parameter $\theta \in \{-\pi/2, \pi/2\}$ for each step.

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- Spatial disorder

$$\left[S\left(\sum_x B(\theta_x)\otimes |x\rangle\langle x|\right)\right]^t |\Psi_{in}\rangle$$

randomly chosen parameters $\theta \in \{-\pi/2, \pi/2\}$ for each position.

Disorder induced by randomized unitary evolution

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randomly chosen parameters $\theta \in \{-\pi/2, \pi/2\}$ for each position.

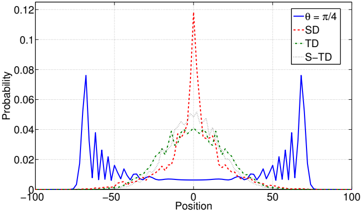
- Spatio-temporal disorder (fluctuating)

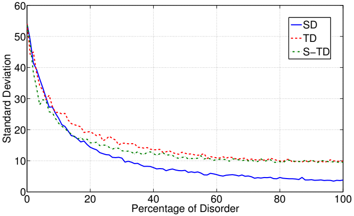
$$S\left(\sum_x B(\theta_{x,t})\otimes |x\rangle\langle x|\right)\cdots S\left(\sum_x B(\theta_{x,j})\otimes |x\rangle\langle x|\right)\cdots S\left(\sum_x B(\theta_{x,1})\otimes |x\rangle\langle x|\right) |$$

randomly chosen parameters $\theta \in \{-\pi/2, \pi/2\}$ for each position and step.

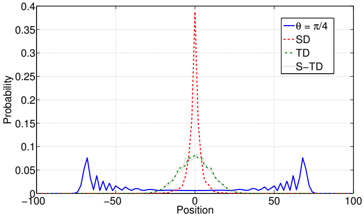
Disorder and localization

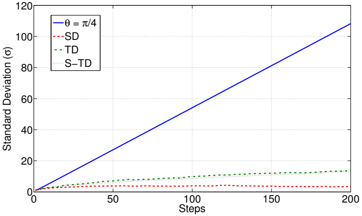
20% disorder





100% disorder





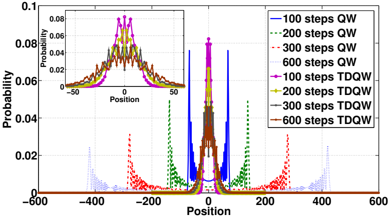
Dynamic localization from temporal disorder

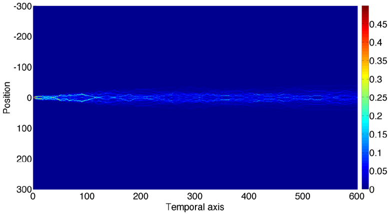
$$v_g(k,\theta_t) = \pm \frac{k\cos(\theta_t)}{\sqrt{k^2\cos(\theta_t) + \mathfrak{A}(\theta,t)}}$$

$$\mathfrak{A}(\theta,t) = 1 + \sin(\theta_t)\Big[\csc(\theta_{t\pm1}) - \cot(\theta_{t\pm1})\Big] - \cos(\theta_t)$$

For each instance of time t the group velocity will be a random value in the range: $-1 \leq v_g(k,\theta_t) \leq 1$. The average group velocity :

$$v_g^{TD}(k,\theta_t,t) = \frac{1}{t}\sum_{t=0}^{t} v_g(k,\theta_t) \approx 0$$

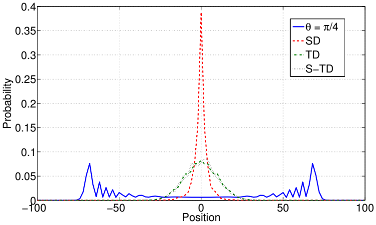


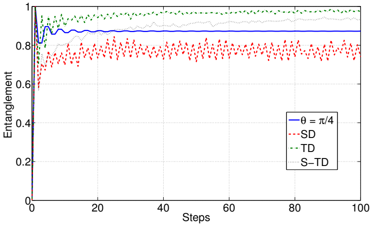


Distribution remains localized near the origin irrespective of the time (t)

Entanglement in localized states

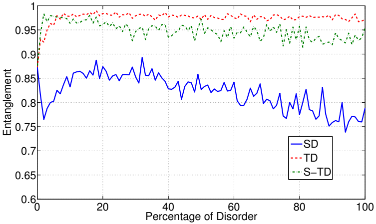
Entanglement in localized states



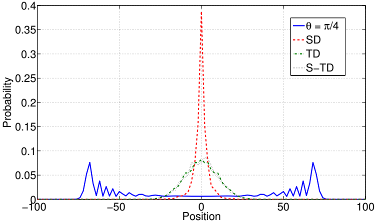


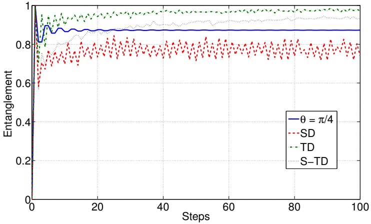
$$\mathcal{N}(\rho) = \sum_i \frac{|\lambda_i| - \lambda_i}{2}$$

where λ_i are the eigenvalues.



Entanglement in localized states





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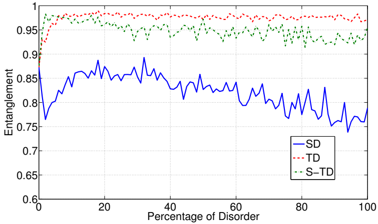
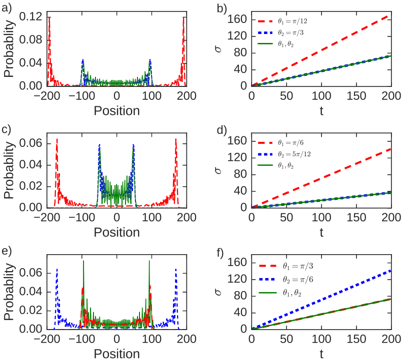
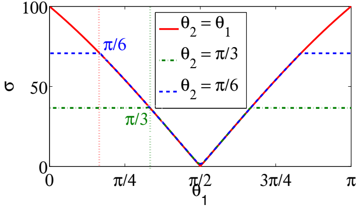


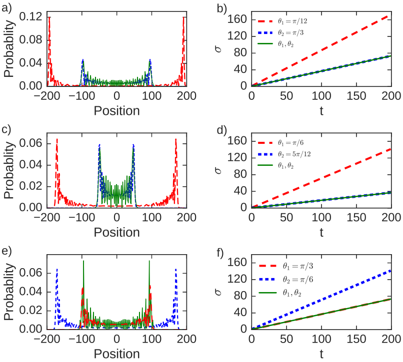
Entanglement is robust against disorder

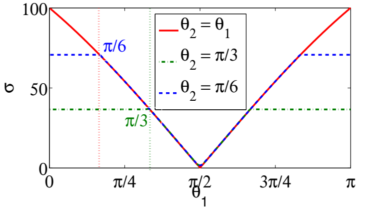
Controlling localization - two period QW

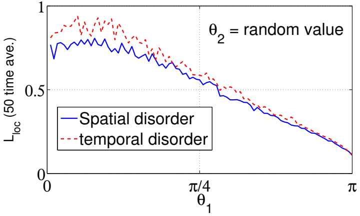


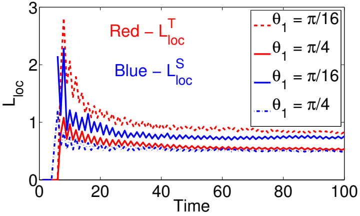


Controlling localization - two period QW









Accelerating quantum walks

Replace θ in coin operation by $\theta(t) = \theta(a, t) = \theta_0 e^{-at}$

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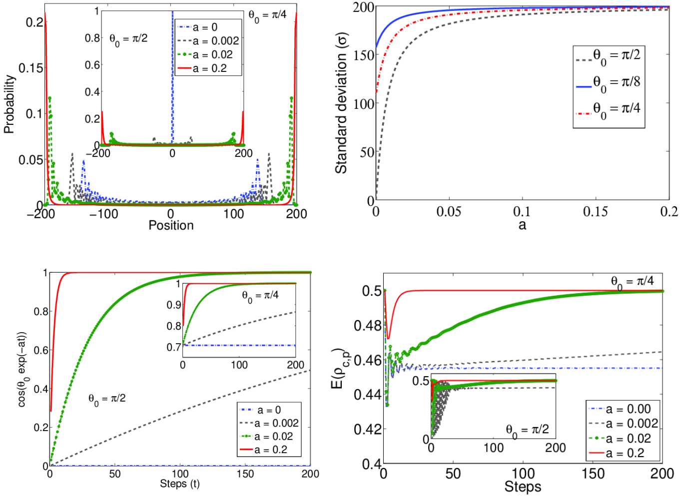
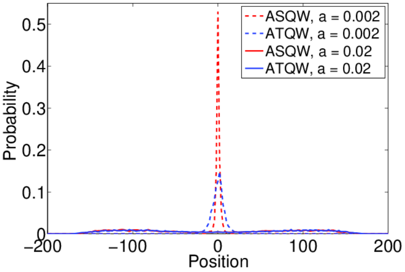


Accelerated quantum walks : disorder and localization

$$\Phi_\phi = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} \otimes \sum_x |x\rangle \langle x|$$

$$W = S_x \Phi(\phi) C(\theta_0) \equiv S_x B(\theta_0, \phi).$$



Accelerated 2 particle quantum walks

- Entanglement

$$|\Psi\rangle_{AB} = |a\rangle_A \otimes |b\rangle_B = |a\rangle_A|b\rangle_B$$

$$|a\rangle_A|b\rangle_B \rightarrow \sum c_{ab}|a\rangle_A|b\rangle_B$$

$$|\Psi\rangle_{AB} = \alpha|0\rangle_A|0\rangle_B \pm \beta|1\rangle_A|1\rangle_B$$

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$|\Psi_{in}\rangle = |00\rangle \to$ localised QW on two particle $\to$ entangled state

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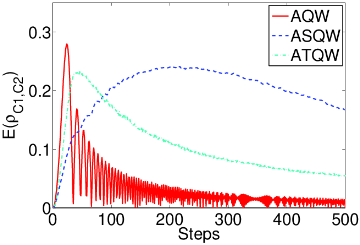
$|\Psi_{in}\rangle = |00\rangle \rightarrow$ localised QW on two particle $\rightarrow$ entangled state
coin operation

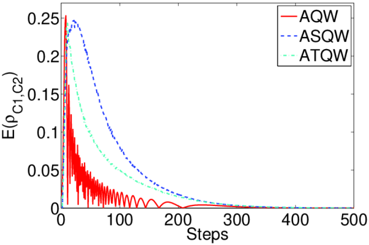
$$C[\theta(t)] = \begin{pmatrix} \cos[\theta(t)] & 0 & 0 & -i\sin[\theta(t)] \\ 0 & \cos[\theta(t)] & -i\sin[\theta(t)] & 0 \\ 0 & -i\sin[\theta(t)] & \cos[\theta(t)] & 0 \\ -i\sin[\theta(t)] & 0 & 0 & \cos[\theta(t)] \end{pmatrix}$$

shift-operator

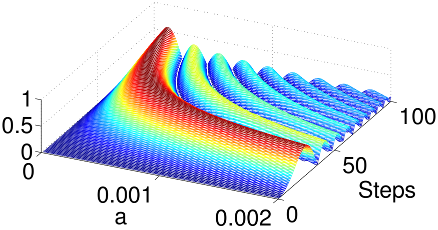
$$S^1 \otimes S^2$$

Two-particle Entanglement Generation





a = 0.002 and a = 0.02



Summary

- With a careful choice of evolution parameters we can engineer the localization of quantum walk for effective use in QIP protocols and simulation of various accessible and inaccessible quantum phenomenon with disorder in both, high energy and low energy physical systems.