
Entanglement mediated by gravity

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Is gravity quantum?

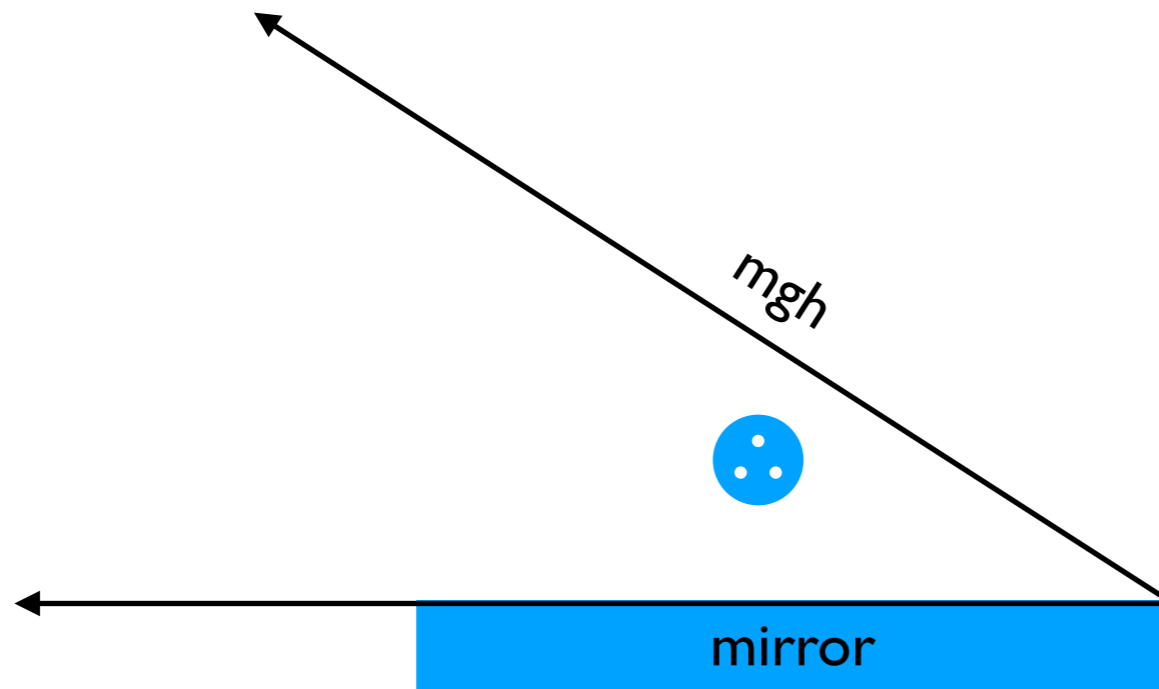
THINGS FALL... COUNTLESS EXPERIMENTAL EVIDENCE



mirror



THINGS FALL... COUNTLESS EXPERIMENTAL EVIDENCE

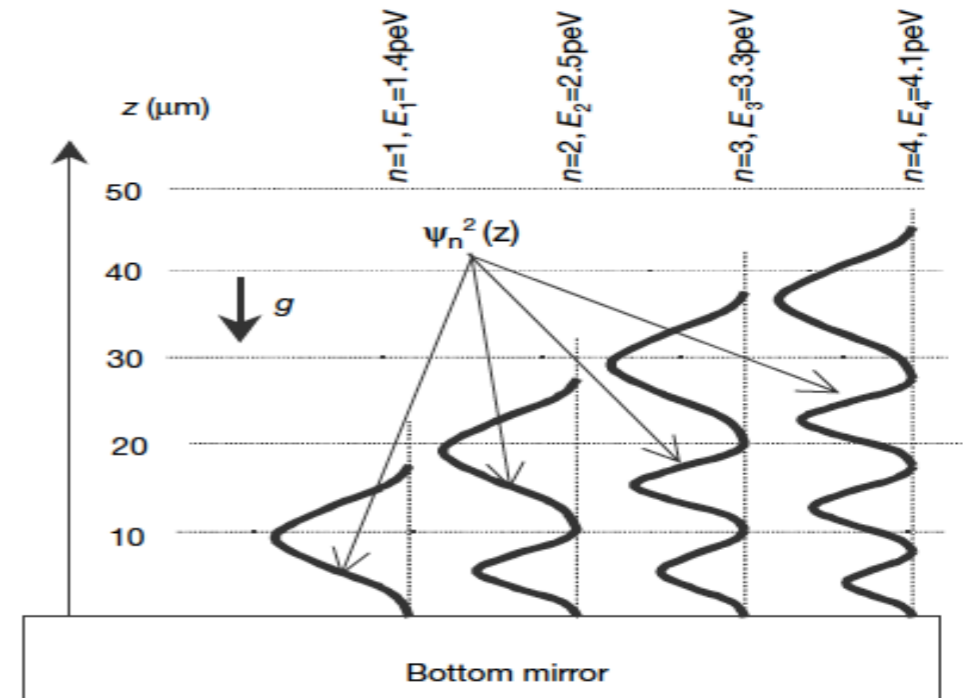
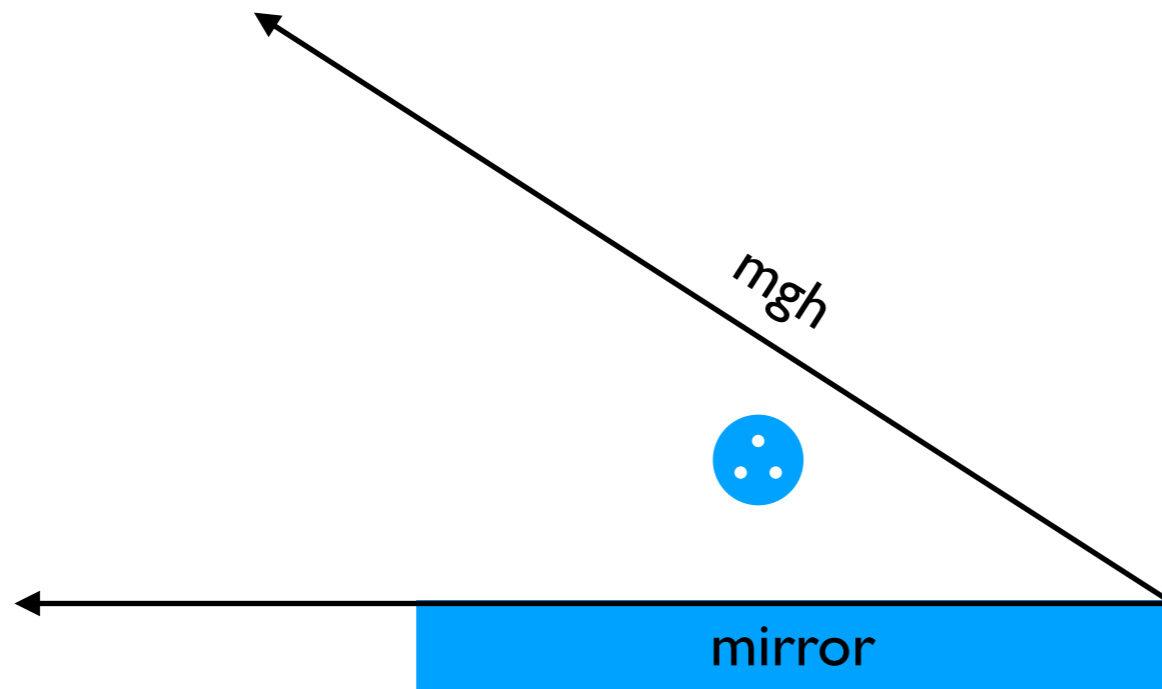


**Quantum states of neutrons
in the Earth's gravitational field**

Valery V. Nesvizhevsky*, **Hans G. Börner***, **Alexander K. Petukhov***,
Hartmut Abele†, **Stefan Baeßler†**, **Frank J. Rueß†**, **Thilo Stöferle†**,
Alexander Westphal†, **Alexei M. Gagarski‡**, **Guennady A. Petrov‡**
& **Alexander V. Strelkov§**

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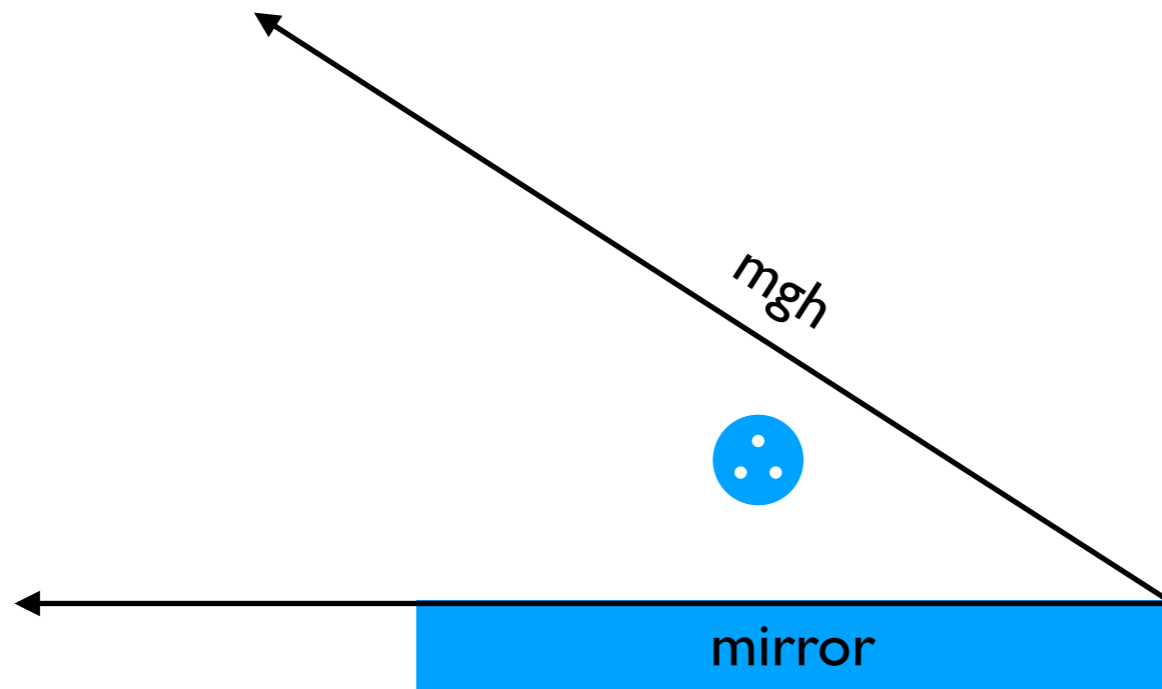
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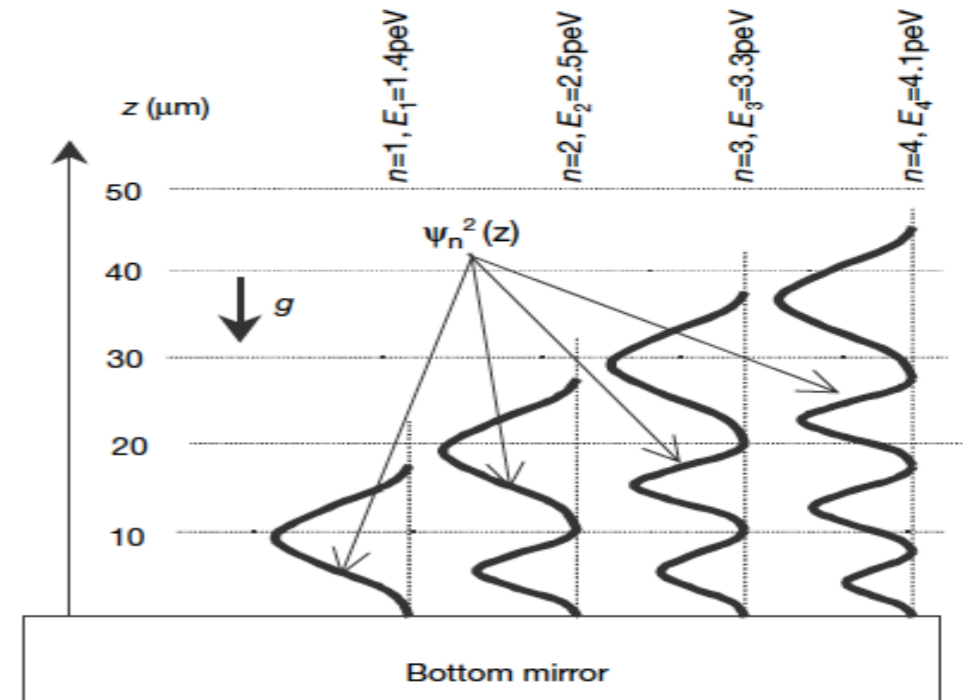
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THINGS FALL... COUNTLESS EXPERIMENTAL EVIDENCE



$$m g (10 \text{ } \mu\text{m}) = 1 \text{ peV}$$

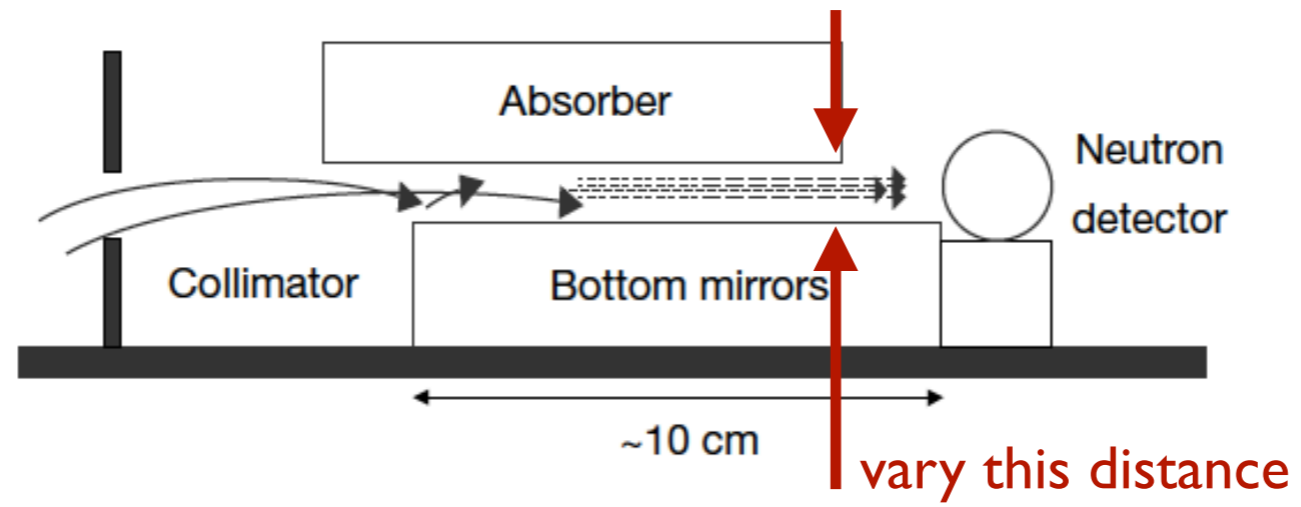


Quantum states of neutrons in the Earth's gravitational field

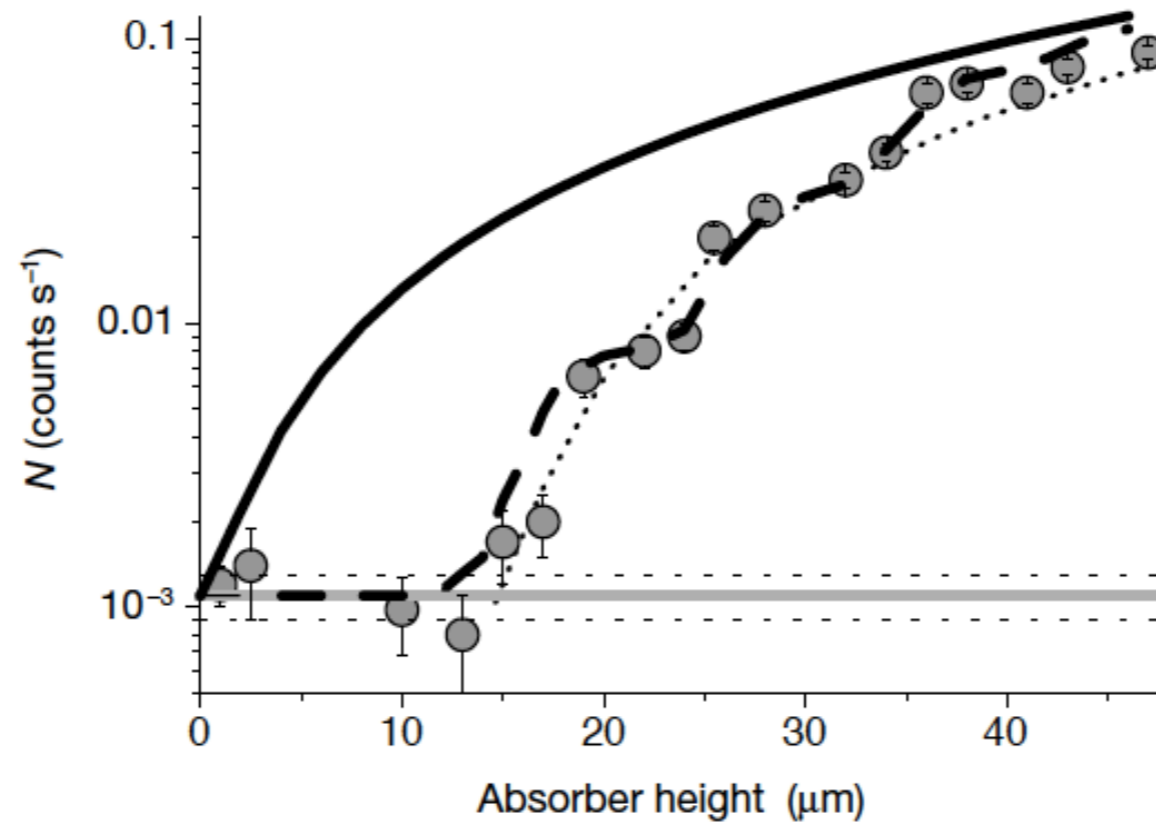
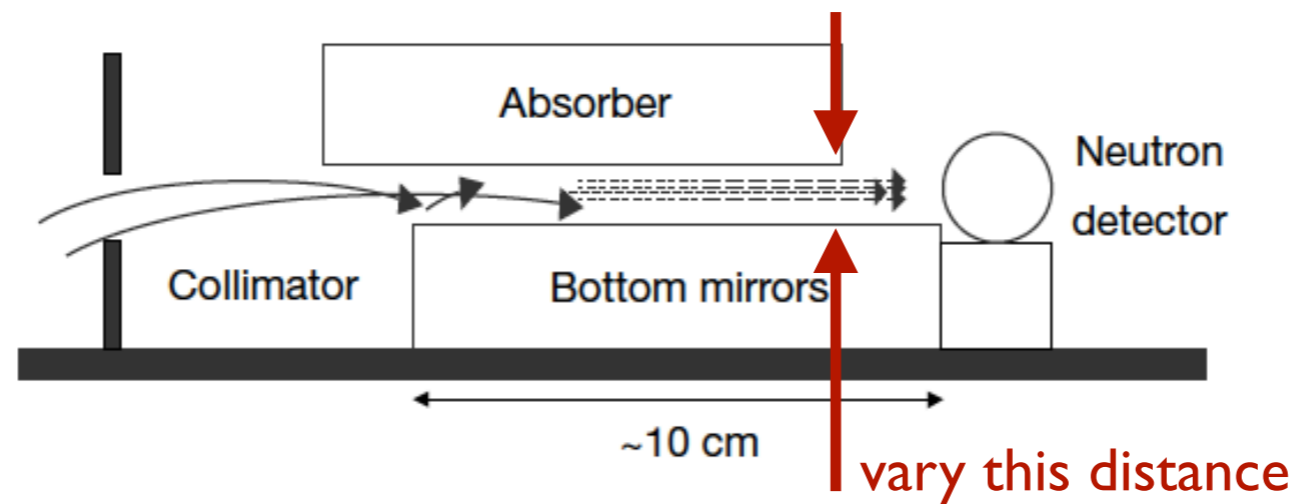
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NEUTRONS IN GRAVITATIONAL BOUND STATES



NEUTRONS IN GRAVITATIONAL BOUND STATES



Newtonian gravity

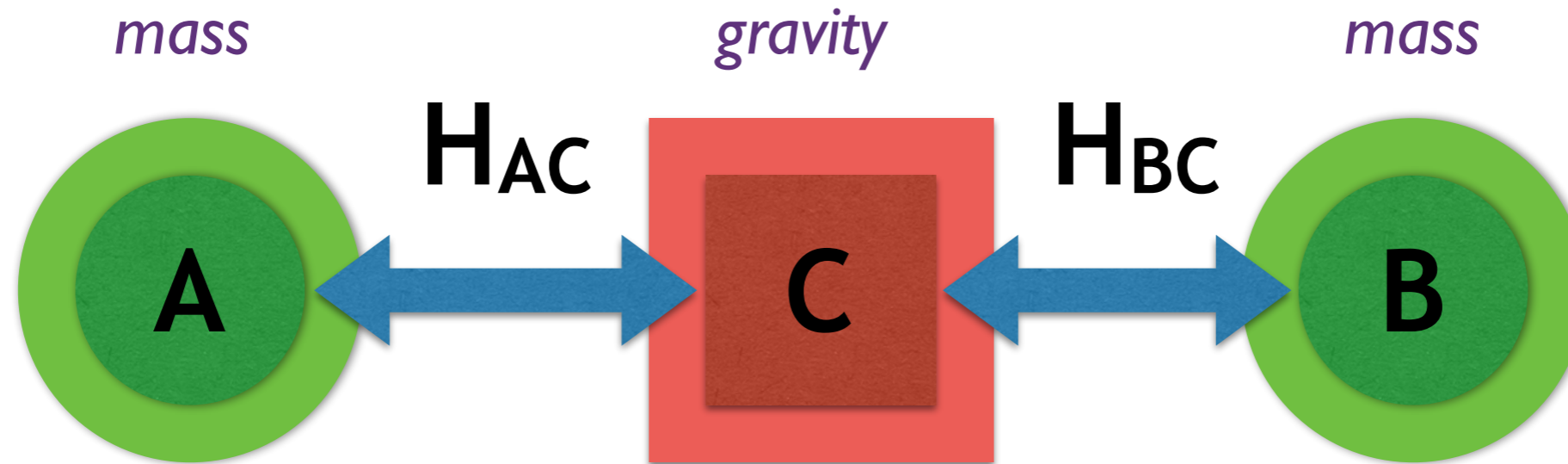
- *neutrons fall* [e.g. Nature **415**, 297 (2002)]
- *vertical interferometers have gravity-dependent phase shift* [PRL **34**, 1472 (1975)]
- *atomic gravimeters* [e.g. Nature **400**, 849 (1999)]

General relativity

- *gravitational redshift* [e.g. PRL **4**, 337 (1960)]
- *atomic clocks tick differently at different heights* [e.g. Science **177**, 168 (1972)]

All explained by **classical** gravity.

IDEA FOR EXPERIMENT INDICATING QUANTUM GRAVITY



PRL 119, 120402 (2017)

PHYSICAL REVIEW LETTERS

week ending
22 SEPTEMBER 2017

Revealing Nonclassicality of Inaccessible Objects

Tanjung Krisnanda,¹ Margherita Zupardo,^{1,2} Mauro Paternostro,³ and Tomasz Paterek^{1,4,5}

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²Science Institute, University of Iceland, Dunhaga 3, IS-107 Reykjavik, Iceland

³School of Mathematics and Physics, Queen's University, Belfast BT7 1NN, United Kingdom

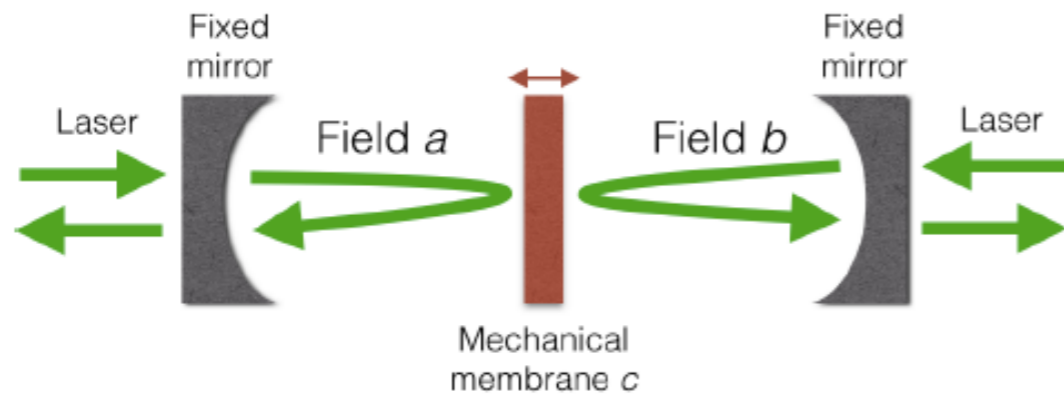
⁴Centre for Quantum Technologies, National University of Singapore, 117543 Singapore, Singapore

⁵MajuLab, CNRS-UNS-NUS-NTU International Joint Research Unit, UMI 3654 Singapore, Singapore

(Received 22 July 2016; revised manuscript received 10 August 2017; published 21 September 2017)

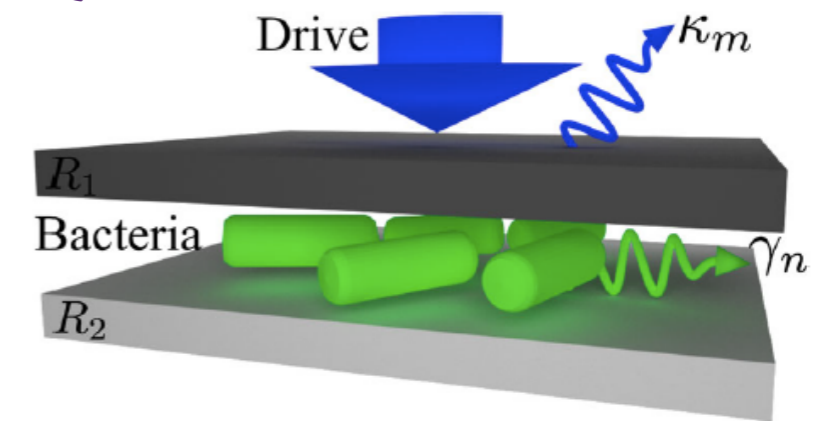
EXEMPLARY APPLICATIONS OF THE METHOD

Opto-mechanics



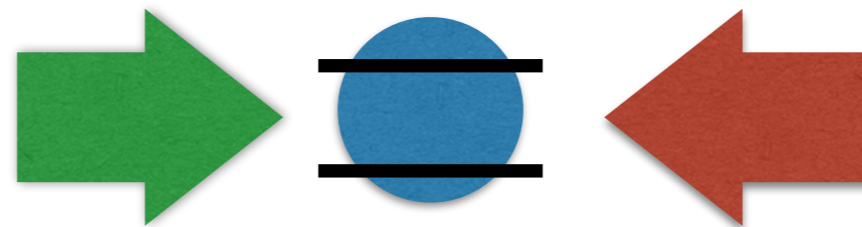
PRL 119, 120402 (2017)

Quantum bio



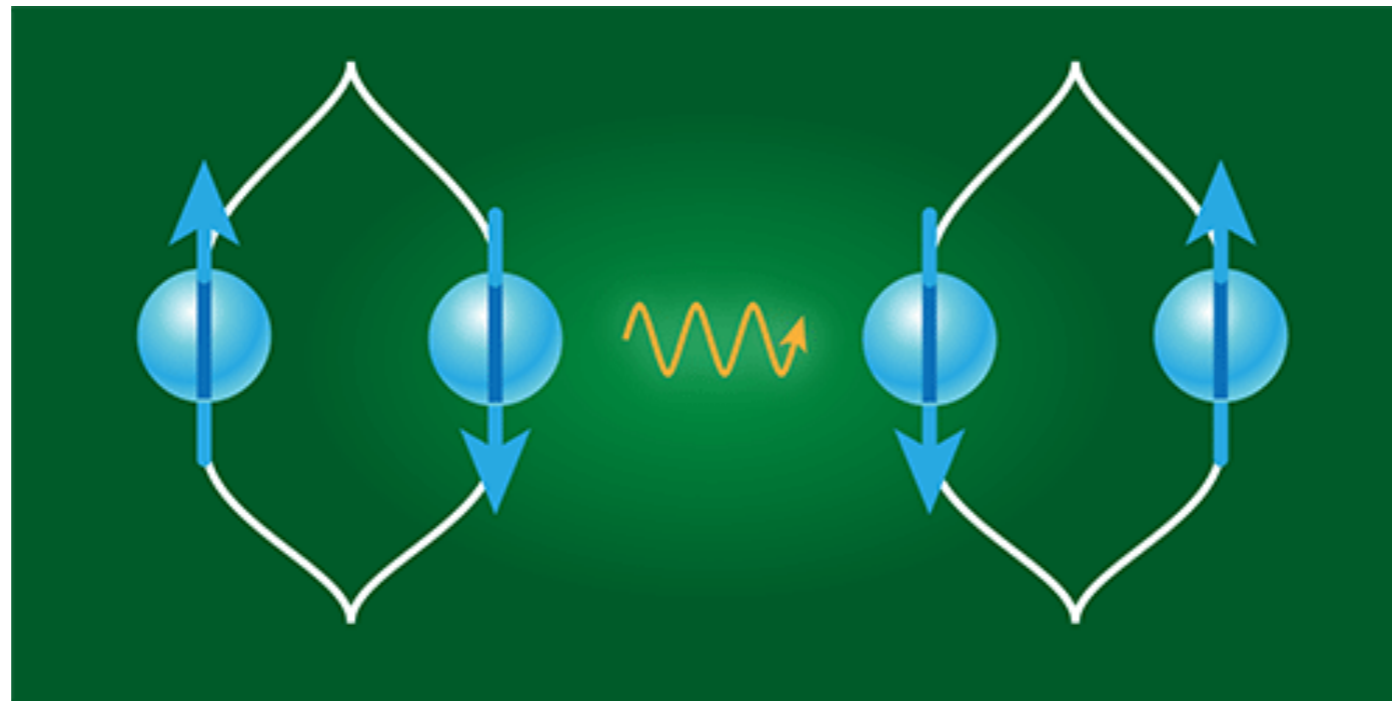
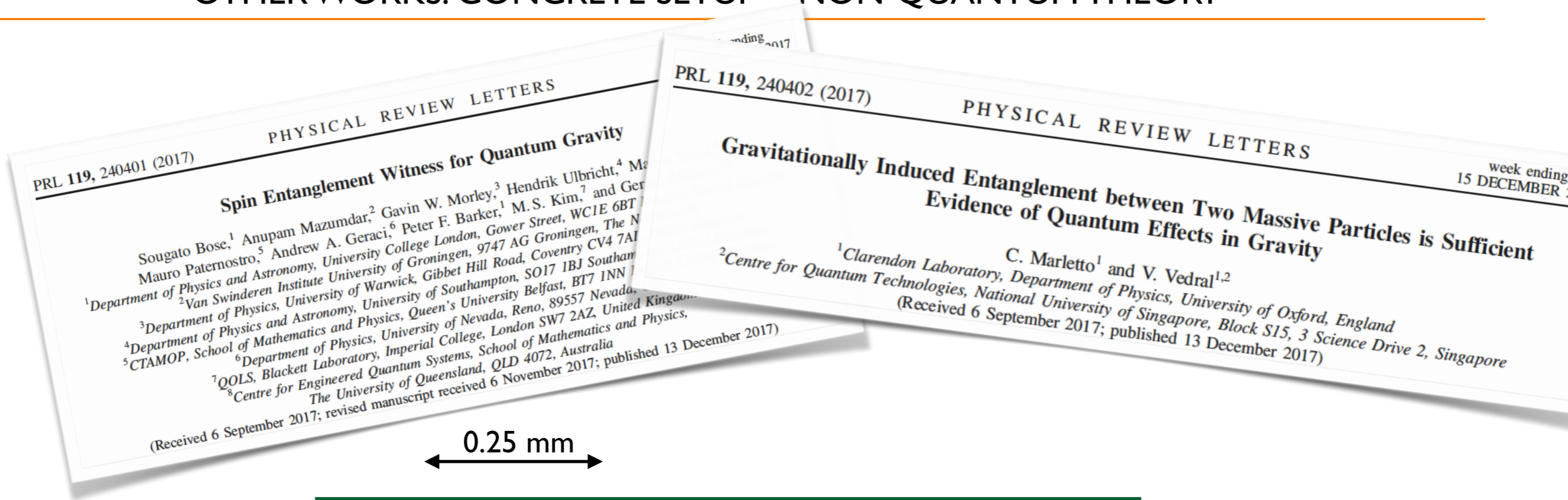
NPJQI 4, 60 (2018)

Non-commutativity



PRA 98, 052321 (2018)

OTHER WORKS: CONCRETE SETUP + NON-QUANTUM THEORY



G.W. Morley / University of Warwick and A. Stonebraker / APS

If there is **no** entanglement initially:

$$E_{A:B}(\tau) > E_{A:B}(0) \text{ then } D_{AB|C}(t) > 0$$



[note minimalistic assumptions about C and its couplings]

NON-CLASSICALITY = DISCORD = COHERENCE

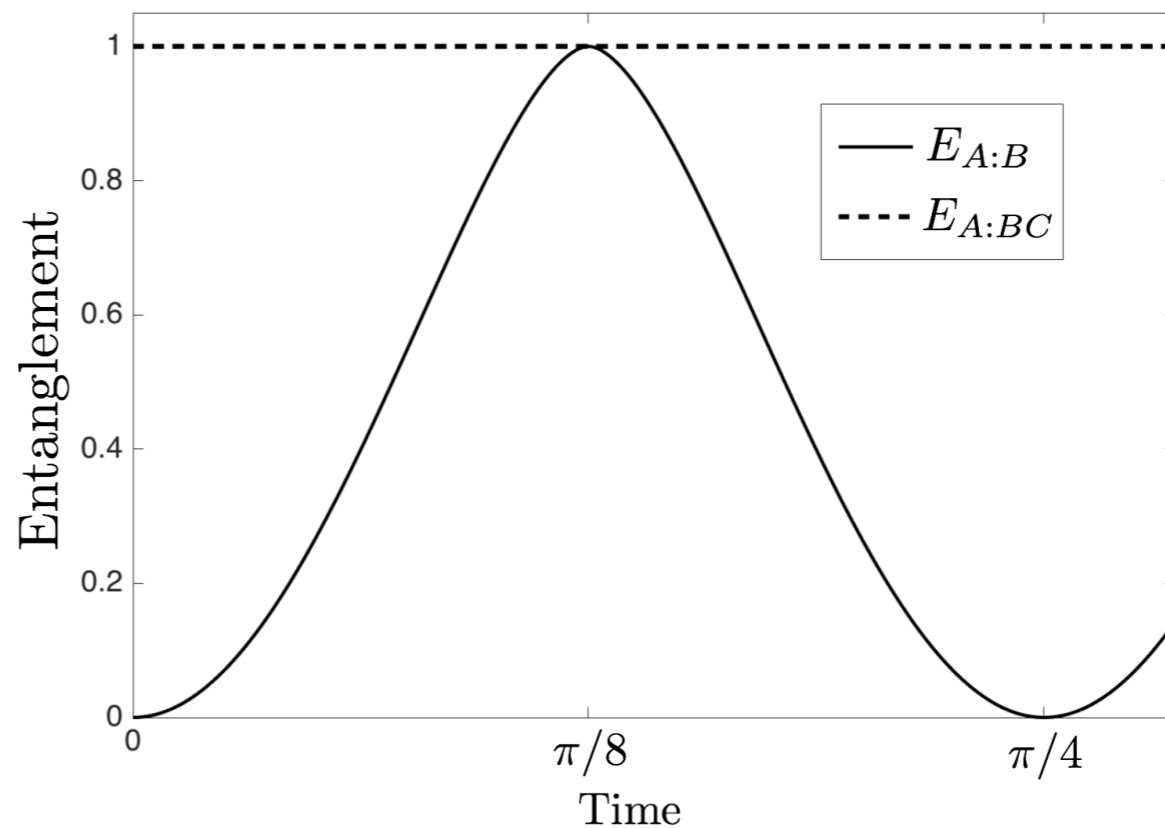
These must be orthogonal

$$\rho_{\text{dis}} \neq \sum_c p_c \rho_{AB|c} \otimes \Pi_c$$


There must be non-orthogonal states of C
By operating on A and B one can prepare them

WARNING: INITIAL STATE IS IMPORTANT

It **is** possible to localise initial entanglement A:BC to subsystem A:B via **classical C**.

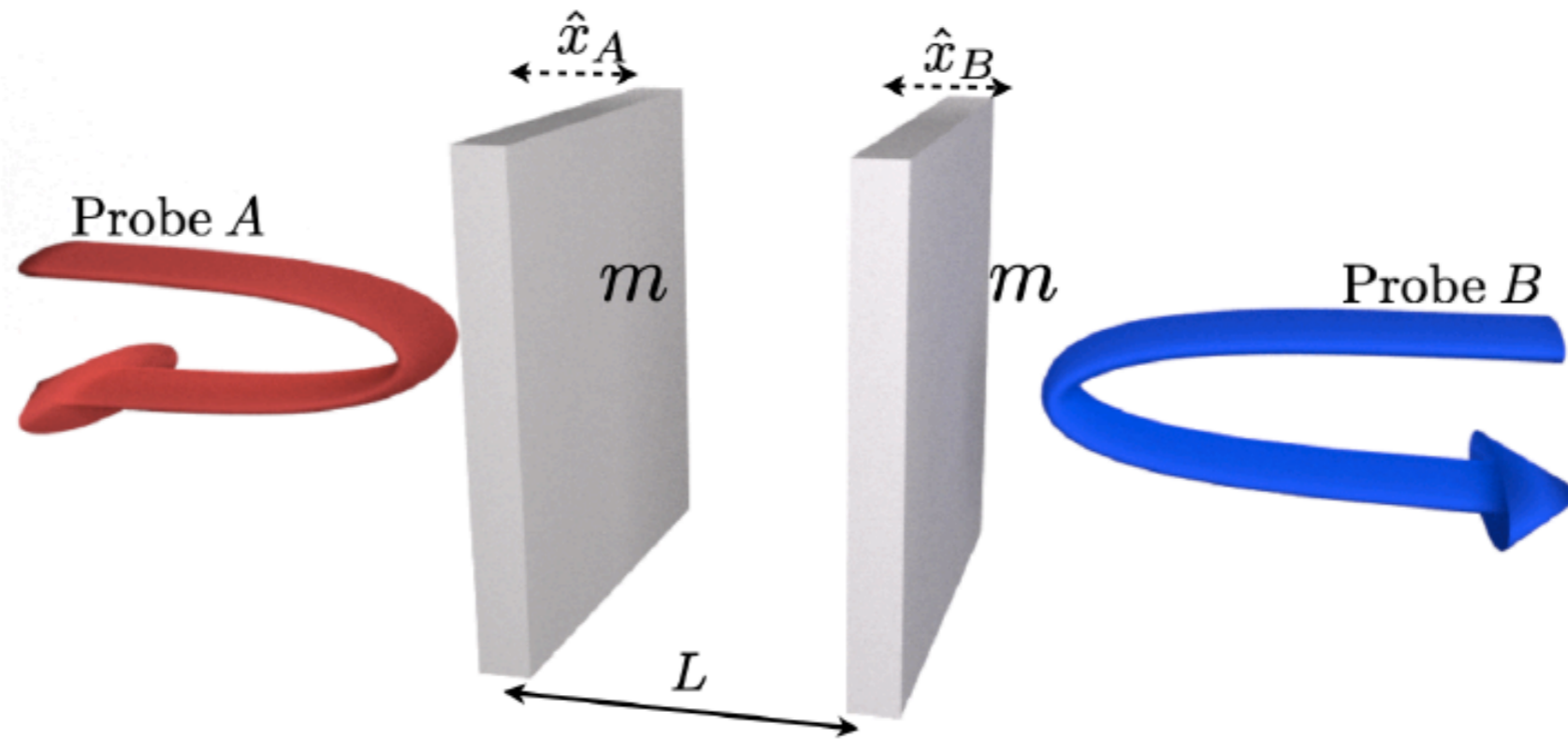


$$\rho_0 = \frac{1}{2} |\psi^+\rangle\langle\psi^+| \otimes |+\rangle\langle+| + \frac{1}{2} |\phi^+\rangle\langle\phi^+| \otimes |-\rangle\langle-|$$

$$H = X \otimes \mathbb{1} \otimes X + \mathbb{1} \otimes X \otimes X$$

Concrete setup

GRAVITY IN OPTO-MECHANICS



Cool down close to ground state

Turn off lasers: gravity + noise + Casimir

Probe entanglement

HAMILTONIAN

$$H = \frac{\hat{p}_A^2}{2m} + \frac{1}{2}m\omega^2\hat{x}_A^2 + \frac{\hat{p}_B^2}{2m} + \frac{1}{2}m\omega^2\hat{x}_B^2 - \frac{Gm^2}{L + (\hat{x}_B - \hat{x}_A)}$$

Each oscillator is spherical.

A and B are directly coupled.

Previous theory not applicable

Incorrect coupling: instantaneous interactions

For two nearby objects very good approximation

Conditions for entanglement the same as with mediator

FIGURE OF MERIT

$$-\frac{Gm^2}{L + (\hat{x}_B - \hat{x}_A)} = -G\frac{m^2}{L} \left(1 - \frac{\hat{x}_A - \hat{x}_B}{L}\right)^{-1}$$
$$\simeq -G\frac{m^2}{L} \left(1 + \frac{\hat{x}_A - \hat{x}_B}{L} + \frac{(\hat{x}_A - \hat{x}_B)^2}{L^2}\right)$$

Energy shift *Local terms* *Possibility of entanglement*

In order to generate appreciable entanglement the last term needs to be comparable to the energy of each oscillator.

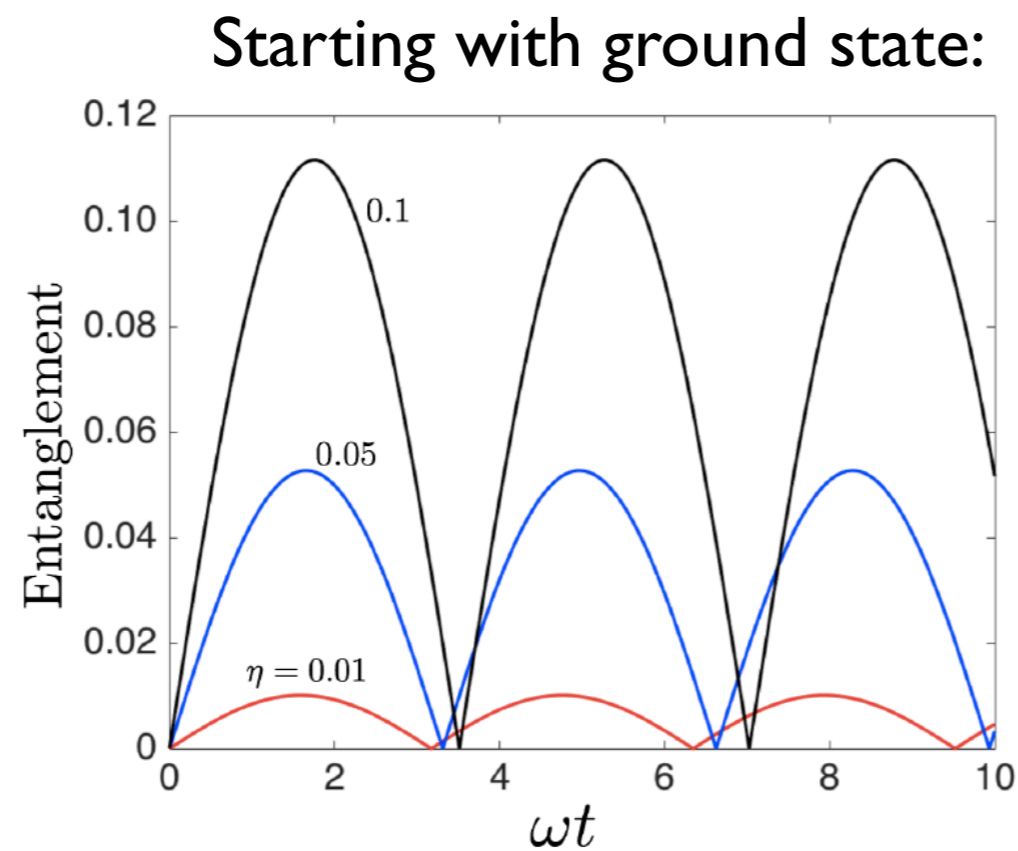
$$\frac{Gm^2(x_A - x_B)^2}{L^3 \hbar \omega} \sim \frac{Gm}{\omega^2 L^3}$$

MAXIMUM ENTANGLEMENT AND ENTANGLING TIME



Analytical solution to the amount of logarithmic negativity.

Assumptions: (i) gravitational interaction up to the second order
(ii) small figure of merit



$$E_{\max} \approx \frac{2Gm}{\omega^2 r^3} \equiv \eta$$

$$\omega t_{\max} \approx \frac{\pi}{2(1 - \eta)}$$

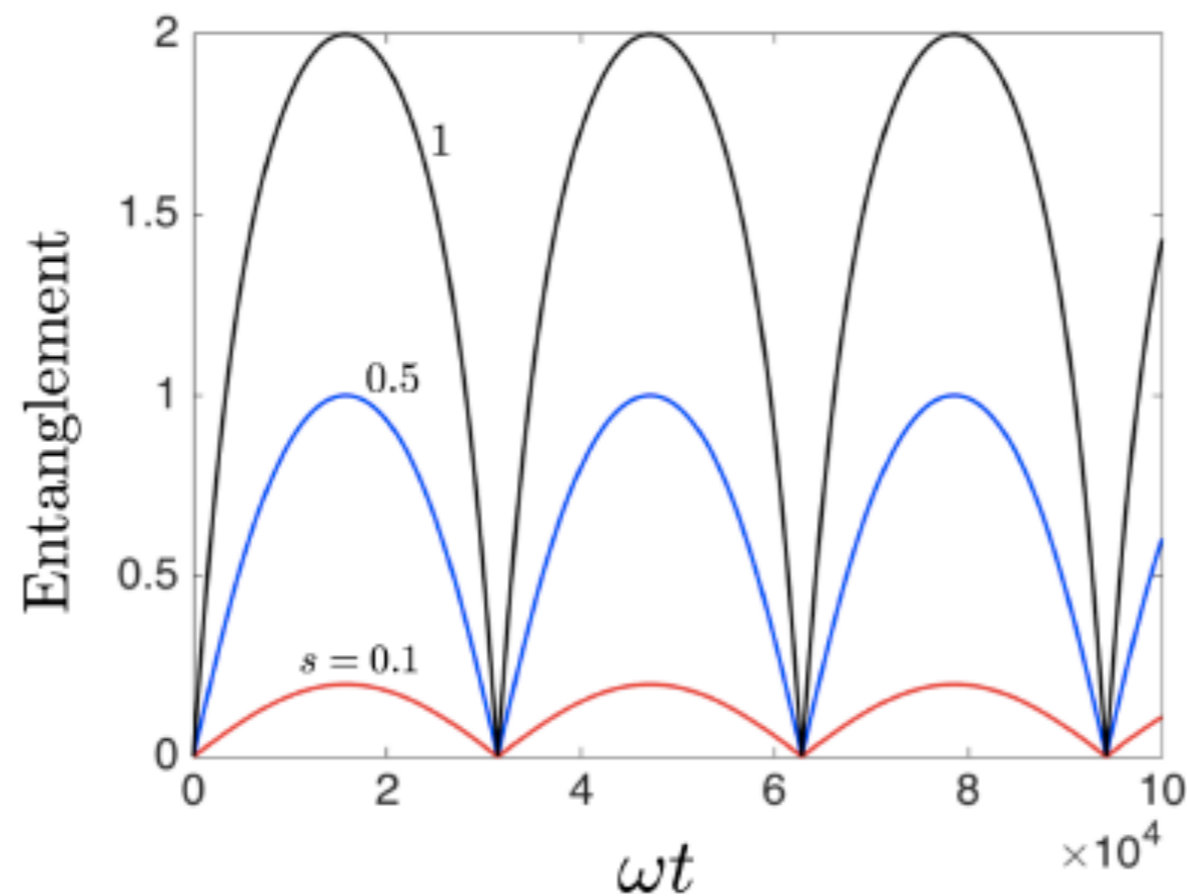
MAXIMUM ENTANGLEMENT AND ENTANGLING TIME



Starting with squeezed vacuum (each oscillator independently):

$$E_{\max} \approx |s_1 + s_2| \quad \text{for} \quad s_1, s_2 \gg \eta$$

$$\omega t_{\max} \approx \frac{\pi}{2\eta}$$



$$\eta = 10^{-4}$$

MAXIMUM ENTANGLEMENT



Starting with thermal state: $E_{\max} \approx \eta - \ln(1 + 2\bar{n})$

Starting with squeezed thermal state: $E_{\max} \approx |s_1 + s_2| - \ln(1 + 2\bar{n})$

As expected temperature kills entanglement.

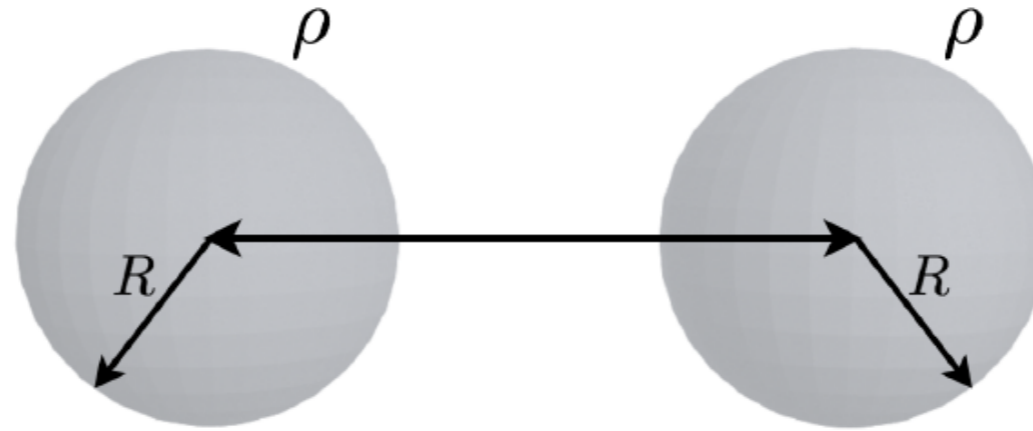
Example: for $\eta = 0.01$, no entanglement for $n > 0.005$.

Hence squeezing is important.

With highest achieved squeezing, $s = 1.7$, no entanglement for $n > 14$.

Can such experiment be done?

OSMIUM OSCILLATORS



Set $L = 2.1 R$

$$\eta = \frac{8\pi}{3(2.1)^3} \frac{G\rho}{\omega^2} \quad (\text{mass independent})$$

$$\eta(\omega) = 1.36 \times 10^{-6} / \omega^2 \quad (\text{with Osmium density})$$

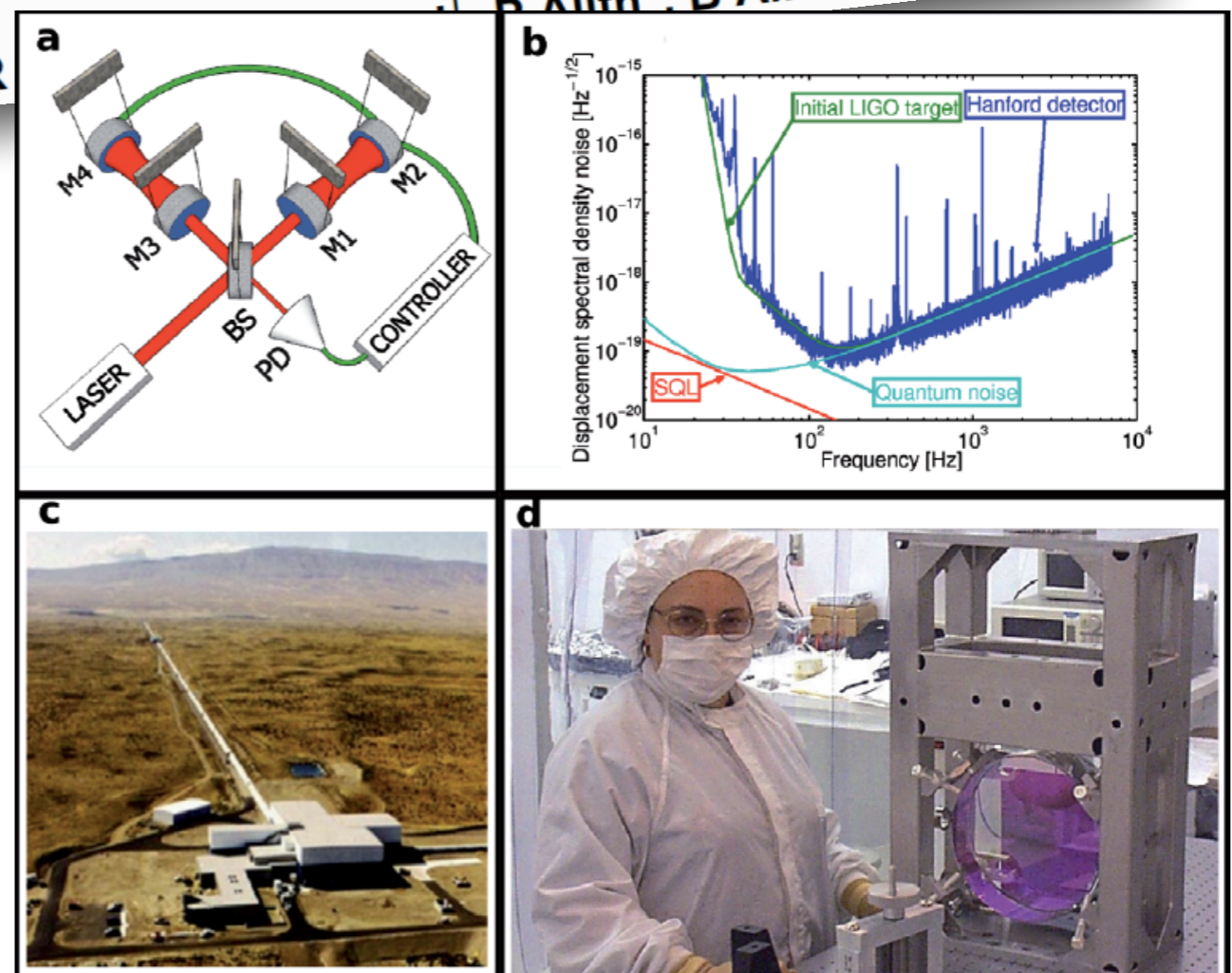
Conclusion: $\eta \sim 0.01$ requires $\omega \sim 0.01$ Hz
 $\eta \sim 0.0001$ requires $\omega \sim 0.1$ Hz

THE LIGO EXPERIMENTS

New Journal of Physics
The open-access journal for physics

Observation of a kilogram-scale oscillator near its quantum ground state

B Abbott¹, R. Aiith², B Allen^{2,3}, G Allen⁴

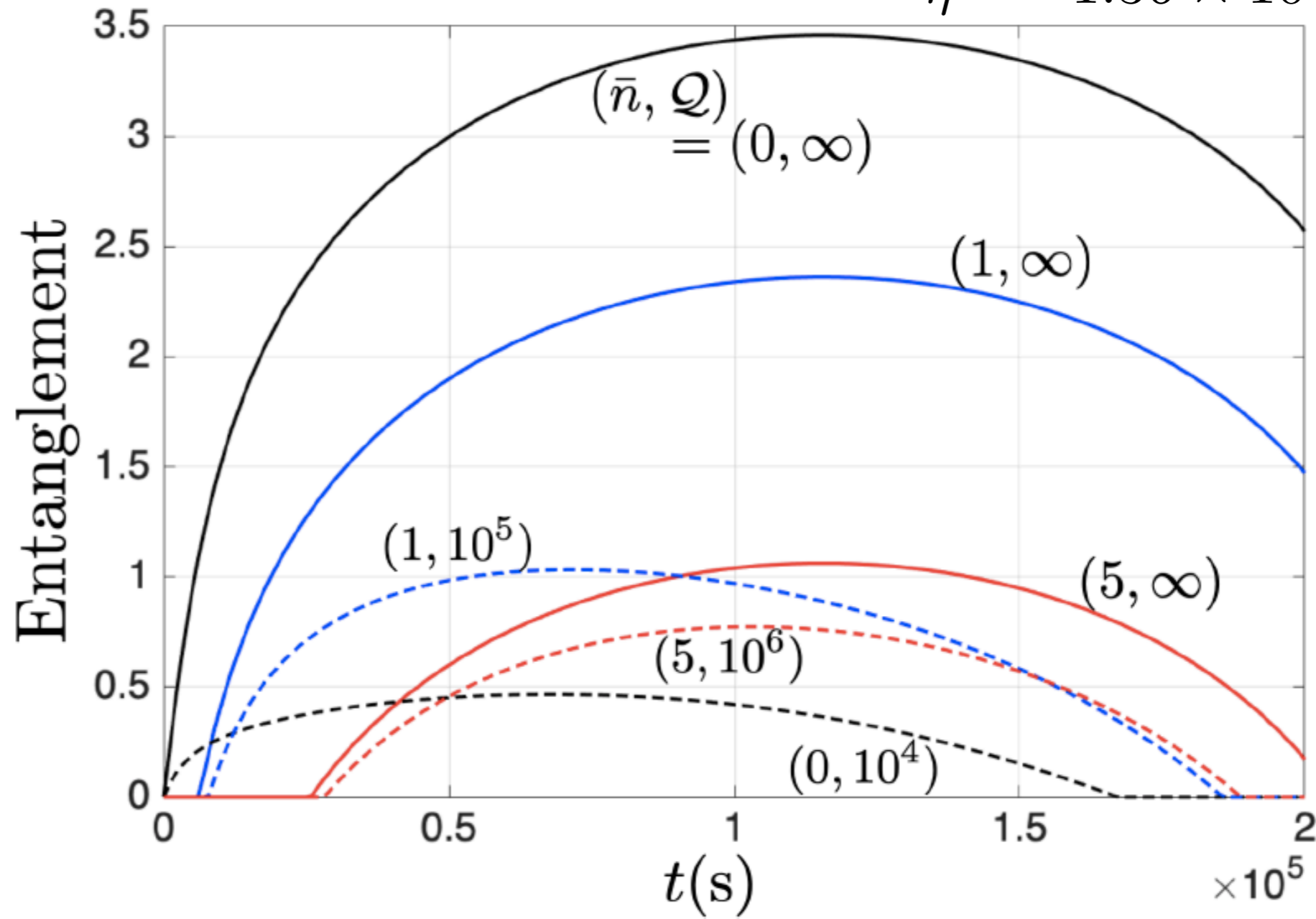


Mirror: 10.8 kg
Frequency: 0.74 Hz
Q factor: 100

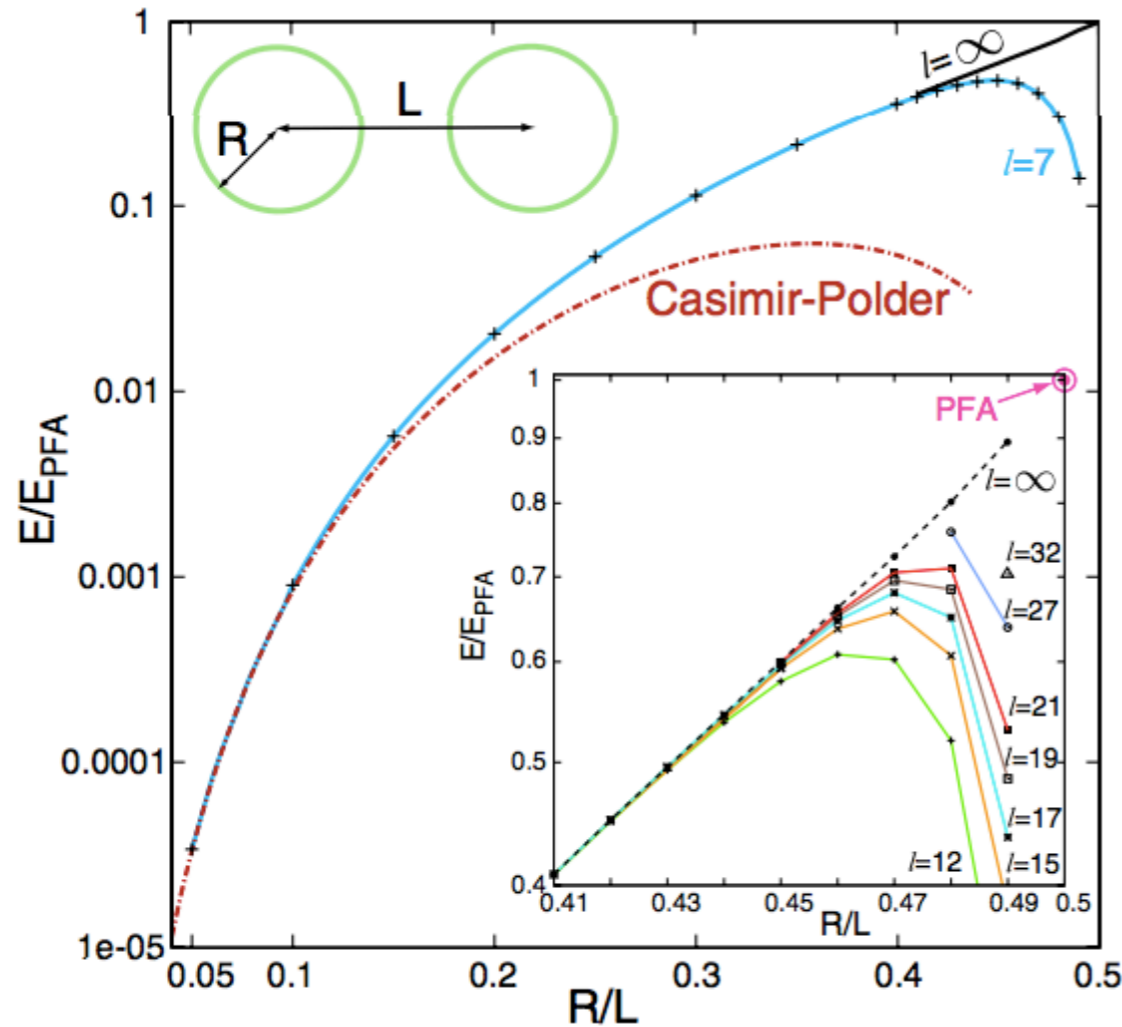
WITH BROWNIAN MOTION

$$\omega = 0.1\text{Hz}$$

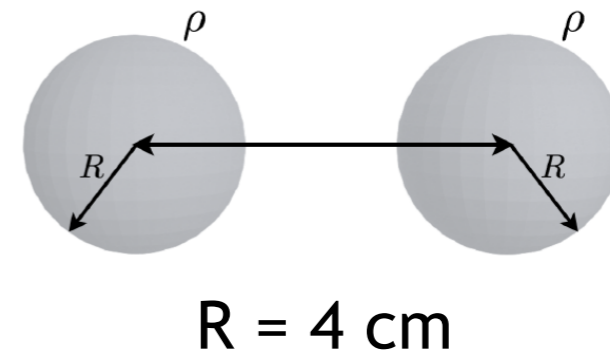
$$\eta = 1.36 \times 10^{-4}$$



CASIMIR FORCES BETWEEN MACROSCOPIC OBJECTS



$$\mathcal{E}_{\text{PFA}} = -\frac{\pi^3}{1440} \frac{\hbar c R}{(L - 2R + (x_B - x_A))^2}$$



week ending
26 OCTOBER 2007

PHYSICAL REVIEW LETTERS

PRL 99, 170403 (2007)

Casimir Forces between Arbitrary Compact Objects

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⁴Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

(Received 12 July 2007; published 25 October 2007)

CASIMIR IRRELEVANT FOR ENTANGLEMENT

Expand Casimir energy:

$$\mathcal{E}_{\text{PFA}} = -\frac{\pi^3}{1440} \frac{\hbar c R}{(L - 2R + (x_B - x_A))^2}$$

$D = L - 2R \equiv 0.1R$

$$\simeq -\frac{\pi^3}{1440} \frac{\hbar c R}{D^2} \left(1 + 2 \frac{x_A - x_B}{D} + 3 \frac{(x_A - x_B)^2}{D^2} \right)$$

Energy shift *Local terms* *Entanglement*

Comparison of entangling Casimir and gravitational terms:

$$\frac{\left(\frac{3\pi^3 \hbar c R}{1440 L^4} \right)}{\left(\frac{G m^2}{r^3} \right)} \approx 3 \times 10^{-12} \frac{1}{m^2}$$

CONCLUSIONS

We need an experiment with an outcome indicating non-classical features of gravity.

Gravity as mediator of entanglement is a possibility.

We provided parameters that are necessary to achieve this for a system of two massive spherical oscillators. LIGO mirrors sound okish, but long coherence time is especially demanding.

Quantitative statements:

How much discord / coherence / ... is needed for entanglement gain?

How much non-commutativity is needed for entanglement gain?

Witnesses of direct / indirect interactions:

Is there a way of proving experimentally that these was (no) mediator?
