Unconditional non-Gaussianity as a resource for quantum computation in optomechanical systems

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QTEQ QUANTUM TECHNOLOGY at QUEEN'S

Outline

- Quantum resource theories
- Resource theory of quantum non-Gaussianity
- Unconditional non-Gaussianity for quantum computation in optomechanical systems

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- Quantum resource theories

Resource theory of quantum non-Gaussianity

Unconditional non-Gaussianity for quantum computation in optomechanical systems

Resource theories

State space	Free states
Allowed operations	**Resources**

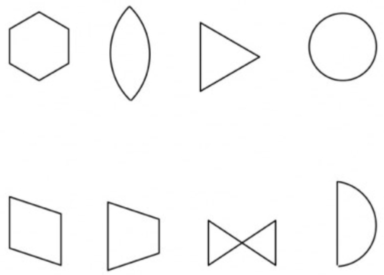
Example: straightedge-and-compass constructions

State space	Free states
Allowed operations	Resources

[Cocke et al., Inf. & Comput. '16]

Example: straightedge-and-compass constructions

State space



Free states

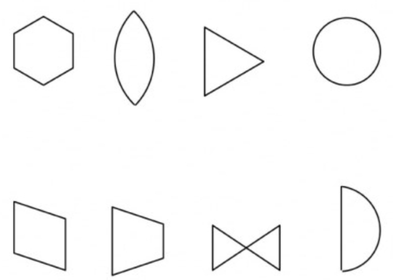
Allowed operations

Resources

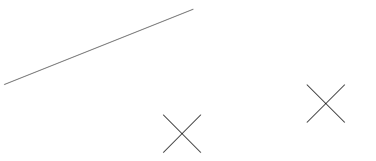
[Cocke et al., Inf. & Comput. '16]

Example: straightedge-and-compass constructions

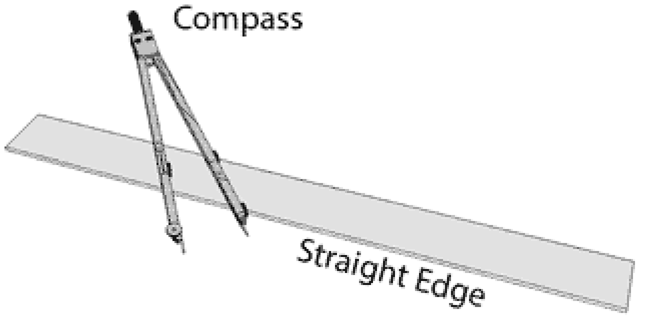
State space



Free states



Allowed operations



Resources

[Cocke et al., Inf. & Comput. '16]

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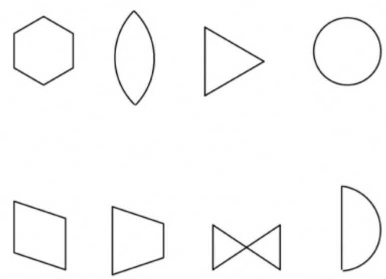
State space



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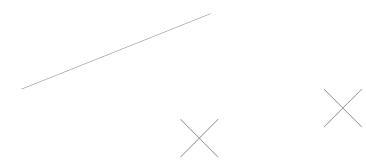

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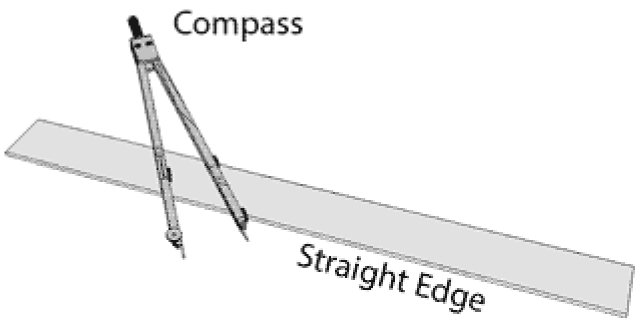


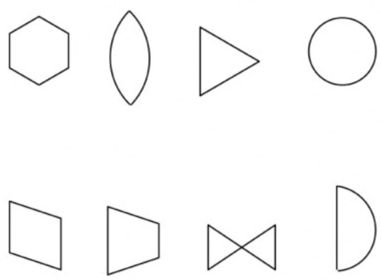
Resources

Not all figures can be drawn, e.g. a square with the same area of a given circle

[Cocke et al., Inf. & Comput. '16]

Example: straightedge-and-compass constructions

State space



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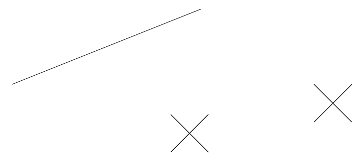

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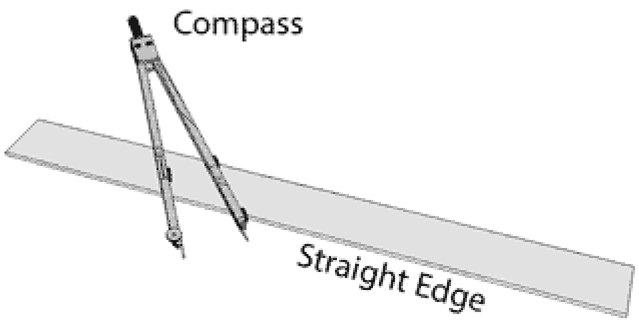

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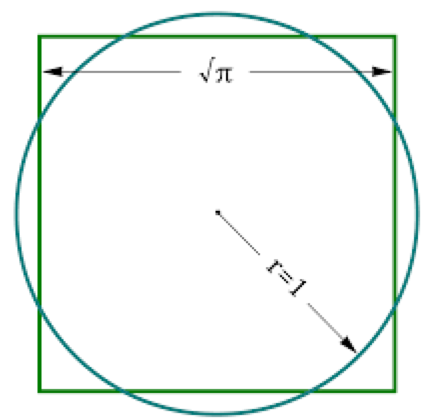

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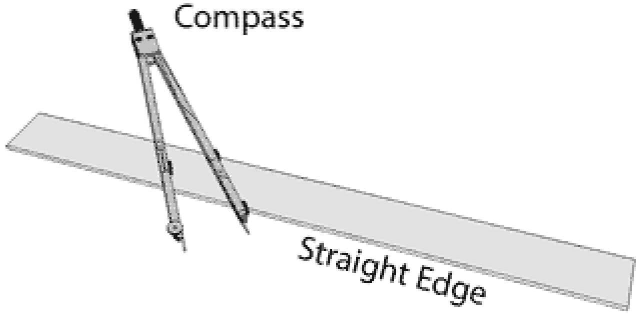

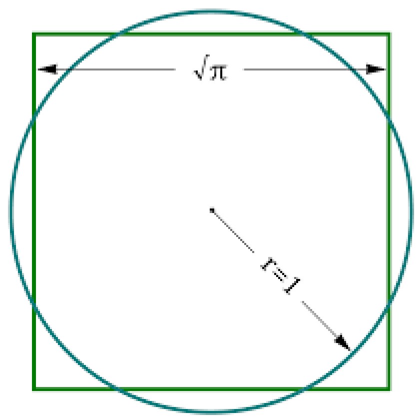

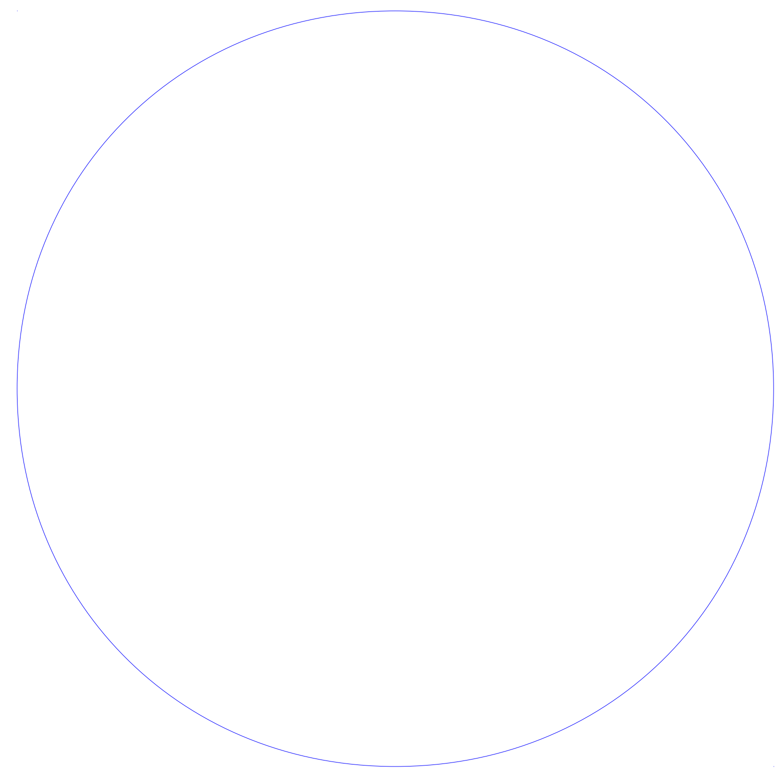

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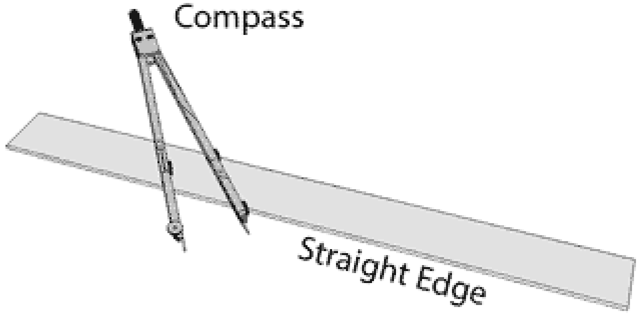

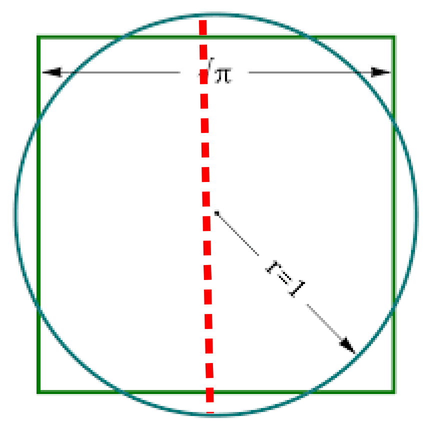

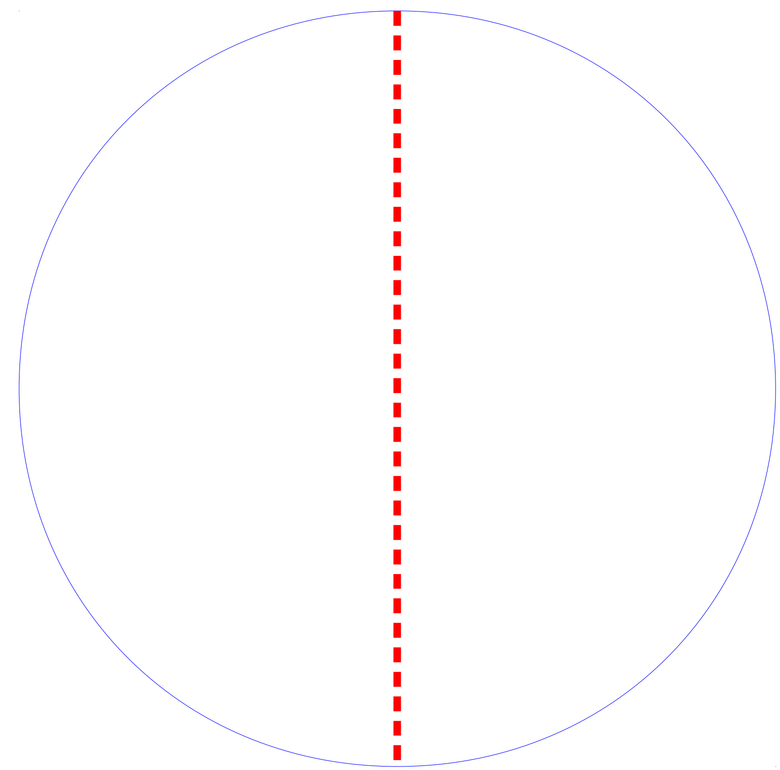

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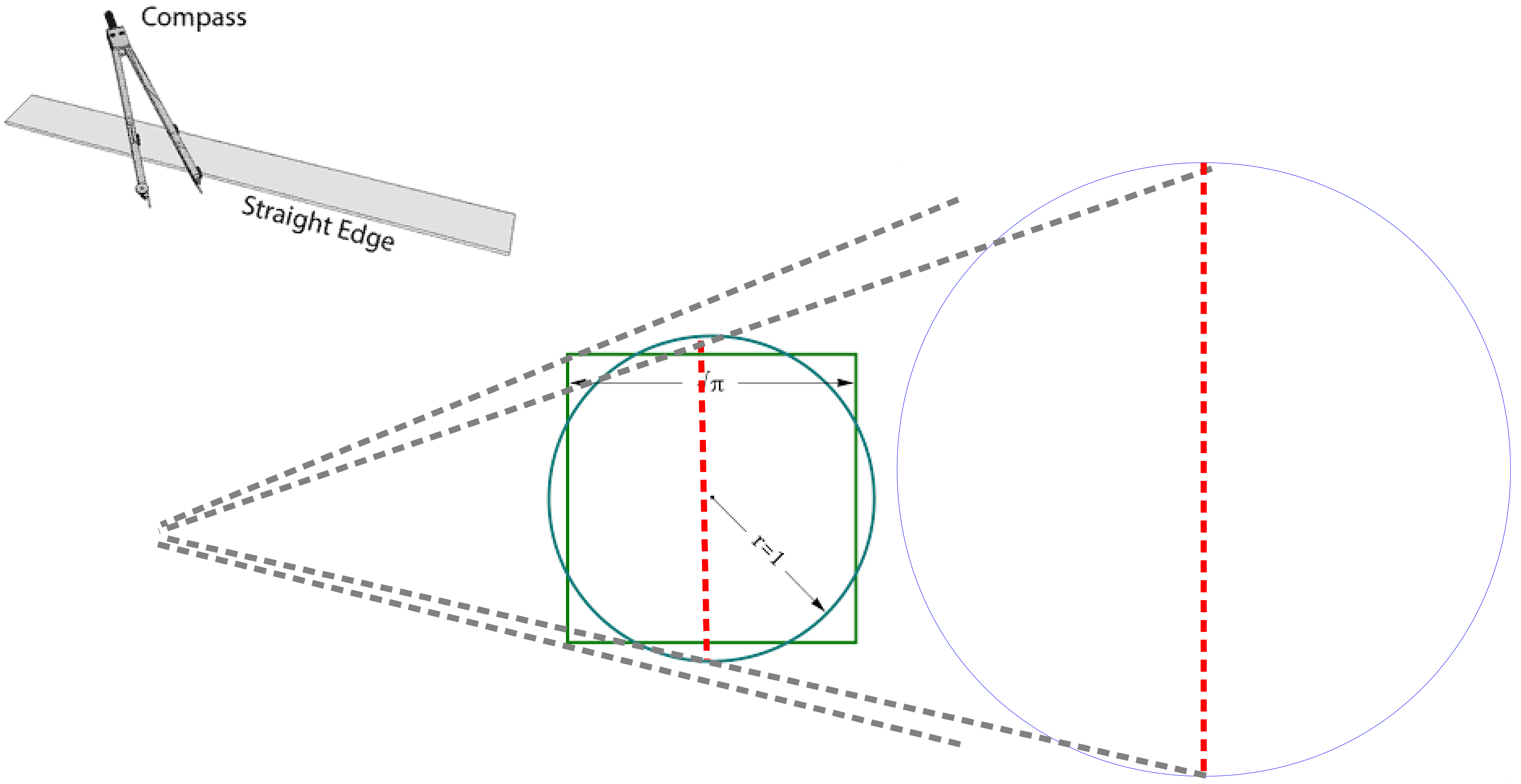

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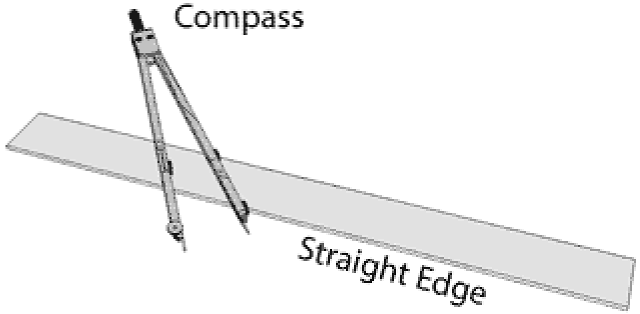

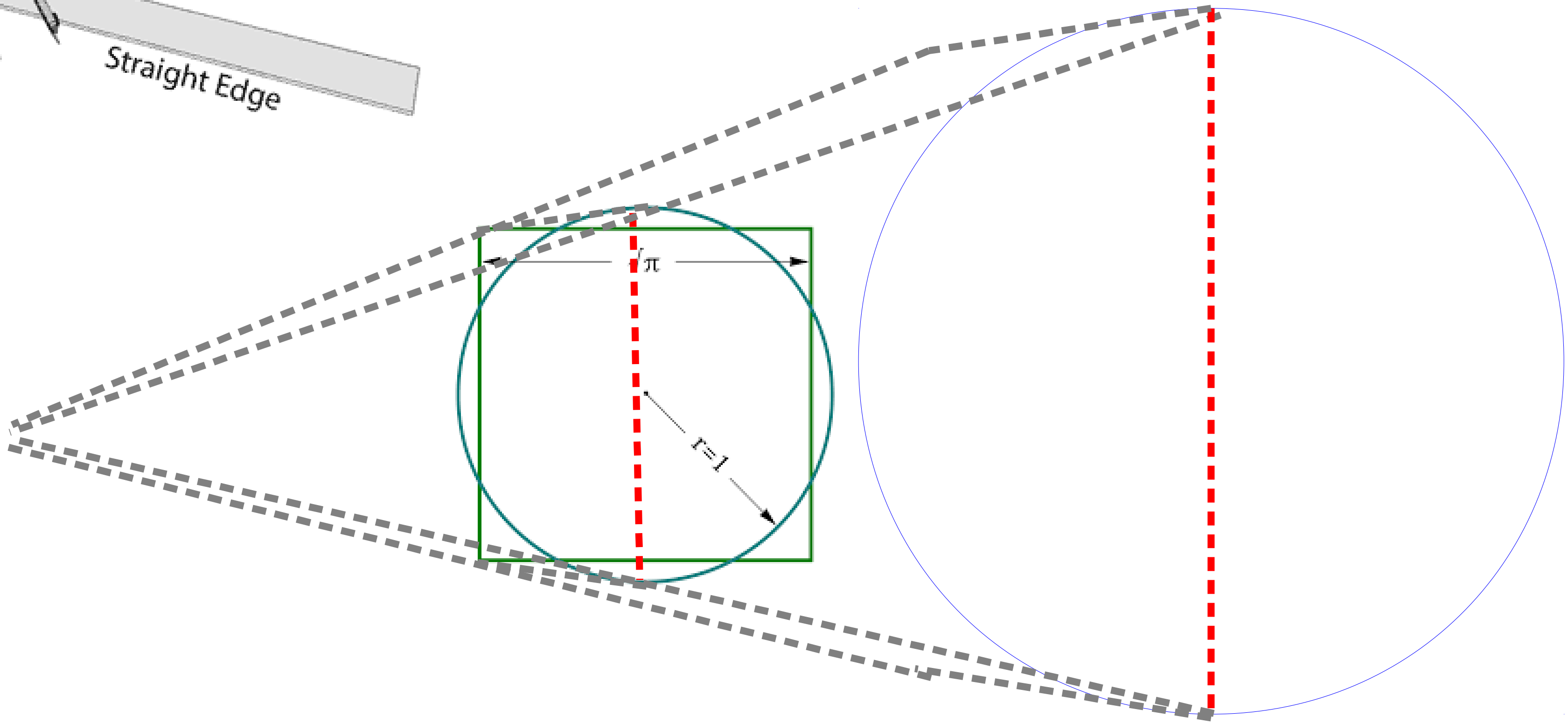

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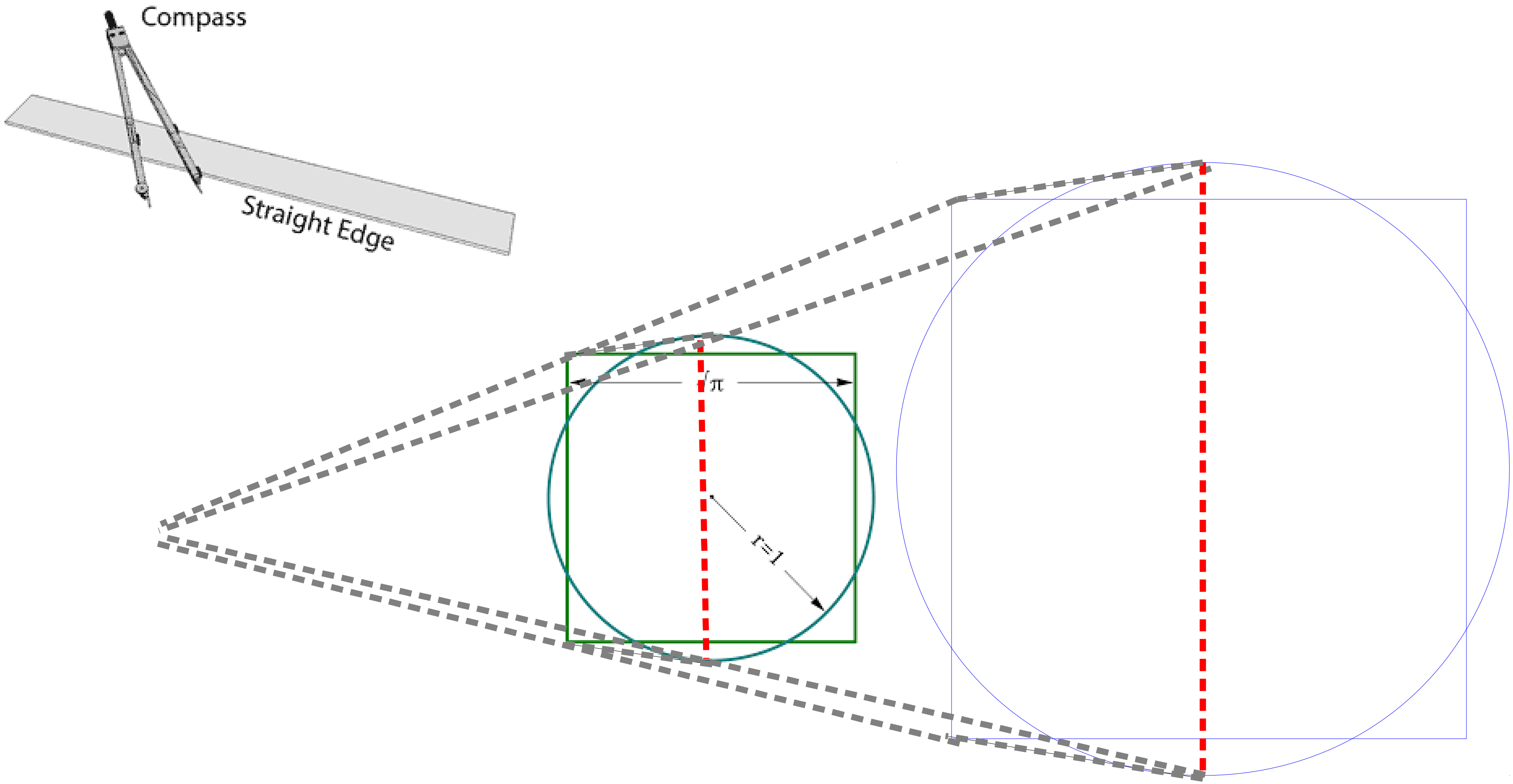

The resource acts as a catalyst, allowing for new figures to be drawn.

[Cocke et al., Inf. & Comput. '16]

Resource theories

	Quantum communication [Horodecki et al., RMP '09]	DV Quantum computation [Veitch et al., NJP'12; NJP'14] [Mari et al., PRL'12; Howard et al. PRL'17]
State space	Bipartite quantum systems	DV quantum register
Allowed operations	Local ops & classical comm (LOCC)	Stabilizer protocols (Clifford gates + basis prep/meas)
Free states	Separable states	Stabilizer states
Resources	(free) entangled states	(free) magic states

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Primary goals:

- Given a state, is it a (maximal) resource?
- Resource **quantification**: how useful is a resource?
- Resource **distillation**: how to obtain more resourceful states?
- State **conversion**: is it possible to convert a resource into another, and at which rate?

Resource theory of entanglement (mixed states)

- Is a state a state a (maximal) resource?
  Difficult to establish whether a state is entangled or not.
  The **singlet** state is maximally resourceful: any other state can be obtained via LOCC.

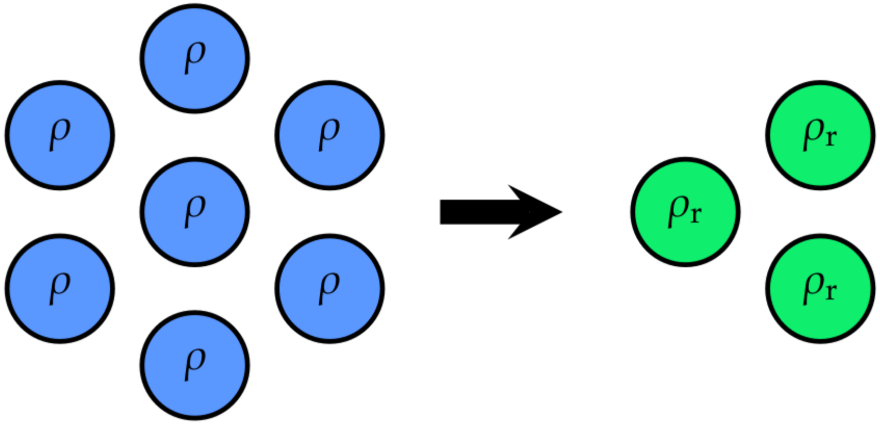
- Resource **quantification**: how useful is a resource?

$$\mathsf{Pure\ \ states:}\quad \mathsf{E}(|\psi\rangle^{\mathsf{AB}}) = \mathsf{S}(\rho^{\mathsf{A}}), \qquad \mathsf{with}\ \ \mathsf{S}(\rho) = -\mathsf{Tr}[\rho \mathsf{log} \rho]$$

$$\mathsf{Mixed\ \ states:}\quad \mathsf{Entanglement\ of\ distillation,\ \ of\ formation,\ \ negativity,}\ \ \ldots$$

- Resource **distillation**: how to obtain the singlet?

Distillation protocols



- State **conversion**: is it possible to convert a resource into another, and at which rate?

Outline

Quantum resource theories

- Resource theory of quantum non-Gaussianity

Unconditional non-Gaussianity for quantum computation
in optomechanical systems

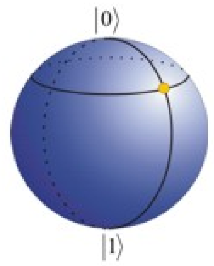
Resource theory of quantum non-Gaussianity

State space	Free states
Allowed operations	Resources

State space: continuous variables

Discrete variables

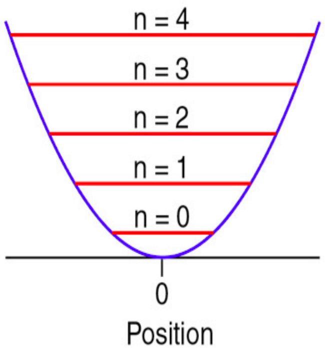
(finite dimension, qubits)



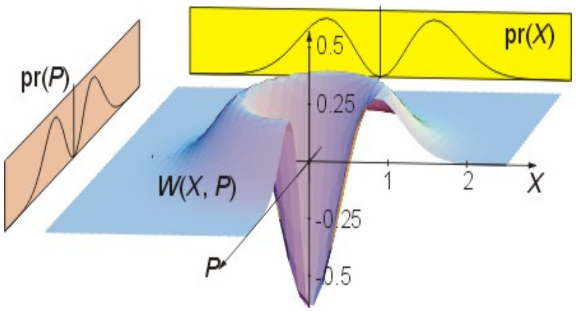
Bloch sphere

Continuous variables

(infinite dimension, canonical c.r., qumodes)

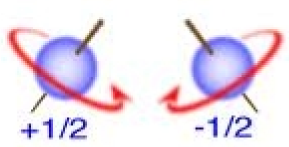


$\hat{q}\,,\hat{p}$

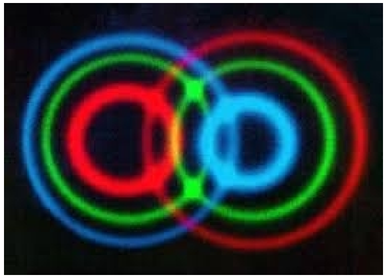


Quantum phase-space
(Wigner function)

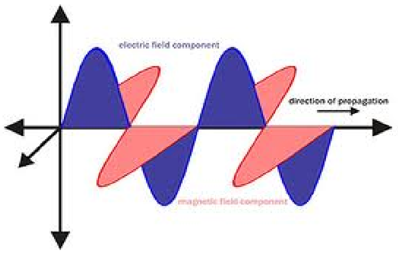
Spin



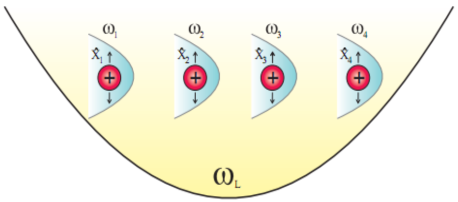
Polarization



Light quadratures



trapped ion motion



Gaussian states

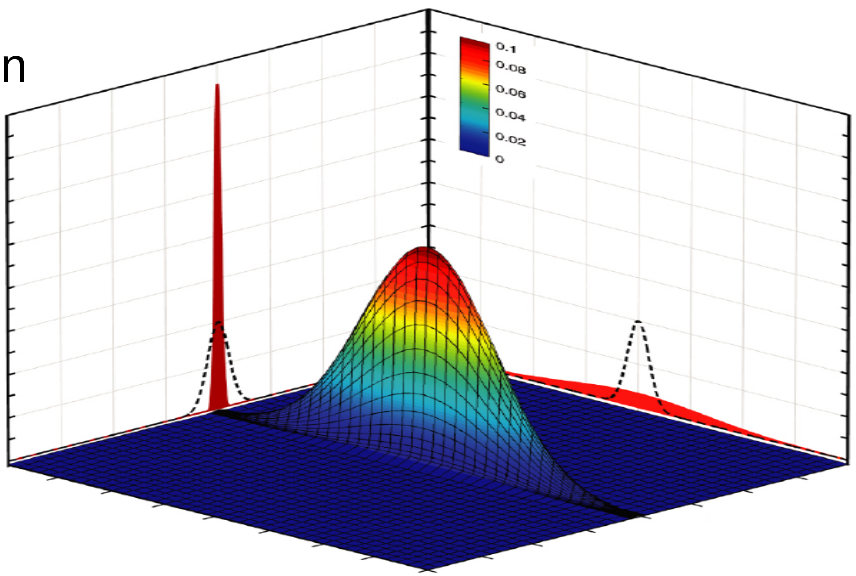
Position and momentum operators

$$q_j = \frac{1}{\sqrt{2}}(b_j + b_j^\dagger) \qquad p_j = \frac{1}{i\sqrt{2}}(b_j - b_j^\dagger) \qquad [q_j, p_k] = i\delta_{j,k}$$

Wigner function

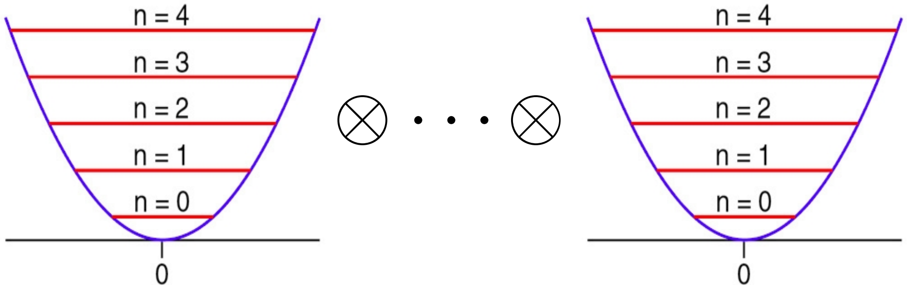
$$\mathcal{W}[\hat{O}](x,y) = \frac{1}{\pi}\int_{\mathbb{R}} dz\, {}_q\langle x+z|\hat{O}|x-z\rangle_q\, e^{-2iyz} \qquad q|x\rangle_q = x|x\rangle_q\,, \quad x \in \mathbb{R}$$

Gaussian states:
Gaussian Wigner function



Resource theory of quantum non-Gaussianity

State space



Free states

Allowed operations

Resources

Allowed operations: Gaussian protocols (GPs)

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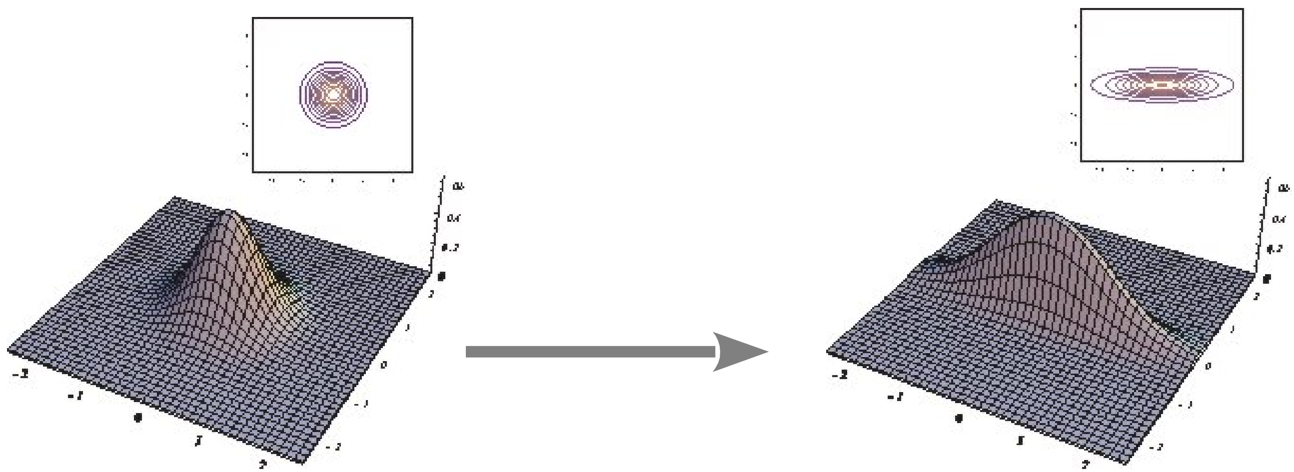
- Gaussian unitaries (e.g., displacement, squeezing, CZ, ...)

Squeezing operator S(s)

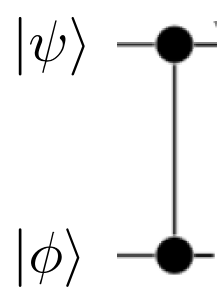
$$|s_\psi\rangle = S_\psi(s)|0\rangle = e^{\frac{s}{2}\left(e^{-i\psi} b^2 - e^{i\psi} b^{\dagger 2}\right)}|0\rangle$$



Position and momentum eigenstates are infinitely squeezed states

Control phase (entangling gate)

$$CZ_{12} \equiv \exp[i q_1 \otimes q_2]$$



$$CZ_{12}[|\psi\rangle \otimes |\phi\rangle]$$

Allowed operations: Gaussian protocols (GPs)

- Gaussian unitaries (e.g., displacement, squeezing, CZ, ...)
- Composition with pure Gaussian states (e.g., squeezed states)

E.g.: composition with a squeezed state

$|\psi\rangle$ —

$$|\psi\rangle \otimes \mathsf{S(s)}|0\rangle$$

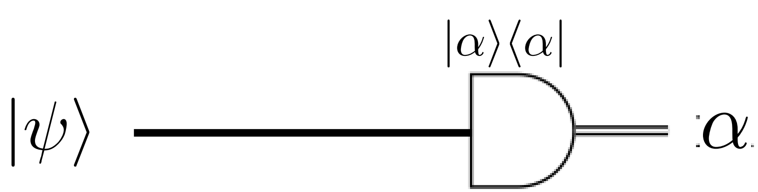
$\mathsf{S(s)}|0\rangle$ —

Allowed operations: Gaussian protocols (GPs)

- Gaussian unitaries (e.g., displacement, squeezing, CZ, ...)
- Composition with pure Gaussian states (e.g., squeezed states)
- Pure Gaussian measurements on subsystems (e.g., homodyne)

E.g.: homodyne measurements (position/momentum ideal projections)

E.g.: heterodyne measurements (coherent-state ideal projections)



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- Gaussian unitaries (e.g., displacement, squeezing, CZ, ...)
- Composition with pure Gaussian states (e.g., squeezed states)
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- Partial trace on subsystems

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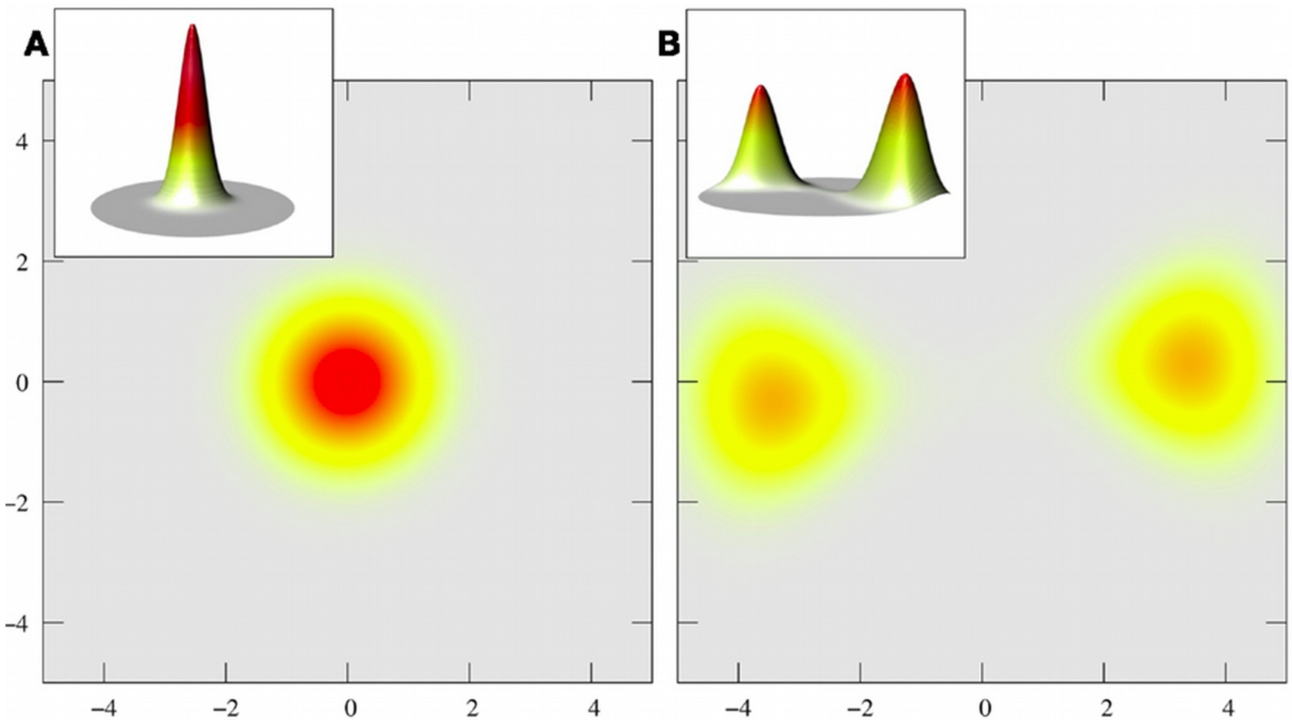
- Partial trace on subsystems
- The above operations conditioned on classical randomness

E.g.: Mixing with Gaussian states



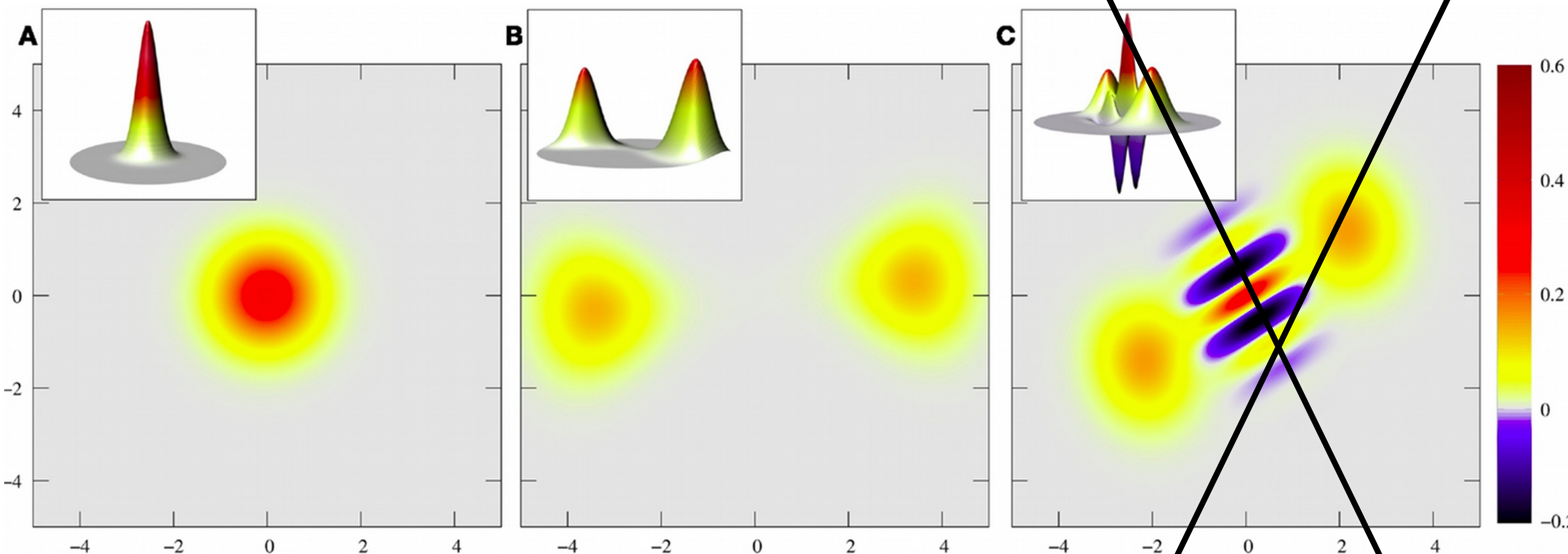
Coherent state

$$|\alpha\rangle$$

Coherent state mixture

$$\frac{1}{2}|\alpha\rangle\langle\alpha| + \frac{1}{2}|\alpha'\rangle\langle\alpha'|$$

E.g.: Mixing with Gaussian states



Coherent state

$$|\alpha\rangle$$

Coherent state mixture

$$\frac{1}{2}|\alpha\rangle\langle\alpha| + \frac{1}{2}|\alpha'\rangle\langle\alpha'|$$

Coherent state superposition
(cat state)

$$\frac{1}{N}(|\alpha\rangle + |\alpha'\rangle)$$

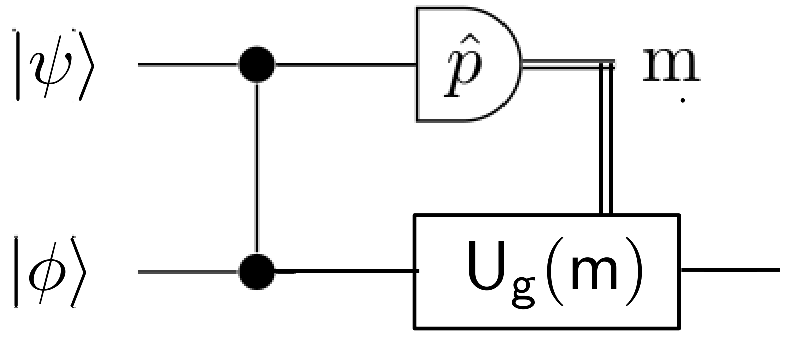
Allowed operations: Gaussian protocols (GPs)

- Gaussian unitaries (e.g., displacement, squeezing, CZ, ...)
- Composition with pure Gaussian states (e.g., squeezed states)
- Pure Gaussian measurements on subsystems (e.g., homodyne)
- Partial trace on subsystems
- The above operations conditioned on classical randomness or

  (a) single measurement outcomes (ideal case)

  (b) measurement outcomes in finite-size intervals (operational case)

E.g.: conditioning on momentum projections



(a) Ideal case:

$$\mathsf{m} = \tilde{\mathsf{m}}$$

(b) Operational case:

$$\mathsf{m} \in [\tilde{\mathsf{m}} - \delta, \tilde{\mathsf{m}} + \delta]$$

Allowed operations: Gaussian protocols (GPs)

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Note:

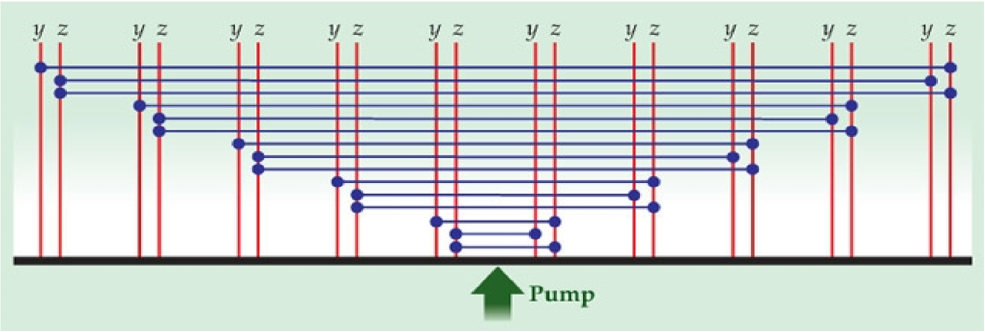
- Classical randomness does not generate a resource
- Ideal GPs are unattainable practically (zero probability)
- Operational GPs have mixed outcome:
  it is not possible to define a resource theory on pure states only

Experimental realizations of Gaussian protocols

60 entangled modes

Frequency encoding

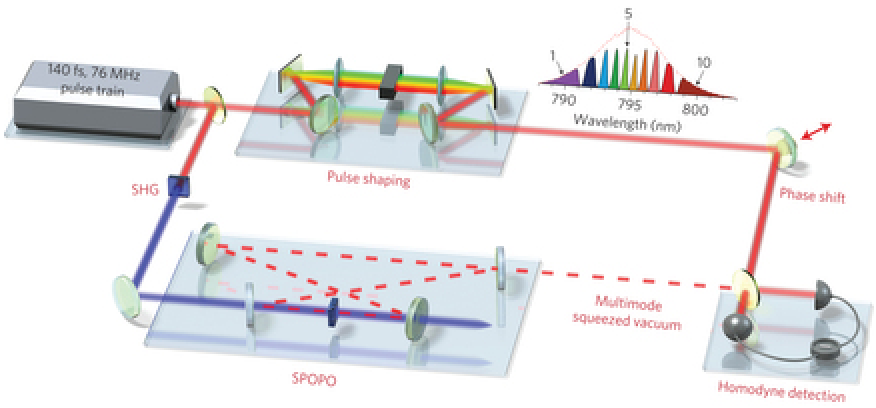
Single crystal & freq comb
[Chen et al., PRL (2014)]



500+ entangled partitions

Frequency encoding

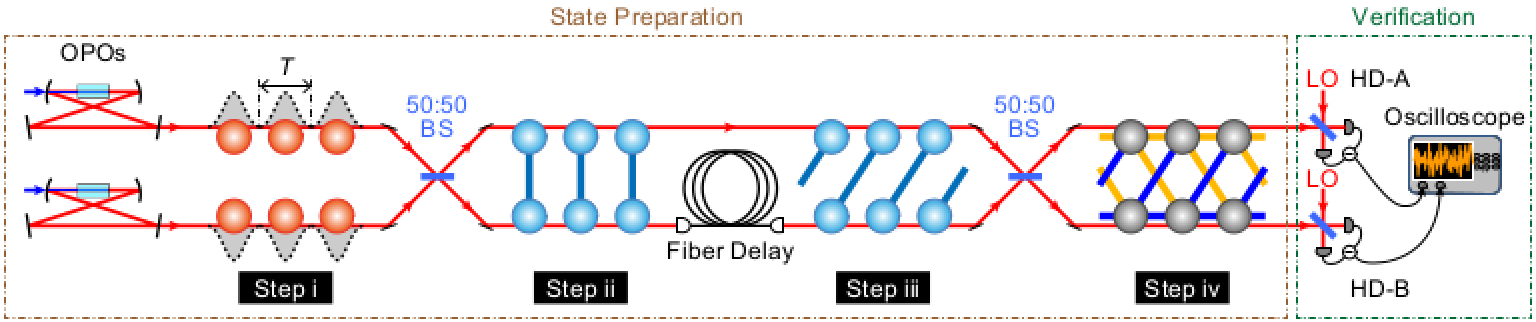
Single crystal & freq comb
[Roslund et al.,
Nat. Photonics (2014)]



10^6 entangled modes

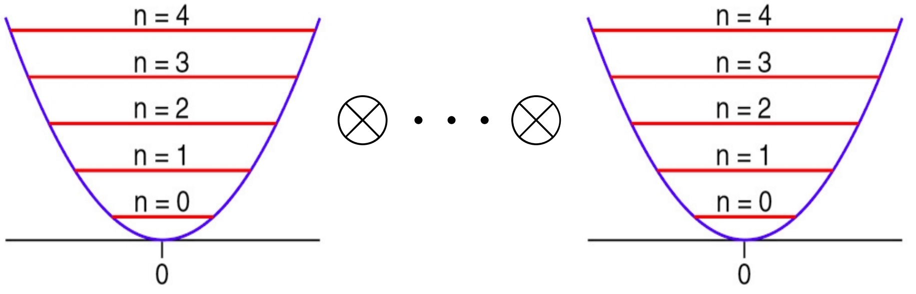
Temporal encoding

Pulsed squeezed states
[Yokoyama et al.,
Nat. Photonics (2013);
Yoshikawa et al.,
APL Photonics (2016)]



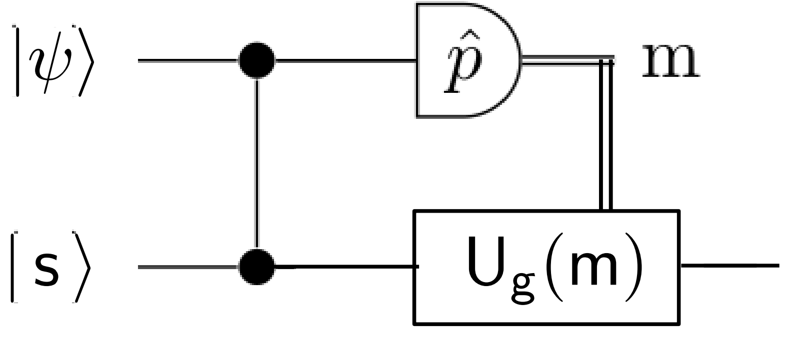
Resource theory of quantum non-Gaussianity

State space



Free states

Allowed operations



Resources

Free states

1) Mixtures of Gaussian states (convex hull)

$$\mathcal{G} = \left\{ \rho \in \mathcal{S}(\mathcal{H}) \mid \rho = \int \mathrm{d}\lambda \, \mathsf{p}(\lambda) \, |\psi_{\mathsf{G}}(\lambda)\rangle\langle\psi_{\mathsf{G}}(\lambda)| \right\}$$

Closed under Gaussian protocols.

States outside this set are called Quantum non-Gaussian states:

resource theory of quantum non-Gaussianity

Free states

1) Mixtures of Gaussian states (convex hull)

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Closed under Gaussian protocols.

States outside this set are called Quantum non-Gaussian states:

resource theory of quantum non-Gaussianity

2) Positive Wigner function

$$\mathcal{W}_{+} = \{ \rho \in \mathcal{S}(\mathcal{H}) \mid \mathsf{W}_{\rho}(\mathsf{r}) \geq 0 \}$$

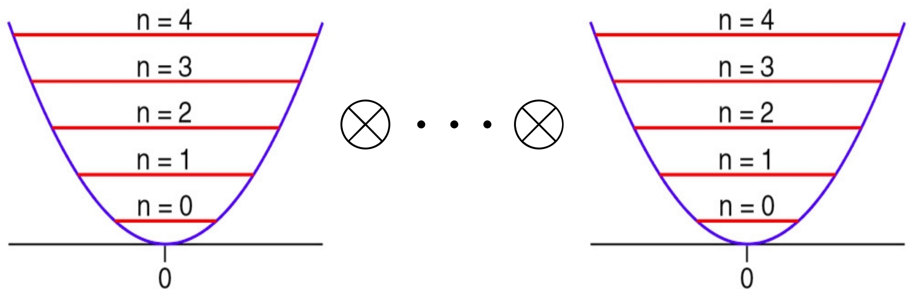
Closed under Gaussian protocols. Note : $\mathcal{G} \subset \mathcal{W}_{+}$

States outside this set are called Wigner-negative states:

resource theory of Wigner negativity

Resource theory of quantum non-Gaussianity

State space



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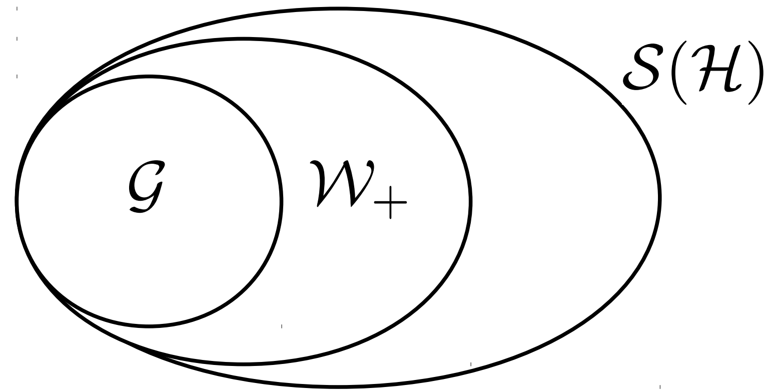

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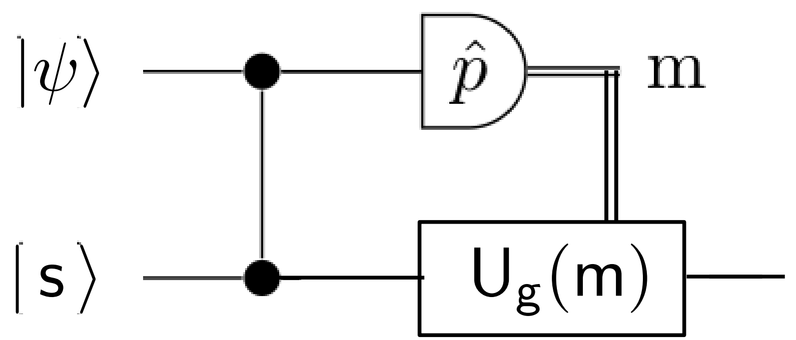


Resources

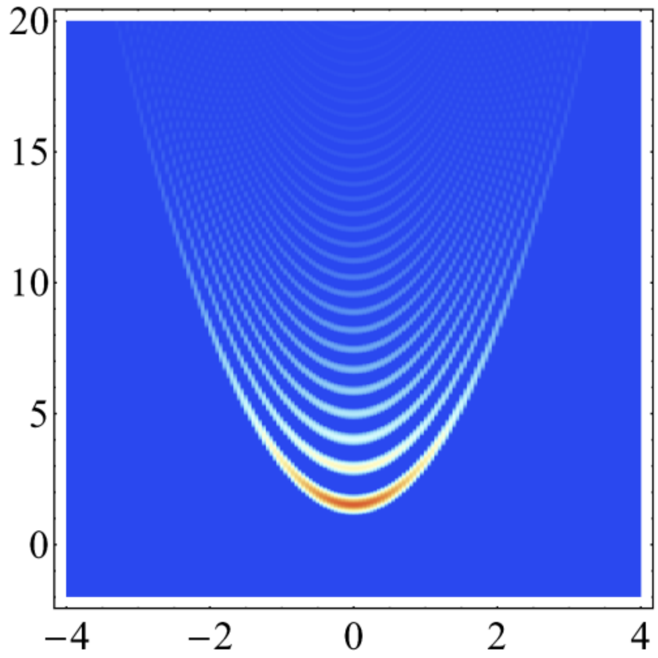
Resources

Example: cubic-phase state

$$|\phi\rangle = |\gamma, s\rangle = \Gamma(\gamma)S(s)|0\rangle = e^{i\gamma(b+b^\dagger)^3} e^{-\frac{s}{2}\left(b^2 - b^{\dagger 2}\right)}|0\rangle$$

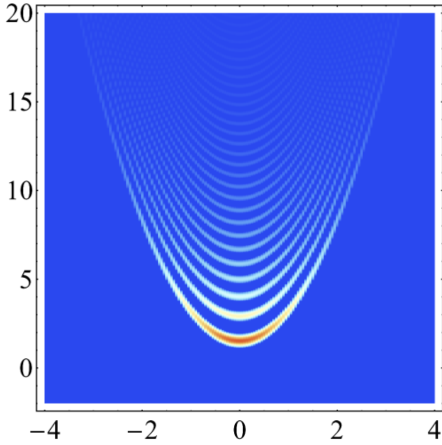
Cubic-phase state

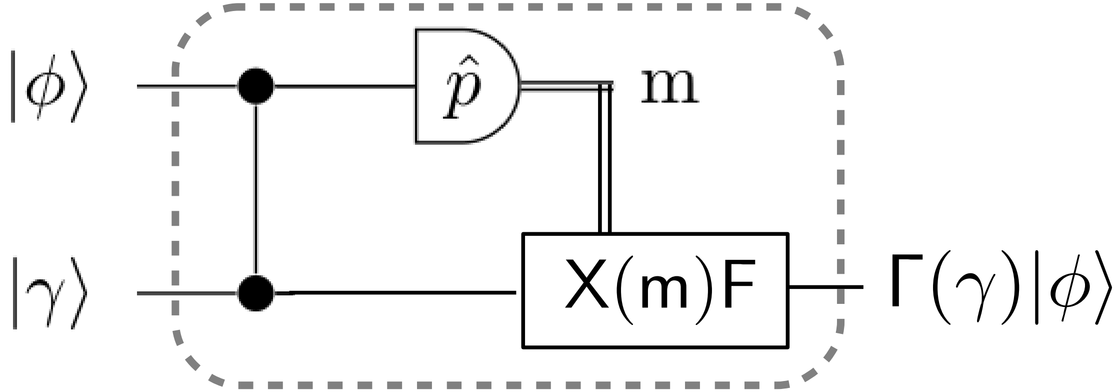
Cubic-phase gate

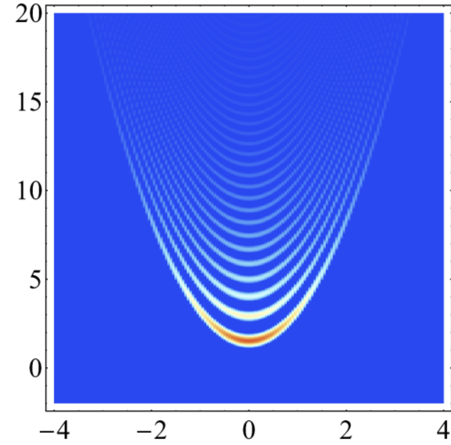


In which sense it is a resource?

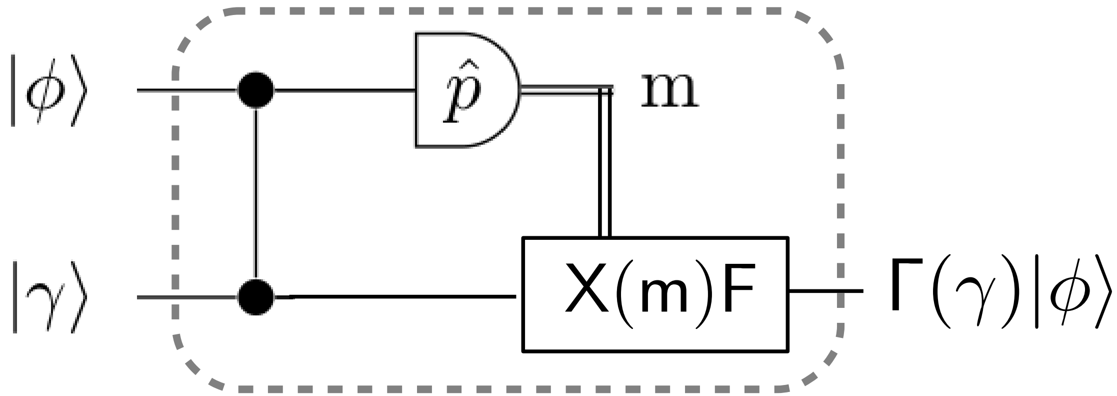
The (ideal) cubic-phase state allows to deterministically implement a cubic-phase gate via an (ideal) Gaussian protocol







The (ideal) cubic-phase state allows to deterministically implement a cubic-phase gate via an (ideal) Gaussian protocol



Theorem

Multimode Gaussian unitaries + any non-Gaussian unitary
=
Arbitrary multimode unitary transformation
=
universal CV quantum computation

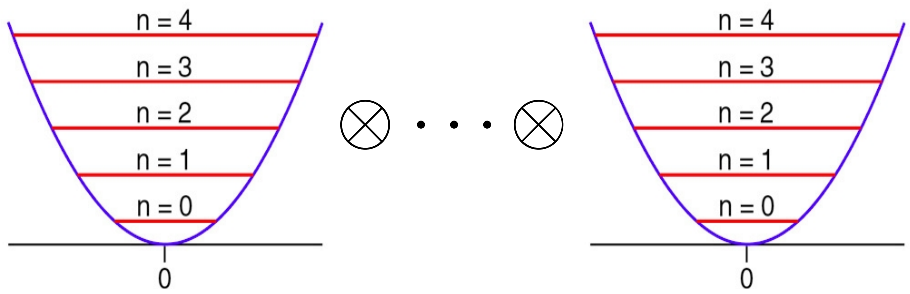
[Braunstein & Lloyd, PRL '99]

Also: non-Gaussian states + Gaussian protocols & quantum supremacy

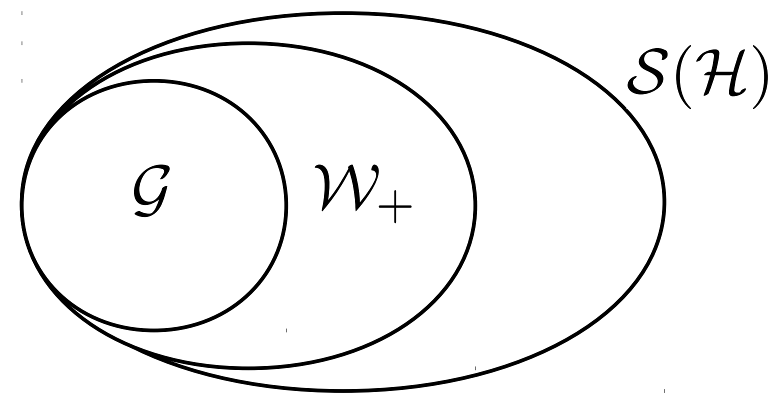
[Douce et al., PRL '17; Douce et al., arXiv:1806.06618]

Resource theory of quantum non-Gaussianity

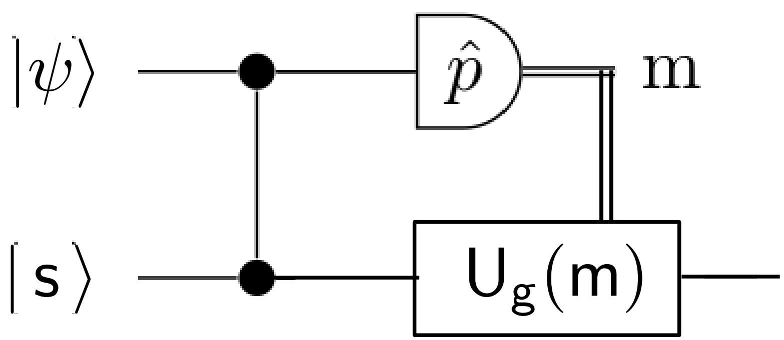
State space



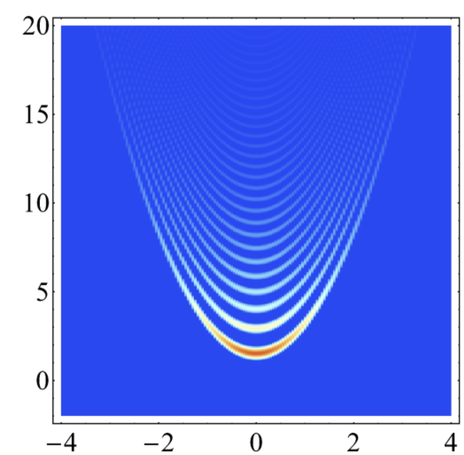
Free states



Allowed operations



Resources



[Albarelli, Genoni, Paris, AF, PRA ('18);
see also Takagi, Zhuang, PRA ('18).]

There exists no maximally resourceful state

No resource state can be transformed via GPs into any other state
(in particular, any other pure states)

Proof

Operational GPs

Output:
- mixed
- pure: Gaussian unitaries on n CVs have finite dimension (of the affine symplectic group $\mathrm{ISp}(2n, \mathbb{R})$: $2n^2 + 3n$)

Ideal GPs

Ideal GPs that map pure inputs into pure outputs are a subset of (non necessarily positive) linearly bounded super-operators that map Gaussian states into themselves. The latter have finite dimension.

Therefore

- No natural unit of QnG exists
- No natural state to distill into or to dilute from
- The cubic-phase state is “sort of” maximally resourceful

Monotones

A Quantum-non-Gaussianity (resp. Wigner Negativity) monotone is a functional from the set of quantum states to non-negative real numbers $\mathcal{M} : \mathcal{S}(\mathcal{H}) \to [0, \infty)$ which satisfies the following properties:

1. $\mathcal{M}(\rho) = 0 \quad \forall \rho \in \mathcal{G}$ (resp. $\mathcal{W}_+$).
2. **Monotonicity under deterministic Gaussian protocols**
   For any trace-preserving GP Λ_{DGP} the monotone must not increase: $\mathcal{M}(\rho) \geq \mathcal{M}(\Lambda_{\mathsf{DGP}}(\rho))$.
3. **Monotonicity on average under probabilistic Gaussian protocols**
   Given a trace-preserving GP Λ_{DGP} we can express its action in terms of free Kraus operators, we require that the monotone must not increase on average:
   (a) Ideal case: $\Lambda_{\mathsf{DGP}}(\rho) = \int \mathrm{d}\boldsymbol{\lambda} \mathsf{p}(\boldsymbol{\lambda}|\rho)\sigma_{\boldsymbol{\lambda}}$, where $\sigma_{\boldsymbol{\lambda}} = \frac{1}{\mathsf{p}(\boldsymbol{\lambda}|\rho)} \mathsf{K}_{\boldsymbol{\lambda}} \rho \mathsf{K}_{\boldsymbol{\lambda}}^{\dagger}$.
   We require that $\mathcal{M}(\rho) \geq \int \mathrm{d}\boldsymbol{\lambda} \mathsf{p}(\boldsymbol{\lambda}|\rho) \mathcal{M}(\sigma_{\boldsymbol{\lambda}})$.
   (b) Operational case: $\Lambda_{\mathsf{DGP}}(\rho) = \sum_{\mathsf{i}} \mathsf{p}_{\mathsf{i}|\rho} \sigma_{\mathsf{i}}$, where $\sigma_{\mathsf{i}} = \frac{1}{\mathsf{p}_{\mathsf{i}|\rho}} \mathsf{K}_{\mathsf{i}} \rho \mathsf{K}_{\mathsf{i}}^{\dagger}$.
   We require that $\mathcal{M}(\rho) \geq \sum_{\mathsf{i}} \mathsf{p}_{\mathsf{i}|\rho} \mathcal{M}(\sigma_{\mathsf{i}})$

A computable monotone: CV-mana (AKA, Wigner Logarithmic Negativity)

The negative volume of the Wigner function is a good candidate:

$$\mathcal{N}[\rho] = \int \mathrm{d}\mathbf{r}|\mathrm{W}_\rho(\mathbf{r})| - 1$$

[A Kenfack, K Życzkowski, J Opt B ('04)]

Define the CV-mana as:

$$\mathrm{M}\left(\rho\right) = \log\left(\int \mathrm{d}\mathbf{r}\ \left|\mathrm{W}_\rho\left(\mathbf{r}\right)\right|\right)$$

The CV-mana is an additive & computable monotone!

Note: not a faithful for Quantum non-Gaussianity

[J Park et al., arXiv:1809.02999]

Examples

Cubic-phase state

The resourcefulness depends on one effective parameter

$$\mathcal{M}\left(|\gamma, \mathsf{r}\rangle\right) = \mathcal{M}\left(|\mathsf{e}^{3\mathsf{r}}\gamma, 0\rangle\right) = \mathsf{f}\left(\mathsf{e}^{3\mathsf{r}}\gamma\right)$$

and it is boosted by the (initial) squeezing

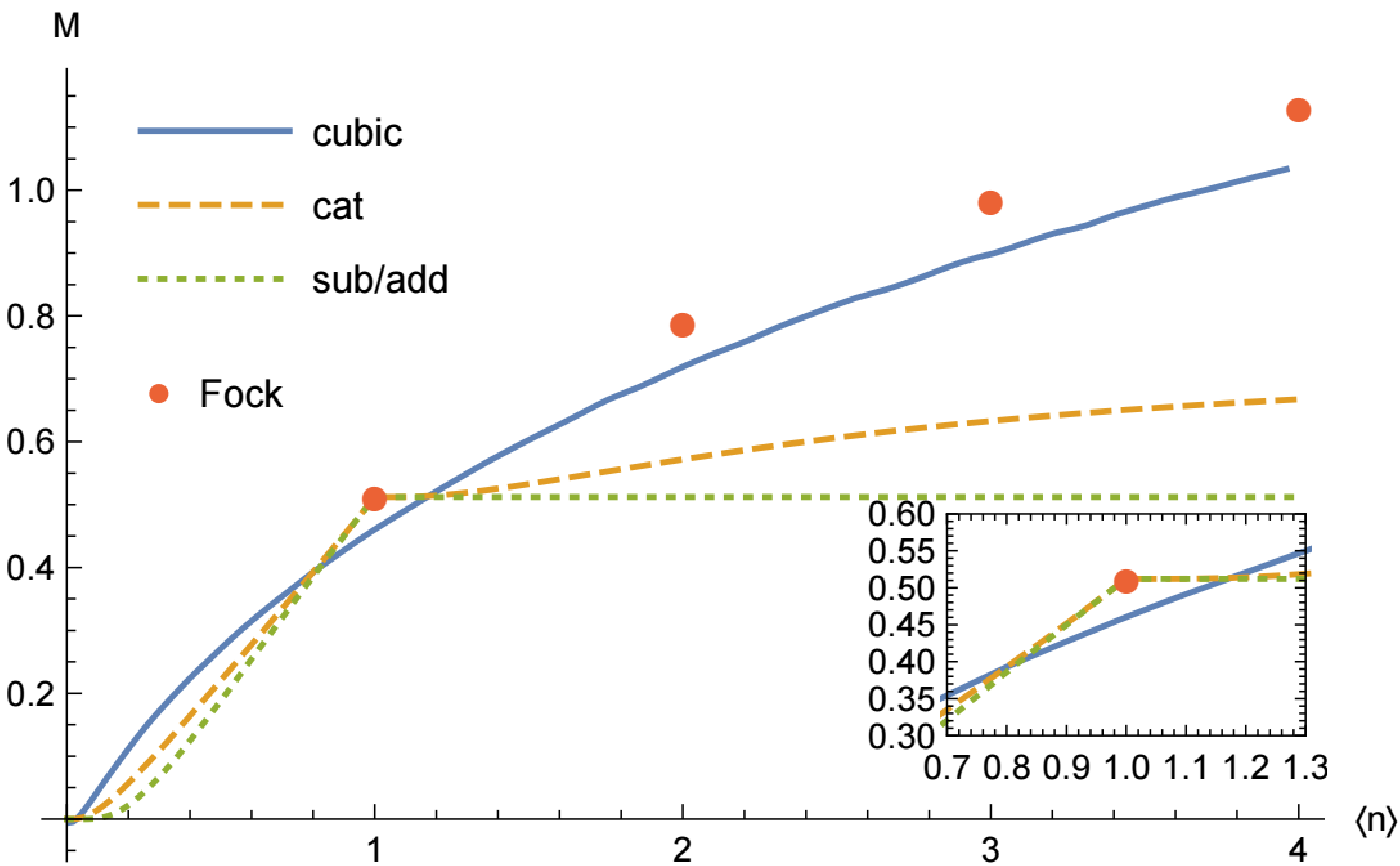
Photon-added and -subtracted states

$$\mathcal{M}\left[|\alpha, \mathsf{r}\rangle_{\mathsf{add}}\right] = \mathcal{M}\left[\mathsf{N}_{\mathsf{add}}^{-1/2}\left(\cosh|\mathsf{r}||1\rangle + \alpha^{*}|0\rangle\right)\right]$$

$$\mathcal{M}\left[|\alpha, \mathsf{r}\rangle_{\mathsf{sub}}\right] = \mathcal{M}\left[\mathsf{N}_{\mathsf{sub}}^{-1/2}\left(\mathsf{e}^{\mathsf{i}\psi}\sinh|\mathsf{r}||1\rangle + \alpha|0\rangle\right)\right]$$

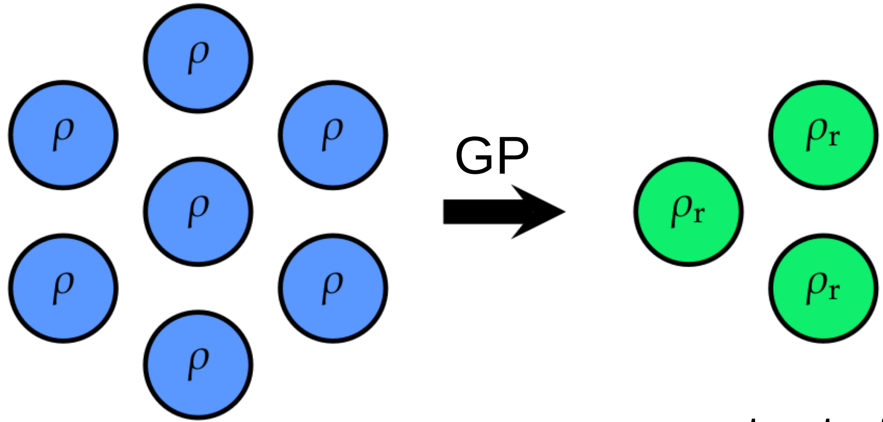
At most as resourceful as the Fock state $|1\rangle$

Resourcefulness comparison (fixed energy)



Fock states are the most resourceful

Resource concentration protocols



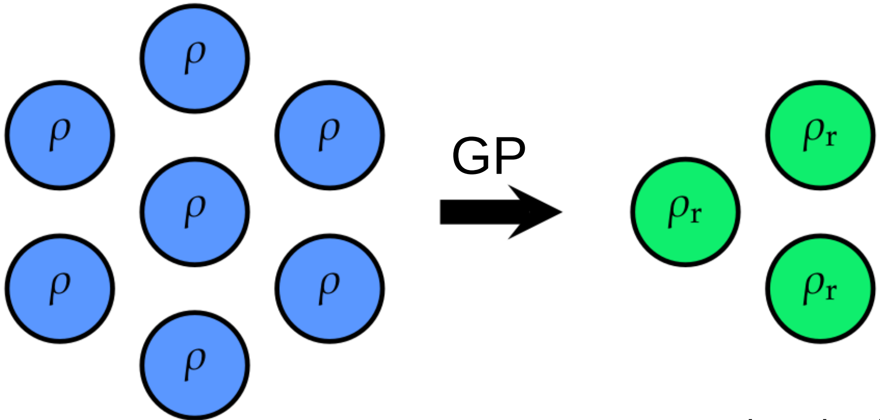
$$\Lambda\left(\rho^{\otimes k}\right) = \sigma^{\otimes m}$$

m output states

k original copies

Probability of success: p

Resource concentration protocols



$$\Lambda\left(\rho^{\otimes \mathrm{k}}\right) = \sigma^{\otimes \mathrm{m}}$$

k original copies

m output states

Probability of success: p

Using the monotonicity on average (any monotone):

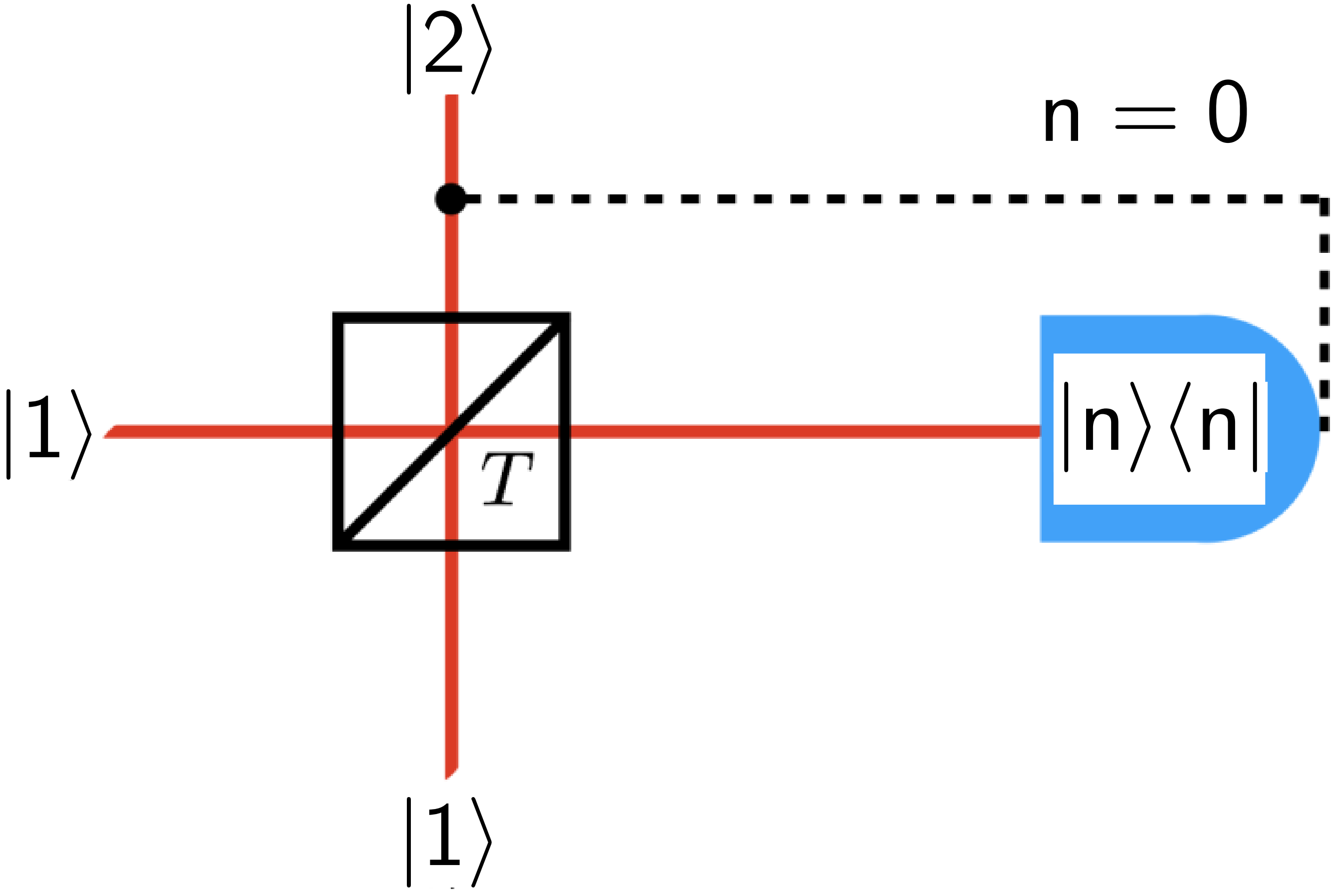
$$\mathcal{M}\left(\rho^{\otimes \mathrm{k}}\right) \geq \mathrm{p}\mathcal{M}\left(\sigma^{\otimes \mathrm{m}}\right)$$

Using the CV-mana additivity:

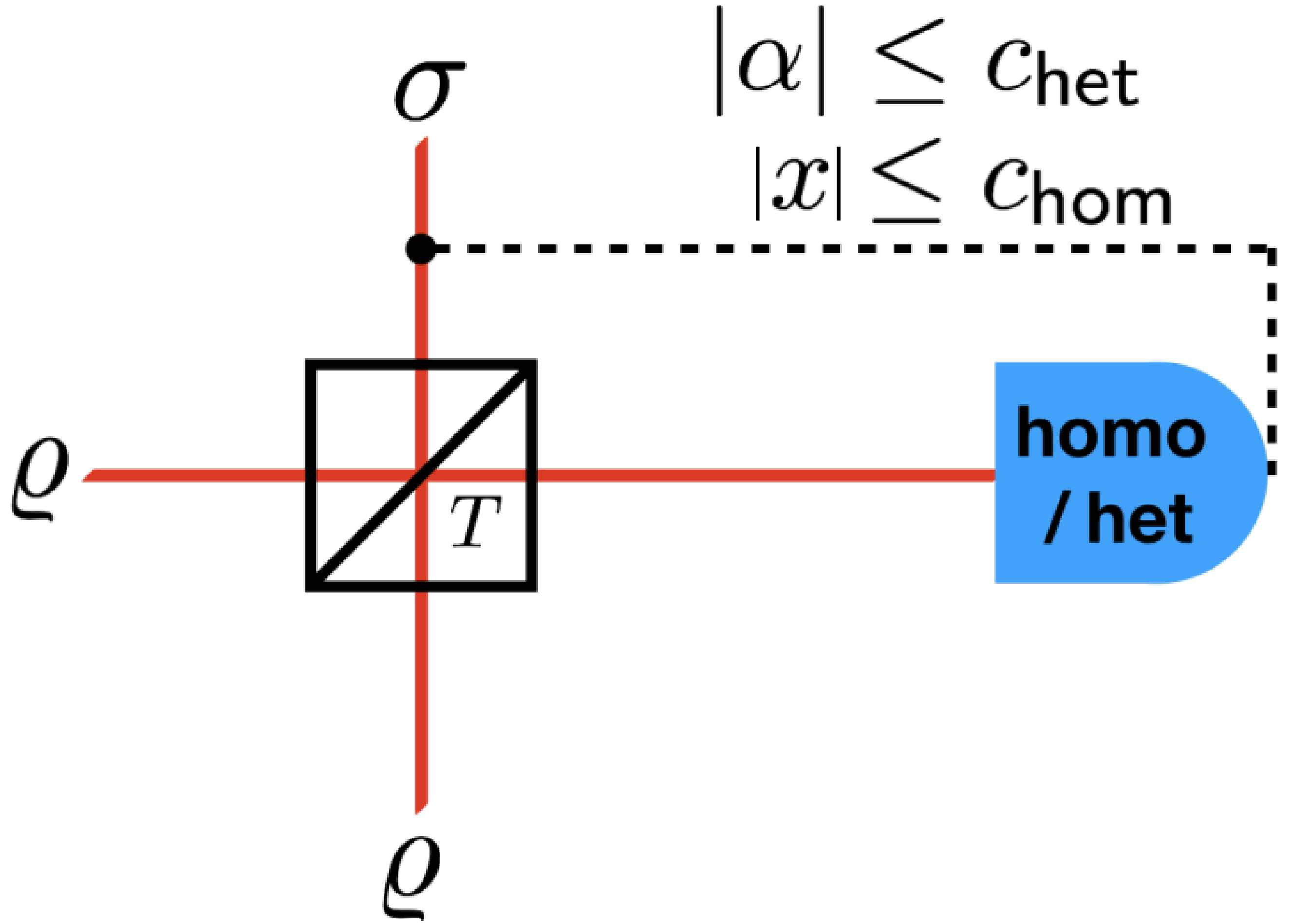
$$\frac{\mathrm{p\ m}}{\mathrm{k}}\frac{\mathrm{M}\left(\sigma\right)}{\mathrm{M}\left(\rho\right)} \leq 1$$

Bound to assess the efficiency of a concentration protocol.

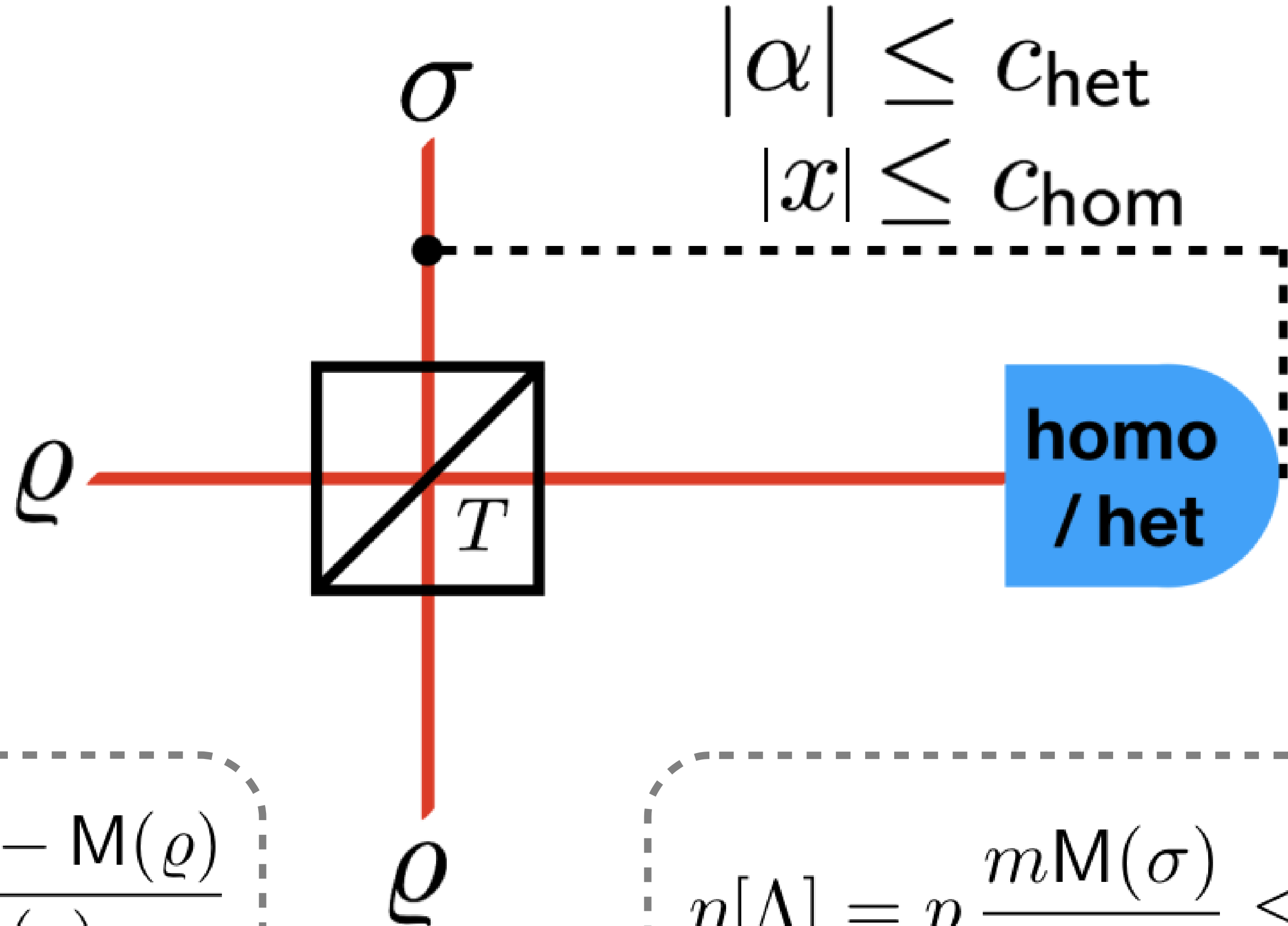
Concentrating the resourcefulness of two single-photon states



Concentrating the resourcefulness of two single-photon states



Concentrating the resourcefulness of two single-photon states



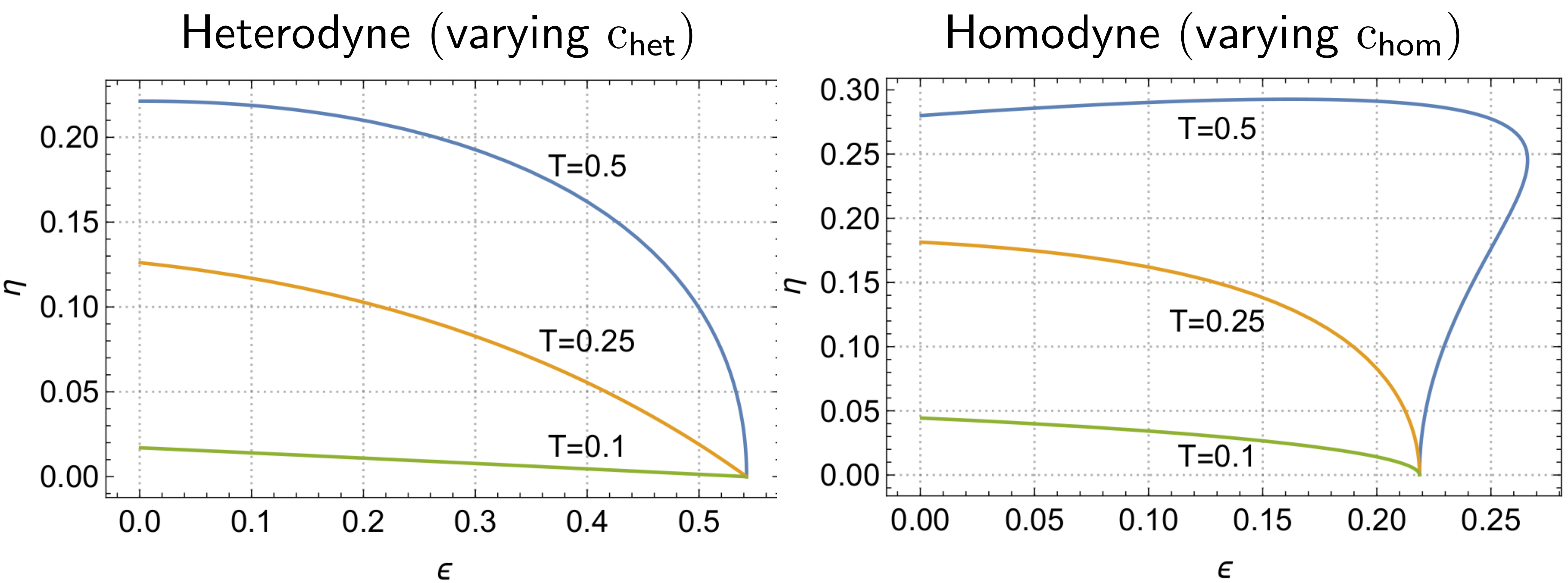
$$\epsilon[\Lambda] = \frac{\mathsf{M}(\sigma) - \mathsf{M}(\varrho)}{\mathsf{M}(\varrho)}$$

gain

$$\eta[\Lambda] = p\,\frac{m\mathsf{M}(\sigma)}{k\mathsf{M}(\varrho)} \leq 1$$

efficiency

Concentrating the resourcefulness of two single-photon states



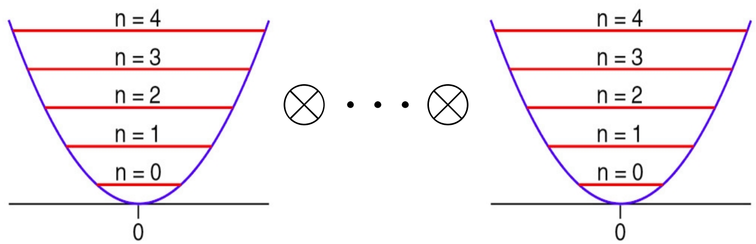
gain $$\epsilon[\Lambda] = \frac{\mathsf{M}(\sigma) - \mathsf{M}(\varrho)}{\mathsf{M}(\varrho)}$$

efficiency $$\eta[\Lambda] = p\,\frac{m\mathsf{M}(\sigma)}{k\mathsf{M}(\varrho)} \leq 1$$

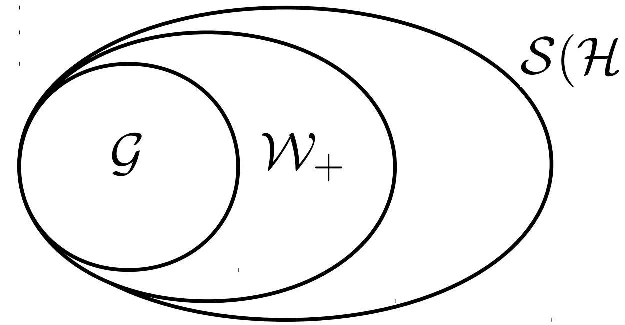
The homodyne-based concentration protocol performs better (optimal working point)

Resource theory of quantum non-Gaussianity

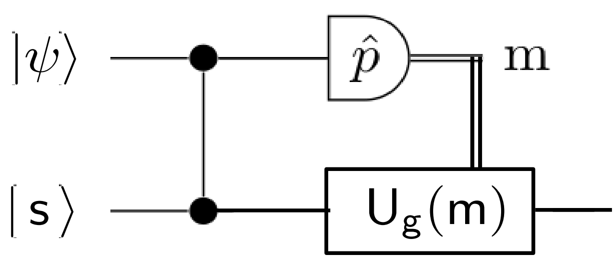
State space



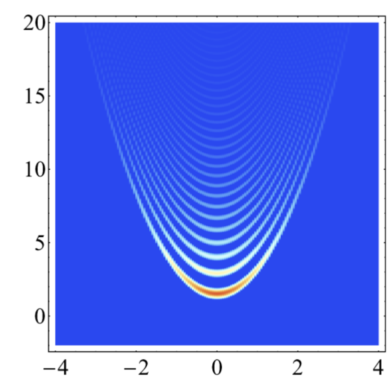
Free states



Allowed operations



Resources



- Is there a maximally resourceful state? No, but the cubic state is asymptotically maximal resourceful.
- Resource **quantification**: computable CV mana.
- Resource **distillation**: bounds to assess the efficiency of protocols.
- State **conversion**: is it possible to convert a resource into another, and at which rate?

[Albarelli, Genoni, Paris, AF, PRA **98** 052350 ('18)]

Outline

Quantum resource theories

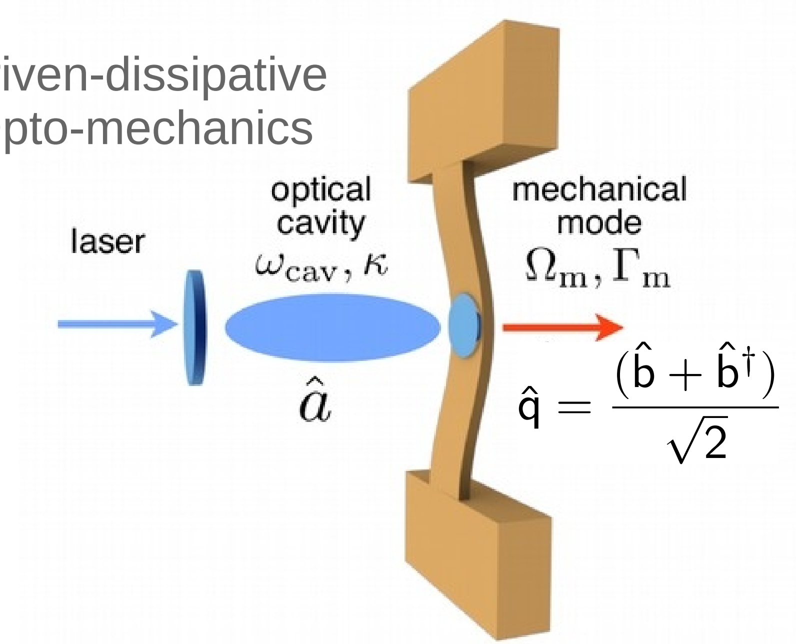
Resource theory of quantum non-Gaussianity

- Unconditional non-Gaussianity for quantum computation in optomechanical systems

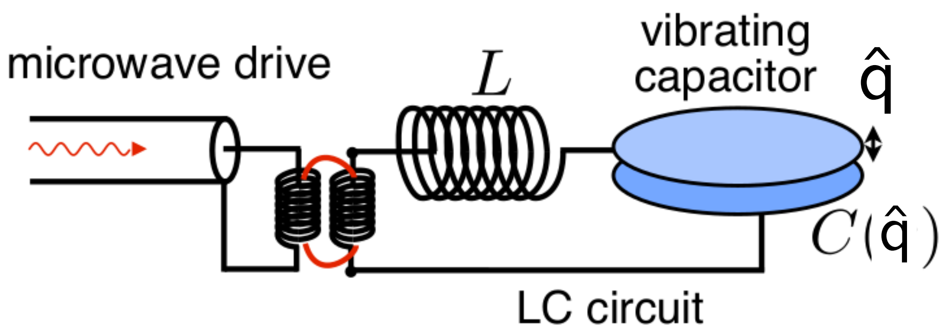
Confined/massive continuous variables

$$\text{Two DoF}: \quad \text{radiation } (\hat{a}) \quad - \quad \text{mechanics } (\hat{b})$$

Driven-dissipative
Opto-mechanics



Driven-dissipative
Electro-mechanics



[Aspelmeyer et al, RMP '13]

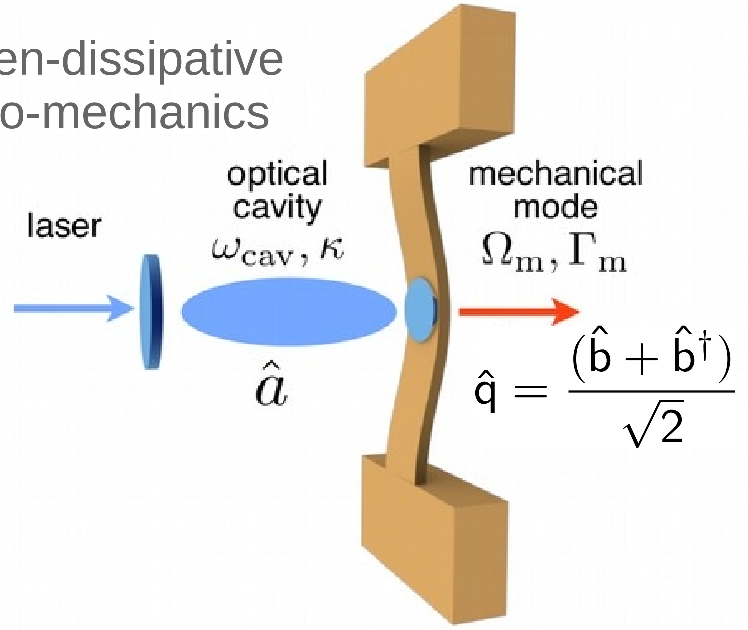
Radiation-pressure interaction

$$\mathcal{H} \approx \omega(\hat{q})\hat{a}^\dagger\hat{a} \approx \left(\omega_c + \omega'\hat{q}\right)\hat{a}^\dagger\hat{a}$$

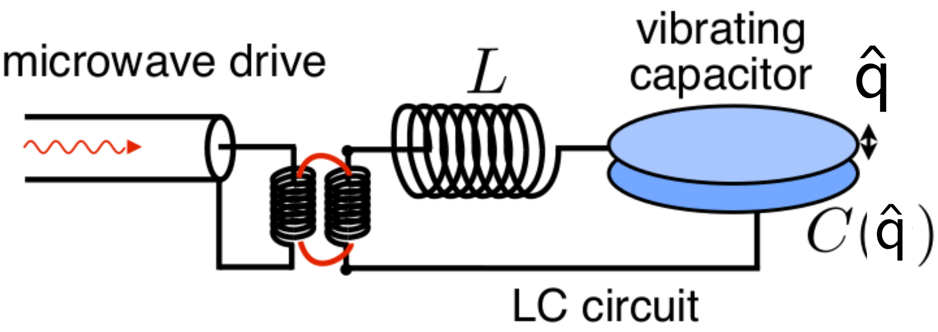
Confined/massive continuous variables

Two DoF : radiation $(\hat{a})$ – mechanics $(\hat{b})$

Driven-dissipative
Opto-mechanics



Driven-dissipative
Electro-mechanics



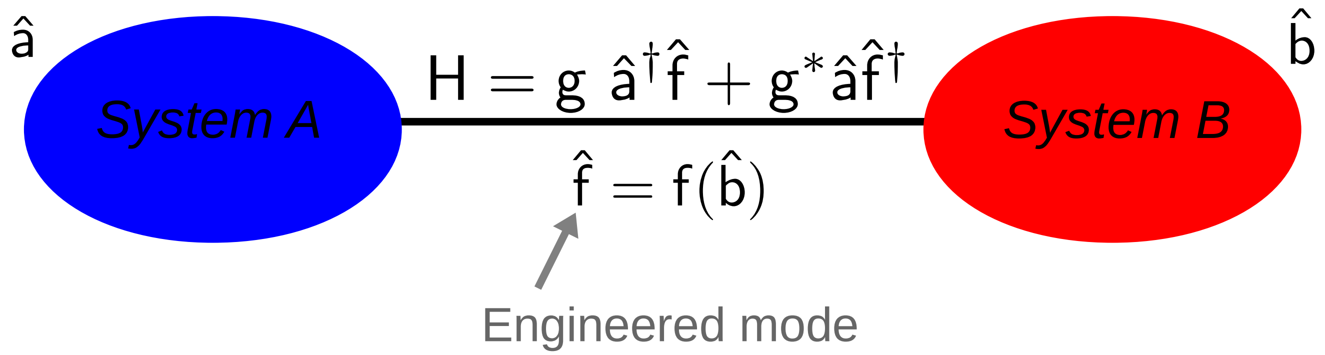
[Aspelmeyer et al, RMP '13]

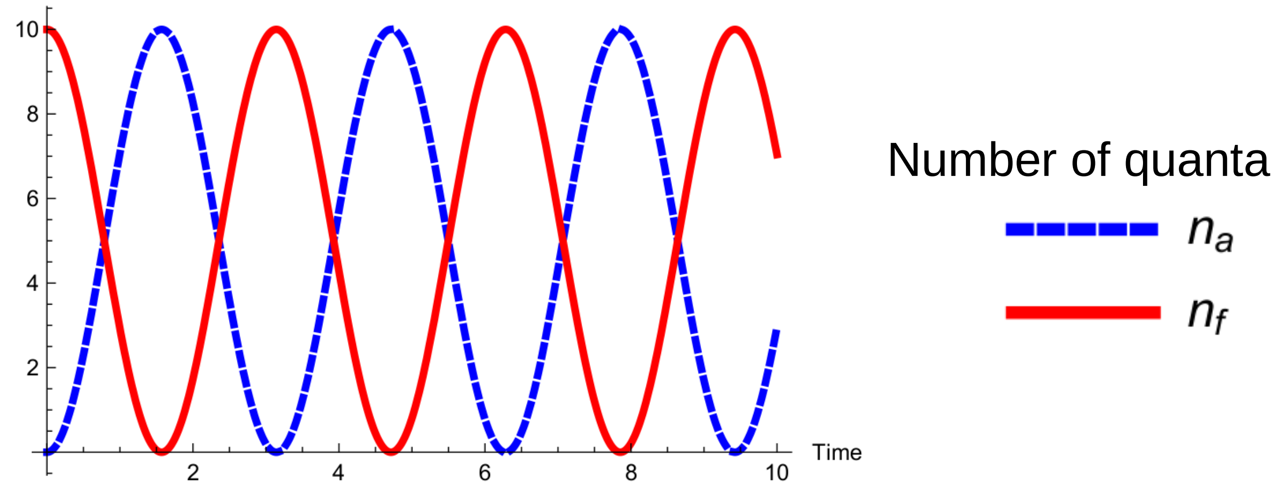
Linear + Quadratic radiation-pressure interaction

$$\mathcal{H} \approx \omega(\hat{q})\hat{a}^\dagger\hat{a} \approx \left(\omega_c + \omega'\hat{q} + \frac{1}{2}\omega''\hat{q}^2\right)\hat{a}^\dagger\hat{a}$$

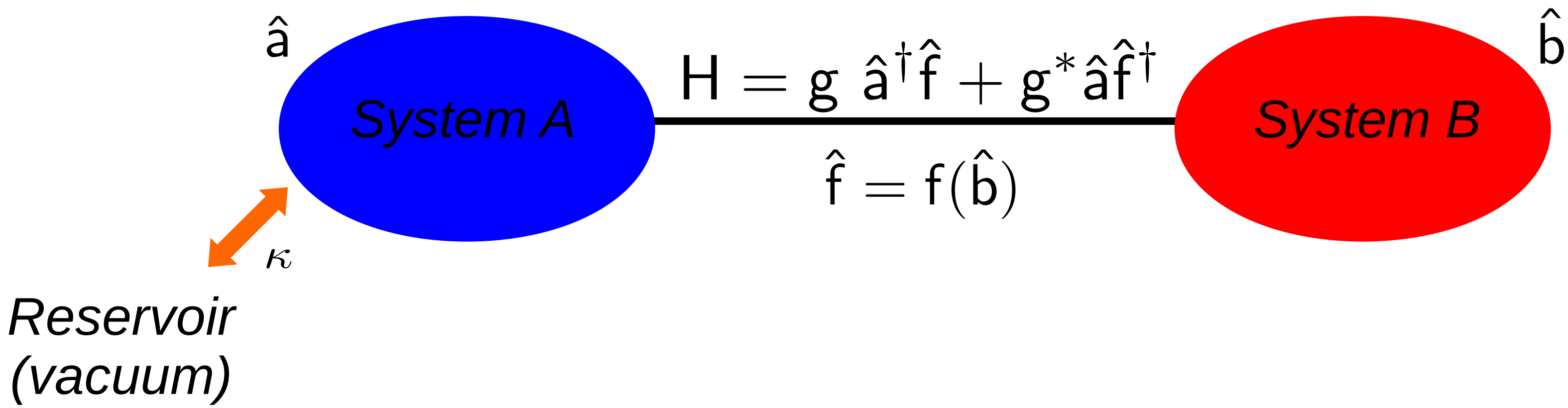
Why interesting? Beyond Gaussian dynamics

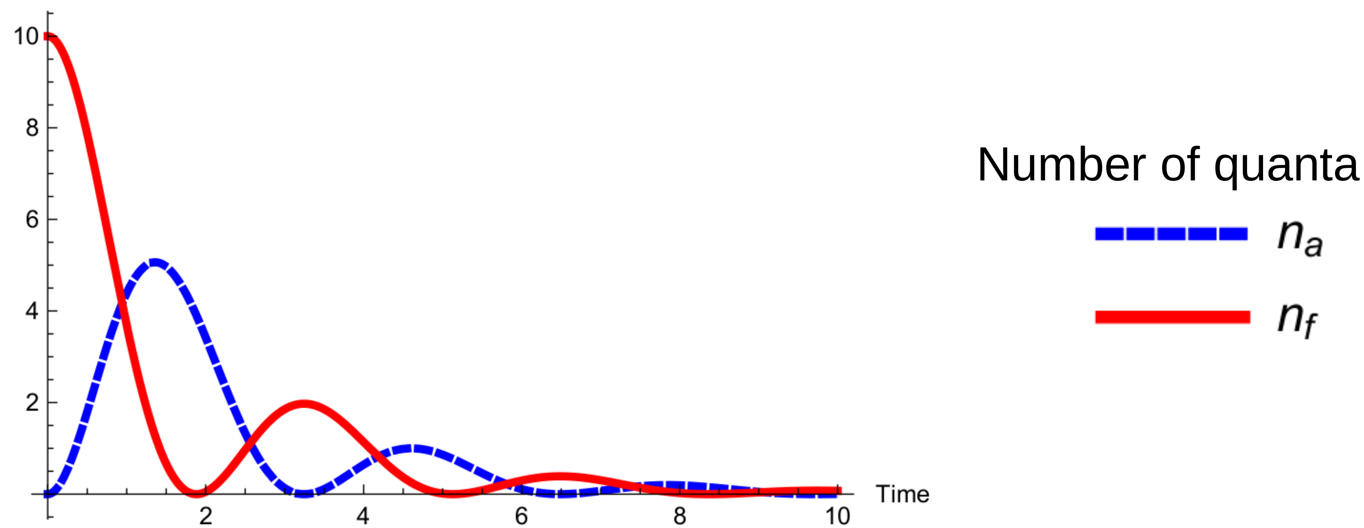
Exploiting the dissipative dynamics for state engineering





Exploiting the dissipative dynamics for state engineering





$$\text{Steady state}: \quad |0\rangle \otimes |\phi\rangle\ , \qquad \text{with} \quad \hat{f}|\phi\rangle = 0$$

The vacuum of *f* can be highly non-trivial

$$f = g_1 b + g_2 b^\dagger + g_3 b^2 + g_4 {b^\dagger}^2 + g_5 \left[b, b^\dagger\right]_+ \qquad g_1, \ldots, g_5 \in \mathbb{C}$$

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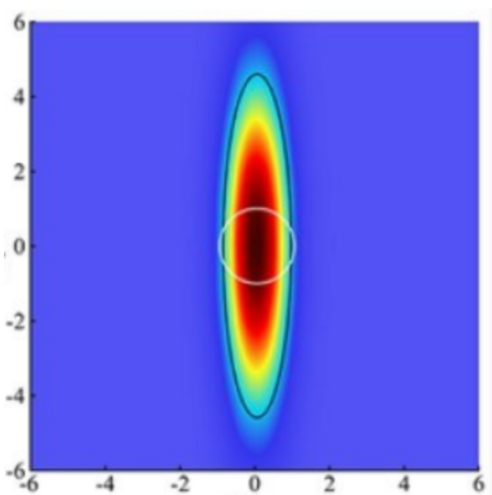
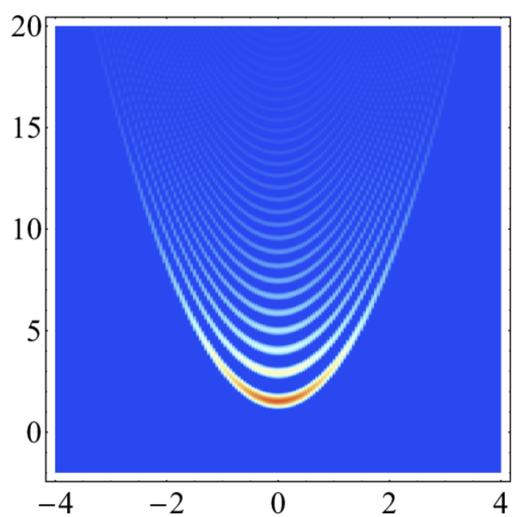
$$f = g_1 b + g_2 b^\dagger + g_3 b^2 + g_4 {b^\dagger}^2 + g_5 \left[b, b^\dagger\right]_+ \qquad g_1, \ldots, g_5 \in \mathbb{C}$$

properly setting g_j

Squeezed states

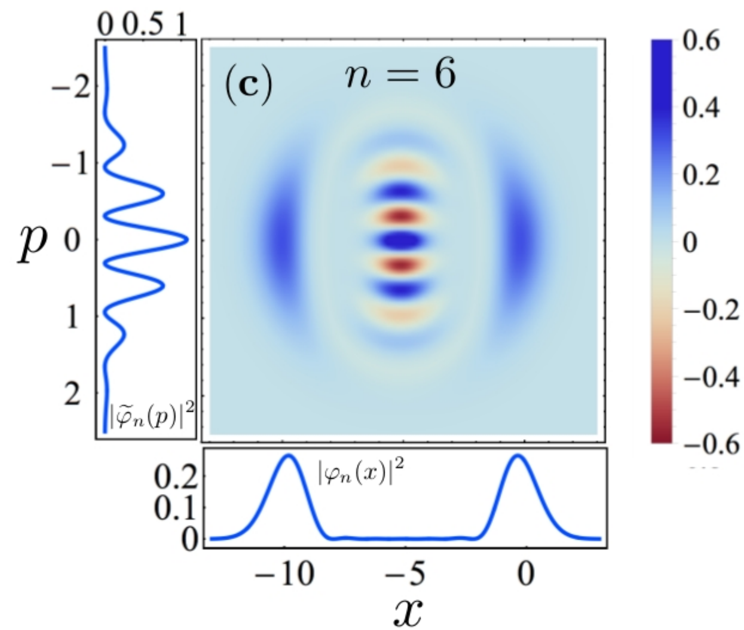


Cubic-phase states



[Houhou, Moore, Bose, AF, arXiv:1809.09733]

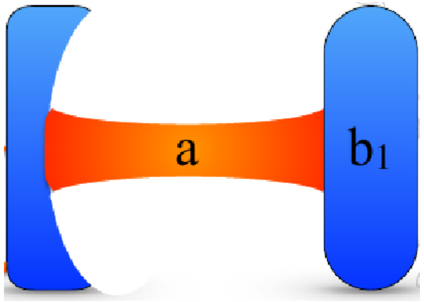
Cat-like states



[Brunelli et al., PRA ('18)]

Hamiltonian engineering in optomechanics

Inspired by linear schemes [Clerk, Hartmann, Marquardt, Meystre, Vitali,...]

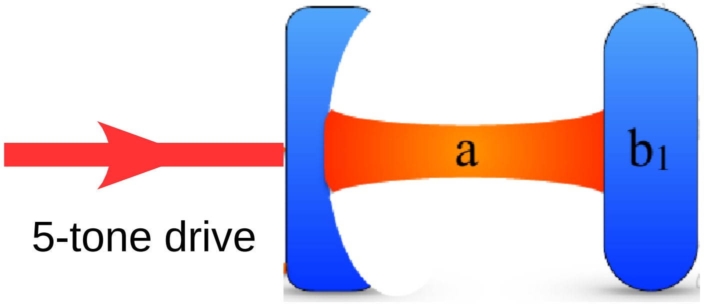


$$\mathcal{H} = \omega_c a^\dagger a + \Omega b^\dagger b + G_L a^\dagger a (b^\dagger + b) + G_Q a^\dagger a (b^\dagger + b)^2$$

Hamiltonian engineering in optomechanics

Inspired by linear schemes [Clerk, Hartmann, Marquardt, Meystre, Vitali,...]

$$\epsilon(t) = \sum_{k=1}^{5} \epsilon_k e^{-i(\omega_c - \Delta_k)t}$$



$$\mathcal{H} = \omega_c a^\dagger a + \Omega b^\dagger b + G_L a^\dagger a (b^\dagger + b) + G_Q a^\dagger a (b^\dagger + b)^2 + \epsilon(t) a^\dagger + \epsilon^*(t) a$$

Hamiltonian engineering in optomechanics

Inspired by linear schemes [Clerk, Hartmann, Marquardt, Meystre, Vitali,...]

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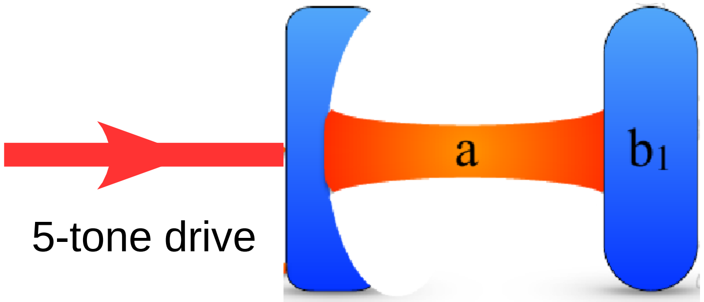

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Driving the first two side-bands
and the central frequency:

$$\Delta_1 = -\Omega \quad \Delta_2 = \Omega$$

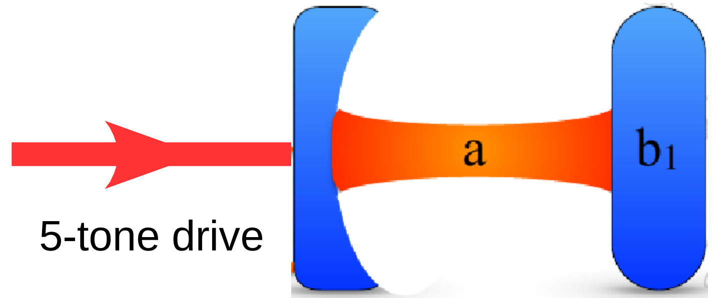
$$\Delta_3 = -2\Omega \quad \Delta_4 = 2\Omega$$

$$\Delta_5 = 0$$

Hamiltonian engineering in optomechanics

Inspired by linear schemes [Clerk, Hartmann, Marquardt, Meystre, Vitali,...]

$$\epsilon(t) = \sum_{k=1}^{5} \epsilon_k e^{-i(\omega_c - \Delta_k)t}$$



$$\mathcal{H} = \omega_c a^\dagger a + \Omega b^\dagger b + G_L a^\dagger a (b^\dagger + b) + G_Q a^\dagger a (b^\dagger + b)^2 + \epsilon(t) a^\dagger + \epsilon^*(t) a$$

Driving the first two side-bands
and the central frequency:

$$\Delta_1 = -\Omega \quad \Delta_2 = \Omega$$

$$\Delta_3 = -2\Omega \quad \Delta_4 = 2\Omega$$

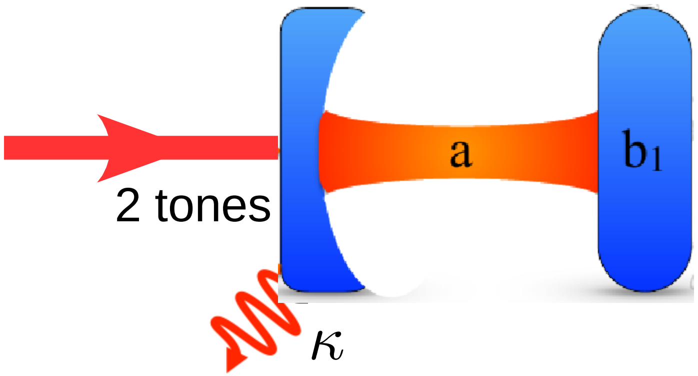
$$\Delta_5 = 0$$

- Linearizing over the mean fields
- Resolved sideband regime
- Weak coupling
- Rotating wave approximation

$$H = g\ a^\dagger f + g^* a f^\dagger\ , \quad f = g_1 b + g_2 b^\dagger + g_3 b^2 + g_4 b^{\dagger 2} + g_5 \left[b, b^\dagger\right]_+$$

g_j depend on the driving fields and can be independently tuned

Squeezed state by dissipation



Setting $g_3 = g_4 = g_5 = 0$:

$$f = g_1 b + g_2 b^\dagger = \cosh(s) b + e^{i\psi} \sinh(s) b^\dagger$$

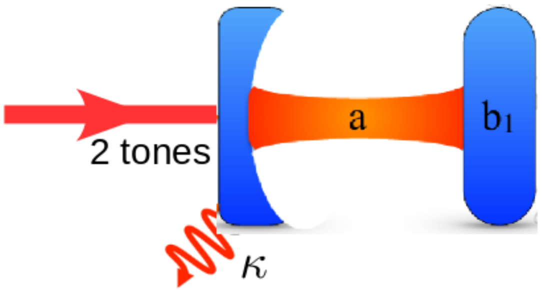
$$|\phi\rangle = S_\psi(s)|0\rangle = e^{\frac{s}{2}\left(e^{-i\psi} b^2 - e^{i\psi} b^{\dagger 2}\right)}|0\rangle$$

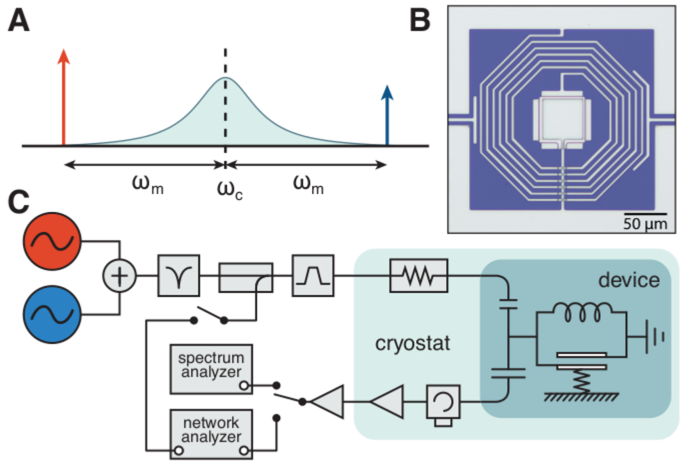
The system is **dissipatively driven to a unique and squeezed steady state**

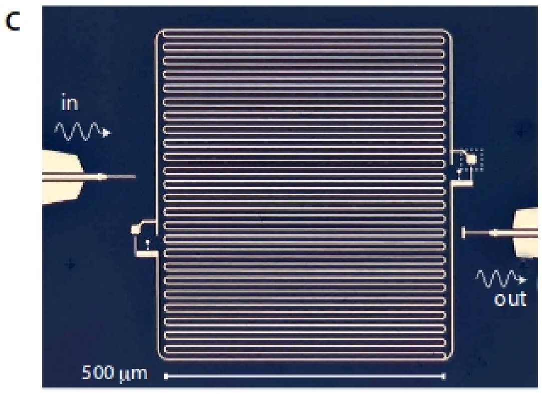
Electro-mechanical experimental implementations

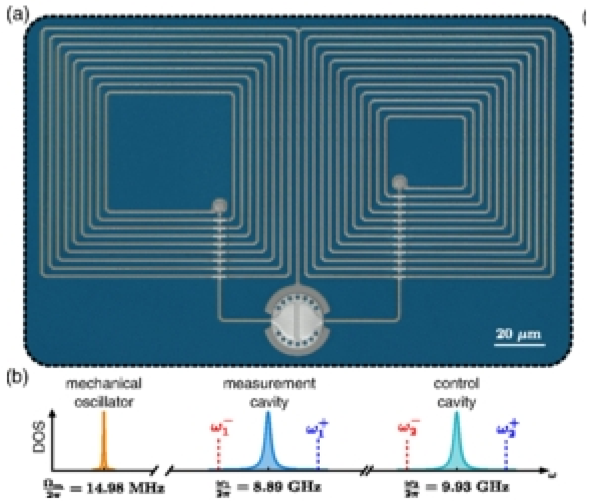
Driving the first mechanical sidebands with two tones

$$H = g\, a^\dagger f + g^* a f^\dagger \;, \quad f = g_1 b + g_2 b^\dagger$$









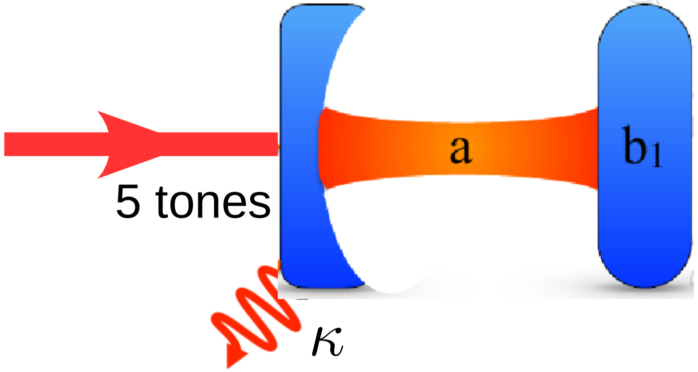
[Woolman et al., Science (2015)]

[Lei et al., PRL (2016)]

[Pirkkallainen et al., PRL 115, 243601 (2015)]

[Lecocq et al., PRX 5, 041037 (2015)]

Cubic-phase state by dissipation



$$f = g_1 b + g_2 b^\dagger + g_3 b^2 + g_4 {b^\dagger}^2 + g_5 \left[b, b^\dagger\right]_+$$

Setting :

$$g_2 = -\tanh(s) g_1$$

$$g_3 = g_4 = g_5 = -\frac{3i}{2\sqrt{2}}\gamma\left[1 + \tanh(s)\right] g_1$$

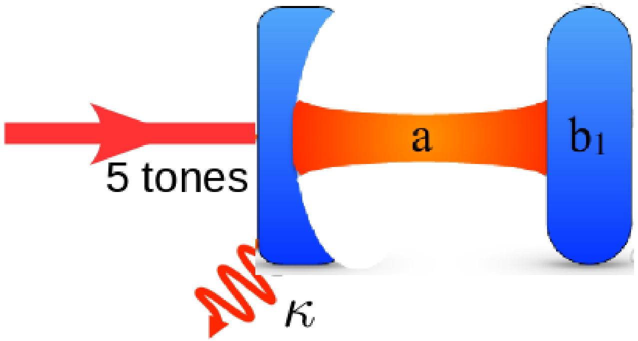
$$|\phi\rangle = |\gamma, s\rangle = \Gamma(\gamma) S(s)|0\rangle = e^{i\gamma(b+b^\dagger)^3} e^{-\frac{s}{2}\left(b^2 - {b^\dagger}^2\right)}|0\rangle$$

Cubic-phase state ($|\phi\rangle$)

Cubic-phase gate ($\Gamma(\gamma)$)

The system is **unconditionally driven to a cubic-phase steady state**

Effect of mechanical noise

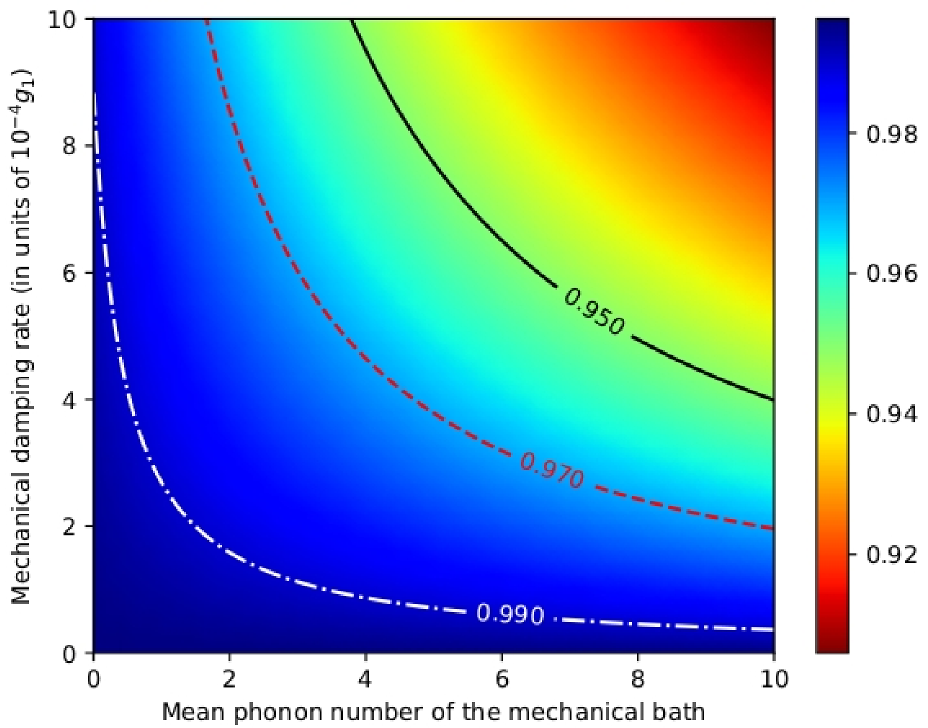


Considering thermal noise:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i[H,\rho] + \kappa D[a]\rho + \gamma_m(\bar{n}+1)D[b]\rho + \gamma_m \bar{n} D[b^\dagger]\rho$$

$$D[A]\rho = A\rho A^\dagger - \frac{1}{2}\left[A^\dagger A, \rho\right]_+$$

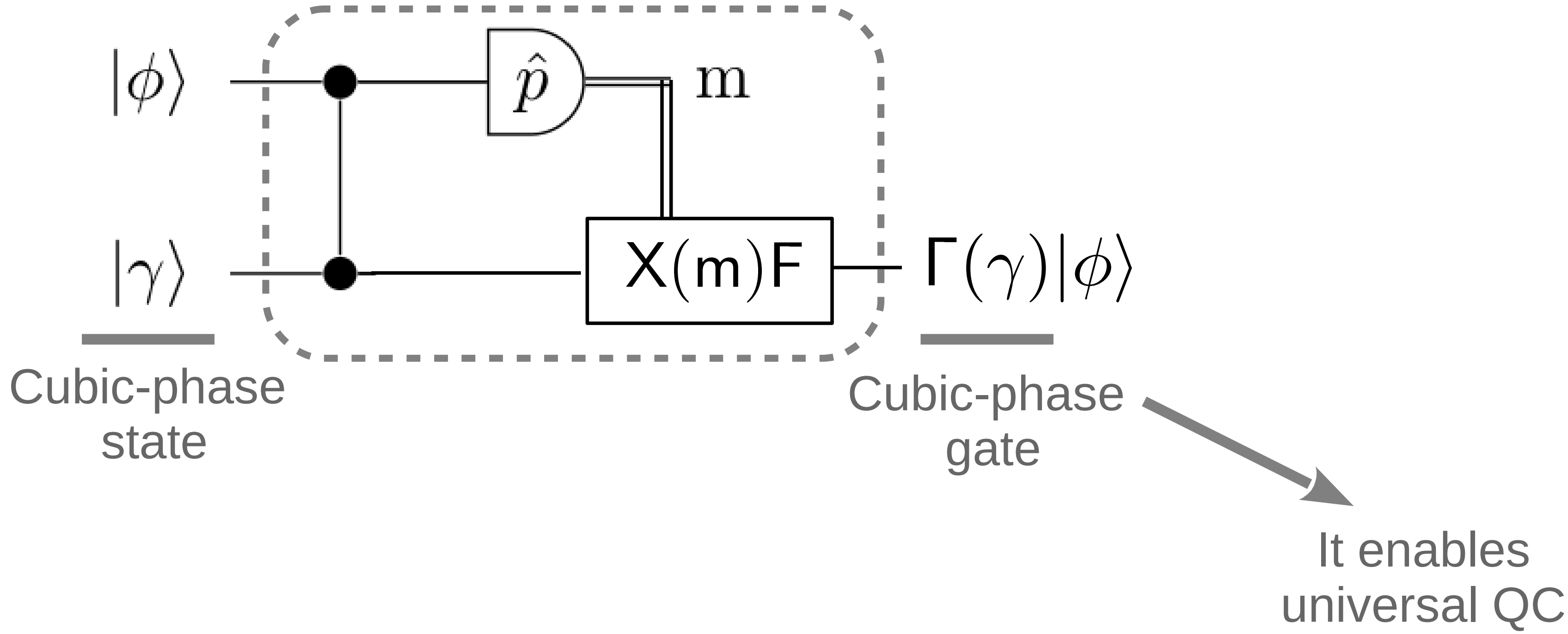
- target state: $\mathrm{e}^{i\gamma(b+b^\dagger)^3}\mathrm{e}^{\frac{-s}{2}(b^2-b^{\dagger 2})}|0\rangle$
- cubicity: $\gamma \approx 0.07$
- squeezing: $s \approx 0.58$ (5dB)



[Houhou, Moore, Bose, AF, arXiv:1809.09733]

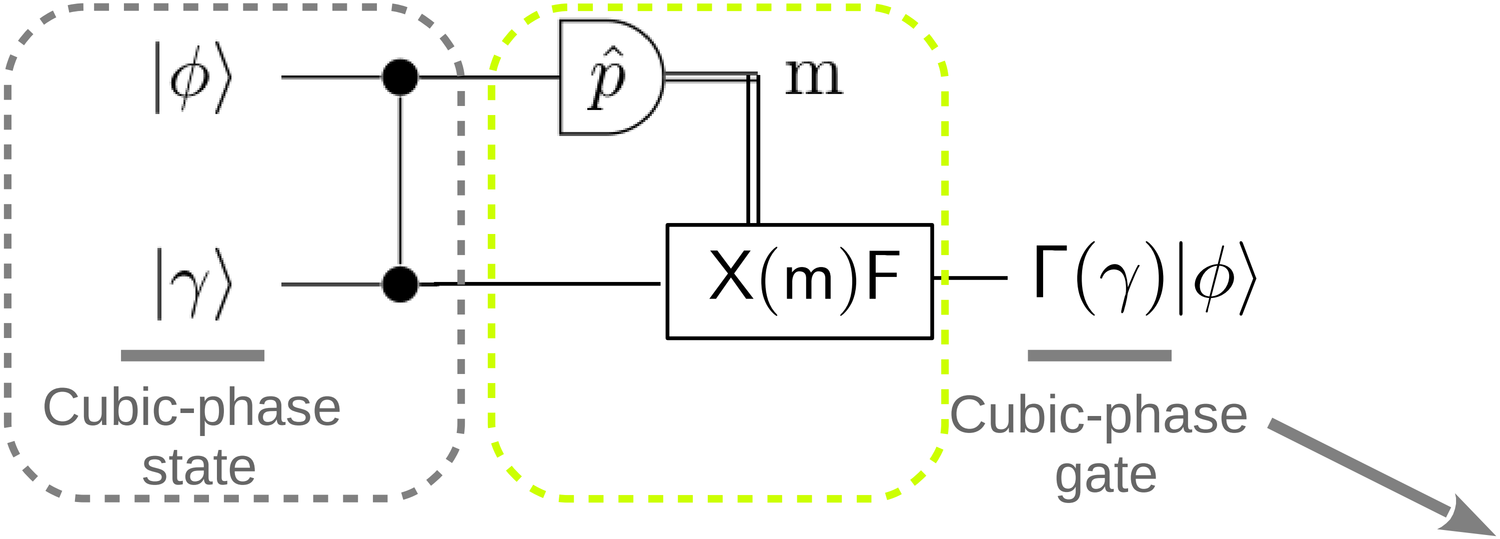
A cubic-phase state enables the cubic-phase gate

Gate teleportation:



A cubic-phase state enables the cubic-phase gate

Gate teleportation:

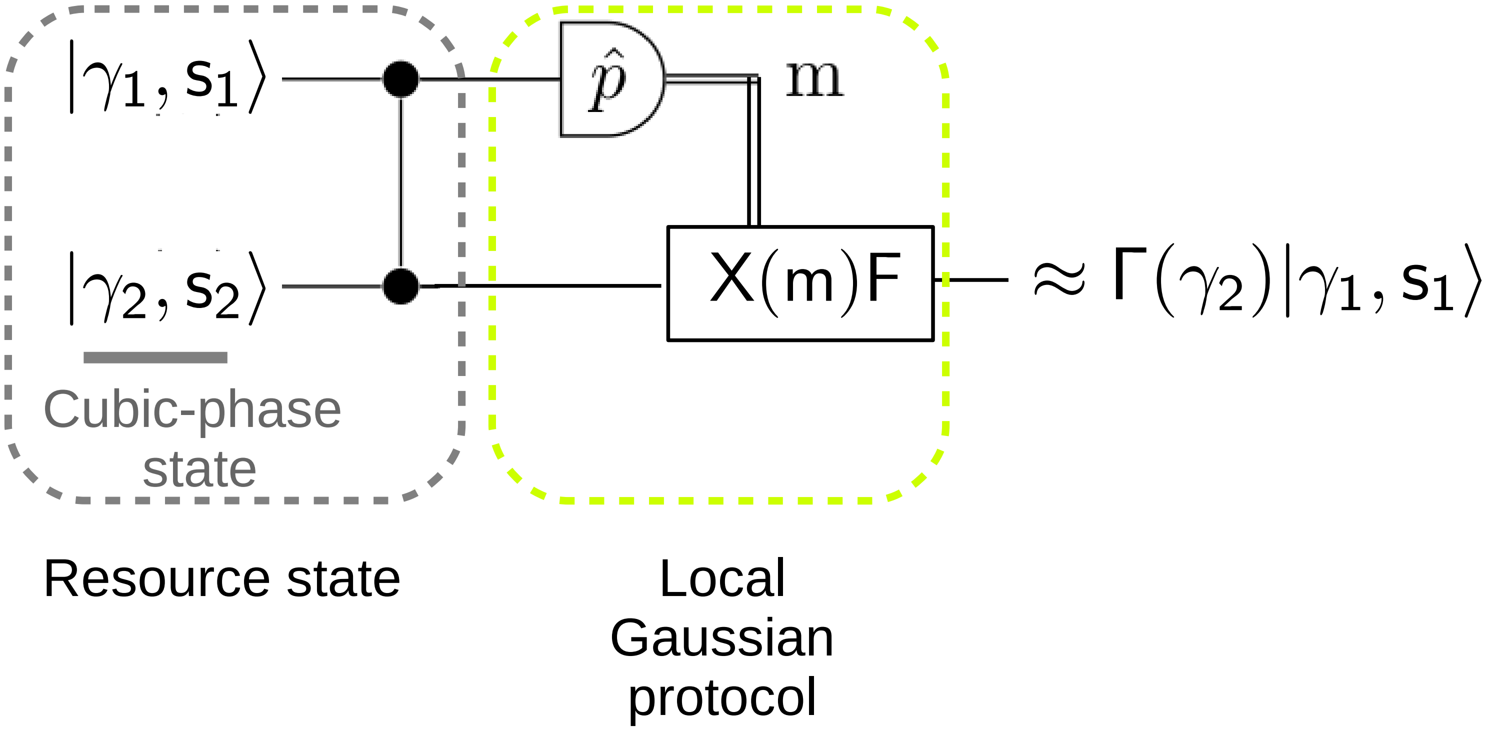


Local Gaussian protocol

It enables universal QC

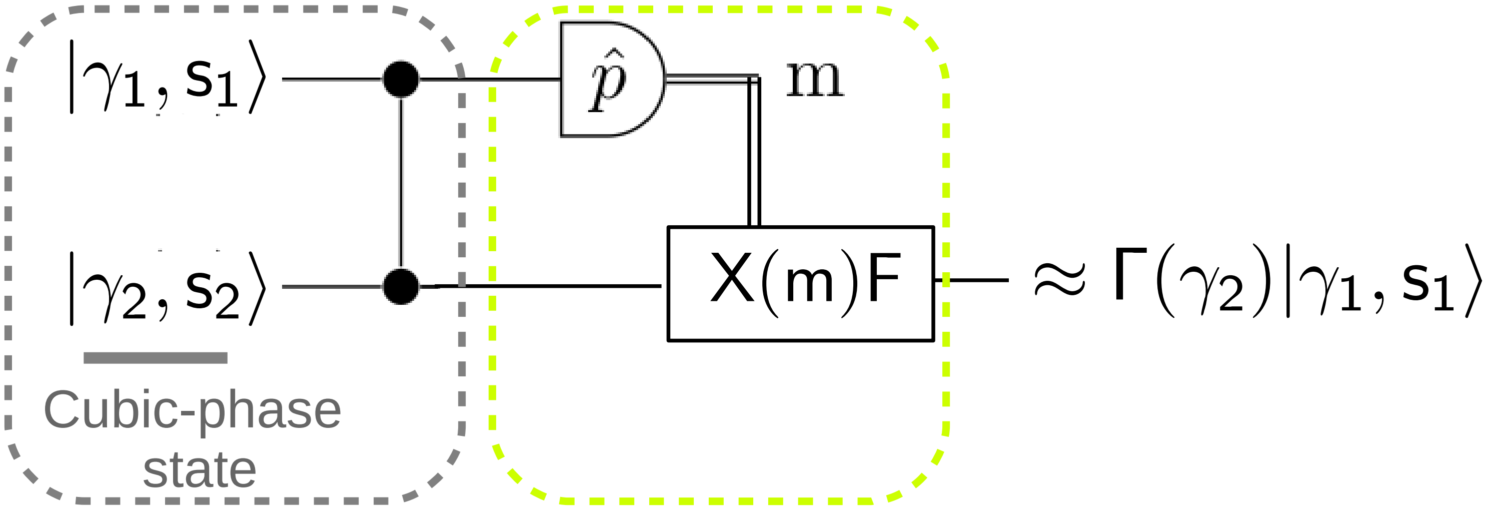
A cubic-phase state enables the cubic-phase gate

Gate teleportation:

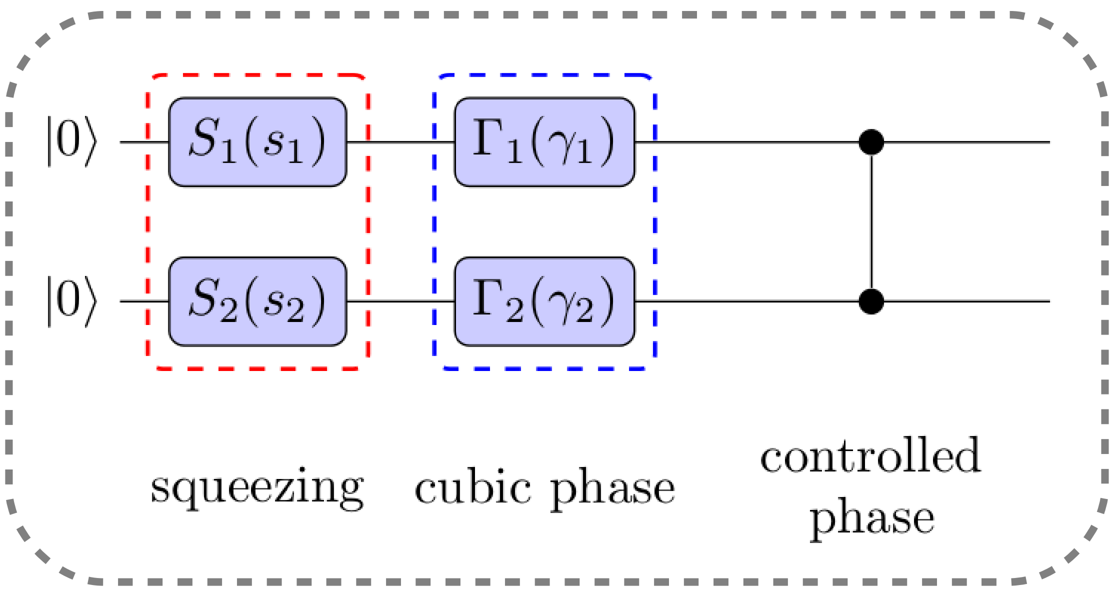


A cubic-phase state enables the cubic-phase gate

Gate teleportation:

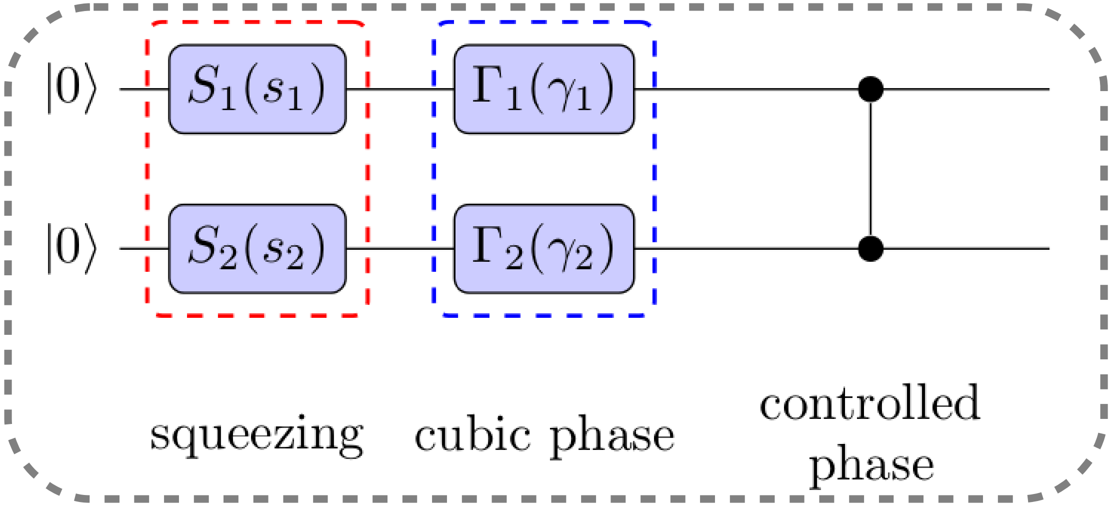


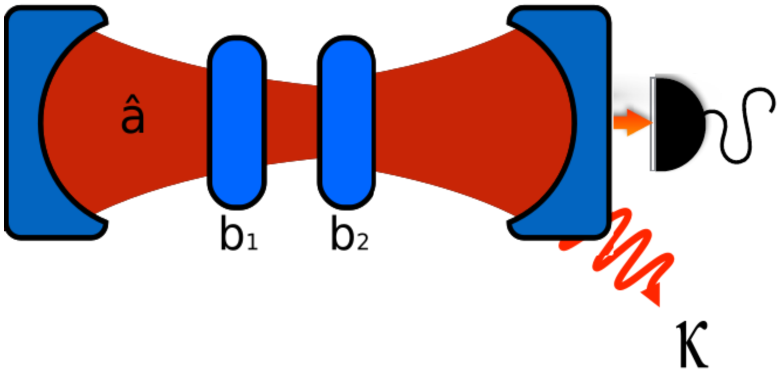
Resource state



Non-Gaussian Two-mode Cluster state

Unconditional generation of the non-Gaussian two-mode cluster state

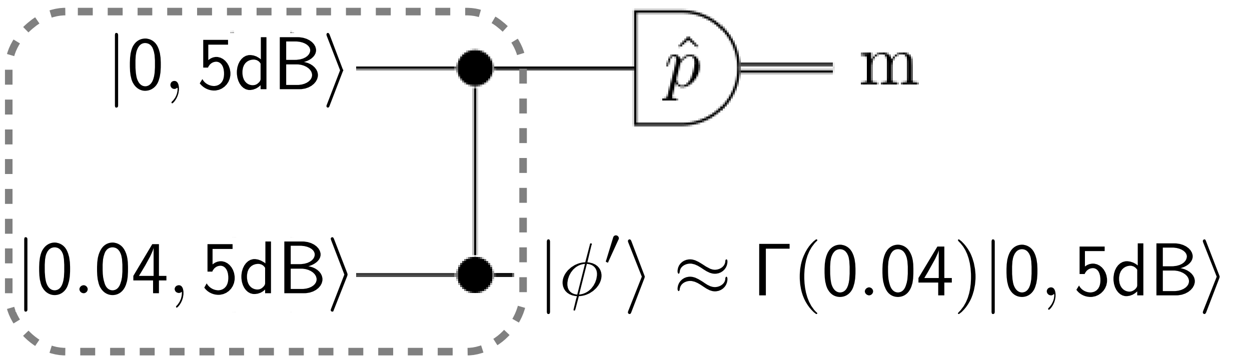


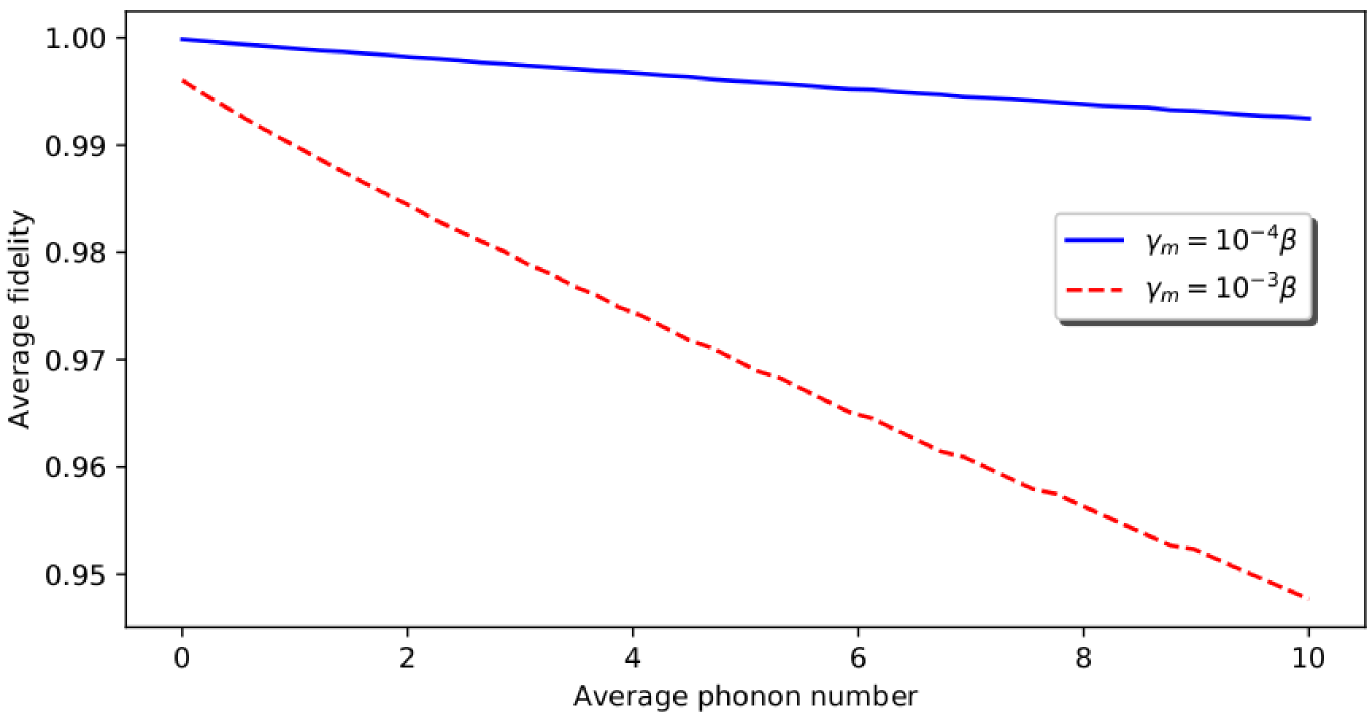


Two-step Hamiltonian engineering:

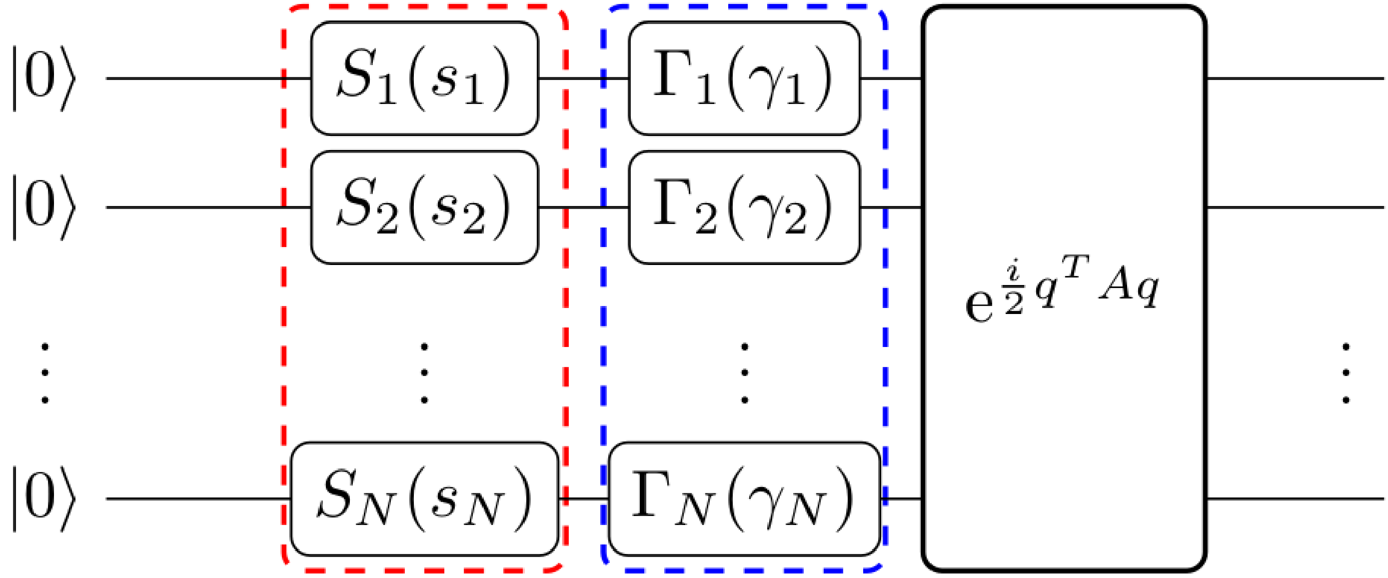
$$
\begin{aligned}
H_1 &= \frac{g}{2} a^\dagger \left[\left(s_1 + \frac{1}{s_1} \right) b_1 - \left(s_1 - \frac{1}{s_1} \right) b_1^\dagger - i s_1 \left(b_2 + b_2^\dagger \right) \right. \\
&\quad \left. - \frac{3 i \gamma_1 s_1}{\sqrt{2}} \left(b_1 + b_1^\dagger \right)^2 \right] + \text{H.c.} \\
H_2 &= \frac{g}{2} a^\dagger \left[- i s_2 \left(b_1 + b_1^\dagger \right) + \left(s_2 + \frac{1}{s_2} \right) b_2 - \left(s_2 - \frac{1}{s_2} \right) b_2^\dagger \right. \\
&\quad \left. - \frac{3 i \gamma_2 s_2}{\sqrt{2}} \left(b_2 + b_2^\dagger \right)^2 \right] + \text{H.c.}
\end{aligned}
$$

Cubic-phase gate teleportation via dissipation





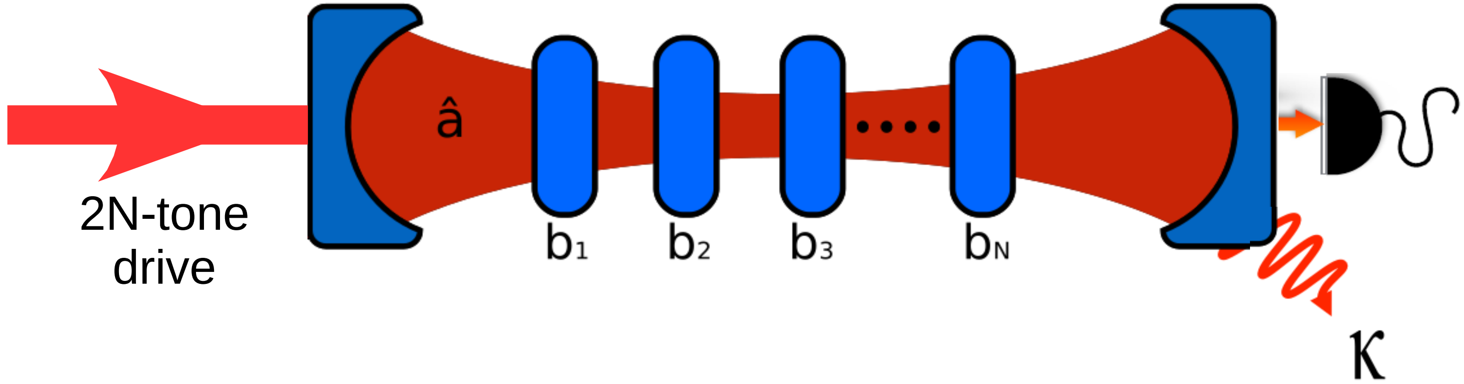
Unconditional generation of a universal multi-mode non-Gaussian cluster state



$$|\gamma, s, A\rangle = E(A)\Gamma(\gamma)S(s)\,|0\rangle$$

$$S(s) = \bigotimes_{j=1}^{N} S_j(s_j) \qquad \Gamma(\gamma) = \bigotimes_{j=1}^{N} \Gamma_j(\gamma_j) \qquad E(A) = e^{\frac{i}{2}q^\top A q}$$

This resource state + (local) Gaussian protocols = universal measurement-based CV quantum computation



Cavity-optomechanics setup with **multiple mechanical oscillators and L+Q coupling**:

1) Generation of cluster states for computation

[Houhou, Aissaoui, AF, PRA '15]

2) Quantum tomography of the resource

[Moore, Tufarelli, Paternostro, AF, PRA '16]

3) Arbitrary Gaussian computation

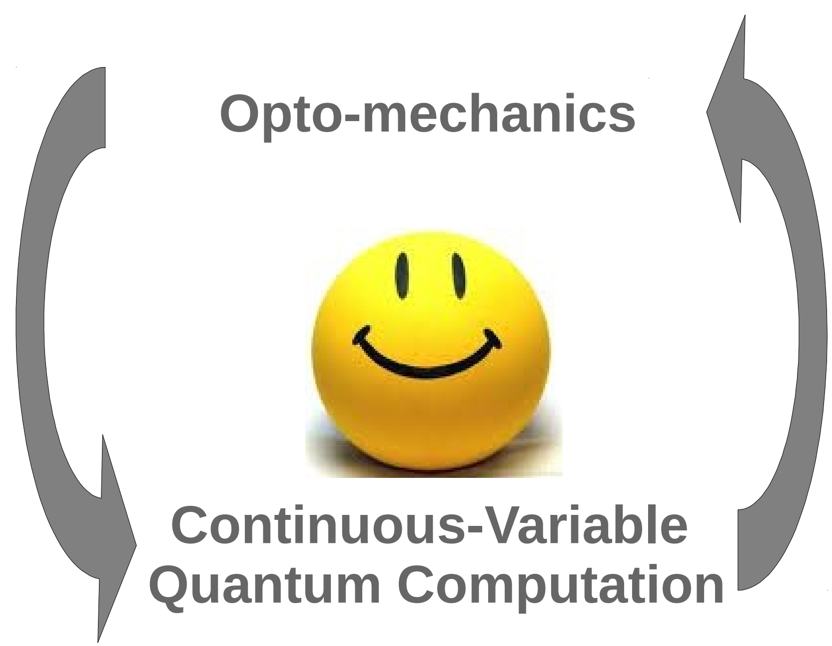
[Moore, Houhou, AF, PRA '17]

4) Unconditional non-Gaussian states generation

[Brunelli, Houhou, Moore, Nunnenkamp, Paternostro, AF, PRA '18]

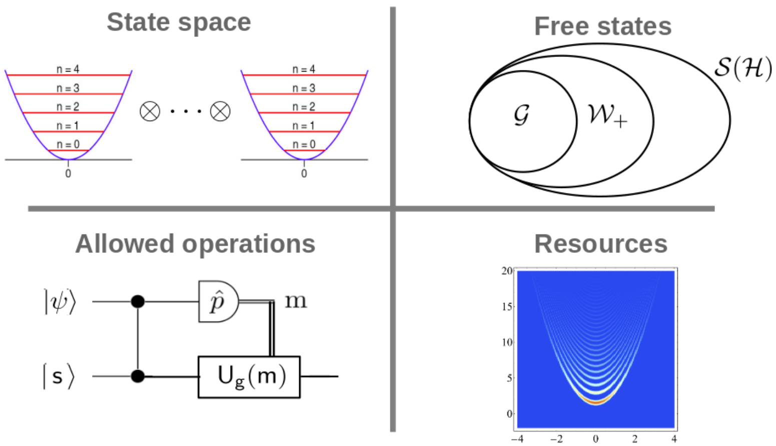
5) Unconditional measurement-based computation

[Houhou, Moore, Bose, AF, arXiv:1809.09733]

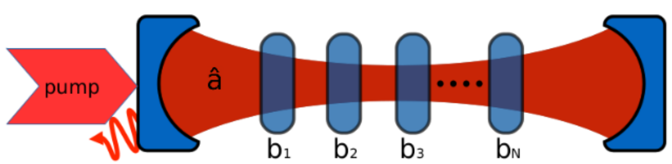


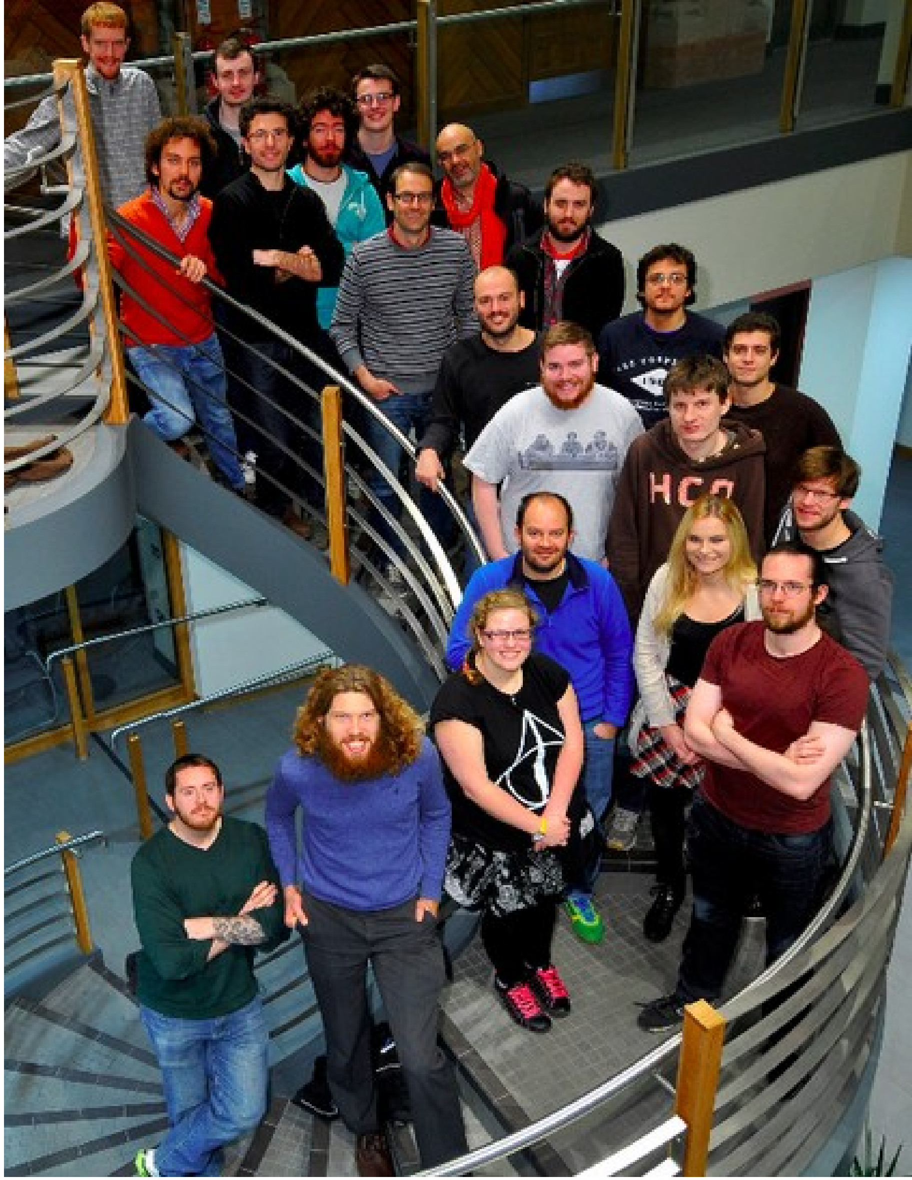
QTεQ
QUANTUM TECHNOLOGY at QUEEN'S

QnG Resource theory



Unconditional quantum computation in optomechanics





European Commission

EPSRC

John Templeton Foundation

O. Houhou (Medea)

D. Moore (Olomuc)

P. McConnell

M. Paternostro

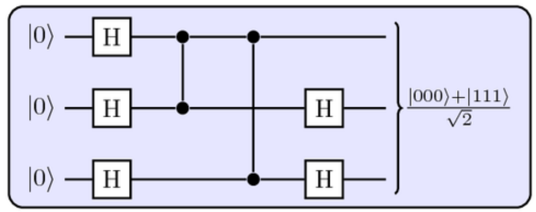
F. Albarelli (Warwick)

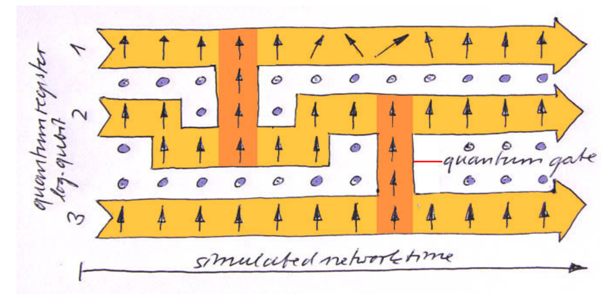
M. Genoni, M. Paris (Milan)

M. Brunelli, A. Nunnenkamp (Cambridge)

S. Bose (UCL)

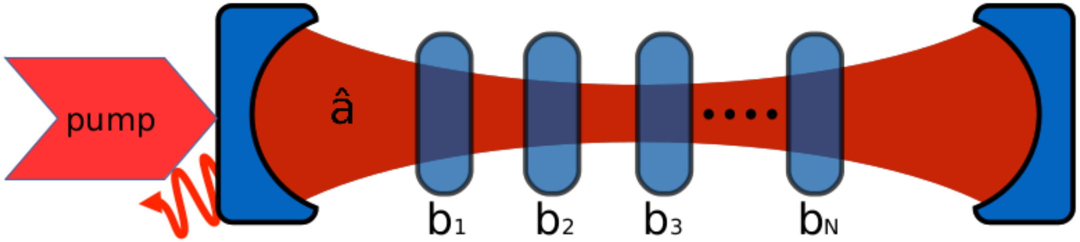
Models of computation





	Circuit-Based Quantum Computation	**Measurement-Based Quantum Computation (MBQC)**
Continuous Variables	Lloyd & Braunstein PRL (1999)	Menicucci et al. PRL (2006)
Fault tolerant (with finite energy)	Gottesman, Kitaev, Preskill PRA (2001) Lund, Ralph, Haselgrove, PRL (2008)	Menicucci PRL (2014)

Implementation - Hamiltonian switching

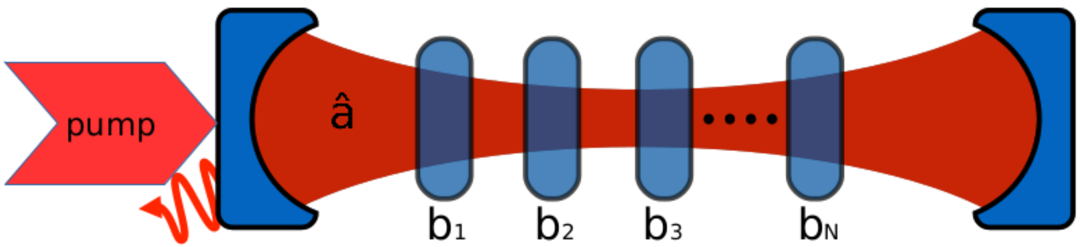


N independent oscillators (b_j) interacting with one damped oscillator (a)

$$\mathcal{H} = g\, a^\dagger \sum_{j=1}^{N} \left(g_1^{(j)} b_j + g_2^{(j)} b_j^\dagger + g_3^{(j)} b_j^2 + g_4^{(j)} {b_j^\dagger}^2 + g_5^{(j)} [b_j, b_j^\dagger]_+ \right) + \text{H.c.}$$

Objective: prepare the state $|\gamma, s, A\rangle = E(A)\Gamma(\gamma)S(s)\,|0\rangle$

Implementation - Hamiltonian switching



N independent oscillators (b_j) interacting with one damped oscillator (a)

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Objective: prepare the state $|\gamma, s, A\rangle = E(A)\Gamma(\gamma)S(s)\,|0\rangle$

Method: N-step preparation protocol:

at step k we implement $E(A)\Gamma(\gamma)S(s)\; b_k \left(E(A)\Gamma(\gamma)S(s)\right)^\dagger \equiv \hat{f}_k$

The new Hamiltonian: $\mathcal{H}_k \equiv g\; a^\dagger \hat{f}_k + \text{H.c.}$

The dynamics obeys: $\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i[\mathcal{H}_k, \rho] + \kappa D[a]\rho.$

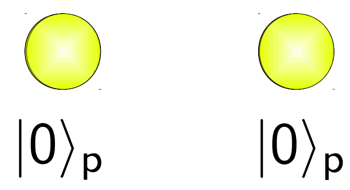
After N steps, the system reaches the target state.

For the case of linear coupling: Gaussian cluster state [Houhou, Aissaoui, AF, PRA '15]

Ideal measurement-based quantum computation

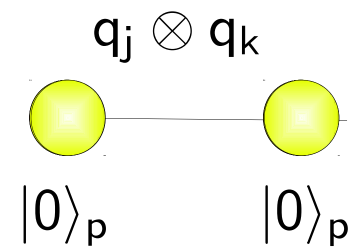
CV cluster state: the universal resource for computation

- Prepare each node in zero-momentum eigenstate



Ideal measurement-based quantum computation

CV cluster state: the universal resource for computation



- Prepare each node in zero-momentum eigenstate

- Entangle connected nodes with

$$\mathsf{CZ}_{jk} \equiv \exp[i q_j \otimes q_k]$$

Ideal measurement-based quantum computation

CV cluster state: the universal resource for computation

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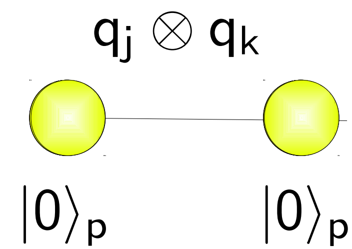

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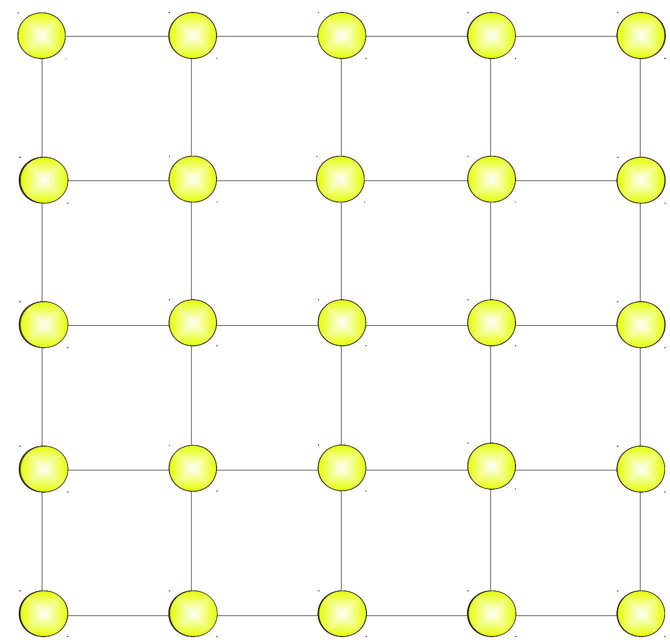
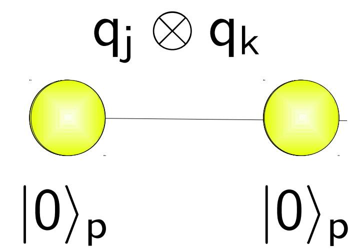


CV cluster state

Ideal measurement-based quantum computation

CV cluster state: the universal resource for computation

- Prepare each node in zero-momentum eigenstate

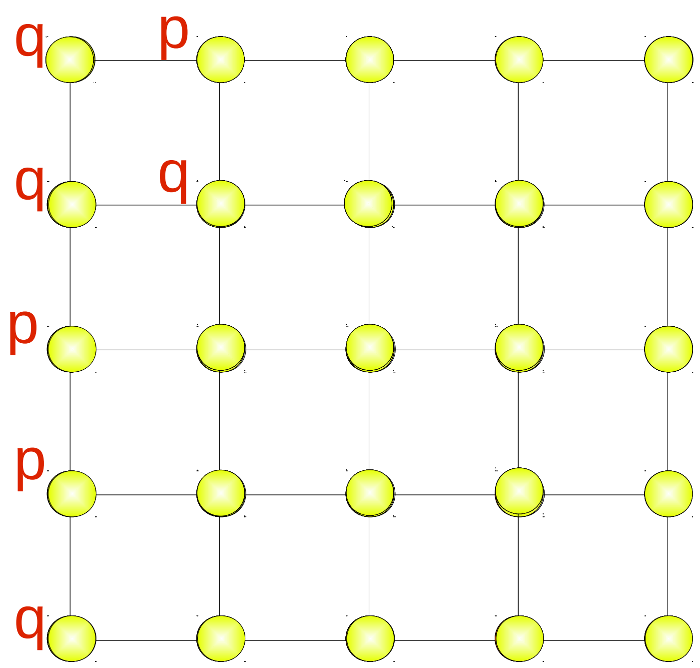


- Entangle connected nodes with

$$CZ_{jk} \equiv \exp[i q_j \otimes q_k]$$

- Measure each node locally

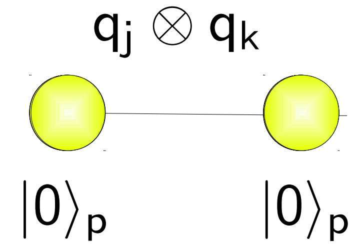
  Quadrature measurements:
  Gaussian computation



Ideal measurement-based quantum computation

CV cluster state: the universal resource for computation

- Prepare each node in zero-momentum eigenstate



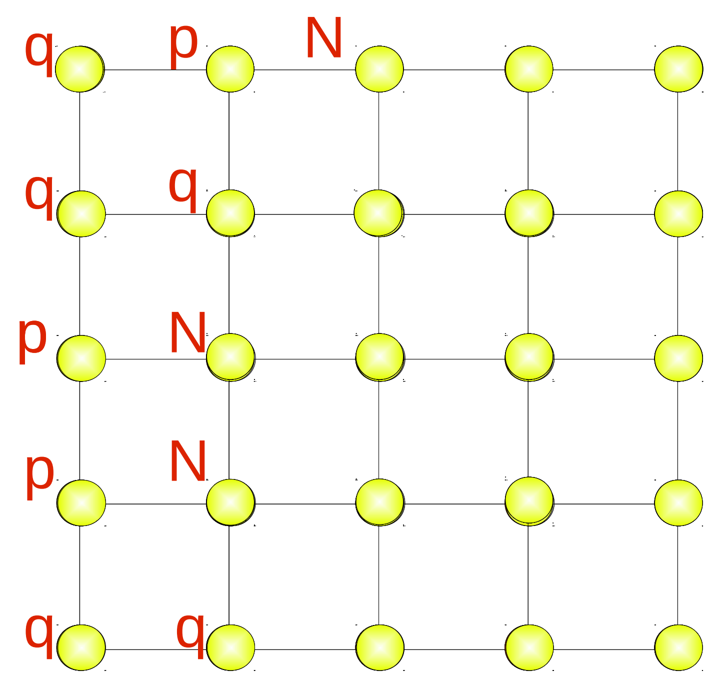
- Entangle connected nodes with

$$CZ_{jk} \equiv \exp[i q_j \otimes q_k]$$

- Measure each node locally

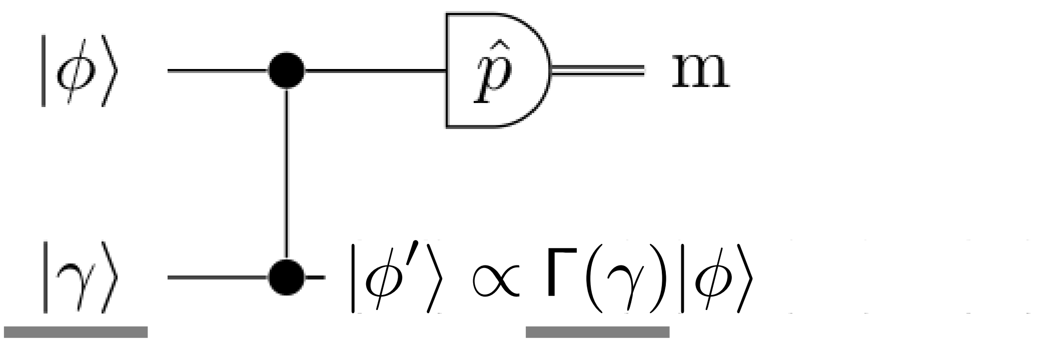
  Quadrature measurements:
  Gaussian computation

  Non-Gaussian measurements:
  Universal computation



Non-Gaussian measurements can be substituted by non-Gaussian states

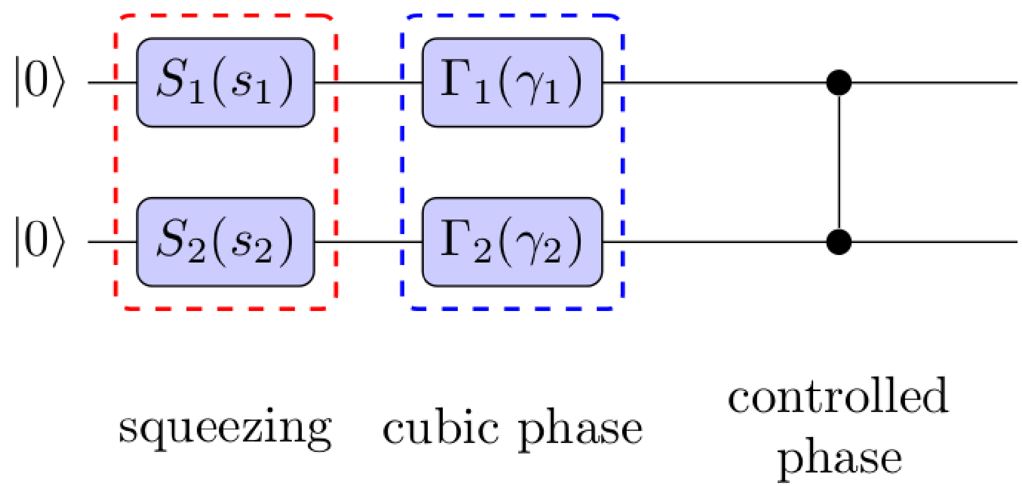
Gate teleportation:



Cubic-phase state

Cubic-phase gate

It enables universal QC



Parameter	Set 1 (realistic)	Set 2 (close to ideal)
η	0.99	1
$\frac{\gamma}{2\pi}$	8 Hz	0 Hz
$\frac{\kappa}{2\pi}$	0.33 MHz	0.1 MHz
τ	0.01κ	0
αg	0.35 MHz	0.35 MHz
T	1 mK	0 K
$r_{\text{post-meas}}$	10 dB	20 dB
r_{cluster}	3 dB	3 dB

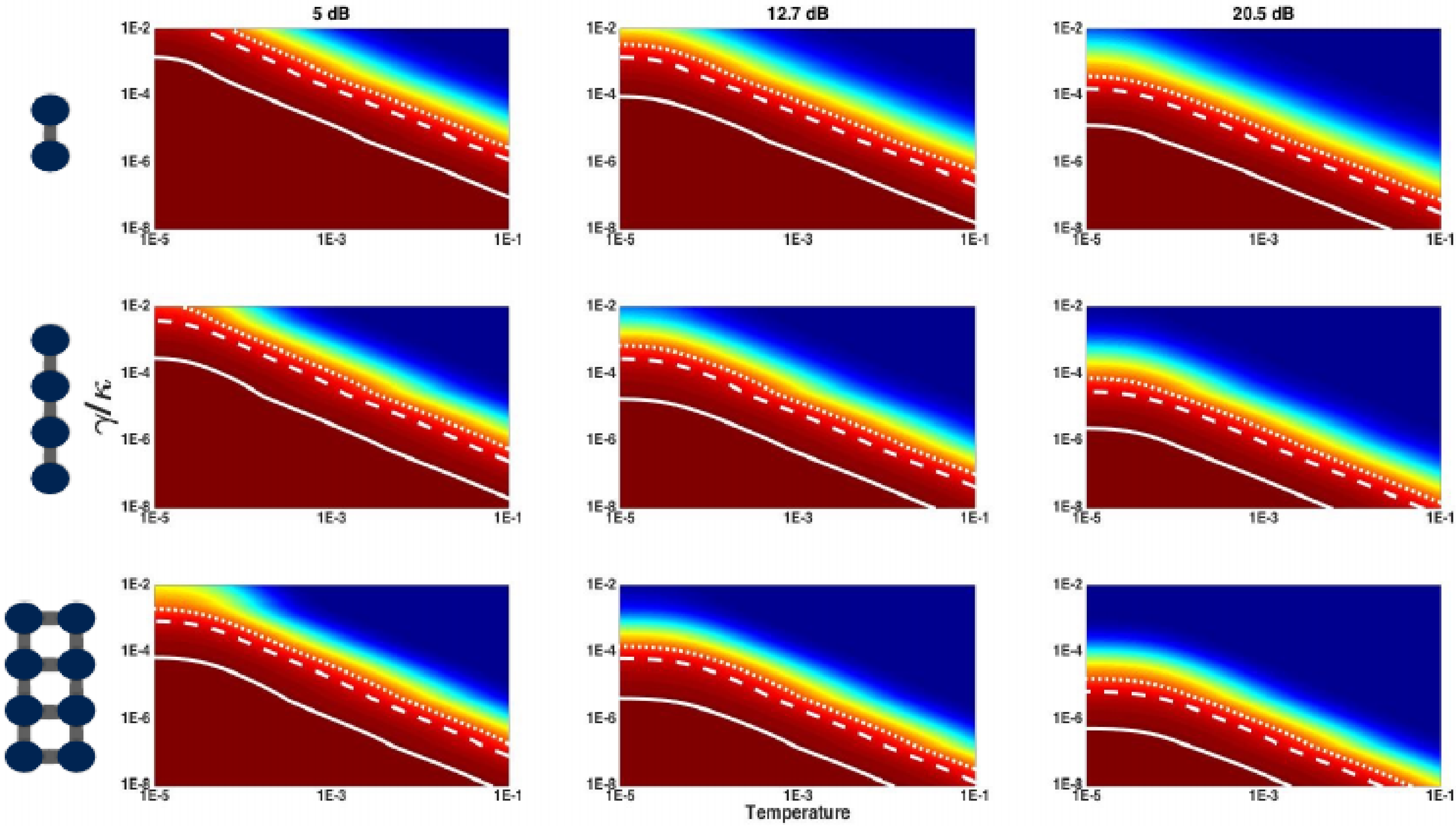
Effects of mechanical noise

Consider mechanical noise at temperature T_j and damping rate γ_j :

$$\frac{d\rho}{dt} = -i[H,\rho] + \kappa(a\rho a^\dagger - \frac{1}{2}a^\dagger a\rho - \frac{1}{2}\rho a^\dagger a) + \mathcal{L}_1 + \mathcal{L}_2$$

with $\gamma_j \, , \kappa \ll \Omega_j$:

$$\begin{aligned}
\mathcal{L}_1 &= \sum_{j=1}^{N} \gamma_j (n_j + 1) \left(b_j \rho b_j^\dagger - \frac{1}{2} b_j^\dagger b_j \rho - \frac{1}{2} \rho b_j^\dagger b_j \right) \\
\mathcal{L}_2 &= \sum_{j=1}^{N} \gamma_j n_j \left(b_j^\dagger \rho b_j - \frac{1}{2} b_j b_j^\dagger \rho - \frac{1}{2} \rho b_j b_j^\dagger \right) \\
n_j &= \left(\exp \frac{\hbar \Omega_j}{K_B T_j} - 1 \right)^{-1}
\end{aligned}$$



- The higher the target squeezing the less the tolerable noise
- The larger the target graph the less the tolerable noise
- Working regime:

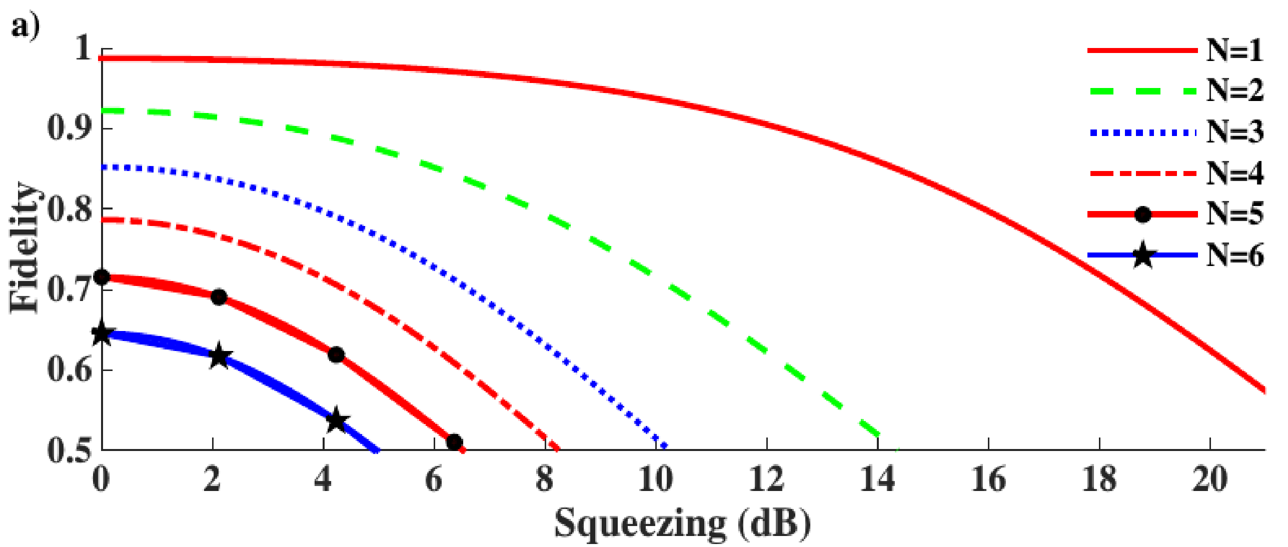
$$\gamma_j \ll \kappa \ll \Omega_j \text{ and low } T_j$$

Experimental feasibility

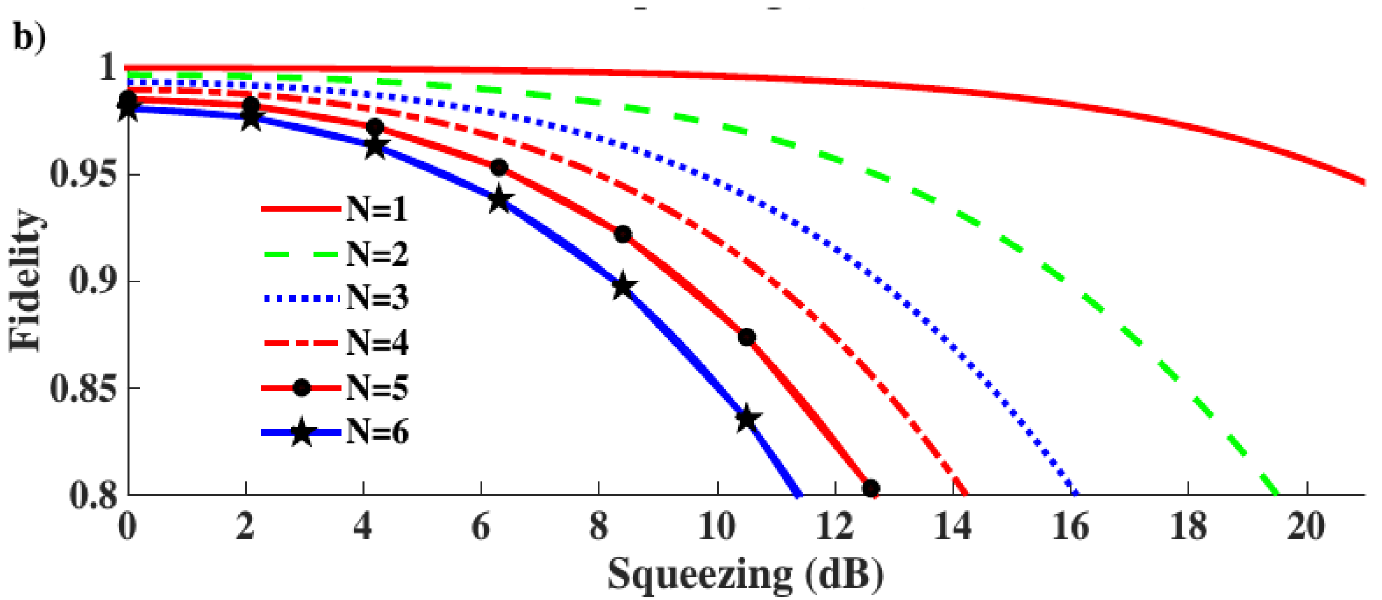
$\Omega_j/2\pi = 11j$ MHz $\gamma/2\pi = 32$ Hz $\kappa/2\pi = 0.2$ MHz

T $= 15$ mK

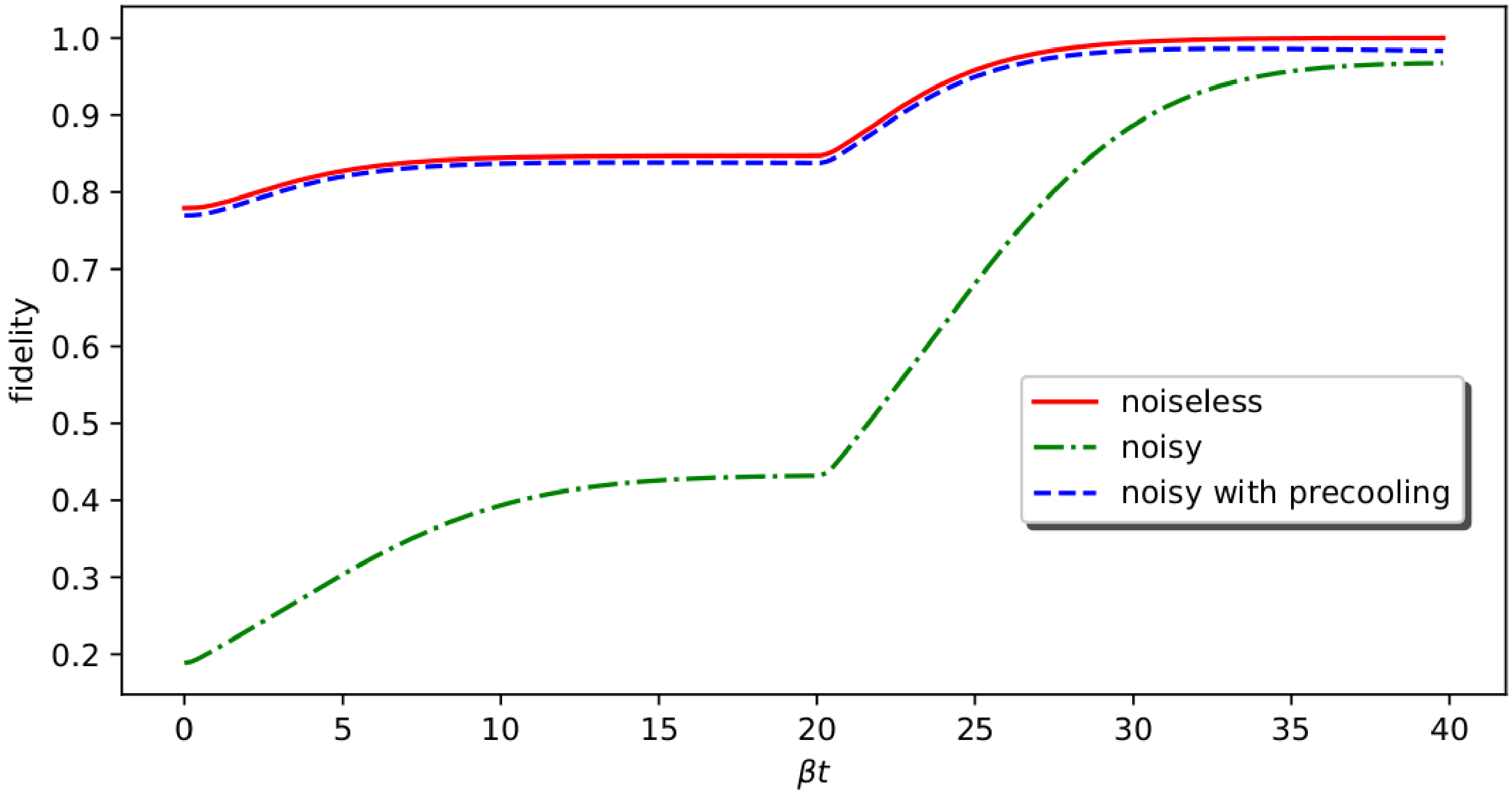
[Teufel et al., Nature (2011)]



T $= 1$ mK



High fidelities can be reached even in the presence of mechanical noise



$$s_1 = s_2 \equiv 5\text{dB}$$

$$\gamma_1 = 0,\ \gamma_2 \approx 0.04$$

The Shuffle

Setting :

$$g_2 = \frac{1}{2}\, g_1$$

$$g_3 = g_5 = \frac{1}{4}\sqrt{\frac{3}{2n+1}}\, g_1 \qquad , \; n \in \mathbb{N}$$

$$g_4 = 0$$

[Brunelli, Houhou, Moore, Nunnenkamp, Paternostro, AF arXiv:1804.00014]

$$\varphi_n(x) \propto e^{-\frac{X_n^2}{4}} H_n(X_n) \qquad X_n = \sqrt{\tfrac{2}{3}}\left(x + \sqrt{4n+2}\right)$$

Fock-like

cat-like