Efficient measurement of high-dimensional quantum states

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Outline

Orbital Angular Momentum of light

- What it means?
- Generation of OAM states of light

Efficient measurement of states of light

- in the orbital angular momentum (OAM) basis.
- in the transverse position basis.

Orbital angular momentum (OAM) of light

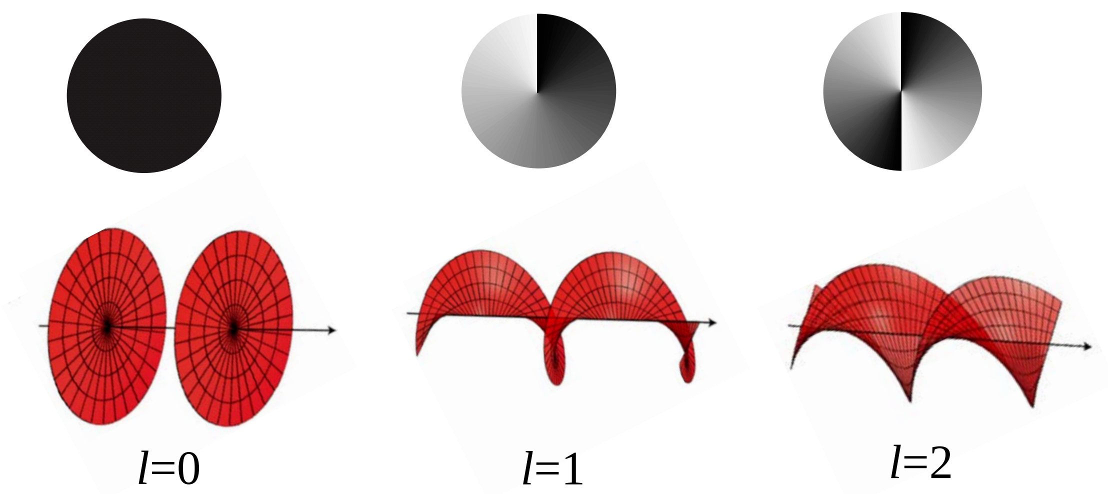
Laguerre-Gaussian (LG) modes are solutions to paraxial Wave Equation

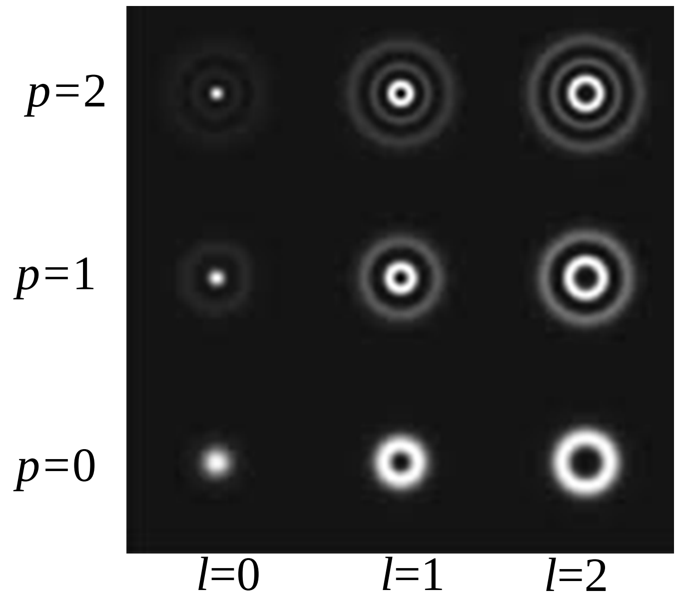
$$LG_p{}^l(\rho,\phi,z) = \frac{C}{\left(1+z^2/z_R{}^2\right)^{1/2}} \exp\left[i(2p+l+1)\tan^{-1}\left(\frac{z}{z_R}\right)\right]\left[\frac{\rho/2}{w(z)}\right]^l L_p{}^l\left[\frac{2\rho^2}{w^2(z)}\right]$$

$$\times \exp\left[-\frac{\rho^2}{w^2(z)}\right]\exp\left[-\frac{ik^2\rho^2 z}{2(z^2+z_R{}^2)}\right]e^{-il\phi}$$

Transverse phase of the LG mode with p=0

Yao and Padgett, Advances in optics and photonics, **3**, 161 (2011)





- **Orbital angular momentum per photon in an LG mode: $\hbar l$** Allen et al., PRA **45**, 8185 (1992)
- **OAM modes $\psi_l(\phi) = \frac{1}{\sqrt{2\pi}}e^{-il\phi}$ form a complete basis & provide an infinite dimensional discrete basis**

(i) higher allowed error rate in cryptography — Phys. Rev. Lett. 88, 127902 (2002).

(ii) higher transmission bandwidth — Phys. Rev. Lett. 90, 167906 (2003).

(iii) Fundamental tests of quantum mechanics — Phys. Rev. Lett. 85, 4418–4421 (2000).

OAM for classical communication

ARTICLES
PUBLISHED ONLINE: 24 JUNE 2012 | DOI: 10.1038/NPHOTON.2012.138
nature photonics

Terabit free-space data transmission employing orbital angular momentum multiplexing

Jian Wang[1,2]*, Jeng-Yuan Yang[1], Irfan M. Fazal[1], Nisar Ahmed[1], Yan Yan[1], Hao Huang[1], Yongxiong Ren[1], Yang Yue[1], Samuel Dolinar[3], Moshe Tur[4] and Alan E. Willner[1]*

nature COMMUNICATIONS

ARTICLE
Received 17 Mar 2014 | Accepted 1 Aug 2014 | Published 16 Sep 2014
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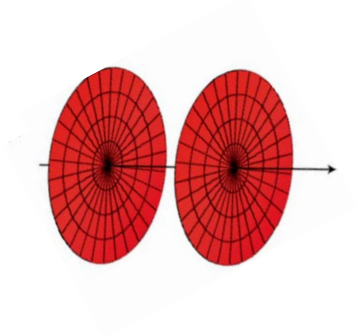
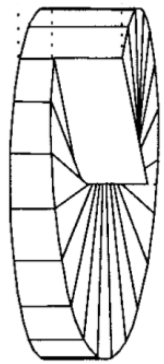
High-capacity millimetre-wave communications with orbital angular momentum multiplexing

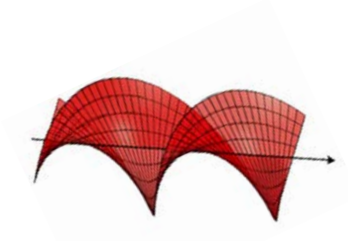
Yan Yan[1,*], Guodong Xie[1,*], Martin P.J. Lavery[2,*], Hao Huang[1,*], Nisar Ahmed[1], Changjing Bao[1], Yongxiong Ren[1], Yinwen Cao[1], Long Li[1], Zhe Zhao[1], Andreas F. Molisch[1], Moshe Tur[3], Miles J. Padgett[2] & Alan E. Willner[1]

How to generate OAM modes?

1. Using an spiral phase plate:

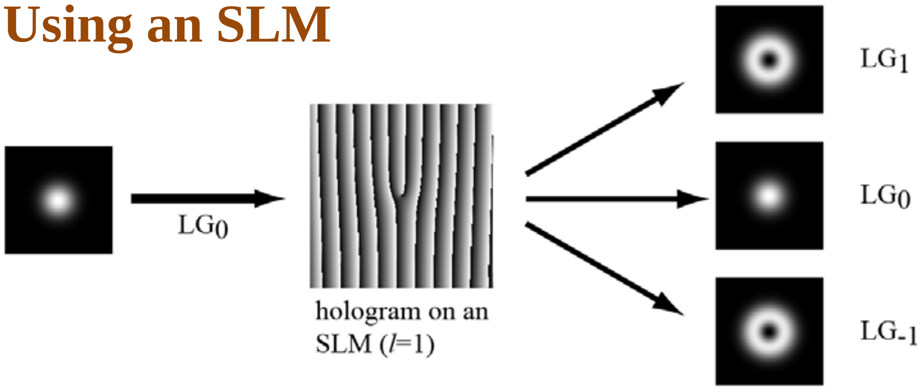




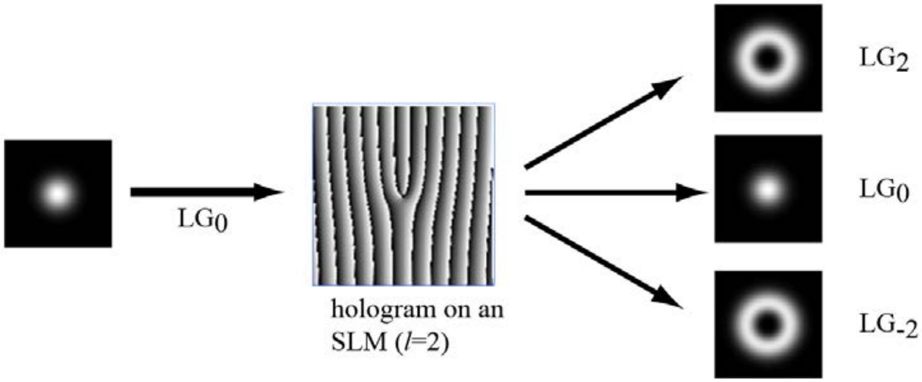


Oemrawsingh et al., Applied Optics, 43, 688 (2004)
Yao and Padgett, Advances in optics and photonics, **3**, 161 (2011)

2. Using an SLM



A Mair *et al*. Nature 412 **313** (2001)
N R Heckenberg *et al*. Opt. Lett. **17**, 221 (1992)



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States in OAM basis : $\langle \boldsymbol{\phi} | \boldsymbol{l} \rangle = e^{-il\phi}$

State in the OAM basis (classical)

State in the OAM basis (quantum)

$$\psi(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi} \longleftrightarrow |\psi\rangle = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l |l\rangle$$

Pure States

$$W(\phi_1, \phi_2) = \langle \psi(\phi_1)\psi^*(\phi_2)\rangle_e \longleftrightarrow \rho = \langle\, |\psi\rangle\langle\psi|\, \rangle_e$$

$$= \frac{1}{2\pi} \sum_{l_1, l_2=-\infty}^{\infty} \langle \alpha_{l_1} \alpha_{l_2}^* \rangle_e \, e^{-i(l_1\phi_1 - l_2\phi_2)} \qquad = \frac{1}{2\pi} \sum_{l_1, l_2=-\infty}^{\infty} \langle \alpha_{l_1} \alpha_{l_2}^* \rangle_e \, |l_1\rangle\langle l_2|$$

Mixed States

When different OAM eigenmodes are uncorrelated. $\langle \alpha_{l_1} \alpha_{l_2}^* \rangle_e = S_{l_1} \delta_{l_1, l_2}$

$$W(\phi_1, \phi_2) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l \, e^{-il(\phi_1 - \phi_2)} \longleftrightarrow \rho = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l |l\rangle\langle l|$$

Diagonal Mixed States

$$W(\Delta\phi) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l \, e^{-il\Delta\phi} \Rightarrow S_l = \int_{-\pi}^{\pi} W(\Delta\phi) e^{il\Delta\phi} d\phi$$

Angular Wiener-Khintchine theorem

Angular correlation function

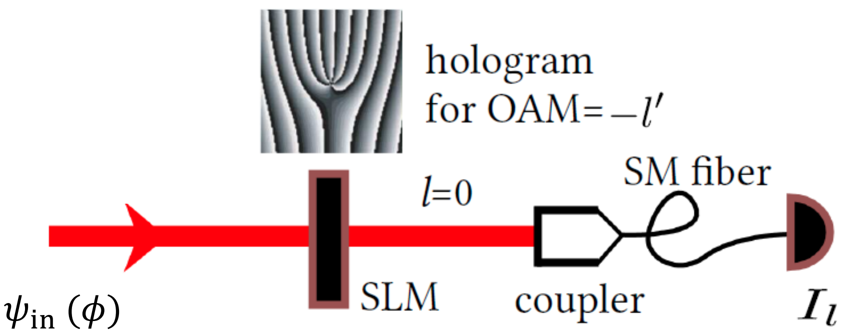
OAM spectrum

A K Jha, G S Agarwal, R W Boyd, PRA **84**, 063847 (2011)

Aim: Measure the angular correlation function $W(\phi_1, \phi_2)$

For diagonal states it yields the OAM spectrum

Existing methods for measuring OAM spectrum of Light



A Mair *et al*. Nature 412 **313** (2001)
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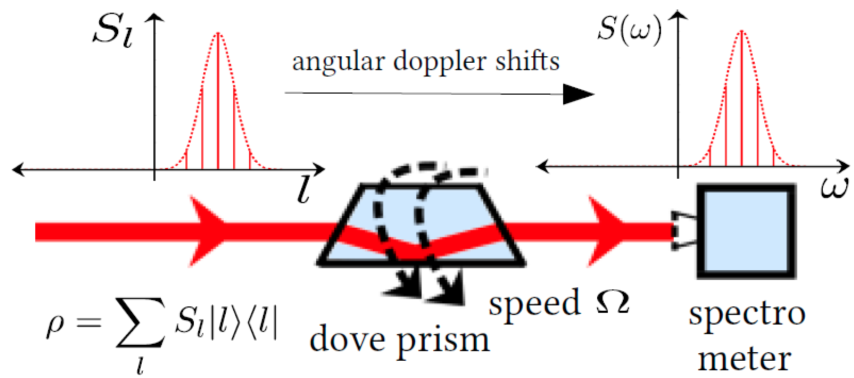
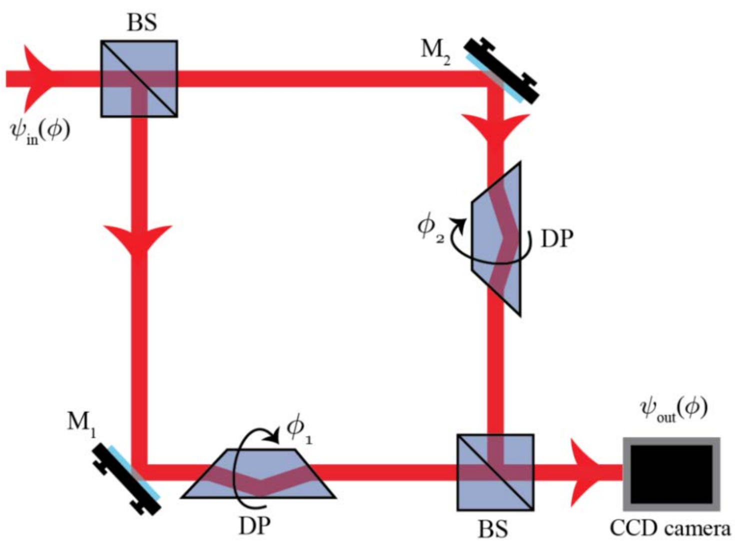
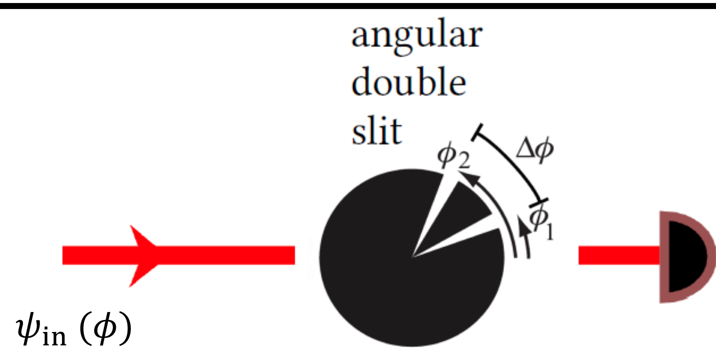


M. Vasnetsov *et al.*, Opt. Lett. 28, 2285 (2003).



H D L Pires *et al*., Opt. Lett., **35**, 889 (2010)
H D L Pires *et al*., Phys Rev Lett **104**, 020505 (2010)



A K Jha, G S Agarwal, R W Boyd, PRA **84**, 063847 (2011)
M Malik *et al*., PRA **86**, 063806 (2012).

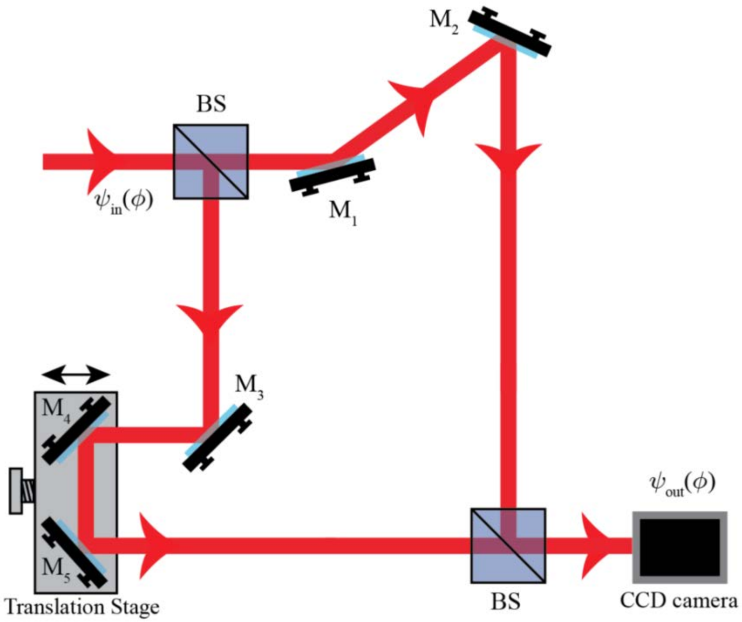
Limitations:
- Requires multiple measurements
- Inefficient or too much loss
- Stringent alignment requirements
- Very sensitive to noise

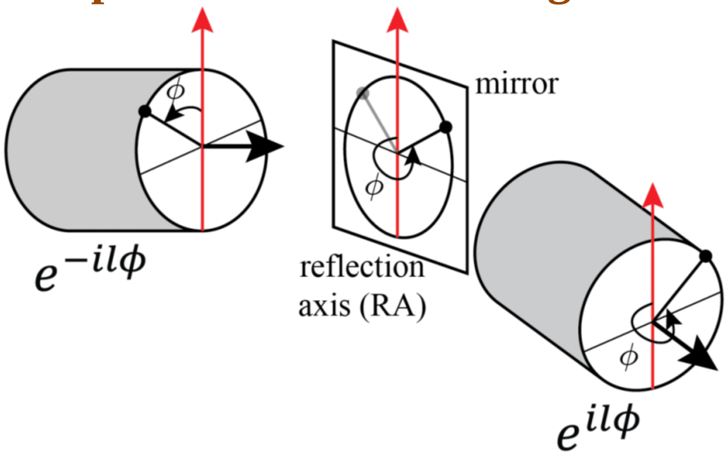
Measuring Orbital Angular Momentum of Light (A new scheme)

Input state in the OAM basis (diagonal mixed state)

$$\psi_{\text{in}}(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi} \quad \text{with} \quad \langle \alpha_l \alpha_{l'}^* \rangle_e = S_l \delta_{l,l'}$$

A reflection flips the wave-front along the reflection axis

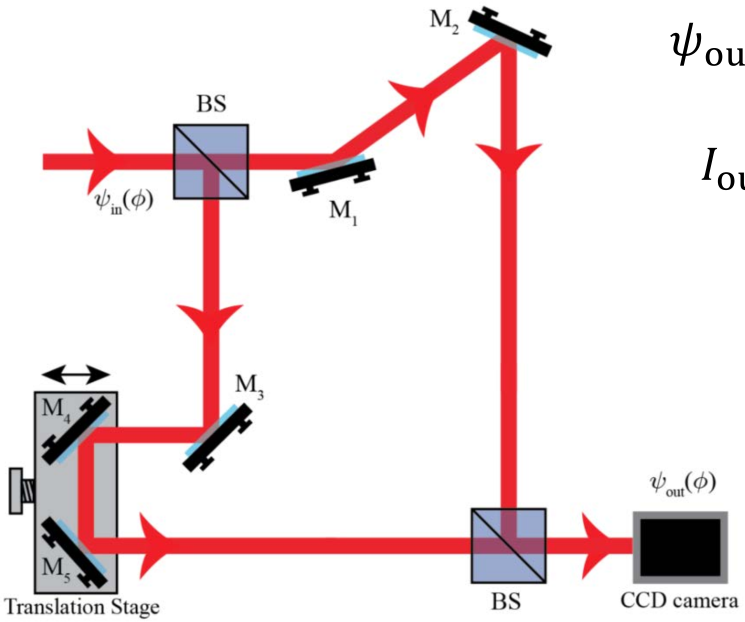




Measuring Orbital Angular Momentum of Light (A new scheme)

Input state in the OAM basis (diagonal mixed state)

$$\psi_{\rm in}(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi} \quad \text{with} \quad \langle \alpha_l \alpha_{l'}^* \rangle_e = S_l \, \delta_{l,l'}$$



$$\psi_{\rm out}(\phi) = \sqrt{\frac{k_1}{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi+i\omega t_1} + \sqrt{\frac{k_2}{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{+il\phi+i\omega t_2}$$

$$I_{\rm out}(\phi) = \langle \psi_{\rm out}(\phi) \psi^*_{\rm out}(\phi) \rangle_e$$

$$= \frac{k_1}{2\pi} + \frac{k_2}{2\pi} + 2\sqrt{k_1 k_2} W(2\phi) \cos\delta$$

where $\delta \equiv \omega(t_1 - t_2)$

$$W(2\phi) \equiv \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l \, e^{-il2\phi}$$

- $W(2\phi)$ is encoded in the interferogram. A single-shot measurement of $I_{\rm out}(\phi)$ yields $W(2\phi)$
- From $W(\Delta\phi)$, S_l can be computed, in a single shot manner. $S_l = \int_{-\pi}^{\pi} W(2\phi) e^{il2\phi} d\phi$
- Still sensitive to background noise and other experimental parameters

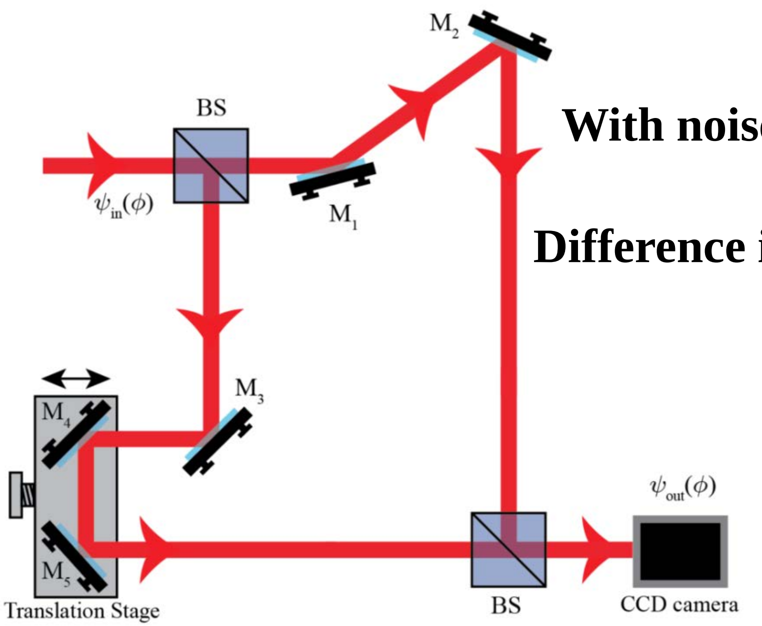
Measuring Orbital Angular Momentum of Light (A new scheme)

Input state in the OAM basis (diagonal mixed state)

$$\psi_{\text{in}}(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{\infty} \alpha_l e^{-il\phi} \quad \text{with} \quad \langle \alpha_l \alpha_{l'}^* \rangle_e = S_l \delta_{l,l'}$$

No noise: $$I_{\text{out}}(\phi) = \frac{k_1 + k_2}{2\pi} + 2\sqrt{k_1 k_2} W(2\phi) \cos\delta$$

With noise: $$I^{\delta}{}_{\text{out}}(\phi) = I^{\delta}{}_{n}(\phi) + \frac{k_1 + k_2}{2\pi} + 2\sqrt{k_1 k_2} W(2\phi) \cos\delta$$



Difference intensity: $$\Delta I_{\text{out}}(\phi) \equiv I^{\delta_c}{}_{\text{out}}(\phi) - I^{\delta_d}{}_{\text{out}}(\phi)$$

$$\Delta I_{\text{out}}(\phi) = \Delta I_{n}(\phi) + 2\sqrt{k_1 k_2} W(2\phi)(\cos\delta_c - \cos\delta_d)$$

If shot-to-shot noise is the same: $\Delta I_{n}(\phi) = 0$ Then:

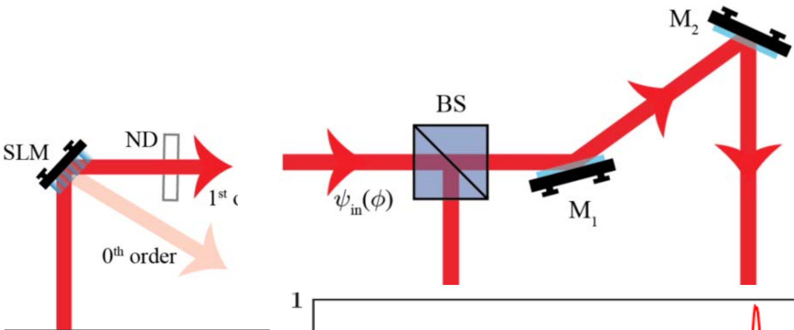
$$\Delta I_{\text{out}}(\phi) = 2\sqrt{k_1 k_2}(\cos\delta_c - \cos\delta_d) W(2\phi) \quad \propto W(2\phi)$$

- $\Delta I_{\text{out}}(\phi)$ has the same functional form as $W(2\phi)$.
- So by measuring $\Delta I_{\text{out}}(\phi)$ the spectrum S_l can be obtained in a single-shot as well as in a noise-insensitive manner

$$S_l = \int_{-\pi}^{\pi} W(2\phi) e^{il2\phi} d\phi$$

G. Kulkarni, R. Sahu, O. S. Magana-Loaiza, R. W. Boyd, and A. K. Jha; **Nature Communications, 8, 1054 (2017)**

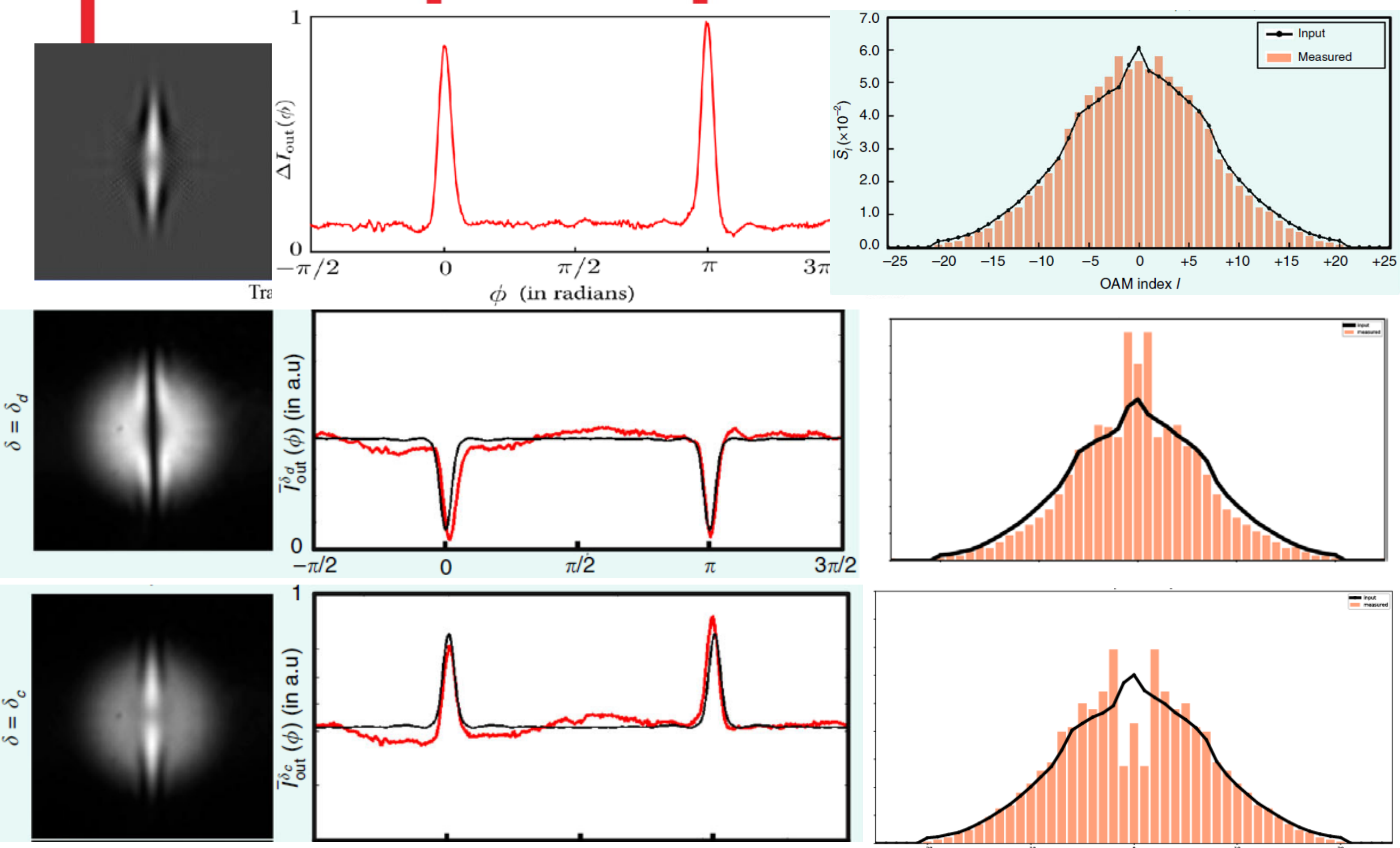
Experimental measurement of OAM spectrum of Light



No noise: $I_{\text{out}}(\phi) = \frac{k_1 + k_2}{2\pi} + 2\sqrt{k_1 k_2} W(2\phi) \cos\delta$

With noise:

$$\Delta I_{\text{out}}(\phi) = 2\sqrt{k_1 k_2}\,(\cos\delta_c - \cos\delta_d) W(2\phi)$$



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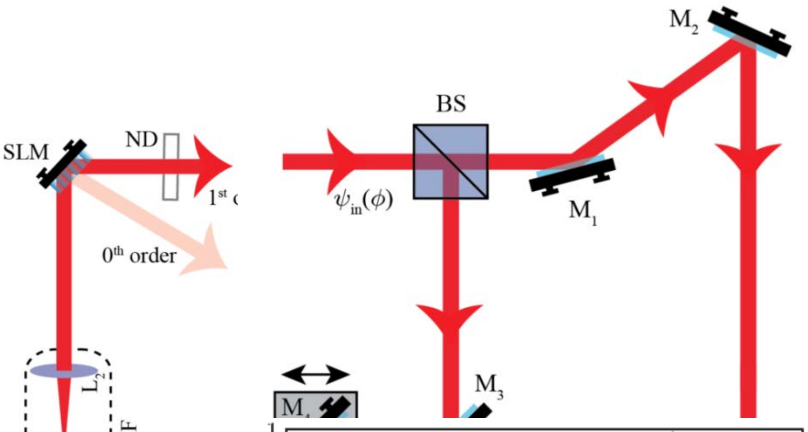

No noise: $I_{\text{out}}(\phi) = \dfrac{k_1 + k_2}{2\pi} + 2\sqrt{k_1 k_2}\, W(2\phi) \cos\delta$

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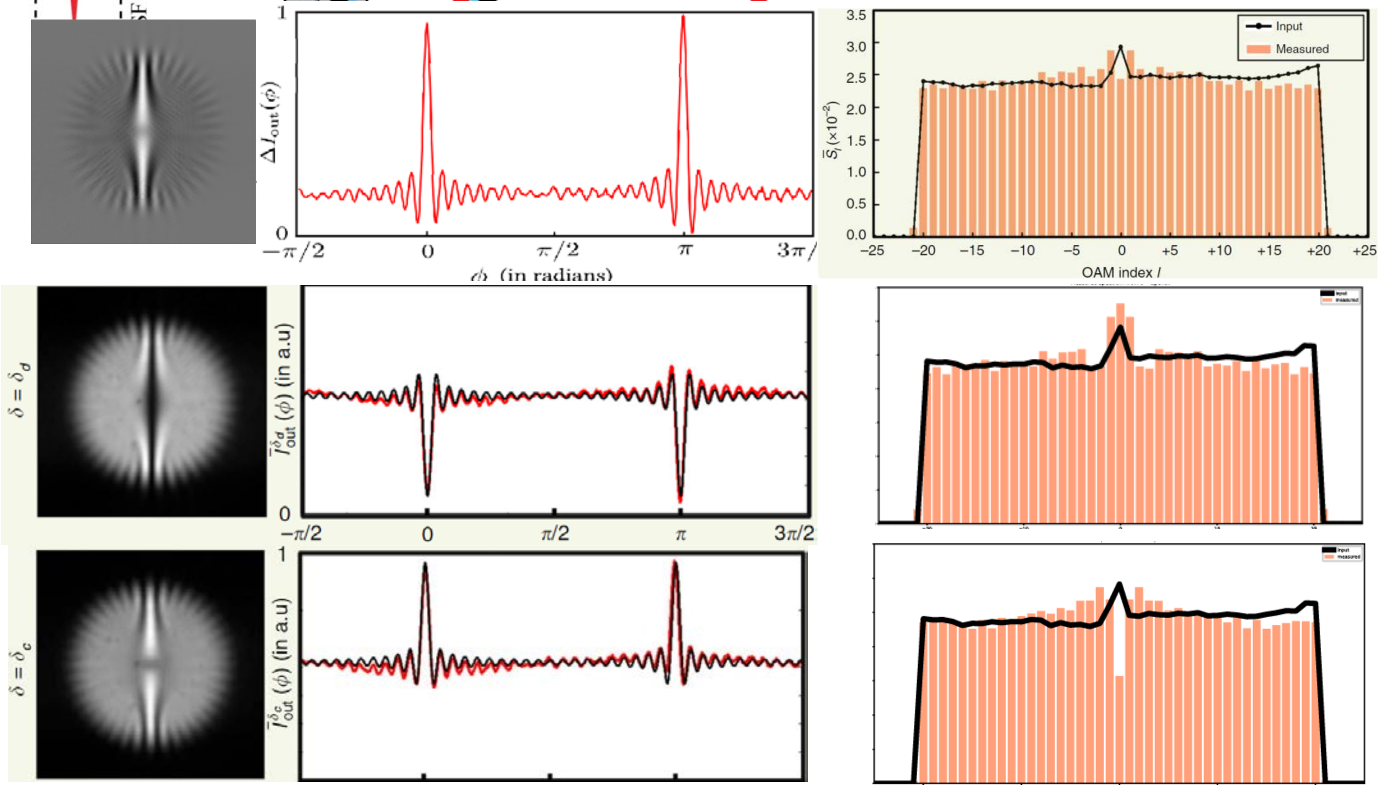
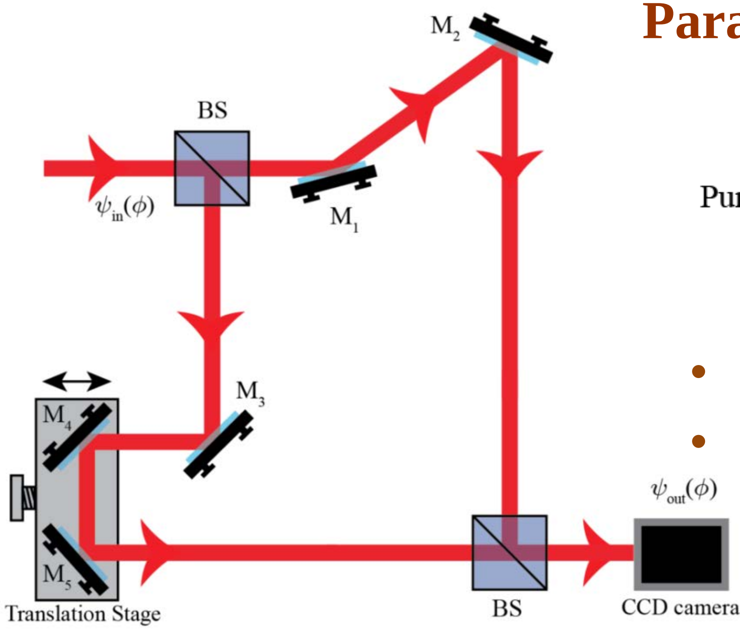
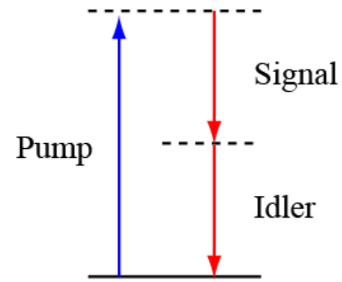


G. Kulkarni, R. Sahu, O. S. Magana-Loaiza, R. W. Boyd, and A. K. Jha; **Nature Communications, 8, 1054 (2017)**

Measuring Orbital Angular Momentum of Light (Quantum)



Parametric down-conversion:



$$|\psi\rangle = \sum_{l=-\infty}^{\infty} \sqrt{S_l}\; |l\rangle_s |-l\rangle_i$$

OAM-Entangled State:

- S_l is called the angular Schmidt spectrum
- Accurate measurement of S_l is very important for quantification of OAM entanglement.
  - The current methods involve coincidence measurements, which is very difficult.

Nature 412 **313** (2001)
Phys Rev A **76**, 042302 (2007)
Phys Rev Lett **104**, 020505 (2010)
New J Phys **14**, 073046 (2012)

The state of the signal photon is

$$\rho_S = \mathrm{tr}_i(\,|\psi\rangle\langle\psi|) = \sum_{l=-\infty}^{\infty} S_l\;\; |l\rangle_{s\;s}\langle l|$$

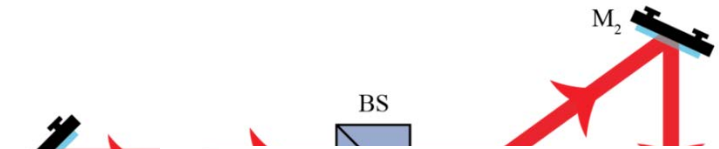
Angular coherence function of the signal photon is

$$W_s(\phi_1, \phi_2) \;\rightarrow W_s(\Delta\phi) \;= \frac{1}{2\pi} \sum_{l\,=-\infty}^{\infty} S_l\; e^{-il\Delta\phi}$$

- The OAM spectrum of the signal photon is same as the angular Schmidt spectrum of the entangled two-photon state.

G. Kulkarni, R. Sahu, O. S. Magana-Loaiza, R. W. Boyd, and A. K. Jha; **Nature Communications, 8, 1054 (2017)**

Measuring Orbital Angular Momentum of Light (Quantum)



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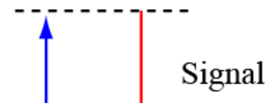

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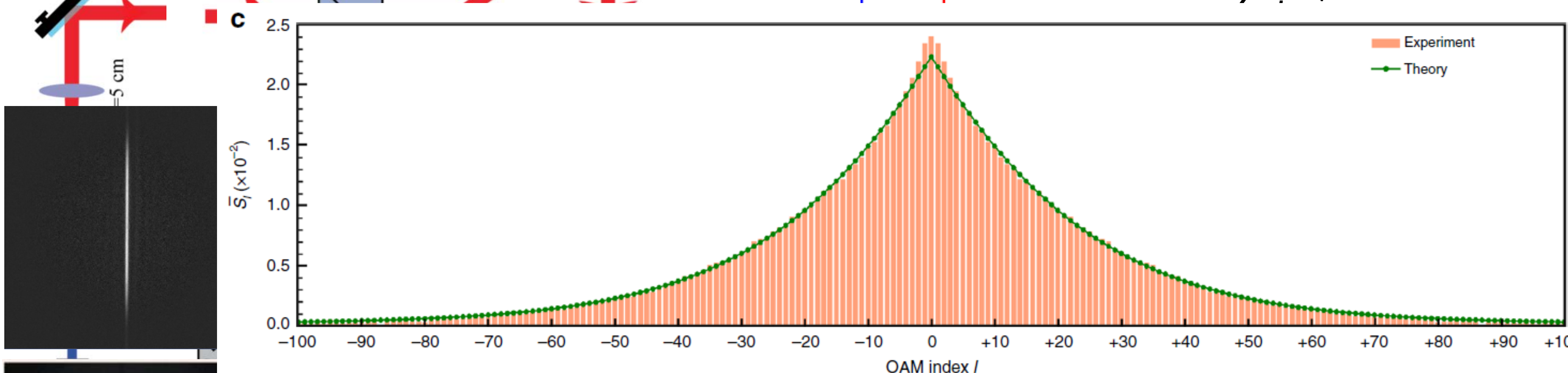
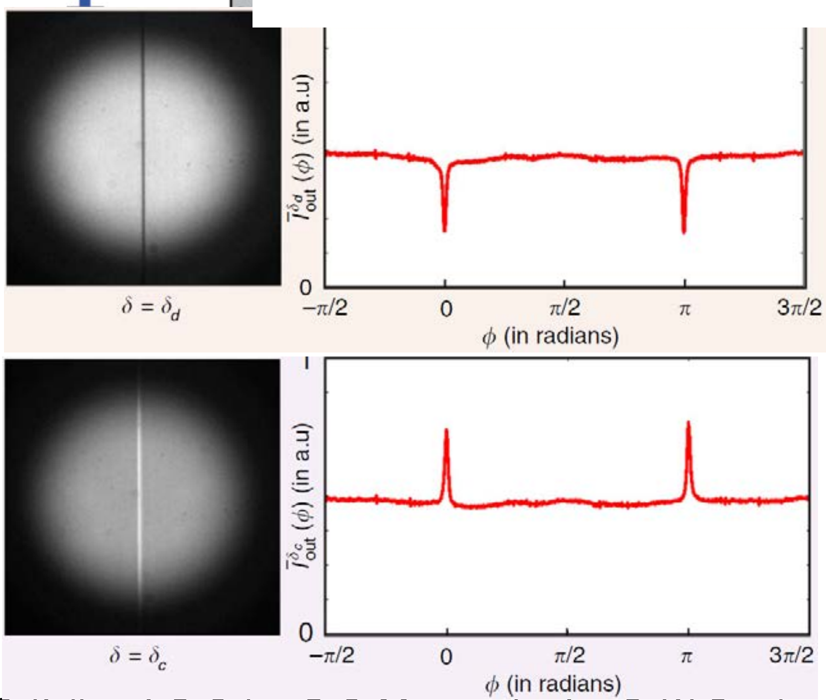
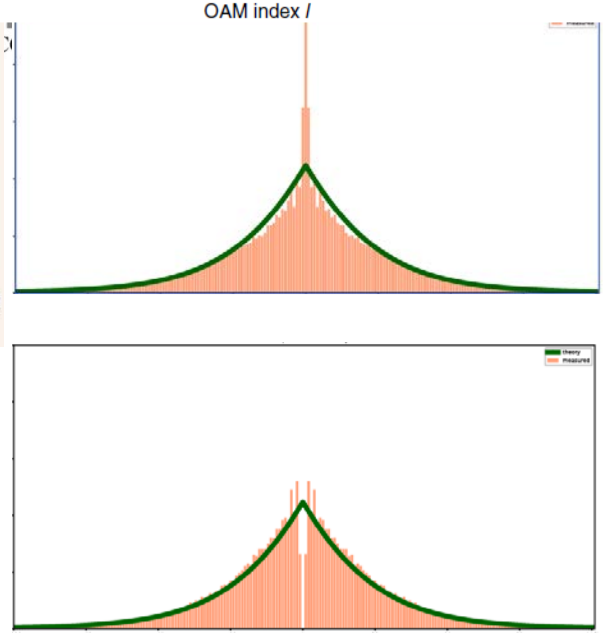






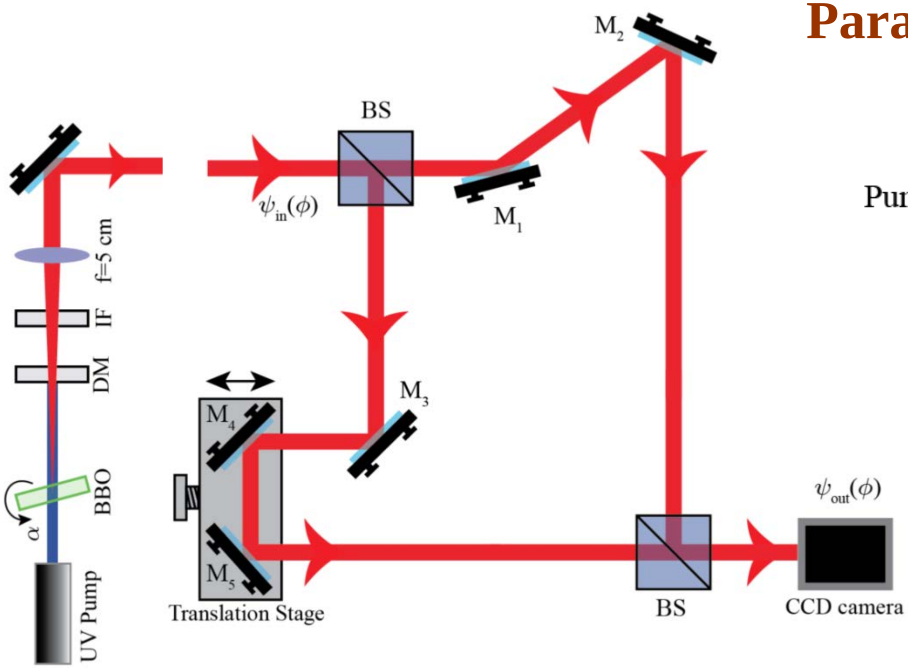
Schmidt Number

$$K = \frac{1}{\sum_l {S_l}^2}$$

$$K = 82.1$$

G. Kulkarni, R. Sahu, O. S. Magana-Loaiza, R. W. Boyd, and A. K. Jha; **Nature Communications, 8, 1054 (2017)**

Measuring Orbital Angular Momentum of Light (Quantum)



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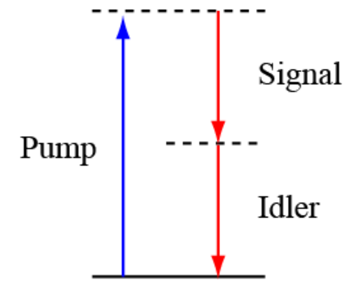
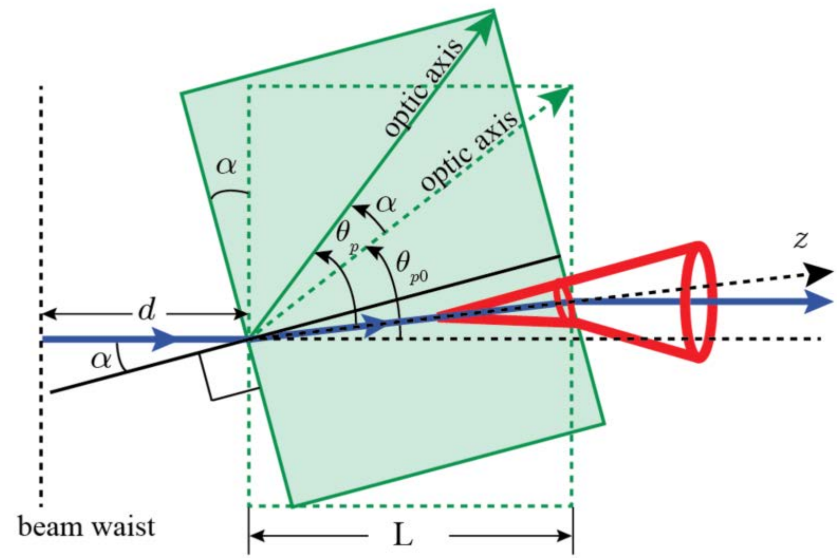


$$|\psi\rangle = \sum_{l=-\infty}^{\infty} \sqrt{S_l}\ |l\rangle_s |-l\rangle_i$$

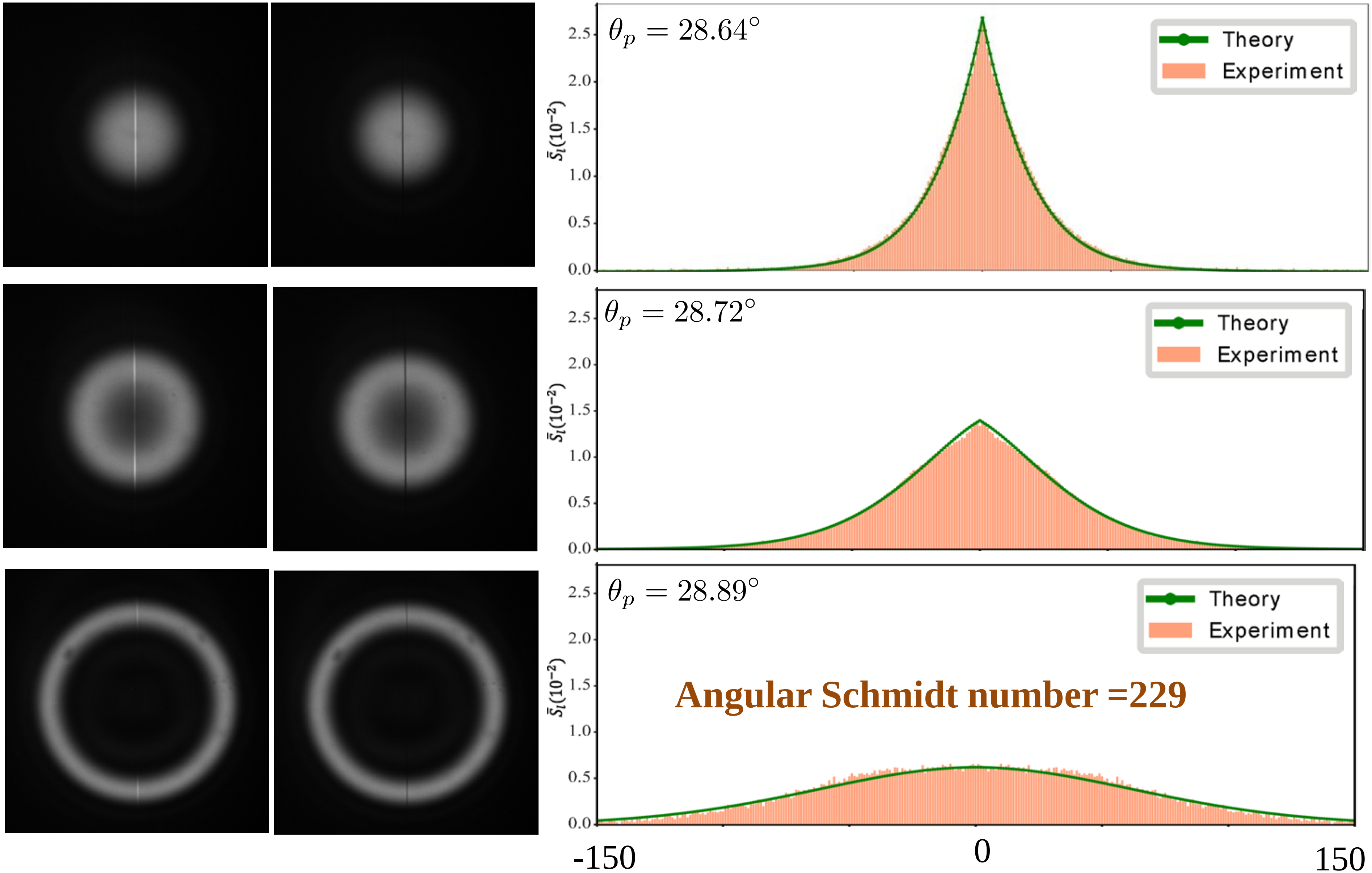
OAM-Entangled State:

$$W_s(\Delta\phi) = \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} S_l\, e^{-il\Delta\phi}$$



G. Kulkarni, S. Aarav, L. Taneja, and A. K. Jha,
Phys. Rev. A 97, 063846 (2018)

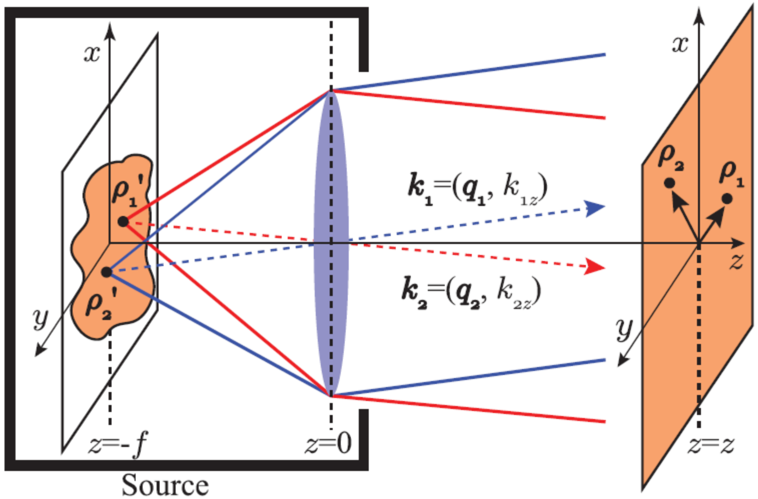
Angular correlation function: Application



G. Kulkarni, S. Aarav, L. Taneja, and A. K. Jha, **Phys. Rev. A 97, 063846 (2018)**

Spatial partially correlated fields

- Partially coherent fields are extremely important for imaging through scattering, etc.



Nature Photonics 6, 355 (2012).

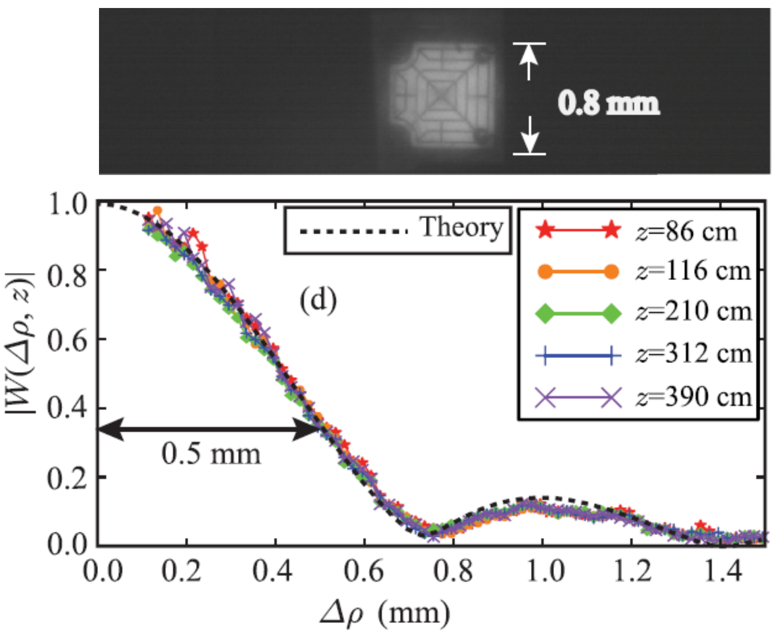
uncorrelated transverse wavevectors

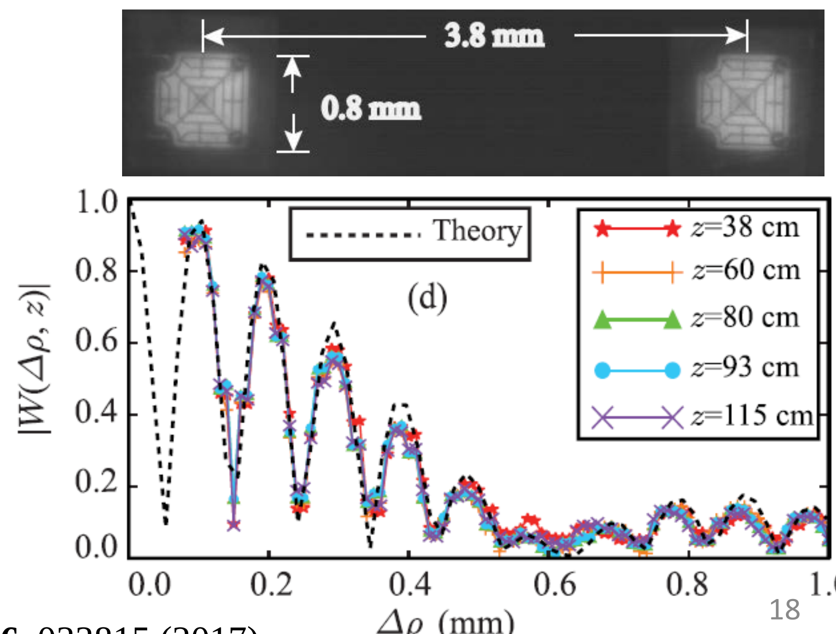
$$\langle a^*(\boldsymbol{q}_1)a(\boldsymbol{q}_2)\rangle_e = I(\boldsymbol{q}_1)\delta(\boldsymbol{q}_1 - \boldsymbol{q}_2)$$

Diagonal mixed state

$$W(\boldsymbol{\Delta\rho}, z) = \int I(\boldsymbol{q})e^{-i\boldsymbol{q}\cdot\boldsymbol{\Delta\rho}}\, d\boldsymbol{q}$$

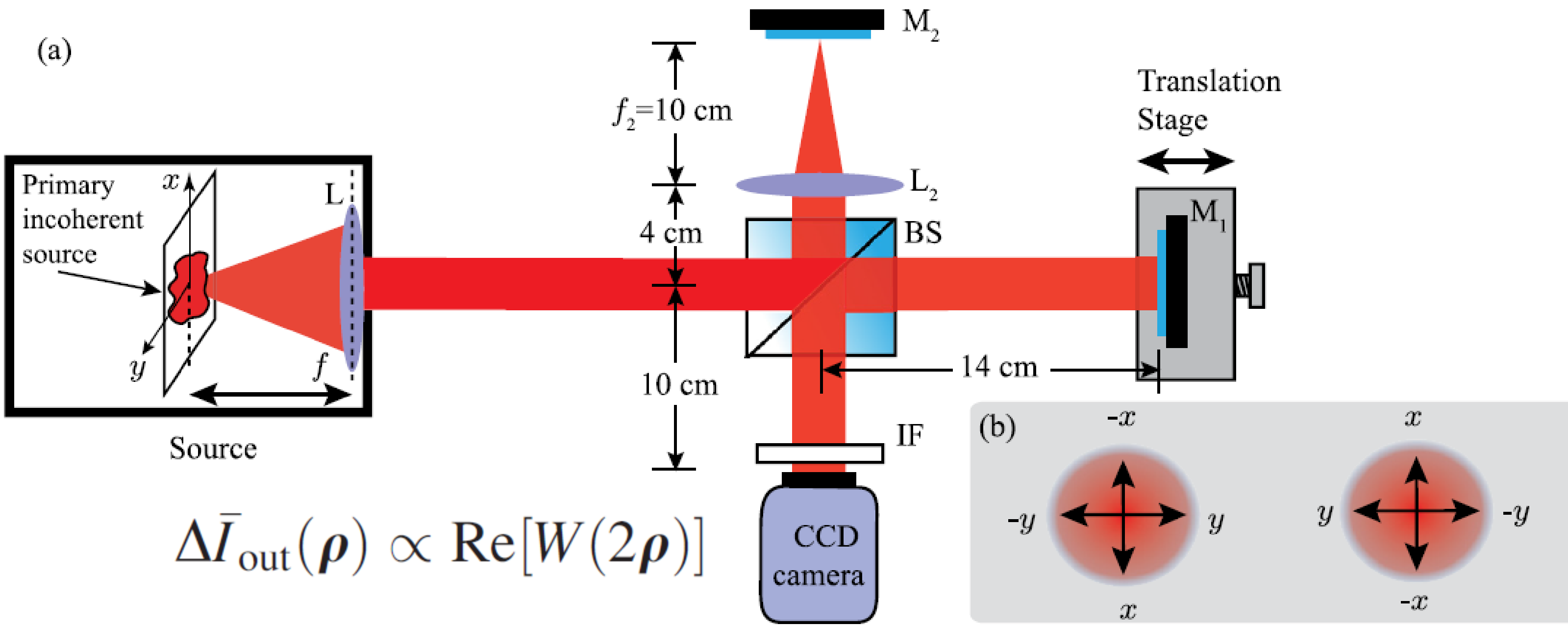
Propagation-invariant spatially partially coherent field



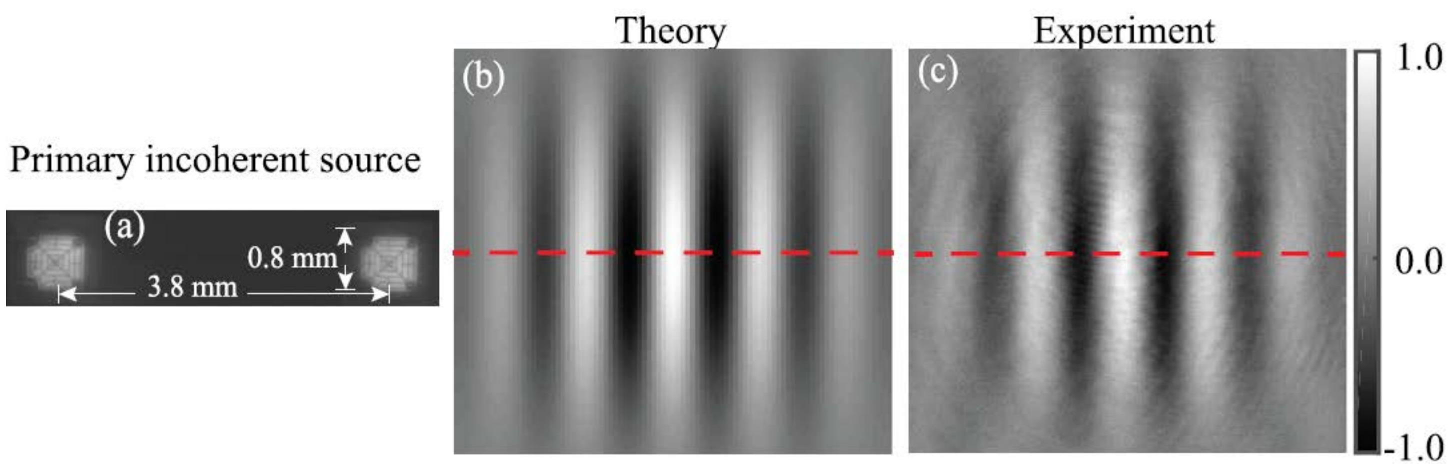


S. Aarav, A. Bhattacharjee, H. Wanare, and A. K. Jha, PRA **96**, 033815 (2017)

Spatial partially correlated fields: efficient measurement

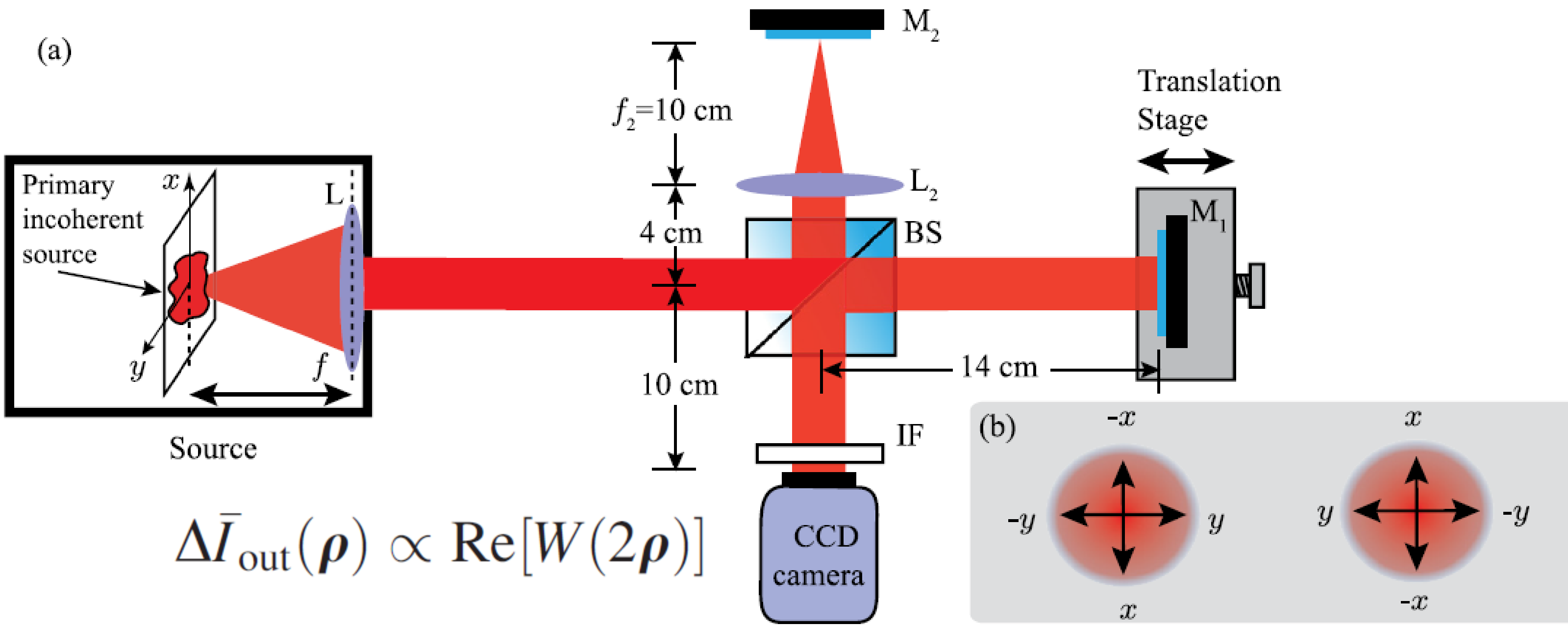


$$\Delta \bar{I}_{\text{out}}(\boldsymbol{\rho}) \propto \text{Re}[W(2\boldsymbol{\rho})]$$

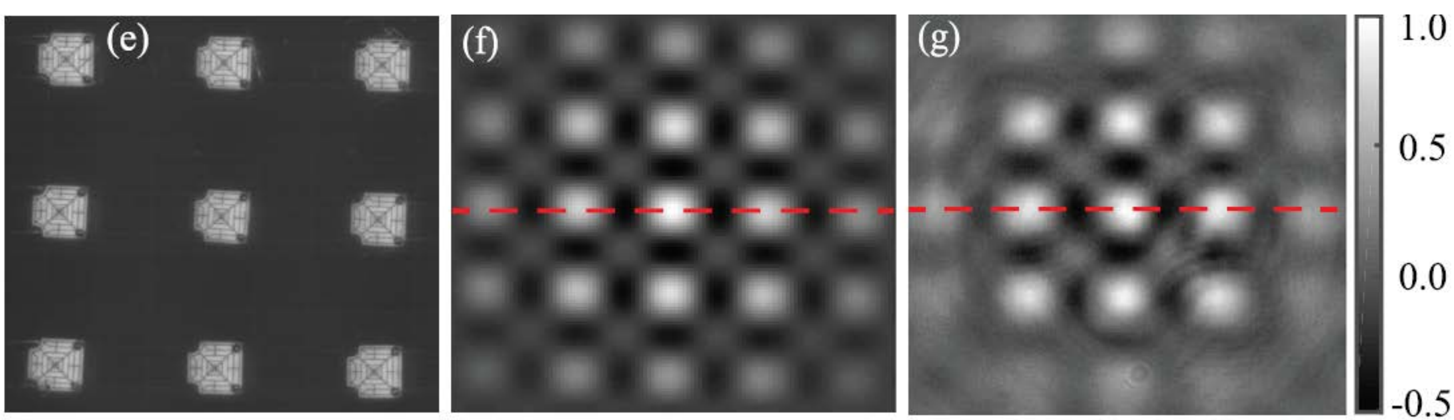


A. Bhattacharjee, S. Aarav, and A. K. Jha, APL **113,** 051102 2018)

Spatial partially correlated fields: efficient measurement

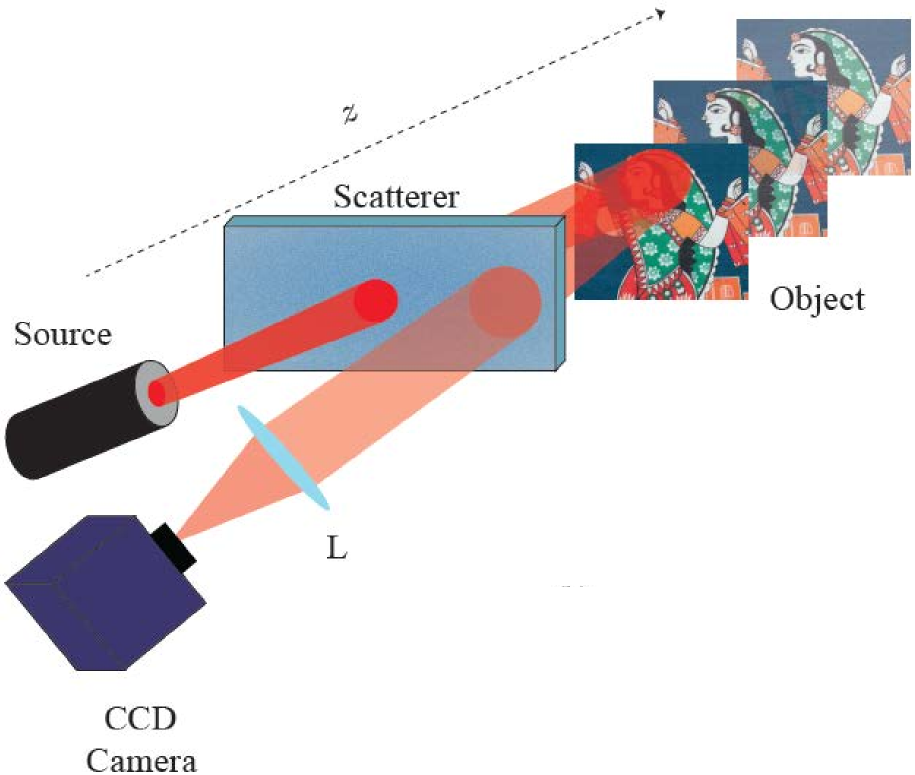


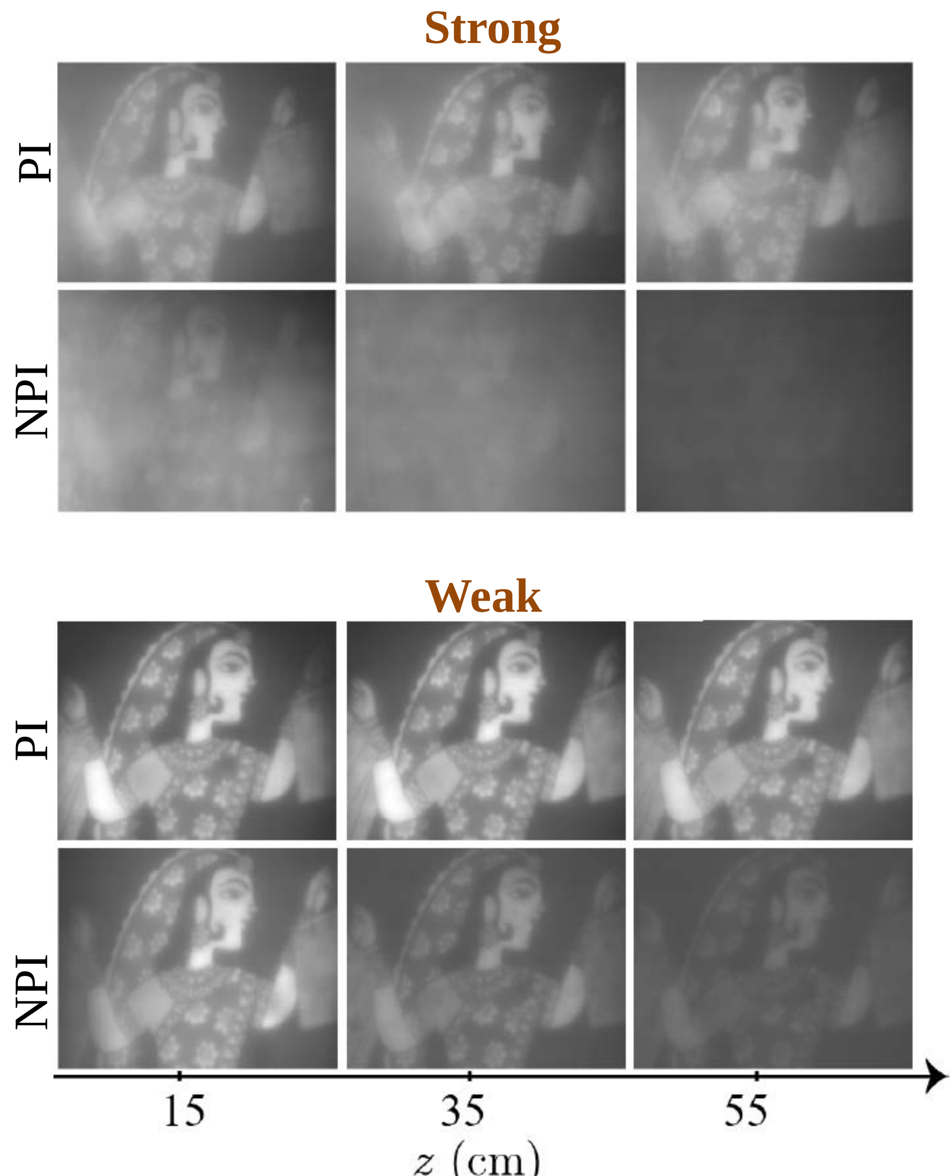
$$\Delta \bar{I}_{\text{out}}(\boldsymbol{\rho}) \propto \text{Re}[W(2\boldsymbol{\rho})]$$



A. Bhattacharjee, S. Aarav, and A. K. Jha, APL **113,** 051102 (2018)

Spatial partially correlated fields: Application





Depth free imaging through scattering medium

A. Bhattacharjee, S. Aarav, and A. K. Jha; **arXiv: 1810.00686**

Conclusions

- Demonstrated a single-shot technique for measuring the angular correlation function.

- For diagonal mixed states, the angular correlation function yields the OAM spectrum through a Fourier transform.

- The technique can be used for measuring the angular Schmidt spectrum of OAM-entangled states in a single-shot manner without requiring coincidence detection.

- The technique can be extended for measuring the spatial correlation functions.

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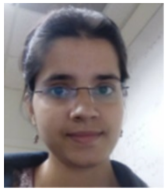

Swati

- Opening for PhD Students (Theory + Experiment)
- Opening for Postdocs (60K p/m **+** ; Deadline 15th Jan 2019)

Thank you for your attention