An alternative multiparty quantum mutual information

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outline-

quantum mutual information (QMI): a brief review

another QMI: motivation and approach

identification/comparison with other quantities

properties and bounds

notation and definition-

ρ_X : quantum state associated with system X

$S(\rho_X) = -\mathrm{tr}(\rho_X \log \rho_x) \equiv S_X$: von Neumann entropy

$$S_X - S_Y \leq S_{XY} \leq S_X + S_Y$$

$S(\rho \parallel \sigma) = \mathrm{tr}(\rho \log \rho - \rho \log \sigma)$: relative entropy

strong sub-additivity relations

$$S_{XYZ} + S_Y \leq S_{XY} + S_{YZ} \qquad S_X + S_Y \leq S_{XZ} + S_{YZ}$$

two-party case-

ρ_{AB}: quantum state of a bipartite system

$I(A:B) = S_A + S_B - S_{AB}$: QMI between A and B

→ nonnegative; sub-additivity of entropy

→ measures total correlation (classical+quantum) between A and B; the amount of work (noise) that is required to erase (destroy) the correlations completely

Groisman, Popescu, Winter, PRA 72, 032317 (2005)

$$\rho_{AB} \to \sum_i p_i \rho_A^i \otimes \rho_B^i \to \sigma_A \otimes \sigma_B$$

multiparty case-

$\rho_{A_1 \cdots A_n}$: multipartite quantum state

- in an n-party system, there can be k-party $(2 \leq k \leq n)$ correlations between the subsystems
- similarly, information can be shared/distributed between two or more parties in a multiparty system

$$A_1 A_2 \cdots A_k \equiv A_1 \cup A_2 \cup \cdots \cup A_k$$

$$\rho_{\overline{A_i}} \equiv \rho_{A_1 \cdots A_{i-1} A_{i+1} \cdots A_n}$$

multiparty case-I

$$C_n(A_1 : \cdots : A_n) = \sum_{k=1}^{n} (-1)^{k+1} \sum_{\{A^{(k)}\}} S_{A^{(k)}}$$

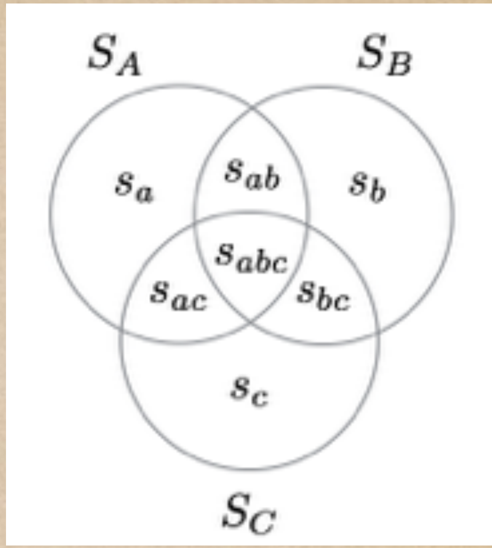
$$= \sum_{k=1}^{n-1} (-1)^{k+1} \sum_{\{A^{(k)}\}} I\left(A_1 : A^{(k)}\right)$$

$$= s_{a_1 \cdots a_n}$$ what is this?

- common (n-party interaction) information
- can be positive, negative or zero

$AB \equiv A \cup B$

three-party system



ρ_{ABC}: tripartite quantum state

$$C_3(A:B:C)$$
$$= S_A + S_B + S_C - (S_{AB} + S_{AC} + S_{BC}) + S_{ABC}$$
$$= I(A:B) + I(A:C) - I(A:BC)$$
$$= s_{abc}$$

→ common information

→ negative value implies monogamy of bipartite QMI

→ vanishes for tripartite pure states

multiparty case-II

$$T_n(A_1 : \cdots : A_n) = \sum_{k=1}^{n} S_{A_k} - S_{A_1 \cdots A_n}$$

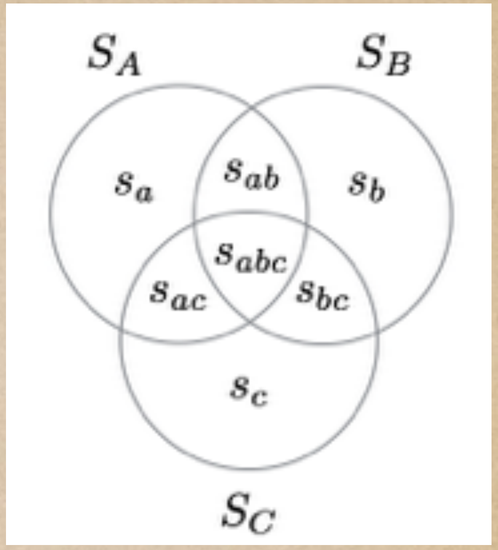
$$= \sum_{k=1}^{n-1} I(A_{k+1} : A_k \cdots A_1)$$

$$= S\left(\rho_{A_1 \cdots A_n} \,\|\, \bigotimes_{k=1}^{n} \rho_{A_k} \right)$$

- straightforward generalisation of bipartite QMI
- non-negative; does not have an interpretation of total correlation unlike the bipartite case

three-party system

$AB \equiv A \cup B$



ρ_{ABC}: tripartite quantum state

$$T_3(A:B:C) = S_A + S_B + S_C - S_{ABC}$$
$$= I(A:B) + I(AB:C)$$
$$= S\left(\rho_{ABC} \,\|\, \rho_A \otimes \rho_B \otimes \rho_C\right)$$
$$= s_{ab} + s_{ac} + s_{bc} + 2s_{abc}$$

why not just $s_{ab} + s_{ac} + s_{bc} + s_{abc}$?

motivation for alternative QMI

- $C_n(A_1 : \cdots : A_n)$

  considers only n-party interaction;
  can vanish for pure states; can be negative

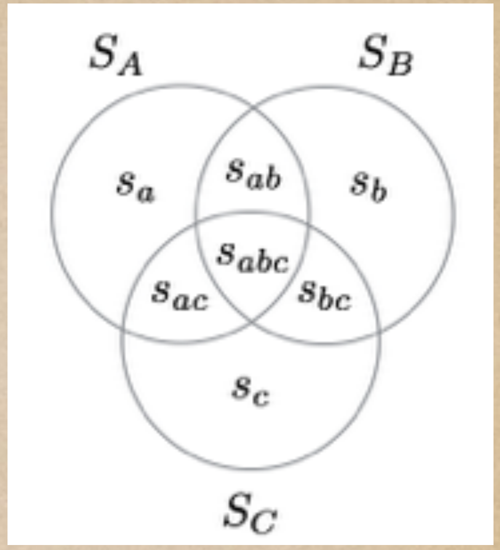
- $T_n(A_1 : \cdots : A_n)$

  some interactions are considered more than once

- $S_n(A_1 : \cdots : A_n)$

  information shared between two and more parties of a multiparty system

$AB \equiv A \cup B$



case-III

ρ_{ABC}: tripartite quantum state

$$S_3(A:B:C) \;= s_{ab} + s_{ac} + s_{bc} + s_{abc}$$

$$= I(A:B) + I(A:C) + I(B:C) - 2C_3(A:B:C)$$

$$= I(A:BC) + I(B:CA) + I(C:AB) - T_3(A:B:C)$$

$$= S_{AB} + S_{AC} + S_{BC} - 2S_{ABC}$$

$$= S\left(\rho_{ABC}^{\otimes 2} \parallel \rho_{AB} \otimes \rho_{AC} \otimes \rho_{BC}\right)$$

multiparty case-III

$$S_n(A_1 : \cdots : A_n) = (s_{a_1 a_2} + \dots) + (\cdots) + s_{a_1 \cdots a_n}$$

$$= \sum_{k=1}^{n} I(A_k : \overline{A_k}) - T_n(A_1 : \cdots : A_n)$$

$$= \sum_{k=1}^{n} S_{\overline{A_k}} - (n-1) S_{A_1 \cdots A_n}$$

dual total correlation

T. S. Han, Inf. Cont. 36, 133 (1978)

secrecy monotone

Cerf, Massar, Schneider, PRA 66, 042309 (2002)

$$= S\left(\rho_{A_1 \cdots A_n}^{\otimes n-1} \,\|\, \bigotimes_{k=1}^{n} \rho_{\overline{A_k}} \right)$$

binding information

AK, PRA 96, 012332 (2017)

properties of $S_n(A_1 : \cdots : A_n)$

- non-negative; does not vanish for pure states; considers all interactions only once; equals T_n for pure states; vanishes for fully product states
- $S_2 = T_2 = C_2$
- additive for product states

  $\rho_{A_1 \cdots A_5} = \rho_{A_1 A_2} \otimes \rho_{A_3 A_4 A_5}$

  $S_5(A_1 : \cdots : A_5) = S_2(A_1 : A_2) + S_3(A_3 : A_4 : A_5)$

properties of $S_n(A_1 : \cdots : A_n)$

- monotonicity under partial trace, and other completely positive maps
- $S_n + T_n = \sum_{k=1}^{n} I(A_k : \overline{A_k})$
- $T_n - (n-2)S_{A_1 \cdots A_n} \leq S_n \leq T_n + 2S_{A_1 \cdots A_n}$
- can S_n exceed T_n? yes, because $C_3 = T_3 - S_3$ can be negative
- measures genuine total correlation!!!

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