Sharing of nonlocal quantum correlations among multiple observers

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- S. Mal, A. S. Majumdar, D. Home, Mathematics 4, 48 (2016)
- S.Mal, D. Das, S. Sasmal, A. S. Majumdar, Phys. Rev. A 98, 012305 (2018)
- S. Datta, A. S. Majumdar, Phys. Rev. A 98, 042311 (2018)

Plan

- Introduction: What are nonlocal quantum correlations ? ……… Definition of the problem
- Information gain versus disturbance trade-off --- Optimality for POVM
- Proof of impossibility of CHSH and analogous steering inequality violation by Alice and more than two Bobs acting sequentially and independently
- Violation of 3-settings steering inequality by Alice and three Bobs
- Violation of NAQC correlations by Alice and one (two) Bob(s)

What are nonlocal quantum correlations ?

[Origin: EPR paradox & Schrodinger’s response]

Violation of Bell (Bell-type) inequalities

Violation of steering inequalities

Other quantum correlations
(e.g., nonlocal advantage of quantum coherence)

EPR Paradox

[Einstein, Podolsky, Rosen, PRA 47, 777 (1935)]

Assumptions: *(i)* Spatial separability & locality: no action at a distance

(ii) reality: *"if without in any way disturbing the system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this quantity."*

EPR considered two spatially separated particles with maximum correlations in their positions and momenta

Measurement of position of 1 implies with certainty the position of 2

(definite predetermined value of position of 2 without disturbing it)

Similarly, measurement of momentum of 1 implies momentum of 2

(again, definite predetermined value of momentum of 2 without disturbing it)

Hence, **particle 2 in a state of definite position *and* momentum.**

Since no state in QM has this property, EPR conclude that QM gives an incomplete description of the state of a particle.

EPR Paradox & Steering

Einstein's later focus on **separability and locality** versus **completeness**

$$|\Psi\rangle = \sum_{n=1}^{\infty} c_n |\psi_n\rangle |u_n\rangle = \sum_{n=1}^{\infty} d_n |\varphi_n\rangle |v_n\rangle$$

Consider nonfactorizable state of two systems:

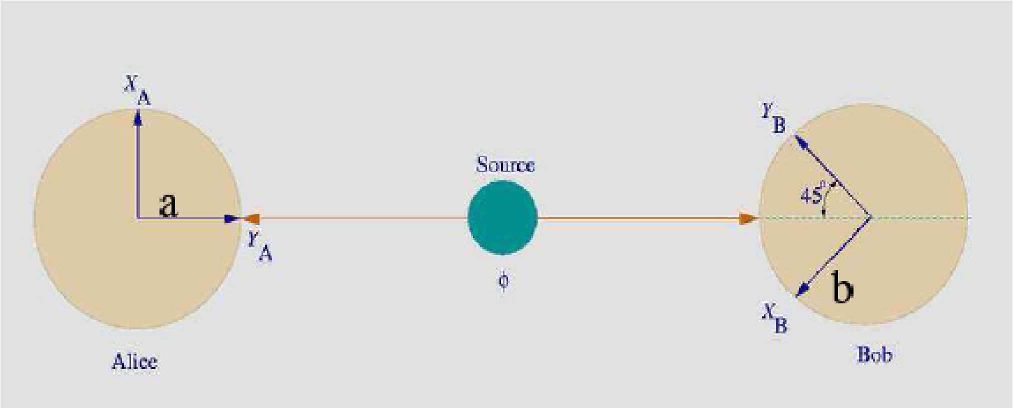
$\{|u_n\rangle\}$ and $\{|v_n\rangle\}$ are two orthonormal bases

If Alice measures in $\{|u_n\rangle\}$ she instantaneously projects Bob's system into one of the states $|\psi_n\rangle$
and similarly, for the other basis.

Since the two systems no longer interact, no real change can take place in Bob's system due to Alice's measurement.
However, the ensemble of $|\psi_n\rangle$s is different from the ensemble of $|\varphi_n\rangle$s

EPR: **nonlocality is an artefact of the incompleteness of QM.**
Schrodinger: ***Steering***: Alice's ability to affect Bob's state through her choice of measurement basis.

Bell Inequalities



Based on idea by Bohm and Aharonov where one measures spin projections of entangled particles.

Measure spins of both particles, but introduce parameter that accounts for the hidden variables: λ

$$
\begin{aligned}
A(\mathbf{a}, \lambda) &= \text{Measurement of } \boldsymbol{\sigma}_1 \cdot \mathbf{a} \\
B(\mathbf{b}, \lambda) &= \text{Measurement of } \boldsymbol{\sigma}_2 \cdot \mathbf{b}
\end{aligned}
$$

If hidden variable theory is reasonable, then the expectation value using λ

$$
P(\mathbf{a}, \mathbf{b}) = \int d\lambda \rho(\lambda) A(\mathbf{a}, \lambda) B(\mathbf{b}, \lambda) \quad \textbf{where} \quad \int d\lambda \rho(\lambda) = 1
$$

should equal the QM result

$$
\langle \boldsymbol{\sigma}_1 \cdot \mathbf{a}\, \boldsymbol{\sigma}_2 \cdot \mathbf{b} \rangle = -\mathbf{a} \cdot \mathbf{b}
$$

Bell Inequalities

- Original Inequality

$$1 + P(\mathbf{b}, \mathbf{c}) \geq |P(\mathbf{a}, \mathbf{b}) - P(\mathbf{a}, \mathbf{c})|$$

- Bell-Clauser-Horne-Shimony-Holt (Bell-CHSH) Version – **More Common**

$$P(\mathbf{a}, \mathbf{b}) + P(\mathbf{a}, \mathbf{b}') + P(\mathbf{a}', \mathbf{b}) - P(\mathbf{a}', \mathbf{b}') \leq 2$$

- Greenberger-Horne-Zeilinger (GHZ) Inequality – **3-body states**

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|+,+,+\rangle - |-,-,-\rangle\right) \quad \longrightarrow \quad \sigma_{1x}\sigma_{2x}\sigma_{3x} = 1_{EPR}$$

- Hardy's Impossibilities
- Bell-Kochen-Specker (Contextuality)

Steering: a modern perspective [Wiseman et al., PRL (2007)]

Steering as an information theoretic task.

Leads to a mathematical formulation

Steering inequalities, in the manner of Bell inequalities

Steering as a task

[Wiseman, Jones, Doherty, PRL 98, 140402 (2007); PRA (2007)]

(Asymmetric task)

Local Hidden State (LHS): Bob's system has a definite state, even if it is unknown to him

$$P(a, b|A, B; W) = \sum_{\xi} \wp(a|A, \xi) P(b|B; \rho_{\xi}) \wp_{\xi}$$

Experimental demonstration: Using mixed entangled states [Saunders et al. Nature Phys. 6, 845 (2010)]

Steering task: (inherently asymmetric)

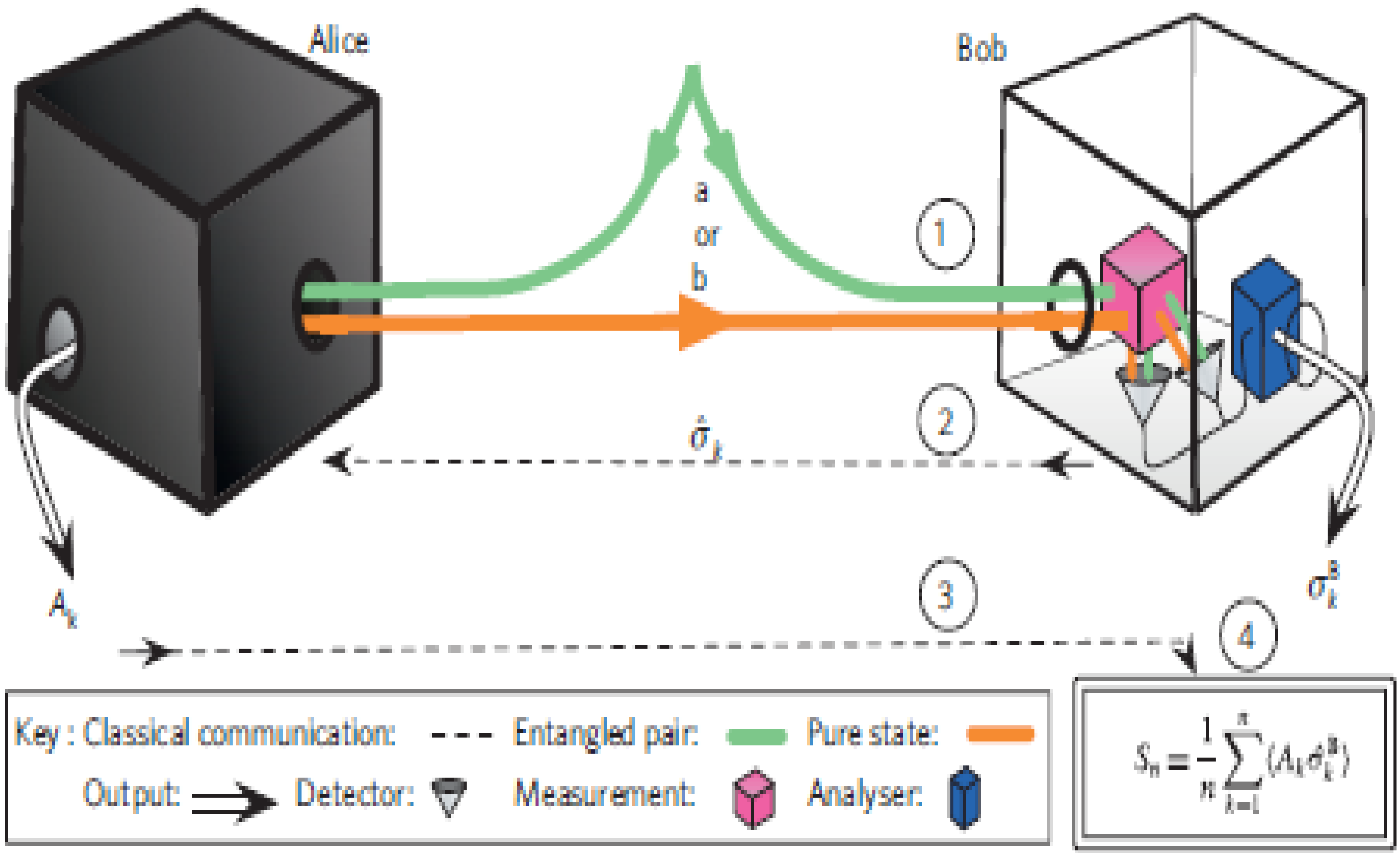
Alice prepares a bipartite quantum state and sends one part to Bob (Repeated as many times)

Alice and Bob measure their respective parts and communicate classically

Alice's taks: To convince Bob that the state is entangled *(If correlations between Bob's measurement results and Alice's declared results can be explained by LHS model for Bob, he is not convinced. – Alice could have drawn a pure state at random from some ensemble and sent it to Bob, and then chosen her result based on her knowledge of this LHS).*

Conversely, if the correlations cannot be so explained, then the state must be entangled.

Alice will be successful in her **task of steering** if she can create genuinely different ensembles for Bob by steering Bob's state.



Saunders et al., Nature Physics (2010)

Three distinct correlations:

LHV ξ $\wp_\xi$: positive probability distribution

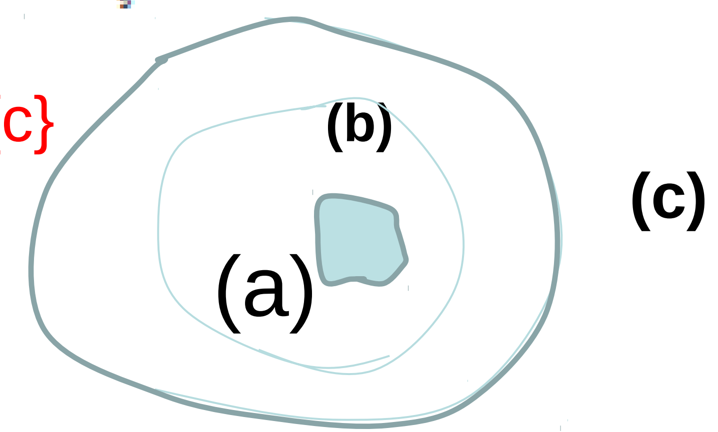
σ_ξ ρ_ξ : quantum states state matrix W

(a) Entangled states: $P(a,b|A,B;W) \neq \sum_\xi P(a|A;\sigma_\xi)P(b|B;\rho_\xi)\wp_\xi$

(b) Steerable states: $P(a,b|A,B;W) \neq \sum_\xi \wp(a|A,\xi)P(b|B;\rho_\xi)\wp_\xi$

(c) Bell-nonlocal state $P(a,b|A,B;W) \neq \sum_\xi \wp(a|A,\xi)\wp(b|B,\xi)\wp_\xi$

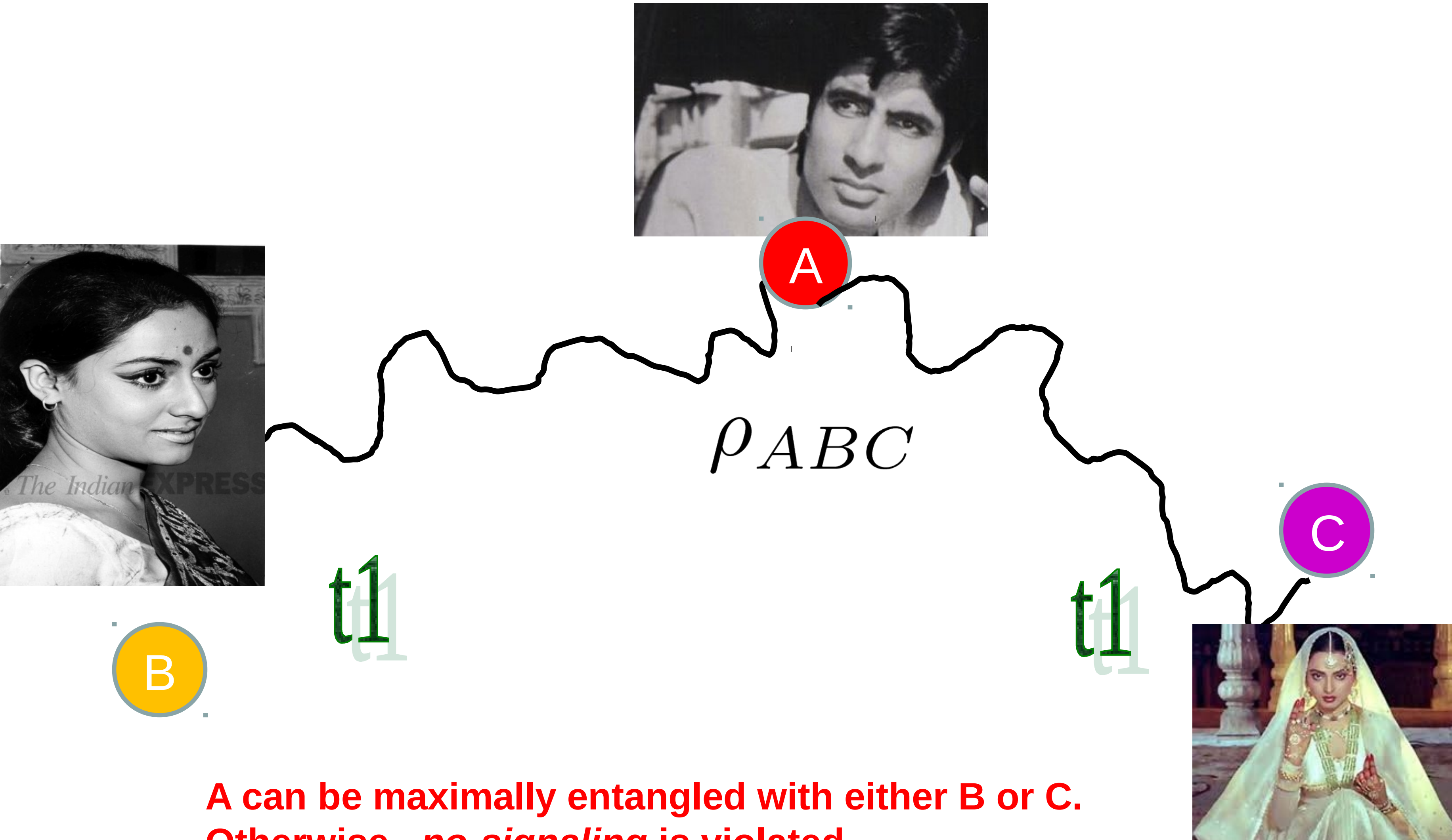
{a} strict subset of {b} strict subset of {c}



What do we mean by sharing of nonlocality ?

Let us first consider sharing of entanglement

Entanglement is Monogamous



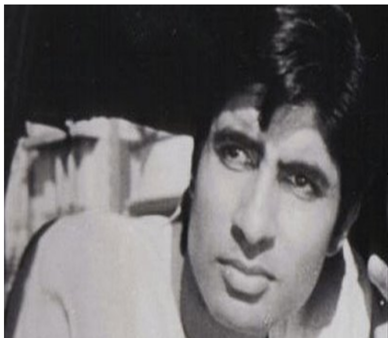
A can be maximally entangled with either B or C.
Otherwise, *no-signaling* is violated.

However, when Alice gets sequentially entangled with Bob <u>and</u> Charlie

No-signaling is not applicable in this situation

Sequential entanglement

ρ_{AC}



A

ρ_{AB}

t2

C

B

t1

ρ_{AC} is conditioned by B's measurement

How is sharing of nonlocal correlations relevant ?

-- fundamental issue of quantum foundations

-- useful in practical information processing

e.g, Device independent certification of random numbers
[c.f., Coyle, Hoban, Kashefi (2018)]

Cryptographic protocols involving multiple parties
[c.f., Mondal et al., arXiv: 1805.11355]

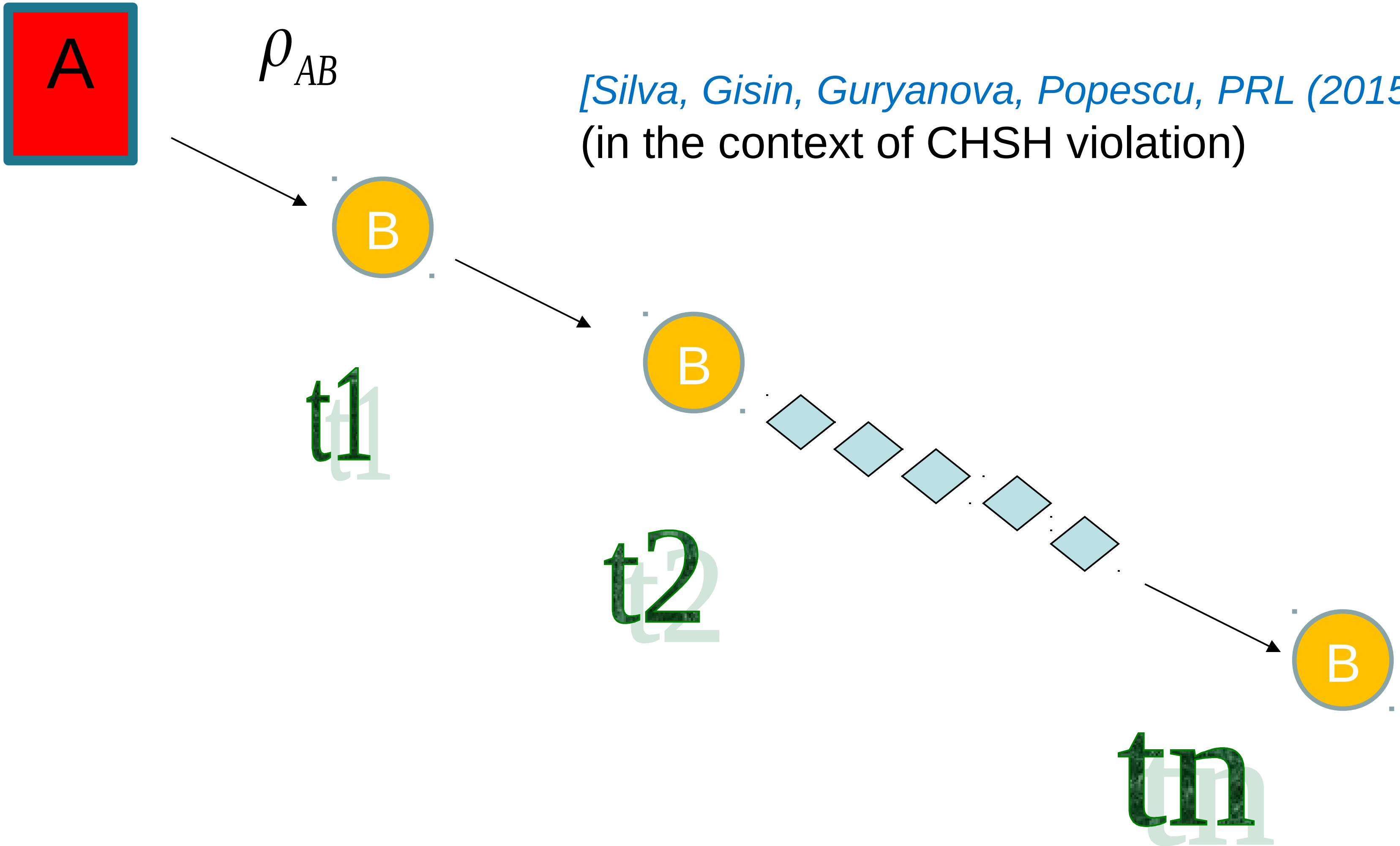
Sharing of nonlocal correlations

- **Q:** How does one reveal nonlocality between Alice of one side and sequential Bobs on the other ?

- **A:** Consider various forms of nonlocality. Check Bell violation / steering by the different pairs, e.g., Alice-Bob(1); Alice-Bob(2),….. Alice-Bob(n)

- **Note:** Bob(1), Bob(2)…..Bob(n-1) must perform POVMs; otherwise, no entangled state remains for the subsequent (n-th) Bob(n).

Sharing of nonlocality
(How many Bobs ?)

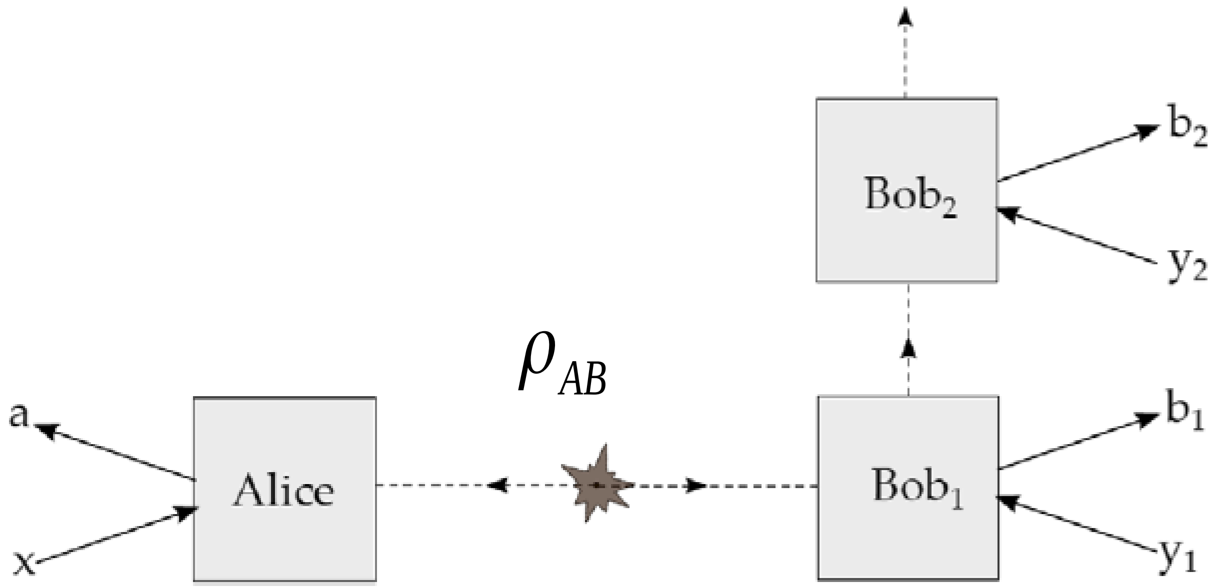


[Silva, Gisin, Guryanova, Popescu, PRL (2015)].
(in the context of CHSH violation)

Definition of the problem

- Alice has access to one particle of a pair of entangled spin ½ particles.
- Series of Bobs (B(1),B(2),….B(n)) on the other side who can access the other particle sequentially but independently of each other (no-signalling condition NOT applicable).
- Alice performs a projective measurement (dichotomic input and output [0,1]).
- B(1),B(2)….B(n-1) perform one-parameter POVMs (dichotomic input and output [0,1]); B(n) may perform a projective measurement.
- Unbiased input settings for Alice and all Bobs, e.g., frequency of receiving input 0 and input 1 is same.

Bell scenario involving Alice and multiple Bobs



One spin-1/2 particle of an entangled pair is accessed by Alice. The other is accessed sequentially by the Bobs. No biasing of measurement inputs is allowed

Definition of the problem……

- To check for how many pairs the CHSH inequality is violated by (Alice-B1), (Alice-B2)…….(Alice-BN).

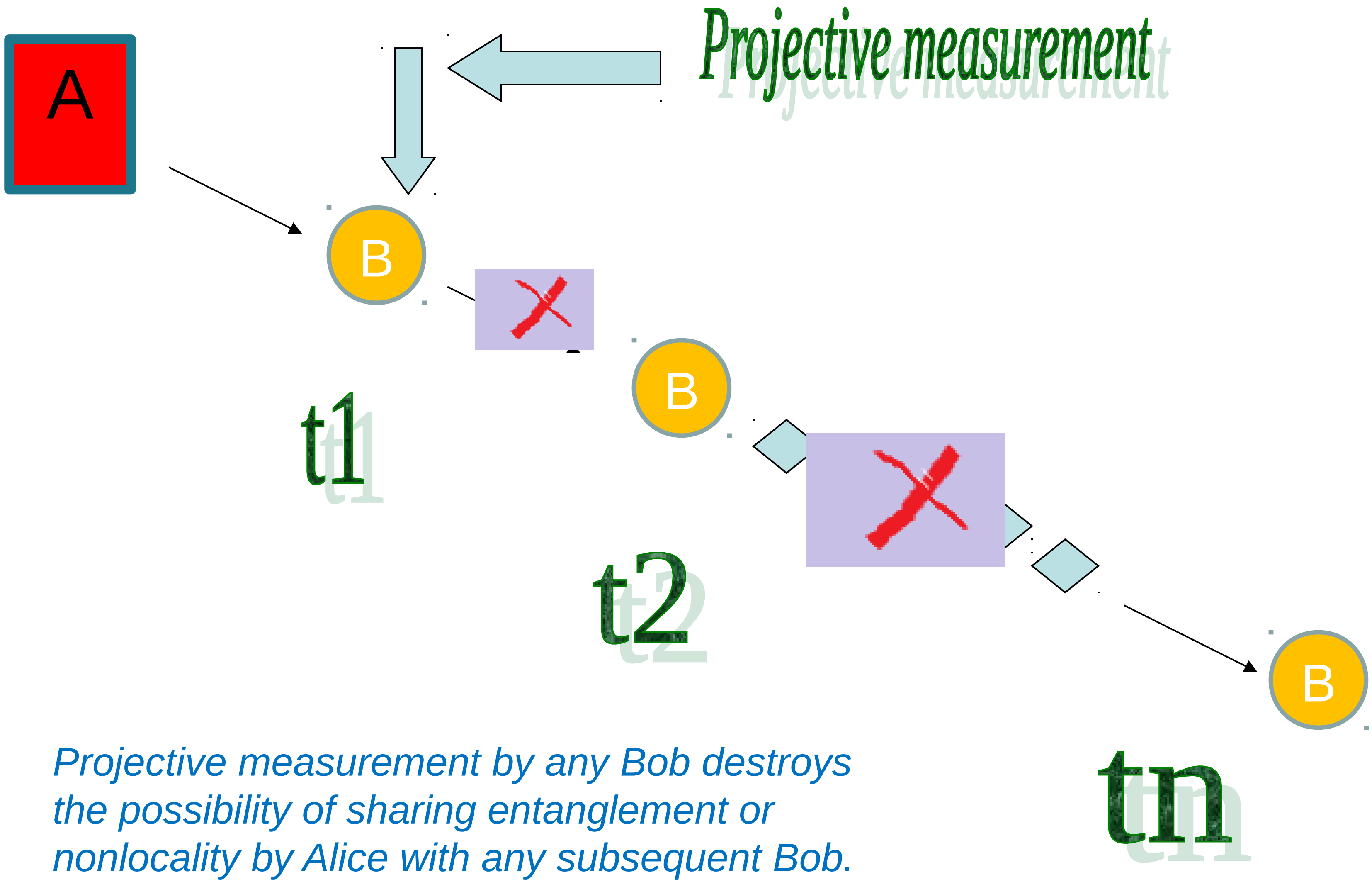
- Upper limit N=2 conjectured numerically for unbiased settings *[Silva, Gisin, Guryanova, Popescu, PRL (2015)].*

  (using constructed models of measurement apparatus)

- We provide analytical proof using optimality of information gain versus disturbance trade-off with 1-parameter POVMs.

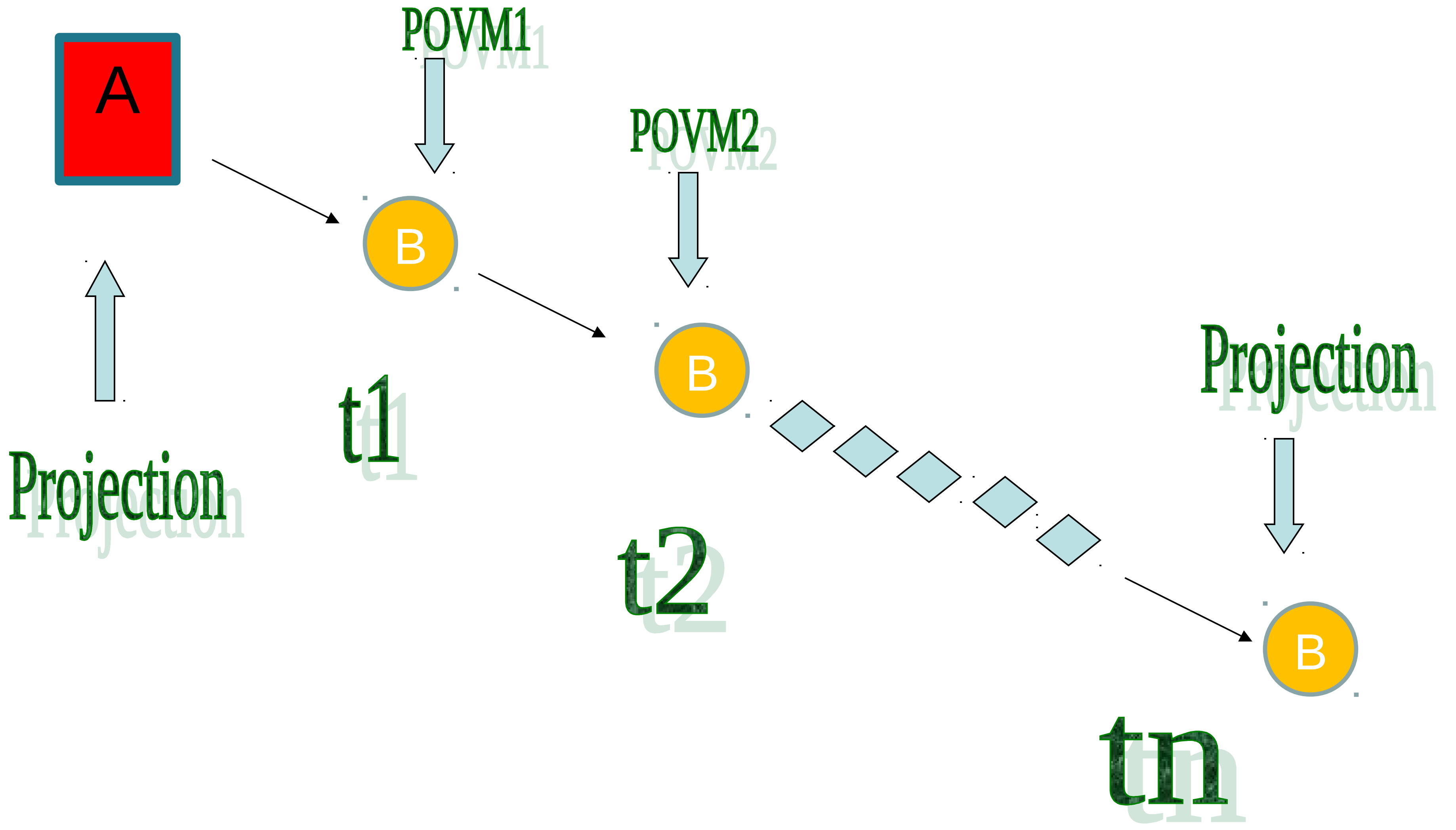
- Utility of the POVM formalism (or unsharp measurement)

Sharing of nonlocality (How many Bobs ?)



Projective measurement by any Bob destroys the possibility of sharing entanglement or nonlocality by Alice with any subsequent Bob.

Sharing of nonlocality
(**How many Bobs** ?)



To determine the max. no. of Bobs we have to apply POVMs

POVMs** provide **optimality** in the **trade-off** between **information gain** and **disturbance** for an **unsharp measurement

Information gain versus disturbance trade-off

System state $|\psi\rangle (= \alpha|0\rangle + \beta|1\rangle)$ Apparatus state $\phi(q)$

Joint system-apparatus state $\alpha|0\rangle \otimes \phi(q-1) + \beta|1\rangle \otimes \phi(q+1)$

Reduced system state: $\rho' = F\rho + (1-F)(\pi^+\rho\pi^+ + \pi^-\rho\pi^-)$

Quality factor: $F(\phi) = \int_{-\infty}^{\infty} \langle \phi(q+1)|\phi(q-1)\rangle dq$

Probability of outcomes: $p(\pm) = G\langle\psi|\pi^{\pm}|\psi\rangle + (1-G)\frac{1}{2}$

Precision of measurement: $G = \int_{-1}^{1} \phi^2(q)dq$

G=1: Sharp measurement

How are G and F related ?

Information gain versus disturbance

Optimality condition: best trade-off
(largest precision G for a given quality factor F)
[Obtained by Silva et al. numerically using various pointer states]

$$F^2 + G^2 = 1$$

One-parameter POVM
(Unsharp measurement with effect parameter):

$$E_{\pm}^{\lambda} = \lambda P_{\pm} + \frac{1-\lambda}{2}\mathbb{I}$$

(*System after pre-measurement)*
Luder transformation

$$\rho' = \sqrt{1-\lambda^2}\rho + (1-\sqrt{1-\lambda^2})(P_+\rho P_+ + P_-\rho P_-)$$

Probabilities for outcomes

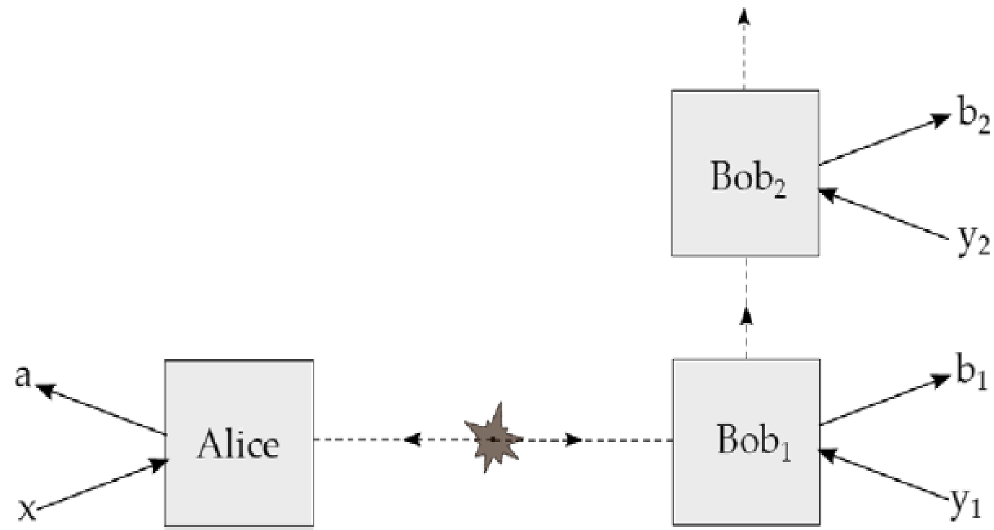
$$p(\pm) = tr[E_{\pm}^{\lambda}\rho] = \lambda tr[P_{\pm}\rho] + \frac{1-\lambda}{2}$$

Relation between G, F and λ $\quad G = \lambda \quad F = \sqrt{1-\lambda^2}$

Limit of sharp measurement: $\quad G = \lambda = 1 \quad$ *F = 0*

Sharing of nonlocality

Two-qubit maximally entangled state



$\frac{1}{\sqrt{2}}(|10> + |01>)$

Bell-CHSH scenario

Alice's setting $\hat{X}$ and $\hat{Z}$

Bobs' setting $\frac{-(\hat{Z}+\hat{X})}{\sqrt{2}}, \frac{-\hat{Z}+\hat{X}}{\sqrt{2}}$ ***(Achieves Tsirelson's bound)***

(orthogonal measurements): ***1-parameter POVMs by Bobs*** *(n-1)*

No bias: Inputs $x, y_1, y_2 \in [0,1]$ with equal frequency

<u>Sharing of nonlocality</u>

Joint probability of getting outcome `a' by Alice and b_n by n-th Bob:

$$p(a,b_n) = p(a)p(b_n|a) = \frac{1}{2}Tr[\frac{\mathbb{I}+\lambda_n b_n \hat{y_n}.\vec{\sigma}}{2}\rho_{n|y_1...y_{n-1}}]$$

<u>Case: Two Bobs</u>

Joint probability:

$$p(a,b_2) = \frac{\sqrt{1-\lambda_1^2}}{2}\frac{1-ab_2\lambda_2\hat{y_2}.\hat{x}}{2} + \frac{1-\sqrt{1-\lambda_1^2}}{2}\frac{1-ab_2\lambda_2\hat{x}.\hat{y_1}\hat{y_1}.\hat{y_2}}{2}$$

First Bob measures weakly:

$$CHSH_{AB_1} = 2\sqrt{2}\lambda_1$$

Second Bob measures sharply:

$$CHSH_{AB_2} = \sqrt{2}(1+\sqrt{1-\lambda_1^2})$$

Precision range for violation by both Bobs:

$$\lambda_1 \in \left[1/\sqrt{2}, \sqrt{2\left(\sqrt{2}-1\right)}\right]$$

Sharing of nonlocality

Three Bobs

1st and 2nd Bob measure weakly; 3rd Bob measures sharply

Joint probability (CHSH correlation between Alice and Bob-3)

$$C_3 = \lambda_3[\sqrt{1-\lambda_1^2}\sqrt{1-\lambda_2^2}\hat{y}_3.\hat{x} + (1-\sqrt{1-\lambda_1^2})\sqrt{1-\lambda_2^2}\hat{x}.\hat{y}_1\hat{y}_1.\hat{y}_3$$
$$+\sqrt{1-\lambda_1^2}(1-\sqrt{1-\lambda_2^2})\hat{x}.\hat{y}_2\hat{y}_2.\hat{y}_3 + (1-\sqrt{1-\lambda_1^2})(1-\sqrt{1-\lambda_2^2})\hat{x}.\hat{y}_1\hat{y}_1.\hat{y}_2\hat{y}_2.\hat{y}_3]$$

Bob is ignorant about inputs of previous Bobs
(average over all possible earlier inputs)
Averaged correlation between Alice and Bob-3

$$\bar{C}_3 = \sum_{y_1 y_2} C_3 P(y_1) P(y_2)$$

Proof of impossibility of sharing nonlocality with more than two Bobs

Averaged CHSH correlation between Alice and Bob-3

$$\mathcal{I}^3 = \frac{(1+\sqrt{1-\lambda_1^2})(1+\sqrt{1-\lambda_2^2})}{\sqrt{2}}$$

CHSH A-B1: $2\sqrt{2}\lambda_1$

CHSH A-B2: $\lambda_2\sqrt{2}(1+\sqrt{1-\lambda^2})$

Range of sharpness for violation: If $\lambda_1 > 1/\sqrt{2}$ and $\lambda_2 > \frac{2}{\sqrt{2}+1}$

$$CHSH_{AB_3} \leq 2$$

$$CHSH_{AB_i} = 1.88$$

Violation not possible by all three Bobs. Max. Violation

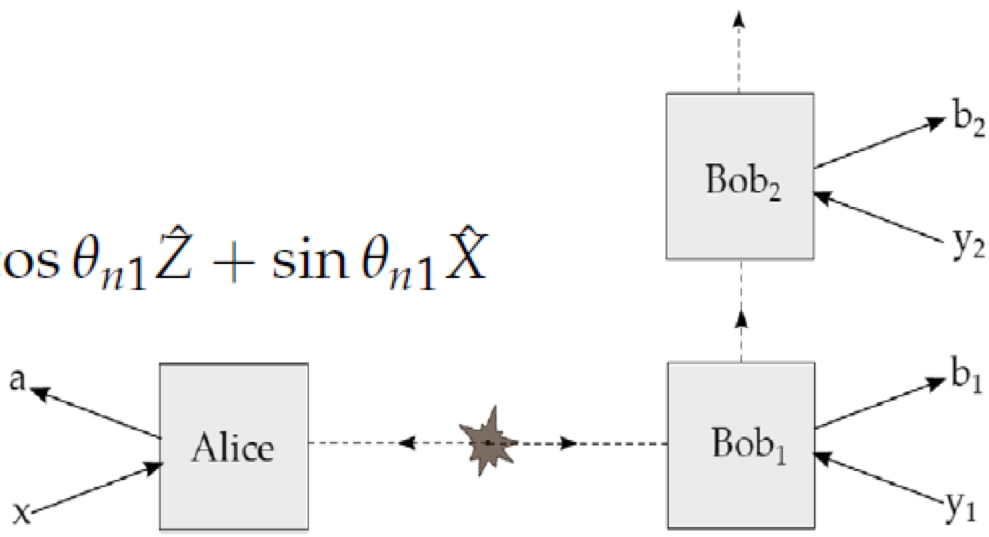
If other two Bobs obtain max violation *(i=1,2,3)*

Non-orthogonal measurements: Is there any advantage ?

Alice's settings: $\hat{X}$ and $\hat{Z}$

Bob's settings:

$$y_n^0 = \cos\theta_{n0}\hat{Z} + \sin\theta_{n0}\hat{X}, y_n^1 = \cos\theta_{n1}\hat{Z} + \sin\theta_{n1}\hat{X}$$



Correlations (Alice-Bob1):

$$\mathcal{I}^1(xy_1) = \lambda_1 \tilde{\mathcal{I}}^1(xy_1)$$

$$\tilde{\mathcal{I}}^1(xy_1) \;=\; (\cos[\theta_{10}] - \cos[\theta_{11}] + \sin[\theta_{10}] + \sin[\theta_{11}])$$

Correlations for non-orthogonal measurement settings:

Alice-Bob2:

$$\mathcal{I}^2(x(y_1)y_2) = \lambda_2[(1-F_1)\tilde{\mathcal{I}}^2(x(y_1)y_2) + F_1\tilde{\mathcal{I}}^1(xy_2)]$$

$$\tilde{\mathcal{I}}^2(x(y_1)y_2) = \frac{1}{2}\sum_{i,j=0}^{1}((-1)^j \cos\theta_{1i} + \sin\theta_{1i})\cos(\theta_{1i} - \theta_{2j})$$

Alice-Bob3:

$$\mathcal{I}^3(x(y_1y_2)y_3) = \frac{F_1+F_2}{2}\tilde{\mathcal{I}}^1(xy_3) + \frac{(1-F_1)F_2}{2}(\tilde{\mathcal{I}}^2(x(y_1)y_3) - \tilde{\mathcal{I}}^2(x(y_1+\frac{\pi}{2})y_3))$$

$$+\frac{(1-F_2)F_1}{2}(\tilde{\mathcal{I}}^2(x(y_2)y_3) - \tilde{\mathcal{I}}^2(x(y_2+\frac{\pi}{2})y_3)) + \frac{(1-F_1)(1-F_2)}{16}\tilde{\mathcal{I}}^3(x(y_1y_2)y_3)$$

$$F_i = \sqrt{1-\lambda_i^2}$$

No improvement in violation:

Example:

$$y_1^0 \approx 0.19\hat{Z} + 0.98\hat{X} \qquad y_1^1 \approx -0.19\hat{Z} + 0.98\hat{X}$$

$$y_2^0 \approx 0.19\hat{Z} + 0.98\hat{X} \qquad y_2^1 \approx -0.19\hat{Z} + 0.98\hat{X}$$

$$y_3^0 \approx 0.04\hat{Z} + 0.99\hat{X} \qquad y_3^1 \approx -0.04\hat{Z} + 0.99\hat{X}$$

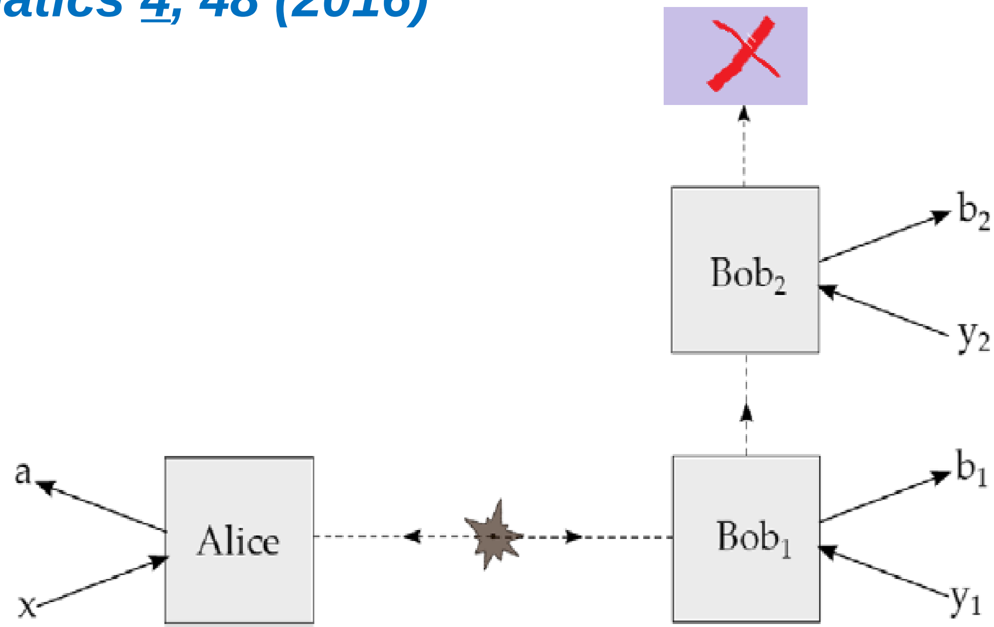
$$I_1 = 2.1 \qquad I_2 = 2.1 \qquad I_3 \to 1.89$$

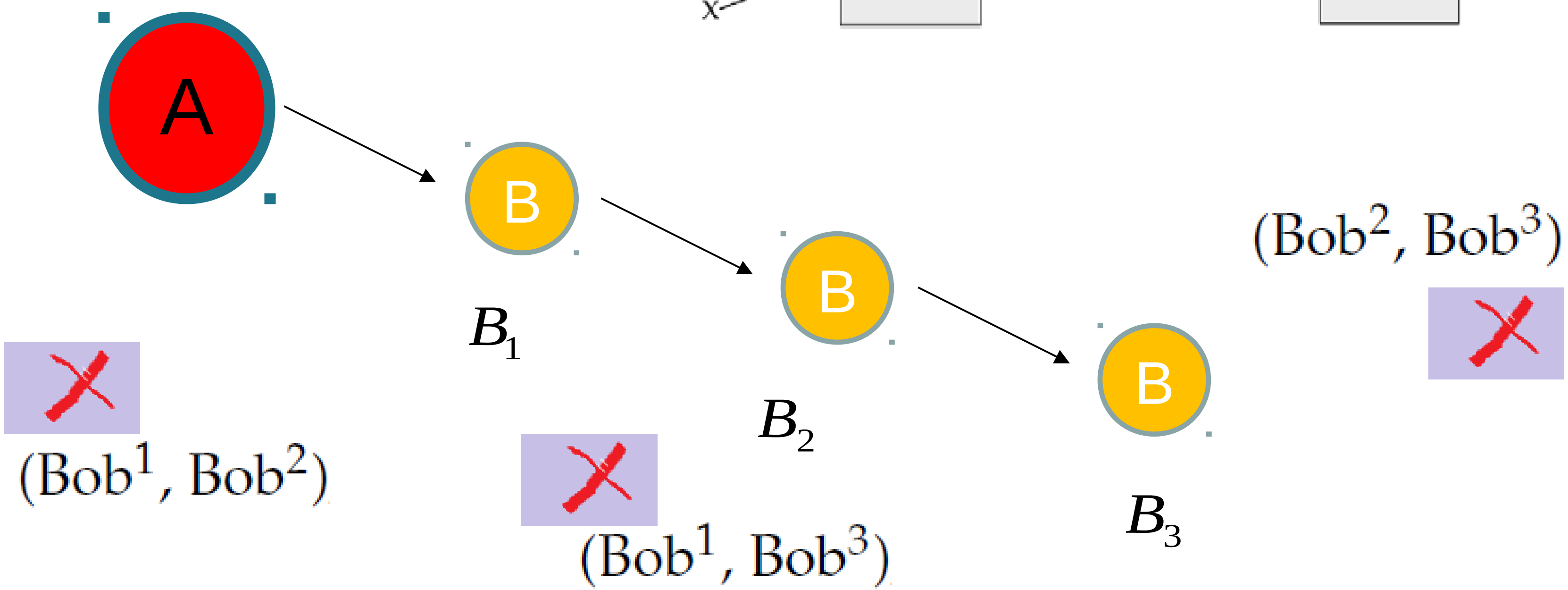
No CHSH violation possible with more than 2 Bobs

S. Mal, ASM, D. Home, Mathematics 4, 48 (2016)

Sequence NOT important

Alice can obtain violations with any two pairs





Experimental verification

Three-observer Bell inequality violation on a two-qubit entangled state

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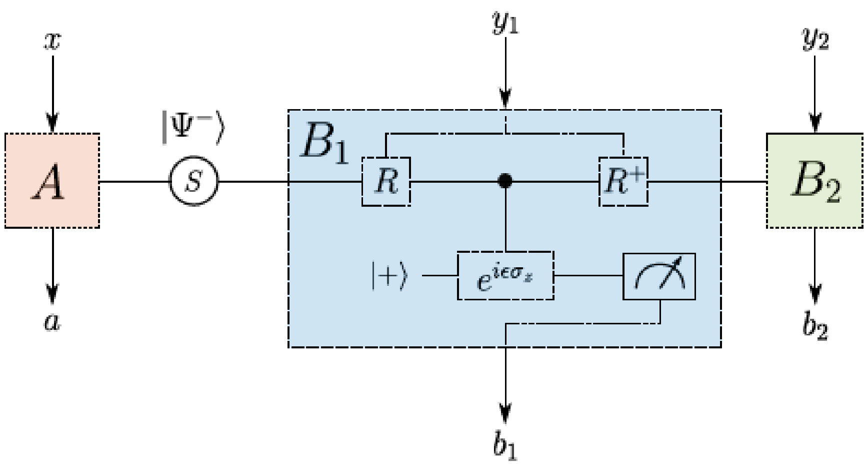
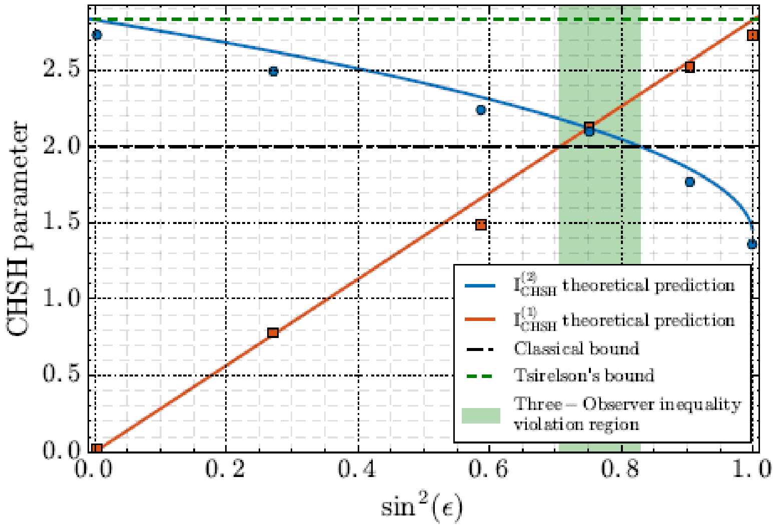
Quantum Sci. Technol. **2,** 015010 (2017)





Experimental verification

Observation of Nonlocality Sharing among Three Observers with One Entangled Pair via Optimal Weak Measurement

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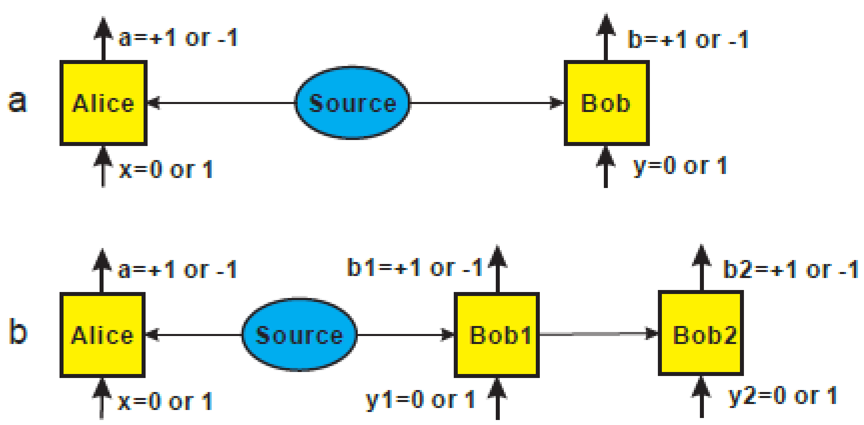
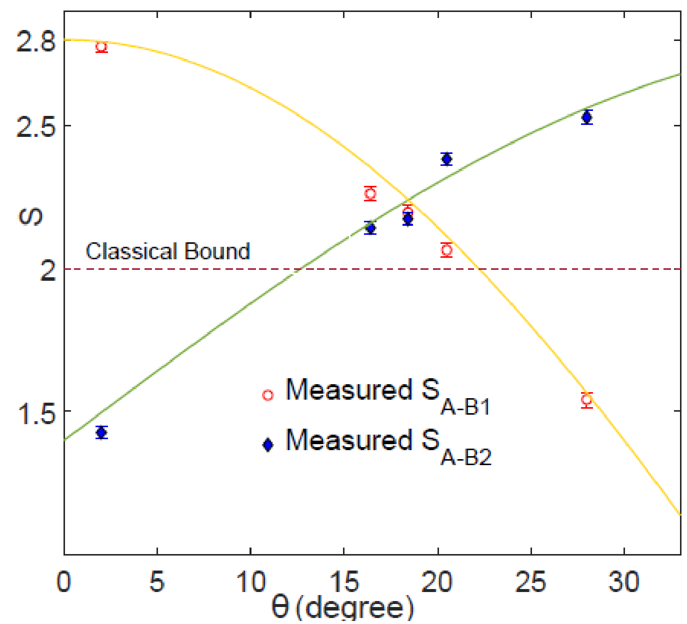
(Dated: November 8, 2018)

arXiv:1609.01863 [quant-ph] (to appear in NJP)





Sharing of steerability

Steering: Weaker form of nonlocality than Bell-violation
(All Bell-violating states are strict subset of all steerable states)

How many Bobs acting sequentially and independently of each other can steer Alice's state ?

Necessary and sufficient steering condition in 2-2-2 scenario
(two-qubit shared state; two parties, two measurement settings per party)

[Cavalcanti, Foster, Fura, Wiseman, (2015)]
CFFW Inequality

$$S_{BA} = \sqrt{\langle (B+B')A\rangle^2 + \langle (B+B')A'\rangle^2} + \sqrt{\langle (B-B')A\rangle^2 + \langle (B-B')A'\rangle^2} \leq 2.$$

Necessary and sufficient state condition

[Cavalcanti, et al., PRA (2016); Quan et al., PRA (2017)]

(a la Horodecki criterion for Bell violation) [S. Mal, ASM et al., arXiv: 1711.00872]

$$\rho = \frac{1}{4}(\mathbb{I} \otimes \mathbb{I} + \vec{r}.\vec{\sigma} \otimes \mathbb{I} + \mathbb{I} \otimes \vec{s}.\vec{\sigma} + \sum_{i,j=1}^{3} t_{ij}\sigma_i \otimes \sigma_j)$$

Correlation matrix: $V = TT^t$ with coefficients $t_{ij} = Tr(\rho\sigma_i \otimes \sigma_j)$

CFFW inequality is violated iff $S(\rho) > 1$ for $S(\rho) = \sqrt{v + \tilde{v}}$.

(v and $\tilde{v}$ are two largest eigenvalues)

Monogamy of steering for tripartite state with parties A, B and C:

[Cheng et al., PRL (2017)]

$$S_{BA}^2 + S_{CA}^2 \leq 8$$

Violation of CFFW inequality by Alice and two Bobs

Alice's settings: $\{\hat{x^0}, \hat{x^1}\}$

Bob's settings: $\{\hat{y_n^0}, \hat{y_n^1}\}$

Correlation between Alice and Bob1 : $C_1^{ji} = -\lambda_1(\hat{y_1^i}.\hat{x^j})$

Average correlation between Alice and Bob2: $\overline{C_2^{jk}} = \sum_{i=0,1} C_2^{jk} P(y_1^i)$

Necessary and sufficient steering condition for 3rd Bob:

$$S_n = \sqrt{(\overline{C_n^{00}} + \overline{C_n^{01}})^2 + (\overline{C_n^{10}} + \overline{C_n^{11}})^2}$$
$$+ \sqrt{(\overline{C_n^{00}} - \overline{C_n^{01}})^2 + (\overline{C_n^{10}} - \overline{C_n^{11}})^2}$$

Violation not possible by more than two Bobs

Violation of 3-settings inequality by Alice and three Bobs

n-settings inequality:
[Cavalcanti-Jones-Wiseman-Reid (2009)]

$$F^n = \frac{1}{\sqrt{n}} \Big| \sum_{i=1}^{n} \langle A_i \otimes B_i \rangle \Big| \leq 1$$

Compute average correlation functions
between Alice and i-th Bob:

$$\overline{C_n^{jk}}$$

CJWR function between Alice
and n-th Bob for 3-settings:

$$F_n^3 = \frac{1}{\sqrt{3}} \Big| \sum_{i=1}^{3} \overline{C_n^{ii}} \Big|$$

Bob1, Bob2 and Bob3 can steer Alice !

What happens if one increases the number of settings ?

Sharing of (NAQC)

Nonlocal Advantage of Quantum Coherence

What is quantum coherence ?

Fundamental measure of quantumness at the level of single particles

[Baumgratz, Kremmer, Plenio, PRL (2014); Girolami, PRL (2014)]

$$C^{l_1}(\rho) = \sum_{\substack{i,j \\ i \neq j}} |\rho_{ij}|$$

Quantum coherence of a state can be steered (NAQC)

[Mondal, Pramanik, Pati, PRA (2017)]

Inequality for NAQC correlations

$$\sum_{i=x,y,z} C_i^{l_1}(\rho) \leq \sqrt{6}$$

Sharing of NAQC correlations

NAQC: Stronger form of nonlocality than Bell-violation

(NAQC states are strict subset of all Bell-violating states) [Hu et al. (2018)]

How many Bobs acting sequentially and independently of each other can steer Alice's state ?

NAQC inequality $$\sum_{i=x,y,z} C_i^{l_1}(\rho) \leq \sqrt{6}$$

Sharing of NAQC by Alice and 2 Bobs

$$\overline{N_{A_2B}^{l_1}} = \frac{2\lambda_2(1+2\sqrt{1-\lambda_1^2})}{1+\lambda_2^2}$$

Violation not possible by more than two Bobs

(For NAQC using relative entropy of coherence, only one Bob !)

Other recent results:

-- *Witnessing entanglement sequentially*
[Bera,Mal, De, Sen, arXiv:1806.01806]
(witnessing entanglement of up to 12 Bobs is possible]

-- *Sharing of tripartite nonlocality*
[Saha et al., arXiv: 1807.08498]
(A-B-C: upto 6 Charlies violation of Mermin inequality; only 2 Charlies for Svetlichny inequality)

-- Sharing of steering in d x d dimenations
[Shenoy,..Gisin, Brunner, arXiv: 1810.06523]
(Search for existence of hidden state model: No. of Bobs: d/log d)

-- Sharing bipartite nonlocality with increased number of measurement settings
[Das, Ghosal, Sasmal, Mal, Majumdar, arXiv: 1811.04813]
(Larger no. of measurement settings provide no advantage– CHSH inequality is optimal; robustness against mixedness)

Sharing of nonlocality for two-qubit state: Summary

[S. Mal, A. S. Majumdar, D. Home, Mathematics 4, 48 (2016); S.Mal, D. Das, S. Sasmal, A. S. Majumdar, Phys. Rev. A 98, 012305 (2018); S. Datta, A. S. Majumdar, ibid. 042311 (2018)]

- With how many sequential Bobs can Alice obtain CHSH violation ?
  [Silva, Gisin, Guryanova, Popescu, PRL (2015)]
- Trade-off between information gain and disturbance in a measurement is optimized by one-parameter POVM.
- **Alice (measuring sharply) on one side cannot obtain CHSH violation with more than two Bobs on other side.** ***(****Result valid for unbiased measurement settings only).*
- Sharing of steerability by Alice and two Bobs using CFFW inequality *(Necessary and sufficient state condition in 2-2-2 scenario)*
- Steerability between Alice and three Bobs using CJWR inequality.
- Sharing of NAQC correlations between Alice & 1 (or 2) Bobs
- ***Open questions****: (i) Search for HV and HS models; (ii) more observers on both sides; (iii) higher dimensional states (qudits, cv states)…….*