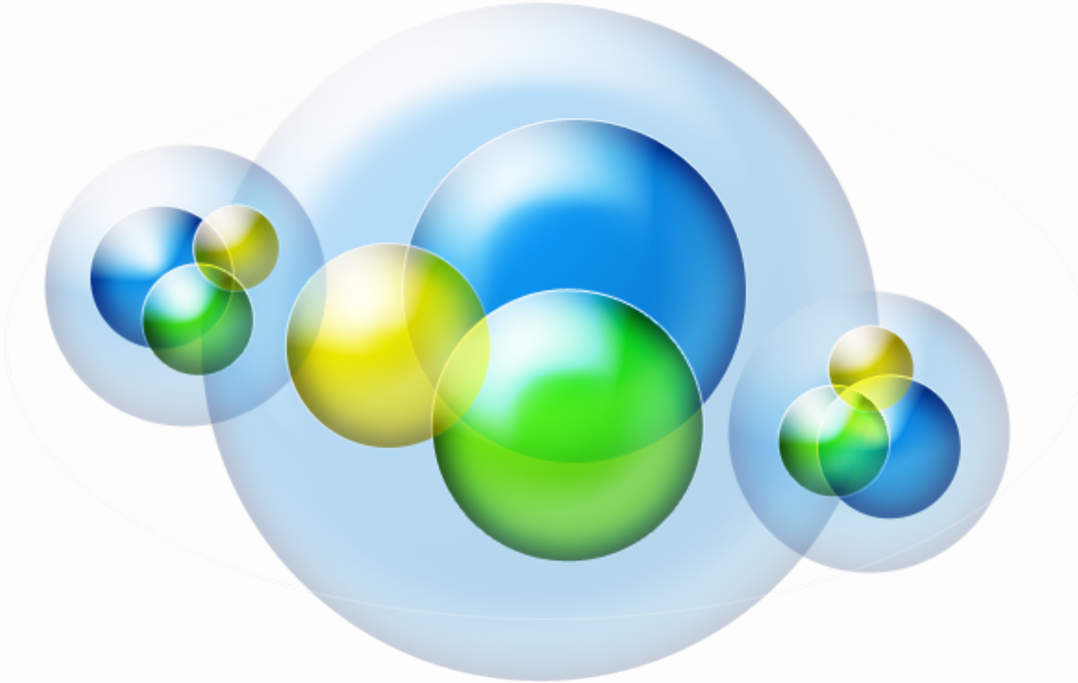
Self-bound liquids in Bose-Fermi mixture in low dimensions



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05-12-2018

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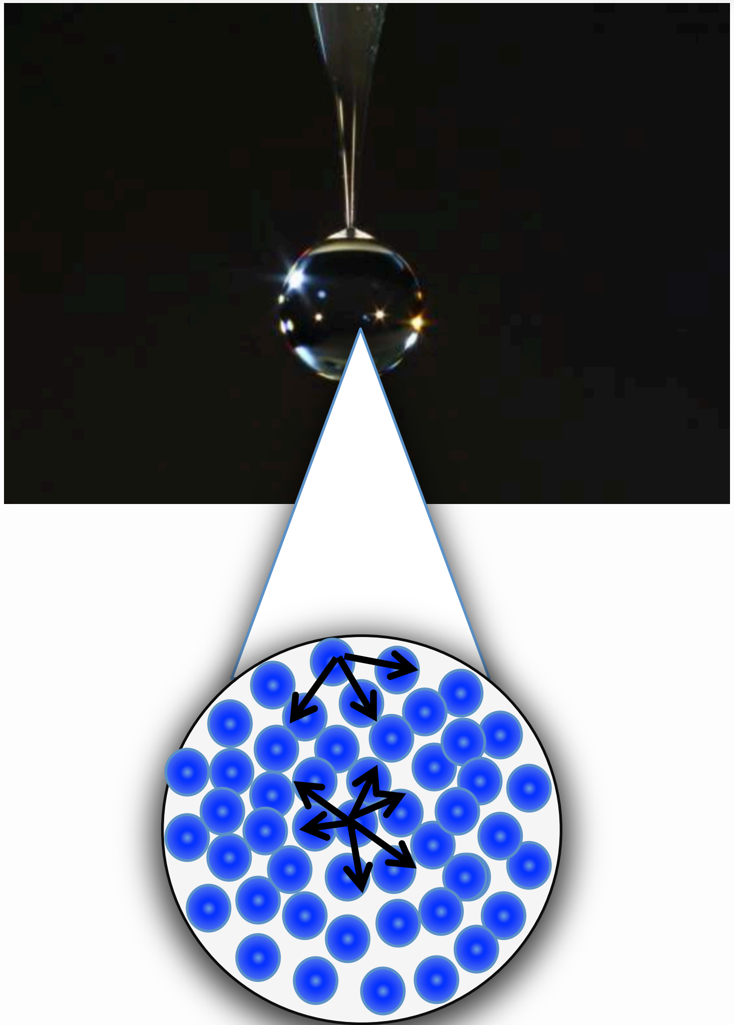
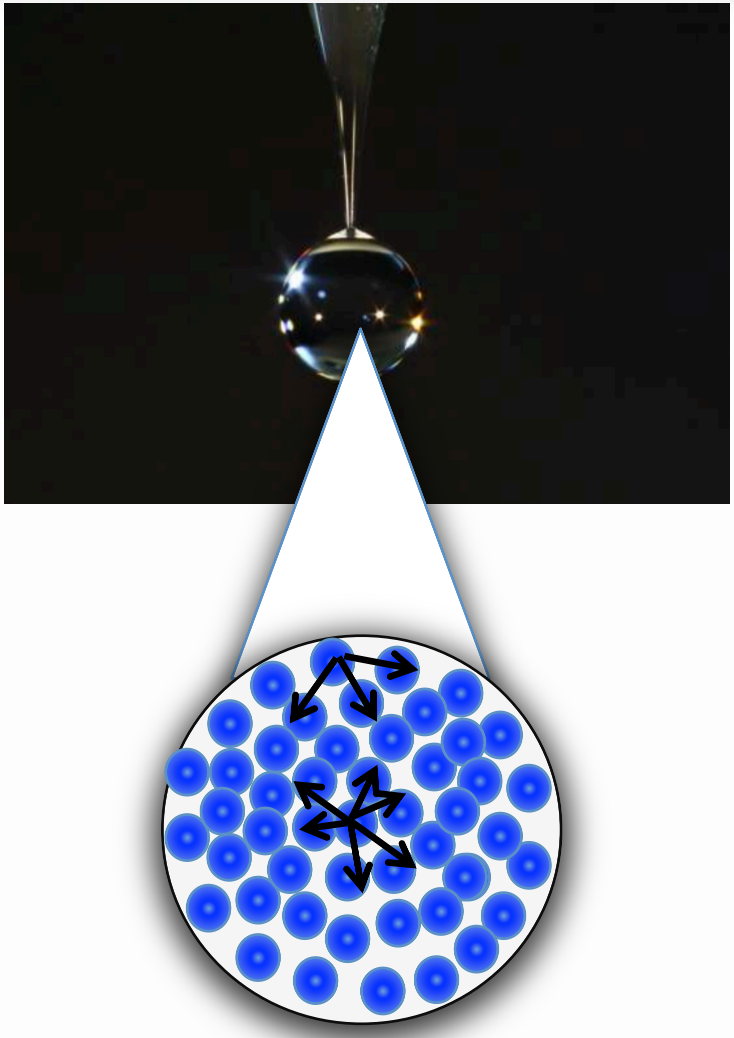
(arXiv:1808:04793)

Droplets

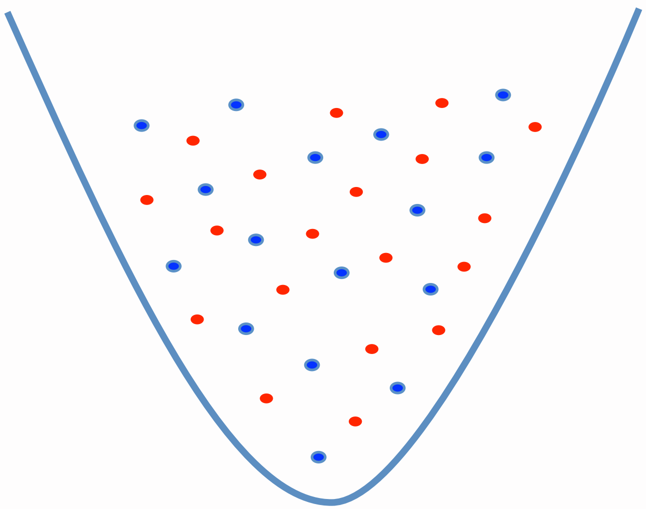


Liquids within van dar Waals paradigm

Droplets

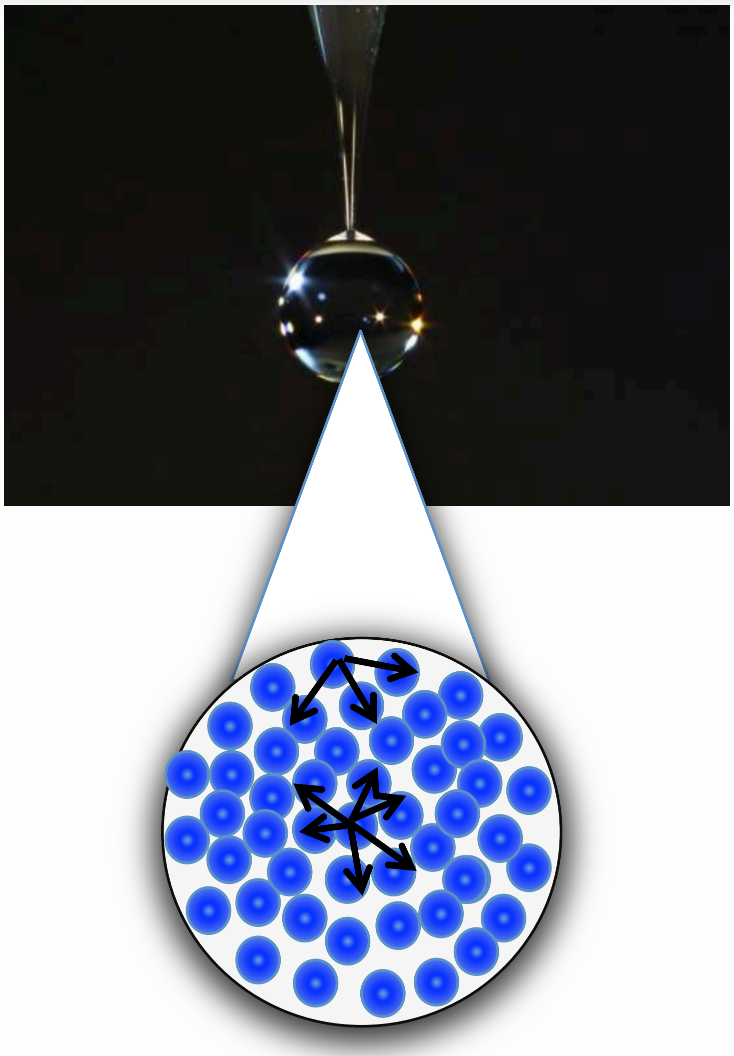


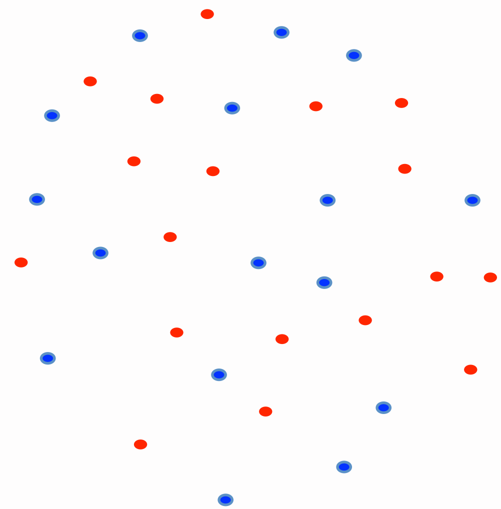
Liquids within van dar Waals paradigm



Low temperature gas mixture in a trap

Droplets





Self-bound!!

Counter-intuitive!!

Most dilute ever!!

Liquids within van dar Waals paradigm

Liquids beyond van dar Waals paradigm

Motivations/Overview

- Story so far

✓ Bose-Bose droplets (D. Petrov, PRL, 2016)
✓ Bose-Fermi droplets in three-dimension (Rakshit et. al., Under review, 2018)

- Low-dimensional droplets

✓ Quantum fluctuations are also strongly modified in reduced dimension (distinct scattering properties)
✓ Reduced three-body loss …
✓ Quadratic scaling of kinetic energy on fermionic density in 2D (a close analogy with Bose-Bose droplets)

- Droplet formation in 2D Bose-Fermi systems

✓ Thermodynamic limit
✓ Finite systems

Neutral atoms at low-temperature

- ✓ They can be cooled at extremely low temperature

- ✓ Dilute atomic gas (n ~ 10^{12}-10^{15} atoms/cm^3)

- Two-body collisions play important role
- Three-body collisions are rare

- ✓ Universality (Average inter-particle spacing $d_{\text{int}} << \lambda_{db}; \lambda_{db} \propto \frac{1}{\sqrt{T}}$)

- At low-temperature de-Broglie wavelength is much larger in comparison to interaction range
- Details of potential do not matter for many applications

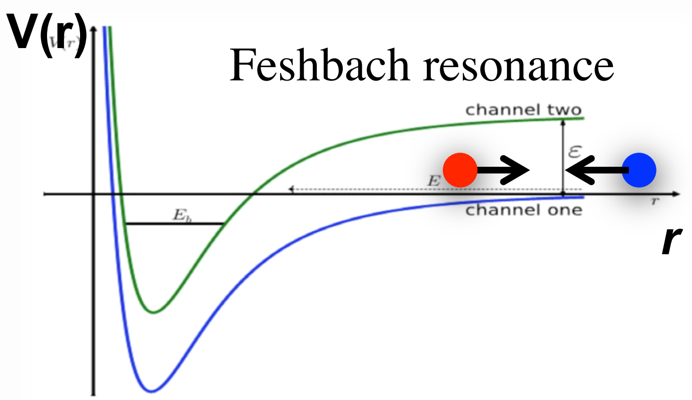
- ✓ Statistics

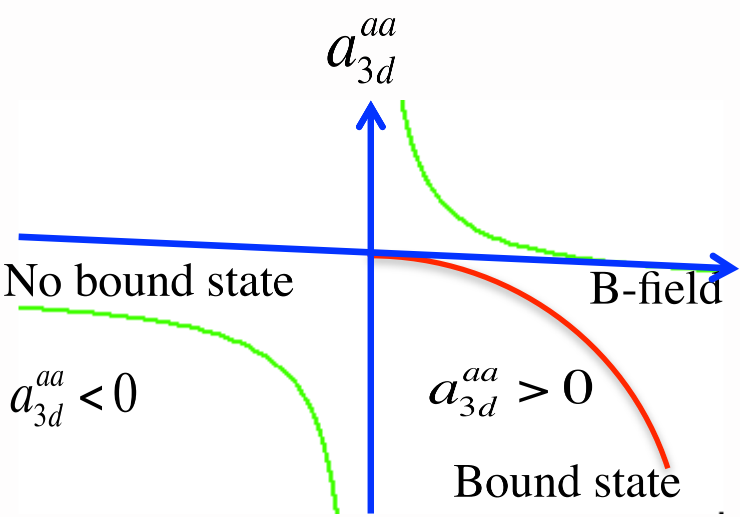
- Neutral atoms can be composite bosons or composite fermions

Neutral atoms at low-temperature

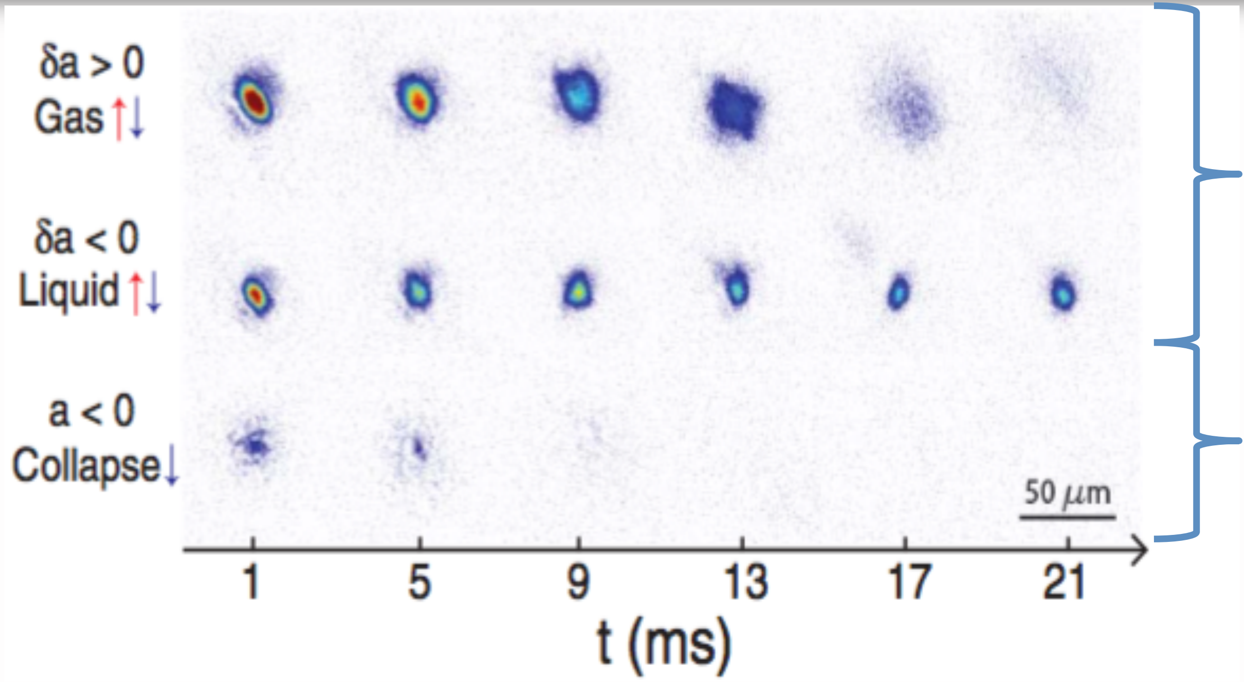
✓ Atom-atom interaction can be manipulated precisely

- s-wave scattering length
- For remainder of talk, we assume, only the s-wave scattering length to have a significant contribution at low temperature





Ultradilute liquid droplets



Bose-Bose Mixture
(39k in two hyperfine states)

Single component
Bose system

$$\delta a = -|a_{12}| + \sqrt{a_{11}a_{22}}$$

INTERSPECIES
(Effective attractive interaction)

INTRASPECIES
(Effective repulsive interaction)

C. R. Cabrera, L. Tanzi, J. Sanz, B. Naylor, P. Thomas, P. Cheiney, and L. Tarruell, **Quantum liquid droplets in a mixture of Bose-Einstein condensates,** Science **359,** 301 (2018)

Liquid Bose-Bose droplets

Mechanical stability: Two component Bose gas in a box with uniform densities n_1 and n_2

$$\varepsilon_{BB} = \frac{1}{2} g_{11} n_1^2 + \frac{1}{2} g_{22} n_2^2 - |g_{12}| n_1 n_2$$

System becomes unstable if $-|g_{12}| + \sqrt{g_{11} g_{22}} < 0; \delta g < 0$

$$\left(\frac{\partial^2 \varepsilon}{\partial n_1^2}\right)\left(\frac{\partial^2 \varepsilon}{\partial n_2^2}\right) - \left(\frac{\partial^2 \varepsilon}{\partial n_1 \partial n_2}\right)^2 \geq 0$$

Liquid Bose-Bose droplets

Mechanical stability:

$$\varepsilon_{BB} = \frac{1}{2} g_{11} n_1^2 + \frac{1}{2} g_{22} n_2^2 - |g_{12}| n_1 n_2 = \lambda_+ n_+^2 + \lambda_- n_-^2$$

$$\lambda_+ = o(\sqrt{g_{11} g_{22}}) > 0$$

$$\lambda_- = o(\delta g) < 0$$

$$\delta g = -|g_{12}| + \sqrt{g_{11} g_{22}}$$

$$n_\pm = (\sqrt{g_{22}} n_1 \mp \sqrt{g_{11}} n_2) / \sqrt{g_{11} + g_{22}}$$

System would prefer to increase densities of both the species while keeping

Collapse!!

Liquid Bose-Bose droplets

Quantum mechanical stabilization of collapsing Bose-Bose mixture:

$$\varepsilon_{BB}^{MF} \propto n^2 \delta g < 0$$

$$\varepsilon_{LHY} \propto n^{5/2} \sqrt{g_{11} g_{22}} > 0$$

They balance at certain ‘equilibrium’ atomic densities and leads to formation of liquid droplets.

Highly dilute systems!!

Liquid Bose-Fermi droplets: 3D case

Bose-Fermi mixture:

$$H = H_B + H_F + H_{BF}$$

$$\varepsilon_{3d} = \kappa_k n_F^{5/3} + \left(\frac{1}{2} g_B n_B^2 + g_{BF} n_B n_F \right) + \left(\varepsilon_{LHY} + \delta\varepsilon_{BF} \right)$$

Mean-field Higher-order corrections

- Mean field treatment does not support formation of a stable droplet
- Mean-field and corrections balance at certain 'equilibrium' atomic densities and leads to formation of liquid droplets (arXiv:1801.00346)
- Higher-order correction in Bose-Fermi interaction plays a crucial role

Highly dilute self-bound systems!! $a_B^3 n_B << 1; a_{BF}^3 n_F << 1$

How to calculate the equilibrium densities of the droplets?

Bose-Fermi mixture

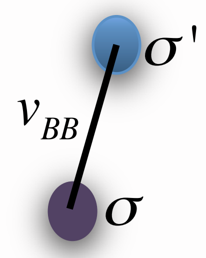
- We consider two-dimensional Bose-Fermi mixture at zero temperature
- Fermions do not interact with each other
- Bosons interact with each other and with fermions

Hamiltonian:

$$H = H_B + H_F + H_{BF}$$

$$H_B = \sum_k \frac{\hbar^2 k^2}{2m_B} \hat{b}_k^+ \hat{b}_k + \frac{1}{2V} \sum_{p,q,k} \hat{b}_p^+ \hat{b}_{p+k} v_{BB}(k) \hat{b}_q^+ \hat{b}_{q-k}$$

Interaction potential



3D-vs-2D

Interaction potential

$$H_B = \sum_k \frac{\hbar^2 k^2}{2m_B} \hat{b}_k^+ \hat{b}_k + \frac{1}{2V} \sum_{p,q,k} \hat{b}_p^+ \hat{b}_{p+k} v_{BB}(k) \hat{b}_q^+ \hat{b}_{q-k}$$

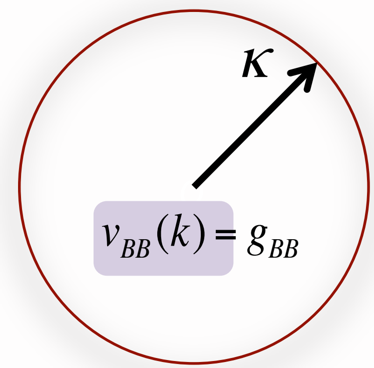
3D

$$v_{\sigma\sigma'}(k) = g_{\sigma\sigma'} \left(1 + \frac{g_{\sigma\sigma'}}{V} \sum_{k \neq 0} \frac{m}{\hbar^2 k^2} \right)$$

$$g_{\sigma\sigma'} = 2\pi\hbar^2 a_{\sigma\sigma'} / \mu_{\sigma\sigma'}$$

$$\mu_{\sigma,\sigma'} = \frac{m_\sigma m_{\sigma'}}{(m_\sigma + m_{\sigma'})}$$

2D



$$g_{\sigma,\sigma'} = \frac{2\pi\hbar^2}{\mu_{\sigma,\sigma'}} \frac{1}{\ln\left(\varepsilon / (a_{\sigma,\sigma'}^2 \kappa^2)\right)}$$

$$\varepsilon = 4\exp(-2\gamma)$$ Popov, Theor. Math. Phys. 11, 565

2D Bose-Fermi mixture: Energy density

Energy contribution due to H_B

$$H_B = \sum_k \frac{\hbar^2 k^2}{2m_B} \hat{b}_k^+ \hat{b}_k + \frac{1}{2V} \sum_{p,q,k} \hat{b}_p^+ \hat{b}_{p+k} V_{BB}(k) \hat{b}_q^+ \hat{b}_{q-k}$$

Consider leading order contributions [collect all possible quadratic terms] and diagonalize via Bogoliubov transformation:

$$\varepsilon_B = g_{BB} n_B^2 + \varepsilon_{LHY}$$

Mean-field LHY correction

$$\varepsilon_{LHY} = \frac{g_{BB}^2 n_B^2 m_B}{8\pi\hbar^2} \ln\left(\frac{g_{BB} n_B \sqrt{e}}{\hbar^2 \kappa^2 / m_B} \right)$$

2D Bose-Fermi mixture: Energy density

Energy contribution due to Bose-Fermi interaction H_{BF}

$$\varepsilon_{BF} = g_{BF} n_B n_F + \delta\varepsilon_{BF}$$

Coupling between density fluctuations in the two species

In 2D:

$$\delta\varepsilon_{BF} = \frac{4\pi m_B n_B g_{BF}^2 k_F^2}{(2\pi)^4 \hbar^2} I_c(\omega,\alpha)$$

$$k_F = \sqrt{4\pi n_F}$$

$$\alpha = 2\omega\left(g_{BB} n_B / \varepsilon_F\right)$$

$$\omega = m_B / m_F$$

Energy contribution due to H_F

$$\varepsilon_F = \frac{1}{2}\beta n_F^2 ; \beta = \frac{2\pi\hbar^2}{m_F}$$

Kinetic energy

2D Bose-Fermi mixture: Energy density

Total energy density of Bose-Fermi mixture :

$$\varepsilon = \frac{\beta}{2} n_F^2 + \frac{g_{BB}}{2} n_B^2 + g_{BF} n_B n_F + \varepsilon_{LHY} + \delta\varepsilon_{BF}$$

Mean-field energy

Beyond mean-field higher-order corrections

Diluteness conditions:

$a_{BB}^2 n_{B,F} << 1$ Weakly repulsive (intraspecies)

$a_{BF}^2 n_{B,F} >> 1$ Weakly attractive (interspecies)

Cut-off dependence

$$\varepsilon = \frac{\beta}{2} n_F^2 + \left(\frac{g_{BB}}{2} n_B^2 + \varepsilon_{LHY} \right) + \left(g_{BF} n_B n_F + \delta\varepsilon_{BF} \right)$$

$$\varepsilon_B = \varepsilon_B(\kappa)$$

$$\varepsilon_{BF} = \varepsilon_{BF}(\kappa)$$

$$\frac{\partial \varepsilon_B(\kappa)}{\partial \kappa^2} \approx 0$$

Negligible dependence
on the cut-off momentum
(can be numerically checked)

Cut-off momentum

$$\varepsilon_{MF} = \frac{\beta}{2} n_F^2 + \frac{g_{BB}}{2} n_B^2 + g_{BF} n_B n_F$$

Choose the momentum cut-off from the bare minimum stability condition of the system

$$\left(\frac{\partial^2 \varepsilon_{MF}}{\partial n_B^2}\right)\left(\frac{\partial^2 \varepsilon_{MF}}{\partial n_F^2}\right) - \left(\frac{\partial^2 \varepsilon_{MF}}{\partial n_B \partial n_F}\right)^2 = 0$$

↓

$$2\ln^2\left(\frac{4\exp(-2\gamma)}{a_{BF}^2 \kappa_c^2}\right) = \frac{(1+\omega)^2}{\omega} \ln\left(\frac{4\exp(-2\gamma)}{a_{BB}^2 \kappa_c^2}\right)$$

Solve for κ_c

Mechanical stabilization of self-bound mixture

Vanishing pressure …

Outside of the droplet the pressure must be equal to zero (free-space).
The pressure related to surface vanishes in thermodynamic limit.

$$E(N_B, N_F, V) = \varepsilon(n_B, n_F)V$$

$$n_{B,F} = \frac{N_{B,F}}{V}$$

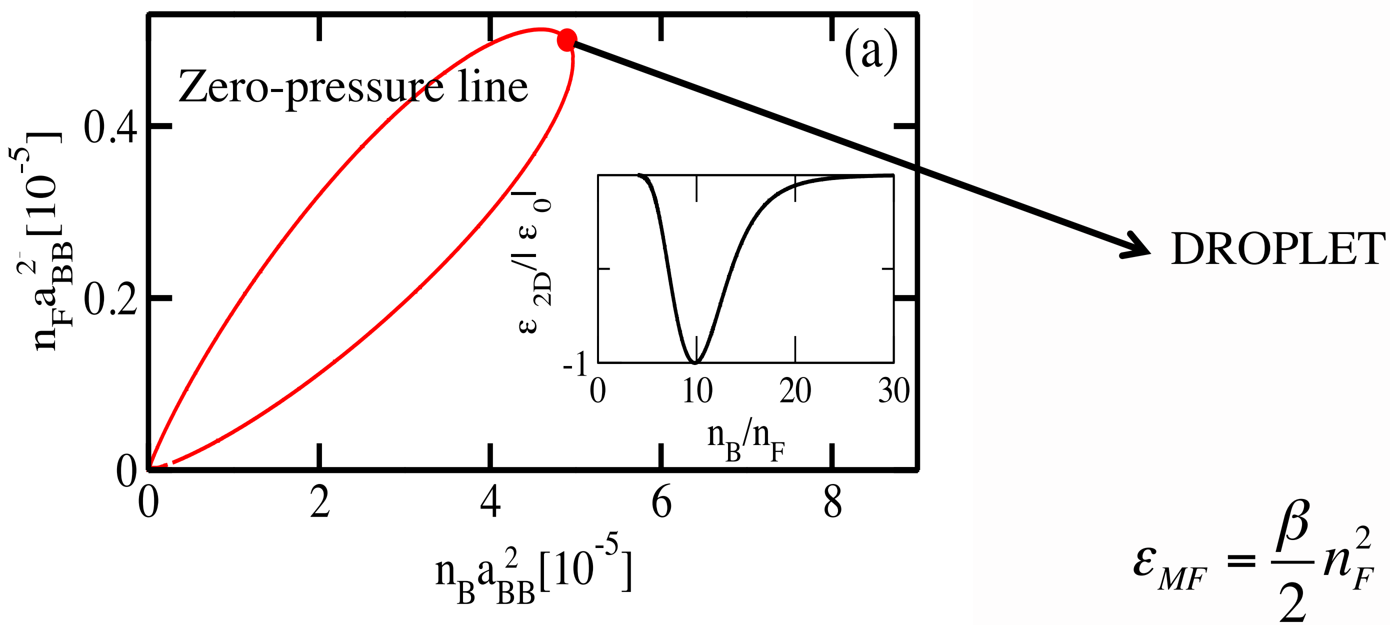
$$P = \left(\frac{dE}{dV}\right)_{N_B, N_F}$$

$$p(n_B, n_F) = n_B \frac{\partial \varepsilon}{\partial n_B} + n_F \frac{\partial \varepsilon}{\partial n_F} - \varepsilon(n_B, n_F) = 0$$

Equilibrium density of Stable Bose-Fermi droplets

^{6}Li - ^{133}Cs mixture (weakly interacting dilute gas)

$$a_{BF} / a_B = 10^4$$



$$\varepsilon_{MF} = \frac{\beta}{2} n_F^2 + \frac{g_{BB}}{2} n_B^2 + g_{BF} n_B n_F$$

Due to the quadratic form of mean-field energy, equilibrium density ratio can be shown to be approximately

$$n_B / n_F \approx \sqrt{\beta / g_{BB}}$$

Mechanical stability – thermodynamic limit

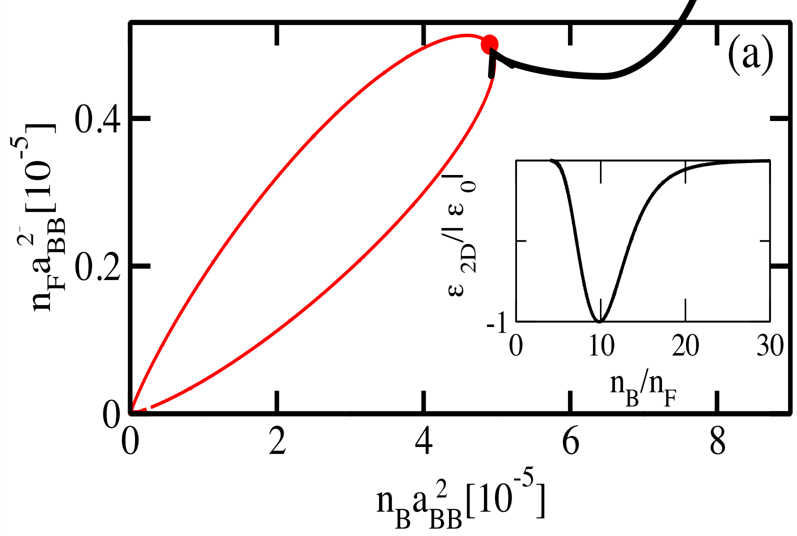
Minimize energy constrained to vanishing pressure

$$\mu_B \frac{\partial p}{\partial n_F} - \mu_F \frac{\partial p}{\partial n_B} = 0$$

$$p(n_B, n_F) = n_B \frac{\partial \varepsilon}{\partial n_B} + n_F \frac{\partial \varepsilon}{\partial n_F} - \varepsilon(n_B, n_F) = 0$$

Solution gives equilibrium densities

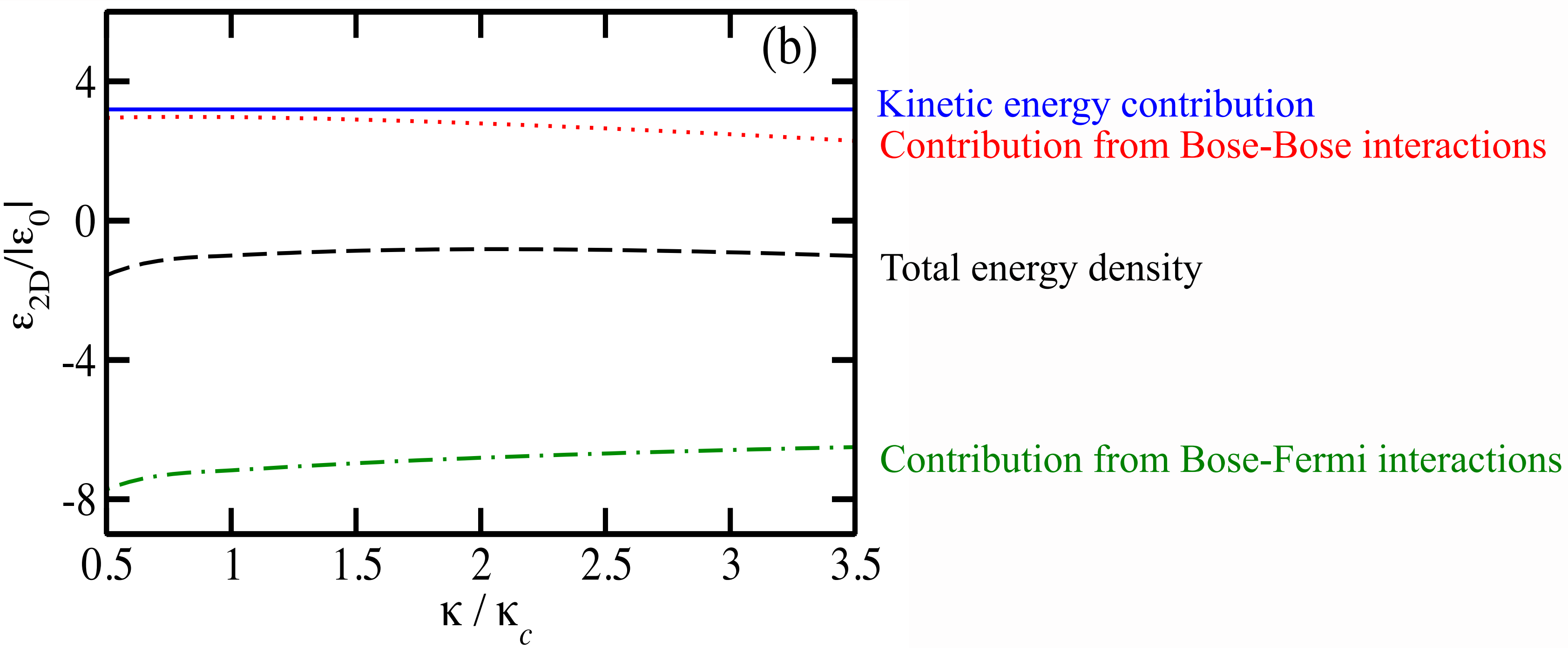
$$\mu_B = \frac{\partial \varepsilon}{\partial n_B}; \mu_F = \frac{\partial \varepsilon}{\partial n_F};$$



Variance with cut-off momentum

Variation of the energy densities as a function of the cut-off momentum around the chosen cut-off momentum

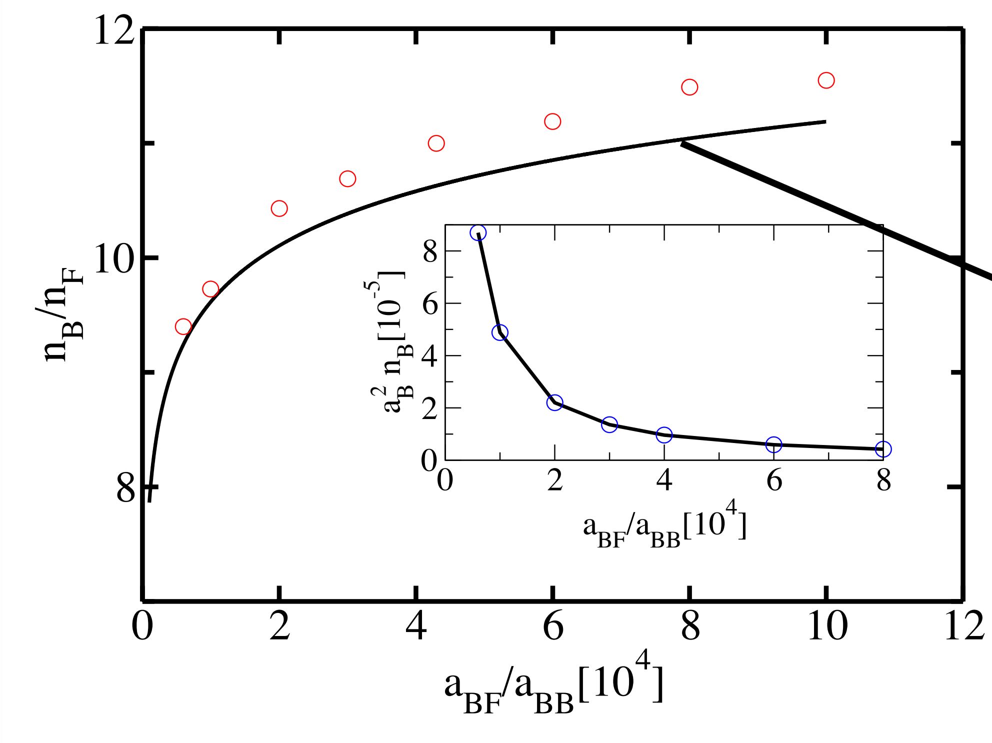
^{6}Li - ^{133}Cs mixture ($a_{BF} / a_B = 10^4$)



Bose-Fermi mixture: Effects of interaction strength

Equilibrium densities as a function of a_{BF}/a_B

^6Li - ^{133}Cs mixture



$$n_B / n_F \approx \sqrt{\beta / g_{BB}}$$

Nonuniform finite droplets

- To include surface effects, energy terms related to density gradient has to be added

- Hydrodynamic equation of motion

$$\frac{\partial}{\partial t} n_F = -\nabla(n_F \vec{v}_F)$$

$$m_F \frac{\partial}{\partial t} \vec{v}_F = -\nabla\left(\frac{\delta T}{\delta n_F} + \frac{m_F}{2}\vec{v}_F^2 + g_{BF} n_B + \frac{\delta \varepsilon_{BF}}{\delta n_F}\right)$$

$$\frac{\partial T}{\partial n_F} = \beta n_F - \xi \frac{\hbar^2}{2m_F} \frac{\nabla^2 \sqrt{n_F}}{\sqrt{n_F}}$$

} Fermionic component

Nonuniform finite droplets

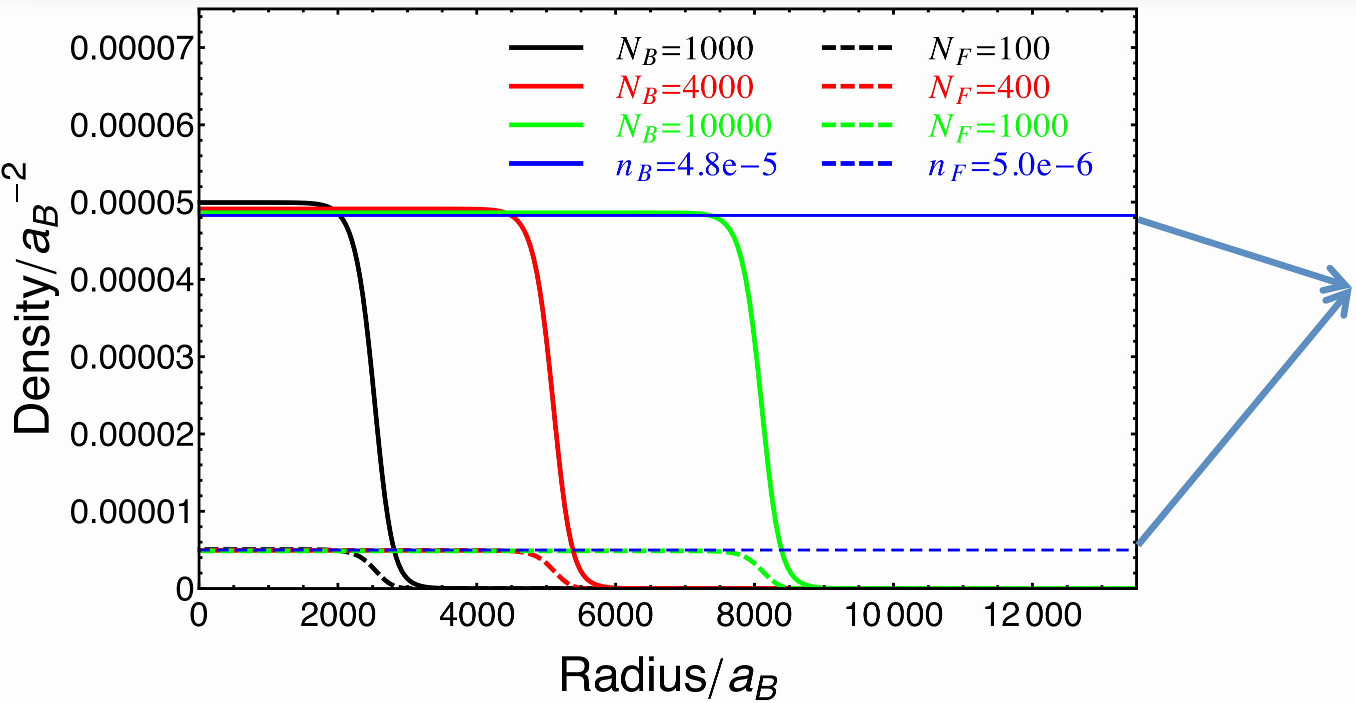
Inverse-Madelung transformation and Schrodinger-like equation of motion

$$i\hbar\frac{\partial\psi_F}{\partial t} = \Big[-\frac{\hbar^2}{2m_F}\nabla^2 + \frac{\xi'\hbar^2}{2m_F}\frac{\nabla^2|\psi_F|}{\psi_F} + \beta|\psi_F|^2 + g_{BF}\,|\psi_B|^2 + \mathcal{C}\mathcal{I}_c(\omega,\alpha) + \mathcal{C}\,|\psi_F|^2\frac{\partial\mathcal{I}_c}{\partial\alpha}\frac{\partial\alpha}{\partial n_F}\Big]\psi_F,$$

$$i\hbar\frac{\partial\psi_B}{\partial t} = \Big[-\frac{\hbar^2}{2m_B}\nabla^2 + \beta|\psi_B|^2 + g_{BF}\,|\psi_F|^2 + \mathcal{A}|\psi_B|^2 + 2\mathcal{A}|\psi_B|^2\ln(\mathcal{B}|\psi_B|^2) + \mathcal{C}\,|\psi_F|^2\frac{\partial\mathcal{I}_c}{\partial\alpha}\frac{\partial\alpha}{\partial n_B}\Big]\psi_B, \quad (3)$$

where $\xi' = 1-\xi = 0.96$ and the coefficients $\mathcal{A} = \frac{g_{BB}^2}{8\pi\hbar^2}$, $\mathcal{B} = \frac{g_{BB}\sqrt{e}}{(\hbar^2\kappa^2/m_B)}$ and $\mathcal{C} = -\frac{(4\pi)^2 m_B g_{BF}^2}{(2\pi)^4\hbar^2}$. The bosonic wave function and the fermionic pseudo-wave function are normalized as $N_{B,F} = \int d\mathbf{r}\,|\psi_{B,F}|^2$.

Nonuniform finite droplets



Equilibrium densities in thermodynamic limit

Solution obtained by reworking hydrodynamic equation into a form of Schrodinger-like equation

The case of ^{133}Cs-^{6}Li mixture for a_{BF}/a_B = 10^4 and the initial number of bosons (fermions) equal to 1000 (100), 4000 (400), and 10000 (1000)

The density does not change, only volume increases …

Summary

- ❑ Low-dimensional Bose-Fermi droplets …
  (We find that a three-dimensional stable Bose-Fermi mixture in *gaseous phase* can be *liquefied* by introducing a transverse confinement)

- ❑ Peculiar scattering processes in low-dimensions significantly modifies the higher-order quantum fluctuation terms, which are crucial ingredients for these droplet formation

- ❑ Quadratic scaling of fermionic kinetic energy makes 2D systems closely analogous to the Bose-Bose droplets

- ❑ In contrary to the 3D case, 2D droplets are formed near vanishing mean-field energy

- ❑ Conditions for droplet formation in 1D were obtained as well (not discussed here; See arXiv:1808.04793)

- ❑ Low-dimensional droplets are most promising from experimental point of view due to reduced three-body losses

Outlook …

- ❑ Across dimensional crossover …
- ❑ Droplets + vapor …
- ❑ Droplet collisions …
- ❑ Droplets in strongly interacting regime …
- ❑ A perfect laboratory for studying various processes – polaron physics, stability of astronomical objects …