Entanglement Witnesses 2.0

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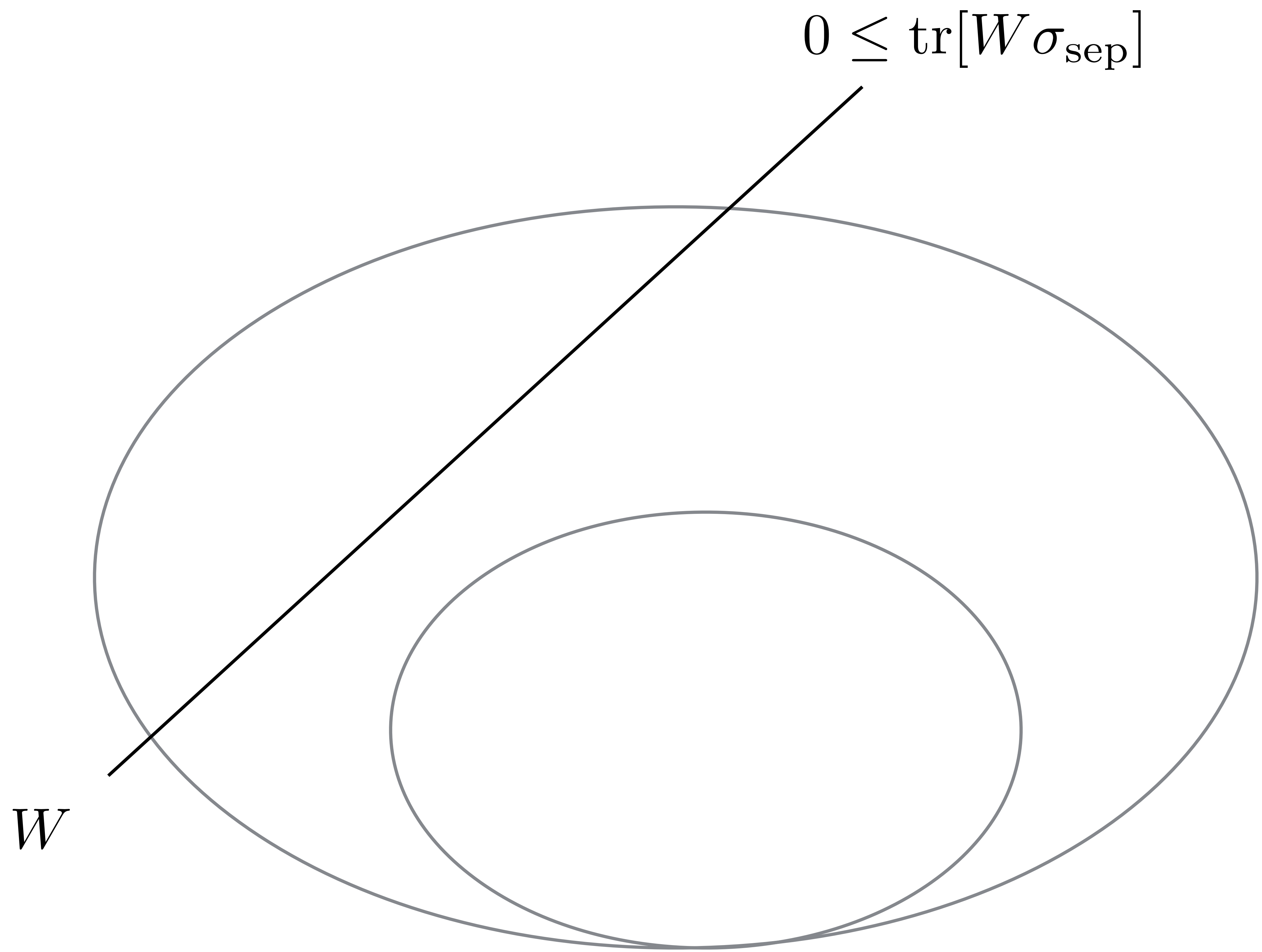
Some results in preparation, work in progress

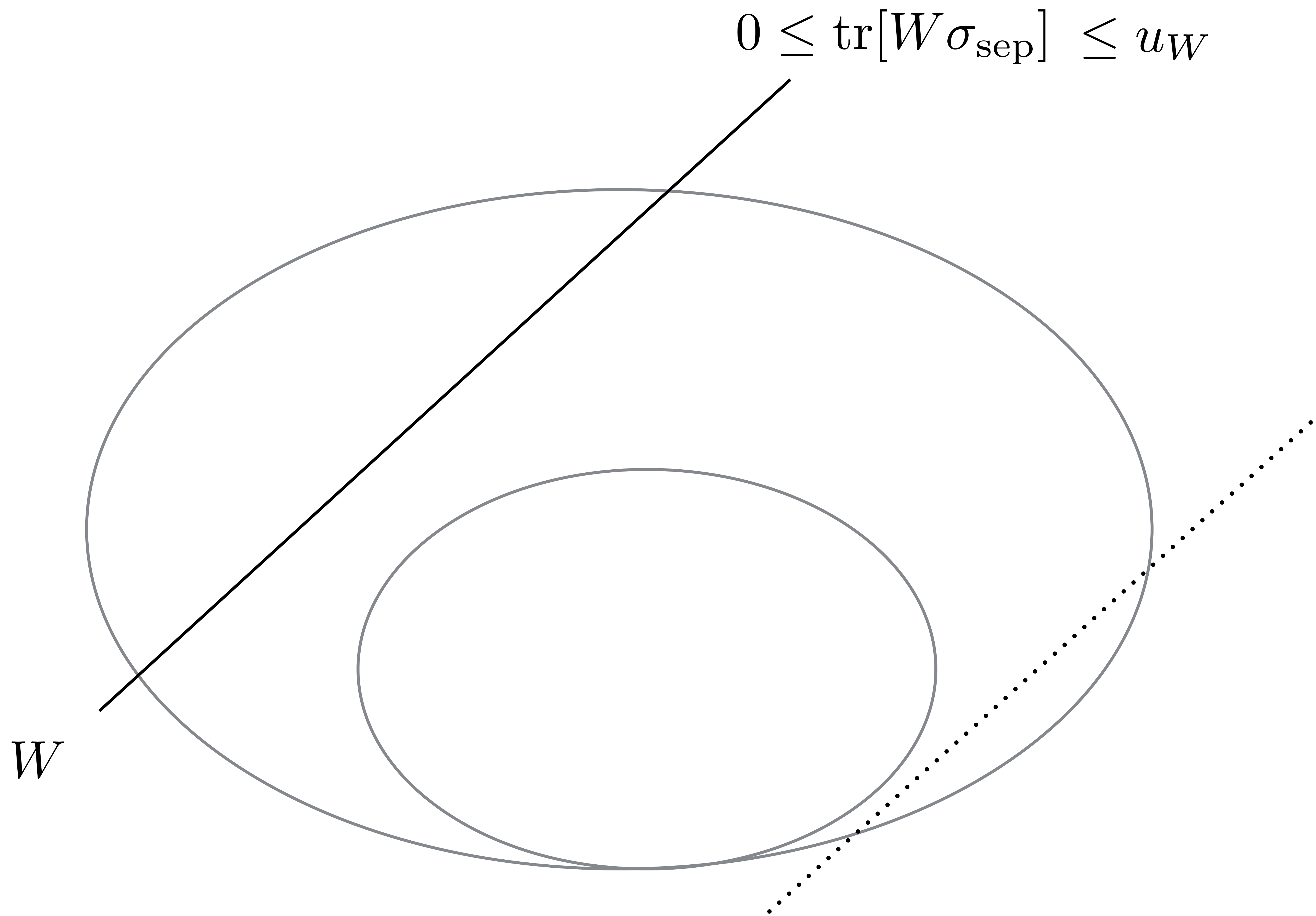
arXiv: 1803.02708
Linking Entanglement Detection and State Tomography via Quantum 2-Designs

arXiv: 1811.09896
Entanglement Witnesses 2.0: Compressed Entanglement Witnesses

Take-home message



Take-home message



Introduction: Entanglement Witnesses (EWs)

Main Question: Entanglement Detection vs. Quantum State Tomography

Our contribution : EWs are more useful than we thought

Theoretical parts: Many hyperplanes can be generated

Experimental proposal: Entanglement detection can be tested many times

Discussions: I no longer need Positive Maps to construct EWs.

Applications : MUBs, the conjecture in d=6, etc.

On-going directions and questions

Entanglement Theory

SEP $\rho_{12} = \sum_i p_i \rho_i^{(1)} \otimes \rho_i^{(2)}$

ENT $\rho_{12} \neq \sum_i p_i \rho_i^{(1)} \otimes \rho_i^{(2)}$

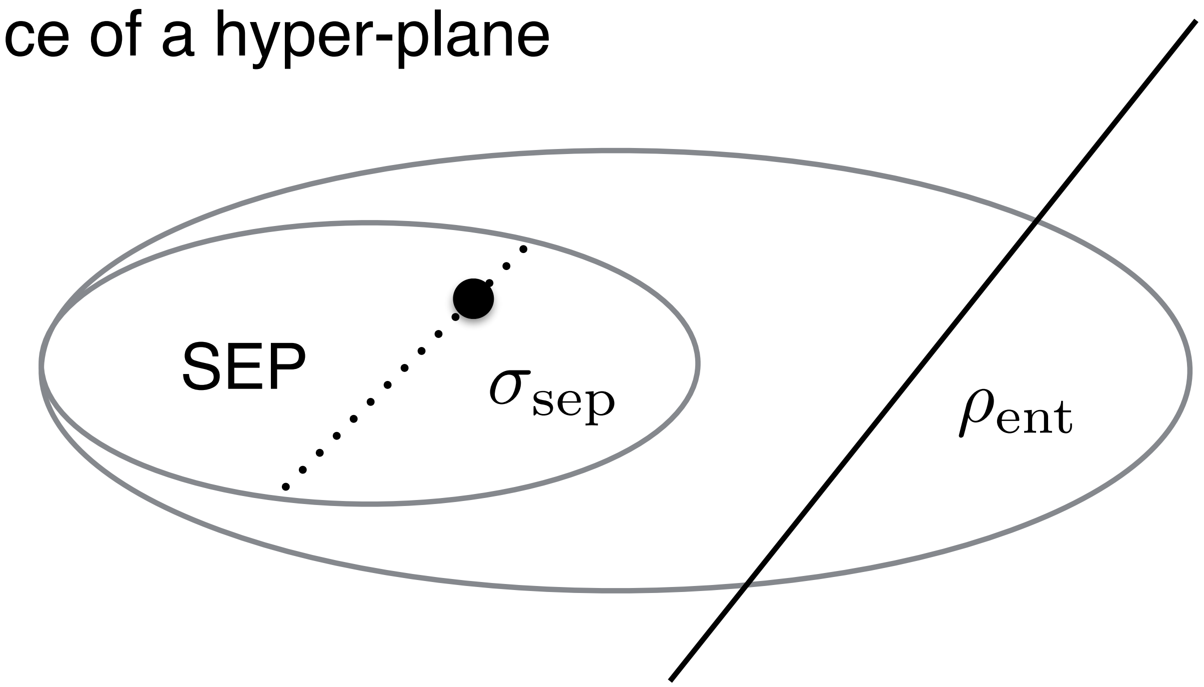
Entanglement Witnesses (EWs)

$$\mathrm{tr}[W\rho_{\mathrm{ent}}] < 0$$

$$\mathrm{tr}[W\sigma_{\mathrm{sep}}] \geq 0 \quad \forall \sigma_{\mathrm{sep}}$$

Hahn-Banach theorem: existence of a hyper-plane

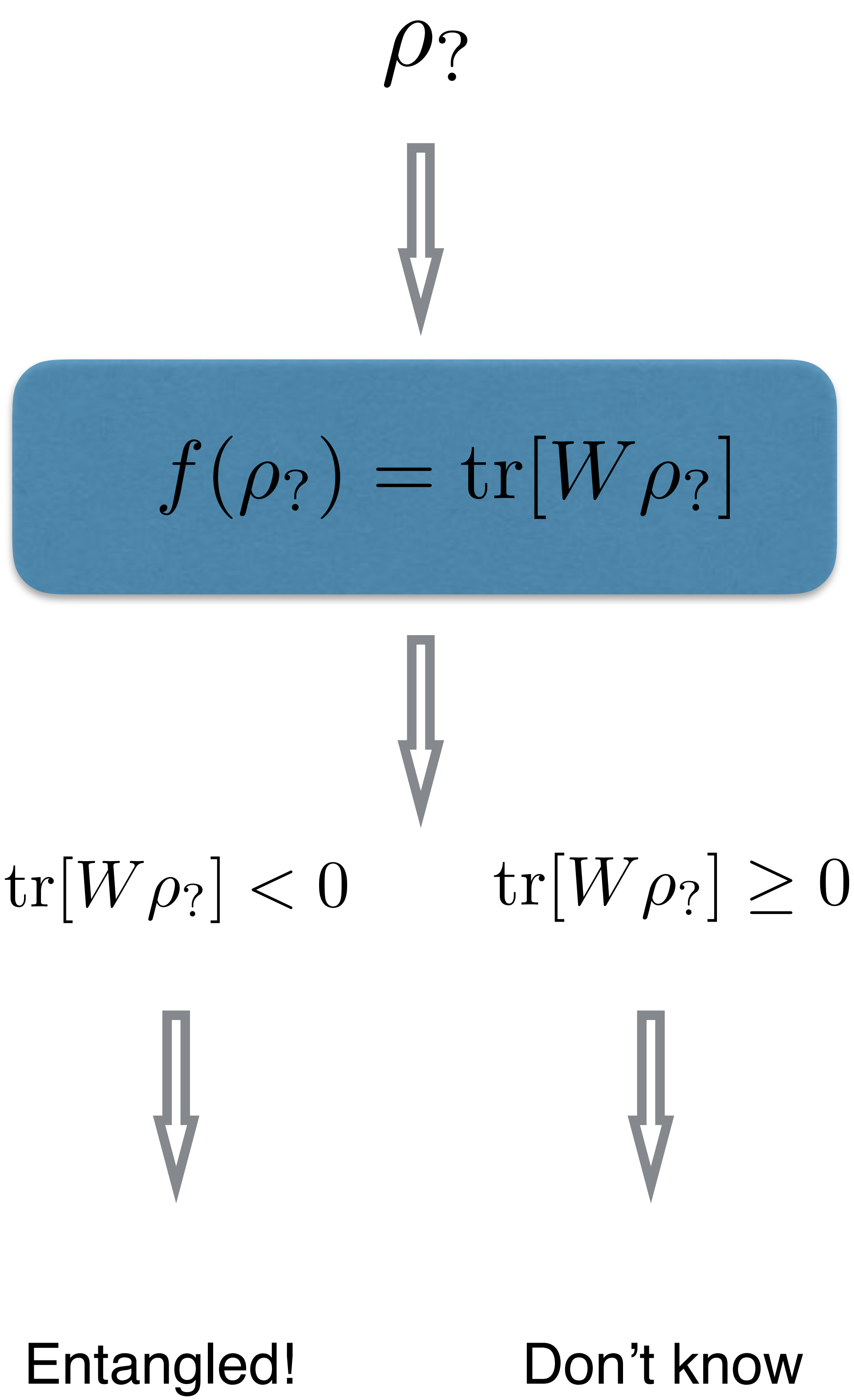
$\exists W$



Entanglement Witnesses (EWs): Hermitian Operators, non-positive $W = W^\dagger \ngeq 0$

ρ must be entangled if $\mathrm{tr}[W\rho] < 0$

The theoretical detection box



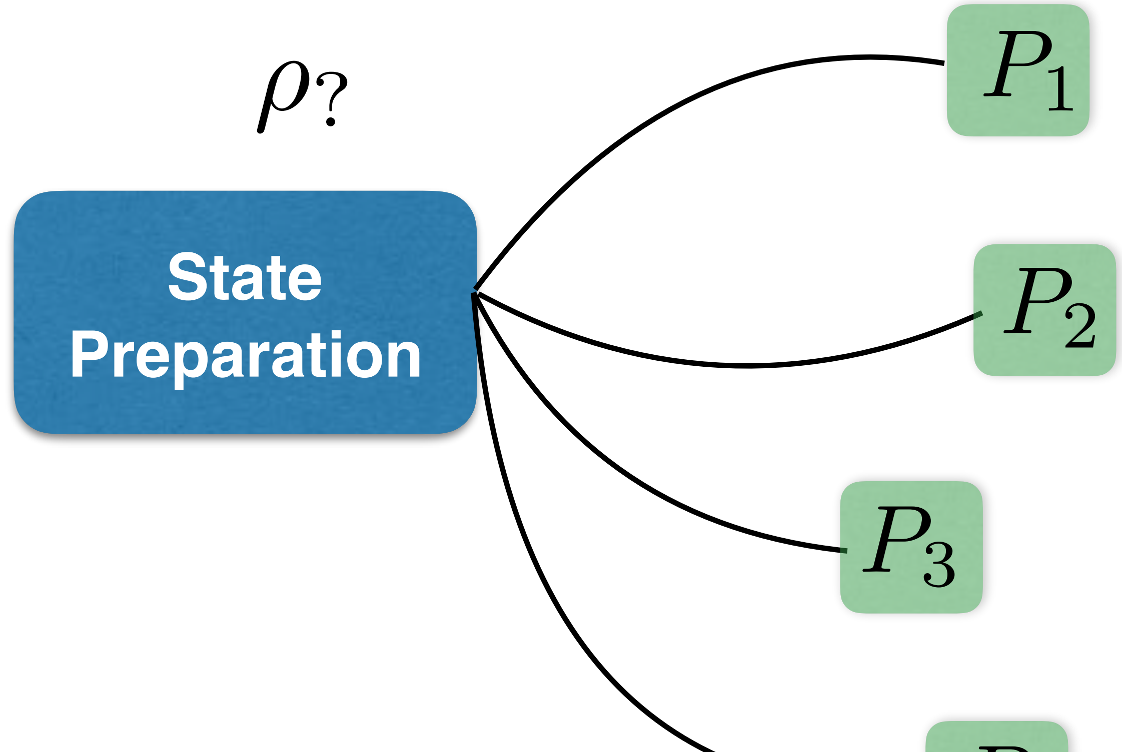
The detection box in reality

$$W = W^\dagger$$

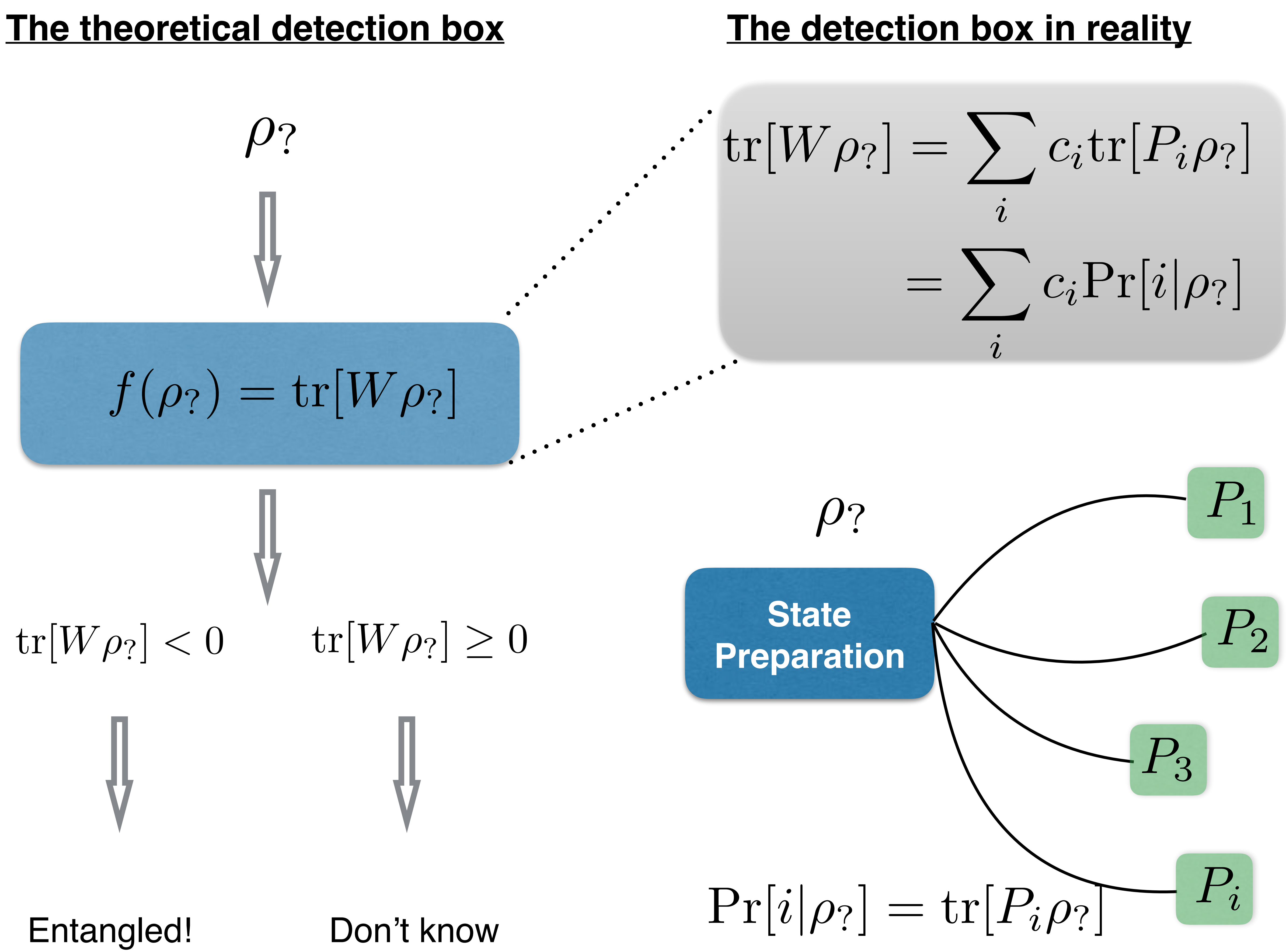
Positive decomposition $W = \sum_{i=1}^{n} c_i P_i$

$$\sum_{i=1}^{n} P_i = I \quad P_i \geq 0 \quad P_i : \ \mathrm{POVM}$$

corresponding to a description of a detecto



$$\Pr[i|\rho_?] = \mathrm{tr}[P_i \rho_?]$$



The theoretical detection box
The detection box in reality
$\rho_?$
$f(\rho_?) = \mathrm{tr}[W\rho_?]$
$\mathrm{tr}[W\rho_?] < 0$
$\mathrm{tr}[W\rho_?] \geq 0$
Entangled!
Don’t know
$\mathrm{tr}[W\rho_?] = \sum_i c_i \mathrm{tr}[P_i\rho_?]$
$= \sum_i c_i \mathrm{Pr}[i|\rho_?]$
$\rho_?$
State Preparation
P_1
P_2
P_3
P_i
$\mathrm{Pr}[i|\rho_?] = \mathrm{tr}[P_i\rho_?]$

Detectors for Entanglement Detection? can they be for QST?

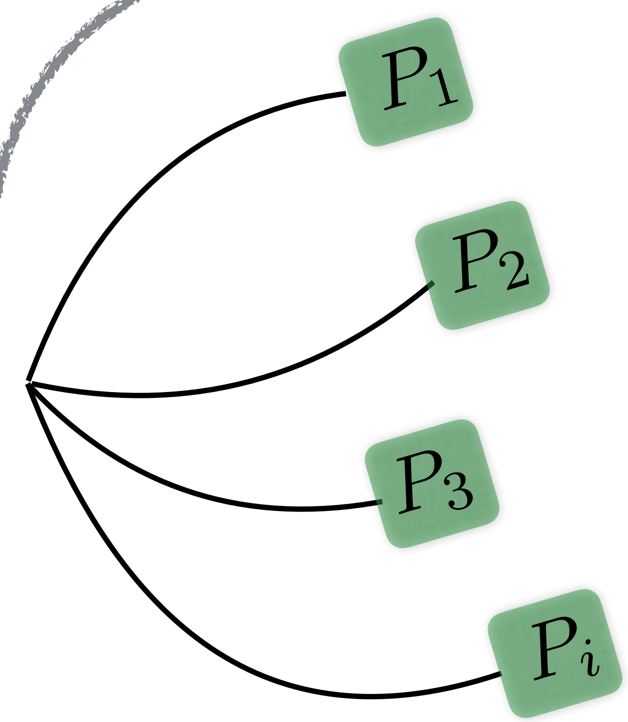
$$\rho_?$$

$$\mathrm{tr}[W\rho_?] = \sum_i c_i \Pr[i|\rho_?]$$

$$\mathrm{tr}[W\rho_?] < 0 \qquad \mathrm{tr}[W\rho_?] \geq 0$$

Entangled! Don't know

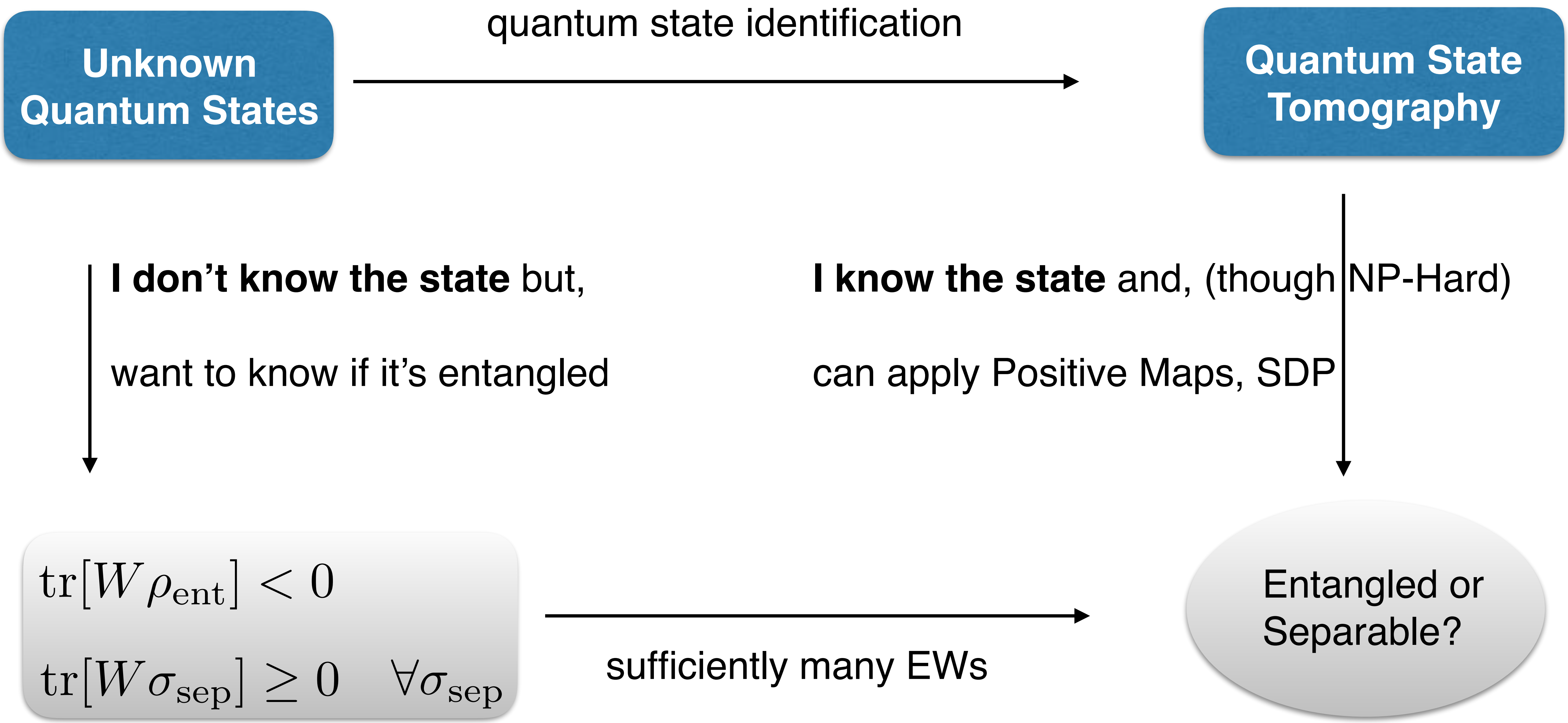
<u>Resources : classical post processing is free</u>



$$\Pr[i|\rho_?] = \mathrm{tr}[P_i\rho_?]$$

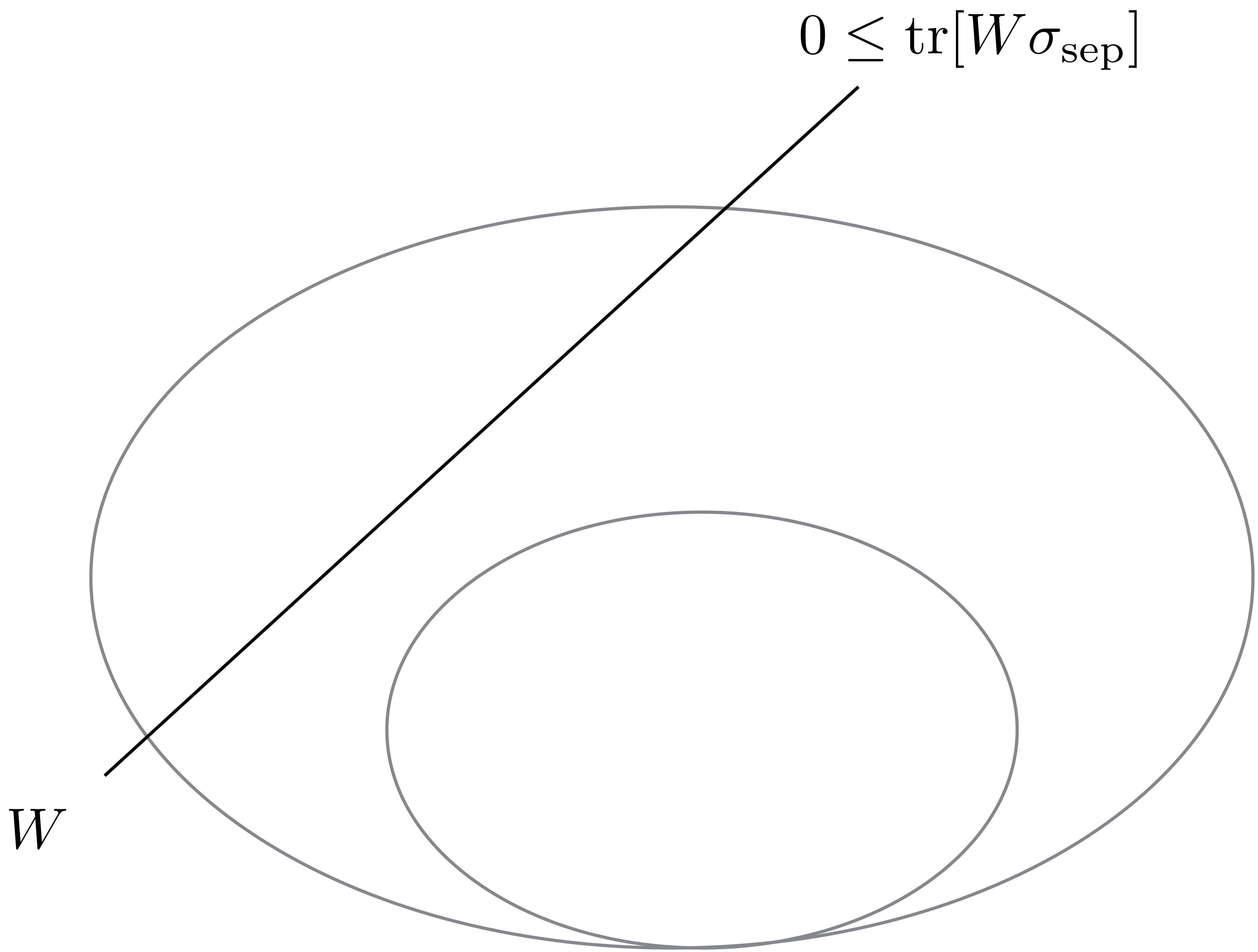
$$\sum_i c_i \Pr[i|\rho_?]$$

<u>General Picture of Entanglement Detection</u>



Advantage of EWs: Entanglement of unknown states can be detected

Is an EW really useful?



Result 1 : EWs to EWs to EWs …

EXPERIMENT FOR ESTIMATION

$$p^*/d^2 \leq \mathrm{tr}[\widetilde{W}\sigma_{\mathrm{sep}}] \leq q^*/d^2$$

$$\mathrm{tr}[W^{(-)}\sigma_{\mathrm{sep}}] \geq 0$$

$$\mathrm{tr}[W^{(+)}\sigma_{\mathrm{sep}}] \geq 0$$

Structural Physical Approximation (SPA)

$$\widetilde{W} = (1-p^*)W + p^*\frac{I\otimes I}{d^2} \qquad p^* = \min_{\widetilde{W}\geq 0} p \qquad p\in[0,1]$$

Detection scheme $\mathrm{tr}[W\sigma_{\mathrm{sep}}] \geq 0 \iff \mathrm{tr}[\widetilde{W}\sigma_{\mathrm{sep}}] \geq p^*/d^2$

Proof. $$\mathrm{tr}[\widetilde{W}\sigma_{\mathrm{sep}}] = (1-p^*)\underbrace{\mathrm{tr}[W\sigma_{\mathrm{sep}}]}_{\geq 0} + p^*\mathrm{tr}[\frac{I\otimes I}{d^2}\sigma_{\mathrm{sep}}]$$

$$\geq p^*\mathrm{tr}[\frac{I\otimes I}{d^2}\sigma_{\mathrm{sep}}] \quad = p^*/d^2$$

positive and negative Structural Physical Approximation (SPA)

pSPA

$$\widetilde{W}^{(+)} = \underbrace{(1-p^*)}_{\geq 0} W + p^* \frac{I \otimes I}{d^2}$$

$$p^* = \min_{\widetilde{W}^{(+)} \geq 0} p$$

$$p \in [0,1]$$

Detection scheme $\mathrm{tr}[W\sigma_{\mathrm{sep}}] \geq 0 \iff \mathrm{tr}[\widetilde{W}^{(+)}\sigma_{\mathrm{sep}}] \geq p^*/d^2$

nSPA

$$\widetilde{W}^{(-)} = \underbrace{(1-q^*)}_{\leq 0} W + q^* \frac{I \otimes I}{d^2}$$

$$q^* = \min_{\widetilde{W}^{(-)} \geq 0} q$$

$$q > 1$$

Detection scheme $\mathrm{tr}[W\sigma_{\mathrm{sep}}] \geq 0 \iff \mathrm{tr}[\widetilde{W}^{(-)}\sigma_{\mathrm{sep}}] \leq q^*/d^2$

Proof.

$$\mathrm{tr}[\widetilde{W}^{(-)}\sigma_{\mathrm{sep}}] = \underbrace{(1-q^*)}_{\leq 0}\underbrace{\mathrm{tr}[W\sigma_{\mathrm{sep}}]}_{\geq 0} + q^*\mathrm{tr}[\frac{I \otimes I}{d^2}\sigma_{\mathrm{sep}}]$$

$$\leq q^*\mathrm{tr}[\frac{I \otimes I}{d^2}\sigma_{\mathrm{sep}}] = q^*/d^2$$

positive and negative Structural Physical Approximation (SPA)

pSPA $\widetilde{W}^{(+)} = (1-p^*)W + p^* \frac{I \otimes I}{d^2}$ $p^* = \min_{\widetilde{W}^{(+)} \geq 0} p$ $p \in [0,1]$

≥ 0

nSPA $\widetilde{W}^{(-)} = (1-q^*)W + q^* \frac{I \otimes I}{d^2}$ $q^* = \min_{\widetilde{W}^{(-)} \geq 0} q$ $q > 1$

≤ 0

Detection scheme $\mathrm{tr}[W\sigma_{\mathrm{sep}}] \geq 0$ $\mathrm{tr}[\widetilde{W}^{(-)}\sigma_{\mathrm{sep}}] \leq q^*/d^2$

$\mathrm{tr}[\widetilde{W}^{(+)}\sigma_{\mathrm{sep}}] \geq p^*/d^2$

positive and negative Structural Physical Approximation (SPA)

pSPA $\widetilde{W}^{(+)} = (1-p^*)W + p^* \frac{I \otimes I}{d^2}$ $\quad p^* = \min_{\widetilde{W}^{(+)} \geq 0} p$ $\quad p \in [0,1]$

$(1-p^*) \geq 0$

nSPA $\widetilde{W}^{(-)} = (1-q^*)W + q^* \frac{I \otimes I}{d^2}$ $\quad q^* = \min_{\widetilde{W}^{(-)} \geq 0} q$ $\quad q > 1$

$(1-q^*) \leq 0$

Given W → $\widetilde{W}^{(+)}$, $\widetilde{W}^{(-)}$

$\mathrm{tr}[W\sigma_{\mathrm{sep}}] \geq 0$ ⇔ $\mathrm{tr}[\widetilde{W}^{(-)}\sigma_{\mathrm{sep}}] \leq q^*/d^2$

$\mathrm{tr}[W\sigma_{\mathrm{sep}}] \geq 0$ ⇔ $\mathrm{tr}[\widetilde{W}^{(+)}\sigma_{\mathrm{sep}}] \geq p^*/d^2$

Question: does the converse work? $\widetilde{W} = \widetilde{W}^{(+)} = \widetilde{W}^{(-)}$ → $W^{(+)}$, $W^{(-)}$

positive and negative Structural Physical Approximation (SPA)

Conversely! $\widetilde{W} = \widetilde{W}^{(+)} = \widetilde{W}^{(-)}$ → $W^{(+)}$, $W^{(-)}$

pSPA

$$p^* = \min_{\widetilde{W}^{(+)} \geq 0} p$$

$$p \in [0, 1]$$

$$\widetilde{W}^{(+)} = \underbrace{(1 - p^*)}_{\geq 0} W + p^* \frac{I \otimes I}{d^2}$$

$$\widetilde{W} = \widetilde{W}^{(+)} = (1 - p^*) W^{(+)} + p^* \frac{I \otimes I}{d^2}$$

$$\mathrm{tr}[W^{(+)} \sigma_{\mathrm{sep}}] \geq 0 \iff \mathrm{tr}[\widetilde{W} \sigma_{\mathrm{sep}}] \geq p^*/d^2$$

nSPA

$$q^* = \min_{\widetilde{W}^{(-)} \geq 0} q$$

$$q > 1$$

$$\widetilde{W}^{(-)} = \underbrace{(1 - q^*)}_{\leq 0} W + q^* \frac{I \otimes I}{d^2}$$

$$\widetilde{W} = \widetilde{W}^{(-)} = (1 - q^*) W^{(-)} + q^* \frac{I \otimes I}{d^2}$$

$$\mathrm{tr}[W^{(-)} \sigma_{\mathrm{sep}}] \geq 0 \iff \mathrm{tr}[\widetilde{W} \sigma_{\mathrm{sep}}] \leq q^*/d^2$$

positive and negative Structural Physical Approximation (SPA)

$$\mathrm{tr}[W^{(+)}\sigma_{\mathrm{sep}}] \geq 0 \iff \mathrm{tr}[\widetilde{W}\sigma_{\mathrm{sep}}] \geq p^*/d^2$$

$$\mathrm{tr}[W^{(-)}\sigma_{\mathrm{sep}}] \geq 0 \iff \mathrm{tr}[\widetilde{W}\sigma_{\mathrm{sep}}] \leq q^*/d^2$$

Detection scheme : Detecting entanglement TWICE

EXPERIMENT FOR ESTIMATION

$$p^*/d^2 \leq \mathrm{tr}[\widetilde{W}\sigma_{\mathrm{sep}}] \leq q^*/d^2$$

$$\mathrm{tr}[W^{(-)}\sigma_{\mathrm{sep}}] \geq 0$$

$$\mathrm{tr}[W^{(+)}\sigma_{\mathrm{sep}}] \geq 0$$

On the level of standard EWs

EXPERIMENT FOR ESTIMATION

$$p^*/d^2 \leq \mathrm{tr}[\widetilde{W}\sigma_{\mathrm{sep}}] \leq q^*/d^2$$

$\mathrm{tr}[W^{(+)}\sigma_{\mathrm{sep}}] \geq 0$ $\mathrm{tr}[W^{(-)}\sigma_{\mathrm{sep}}] \geq 0$

⇕

EXPERIMENT FOR ESTIMATION

$$0 \leq \mathrm{tr}[W^{(-)}\sigma_{\mathrm{sep}}] \leq \frac{1}{d^2}\frac{q^* - p^*}{q^* - 1}$$

$\mathrm{tr}[W^{(+)}\sigma_{\mathrm{sep}}] \geq 0$

arXiv.org > quant-ph > arXiv:1811.09896

Entanglement Witness 2.0: Compressed Entanglement Witnesses

Joonwoo Bae, Dariusz Chruściński, Beatrix C. Hiesmayr

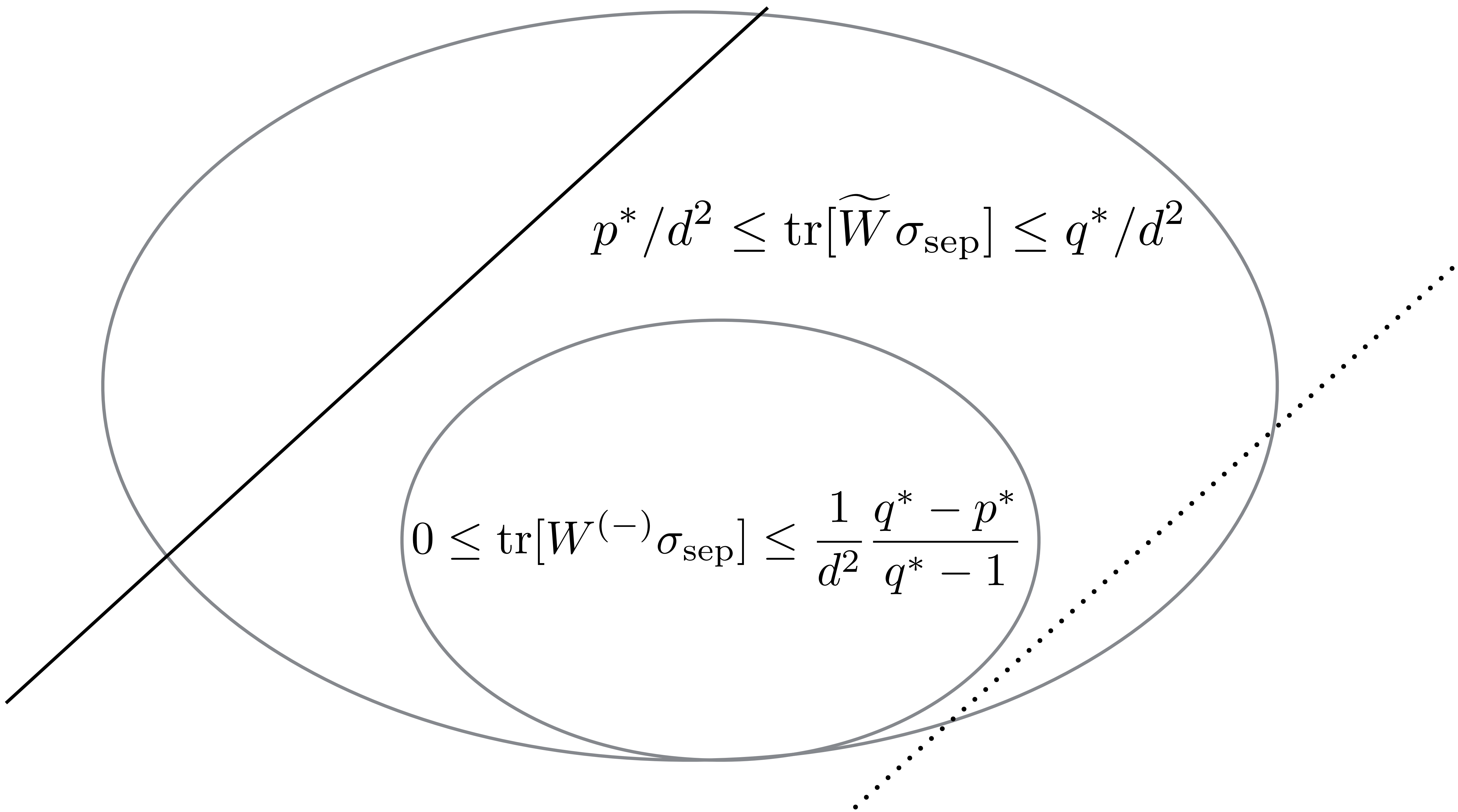
(Submitted on 24 Nov 2018)

An entanglement witness is an observable detecting entanglement for a subset of states. We present a framework that makes an entanglement witness twice as powerful due to the general existence of a second (lower) bound, in addition to the (upper) bound of the very definition. This second bound, if non-trivial, is violated by another subset of entangled states. Differently stated, we prove via the structural physical approximation that two witnesses can be compressed into a single one. Consequently, our framework shows that any entanglement witness can be upgraded to a witness 2.0. The generality and its power are demonstrate by applications to bipartite and multipartite qubit/qudit systems.

Remarks.

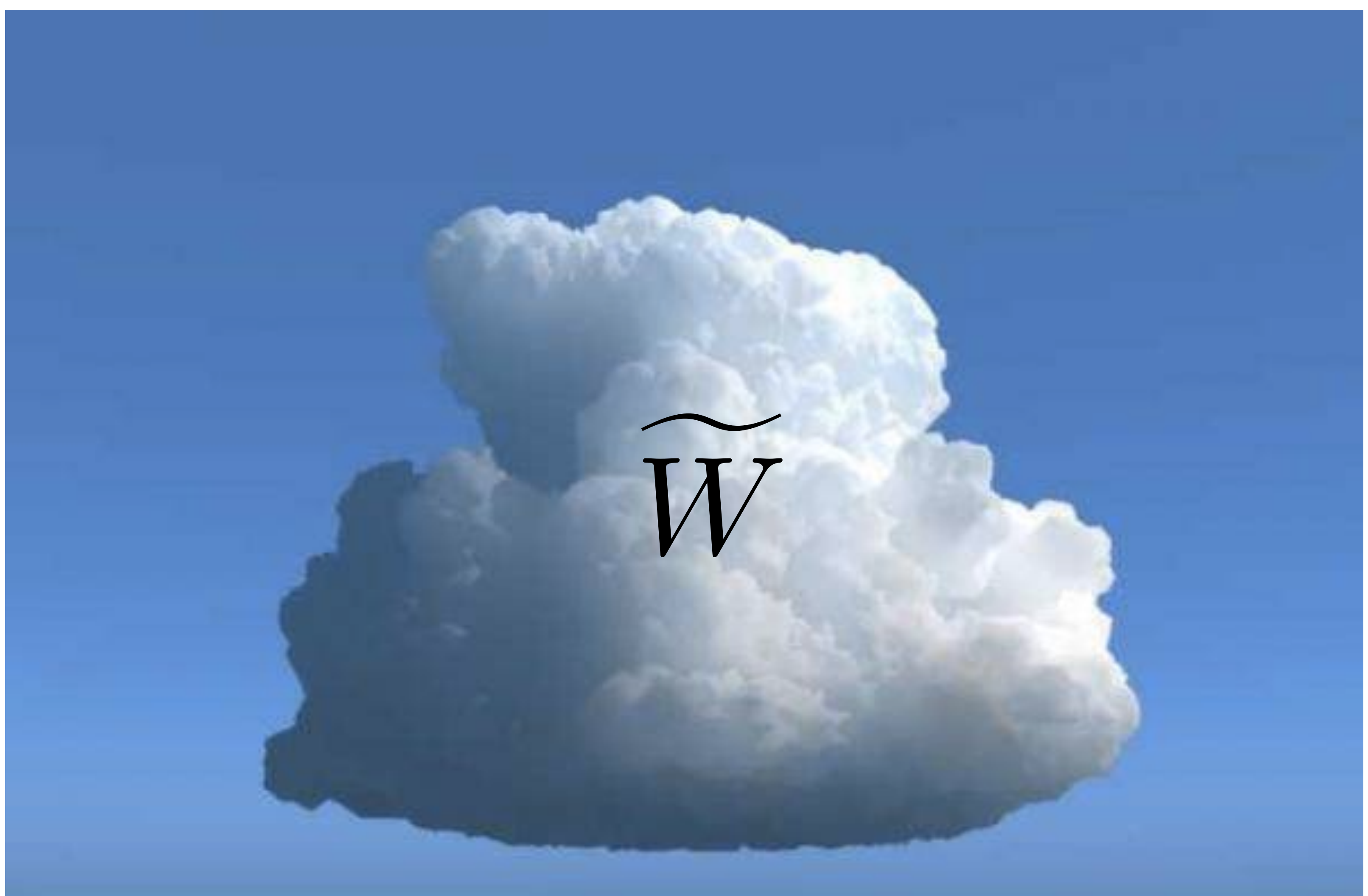
1. p &n SPA are valid in multipartite systems

2. To detect entangled states, I don't need EWs nor positive operators



$$p^*/d^2 \leq \mathrm{tr}[\widetilde{W}\sigma_{\mathrm{sep}}] \leq q^*/d^2$$

Result 2 : POVM Cloud = EWs



How can I detect entangled states?

mathoverflow

Questions Tags Users

positive not completely positive maps

14

In extension to this question Positive but not completely positive? I'd like to know, fo
examples of k-positive linear maps of a matrix algebra into itself that are not $k+1$-p
know a single one.) By M.D. Choi's theorem the size of the matrices involved must g
how fast?

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How can I detect entangled states?

$$p^*/d^2 \leq \mathrm{tr}[\widetilde{W}\sigma_{\mathrm{sep}}] \leq q^*/d^2$$

$$L = \min_{\sigma_{\mathrm{sep}}}[\widetilde{W}\sigma_{\mathrm{sep}}] \leq \widetilde{W} \leq U = \max_{\sigma_{\mathrm{sep}}}[\widetilde{W}\sigma_{\mathrm{sep}}]$$

$$p^*/d^2 \leq \mathrm{tr}[\widetilde{W}\sigma_{\mathrm{sep}}] \leq q^*/d^2$$

Result 3 : POVM Cloud = MUBs and SICs (for tomography)

$$W = (I \otimes T)(|\phi^+\rangle\langle\phi^+|)$$

pSPA $\Longrightarrow$ $$\widetilde{W} = (1-p^*)W + p^*\frac{I \otimes I}{d^2}$$

Quantum 2-design $\Longleftrightarrow$ $$\widetilde{W} = \mathrm{Sym}_{\mathcal{H}\otimes\mathcal{H}}$$

$$= \mathrm{supp}\{\mathrm{MUBs\ or\ SICs}\}$$

POVM cloud with quantum 2-designs

$$P_k^{(\mathrm{MUB})}(i,i) = \mathrm{tr}[|b_i^k\rangle\langle b_i^k|^{\otimes 2}\rho]$$

$$P^{(\mathrm{SIC})}(j,j) = \mathrm{tr}[|s_j\rangle\langle s_j|^{\otimes 2}\rho]$$

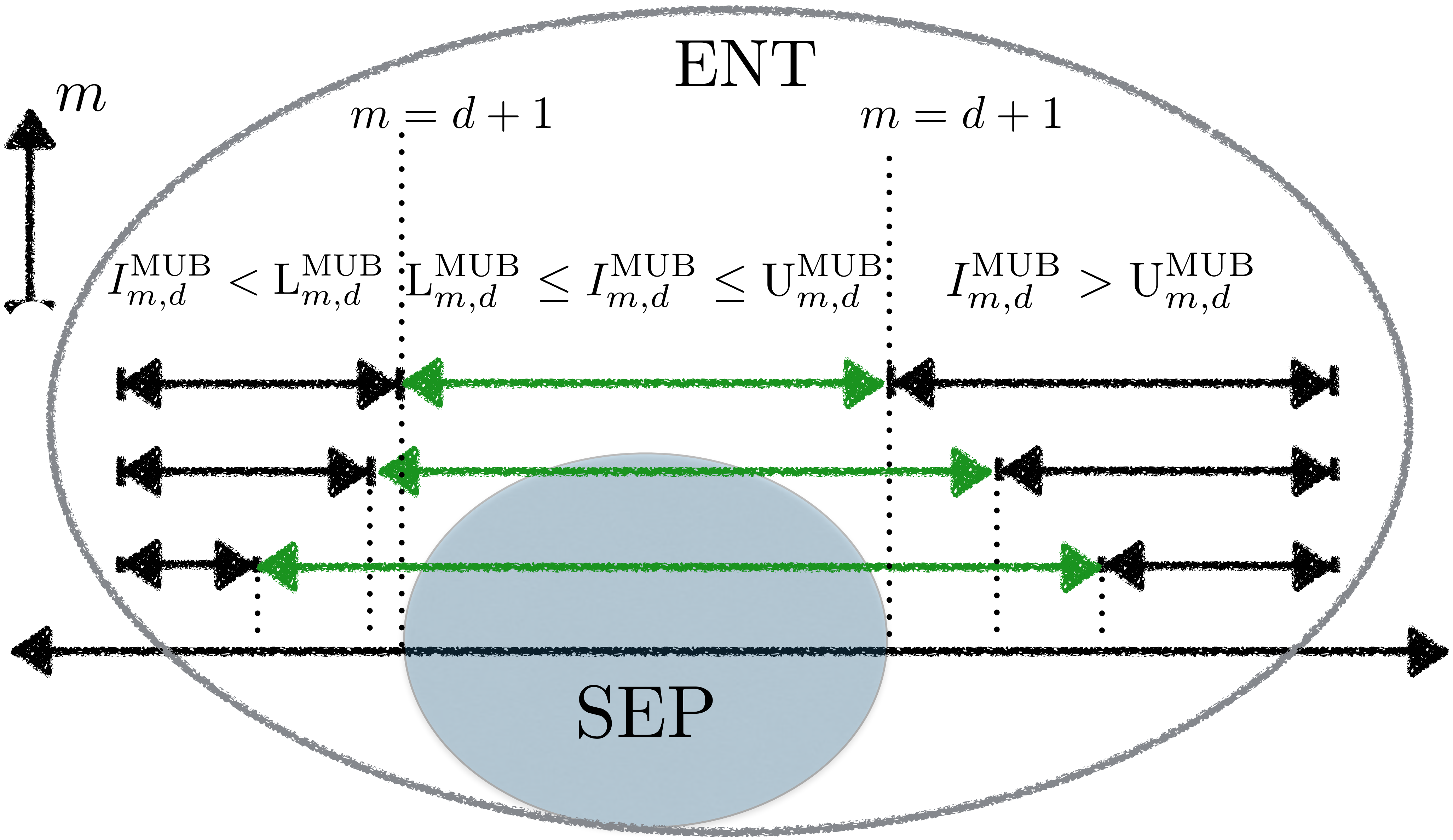
$$\Rightarrow \quad I_m^{(\mathrm{MUB})}(\rho) = \sum_{k=1}^{m}\sum_{i=1}^{d} P_k^{(\mathrm{MUB})}(i,i)$$

$$I_m^{(\mathrm{SIC})}(\rho) = \sum_{j=1}^{m} P^{(\mathrm{SIC})}(j,j)$$

$$\Rightarrow \quad L_m^{(\mathrm{MUB})} \leq I_m^{(\mathrm{MUB})}(\sigma_{\mathrm{sep}}) \leq U_m^{(\mathrm{MUB})}$$

$$L_m^{(\mathrm{SIC})} \leq I_m^{(\mathrm{SIC})}(\sigma_{\mathrm{sep}}) \leq U_m^{(\mathrm{SIC})}$$

Our Scheme of Detecting Entanglement: *TWICE*



Upper bounds detect entangled isotropic states

Lower bounds detect entangled Werner states

Throughout technical parts, main results: a number of inequalities

d=2

$$0.5 \le I_2^{(\mathrm{MUB})}(\sigma_{\mathrm{sep}}) \le 1.5$$

$$1 \le I_3^{(\mathrm{MUB})}(\sigma_{\mathrm{sep}}) \le 2$$ **QST can be applied**

d=3

$$0.211 \le I_2^{(\mathrm{MUB})}(\sigma_{\mathrm{sep}}) \le 1.333$$

$$0.5 \le I_3^{(\mathrm{MUB})}(\sigma_{\mathrm{sep}}) \le 1.666$$

$$1 \le I_4^{(\mathrm{MUB})}(\sigma_{\mathrm{sep}}) \le 2$$ **QST can be applied**

Throughout technical parts, main results: a number of inequalities

d=4

$$0 \leq I_2^{(\mathrm{MUB})}(\sigma_{\mathrm{sep}}) \leq 1.25$$

$$0.5 \leq I_3^{(\mathrm{MUB})}(\sigma_{\mathrm{sep}}) \leq 1.5$$

$$0.5 \leq I_4^{(\mathrm{MUB})}(\sigma_{\mathrm{sep}}) \leq 1.75$$

$$1 \leq I_5^{(\mathrm{MUB})}(\sigma_{\mathrm{sep}}) \leq 2$$

QST can be applied

Remarks. MUBs vs. Capability of Entanglement Detection

Entanglement vs. properties of MUBs

Throughout technical parts, main results: a number of inequalities

d=2

$$0 \le I_{2,2}^{(\mathrm{SIC})} \le \frac{(\sqrt{3}+1)^2}{6}$$

$$\frac{4}{15} \le I_{3,2}^{(\mathrm{SIC})} \le \frac{4}{3}$$

$$\frac{2}{3} \le I_{4,2}^{(\mathrm{SIC})} \le \frac{4}{3}.$$

QST can be applied

Throughout technical parts, main results: a number of inequalities

d=3

$$0 \leq I_{3,3}^{(\mathrm{SIC})} \leq \frac{9}{8}$$

$$0 \leq I_{4,3}^{(\mathrm{SIC})} \leq 1.25414$$

$$0 \leq I_{5,3}^{(\mathrm{SIC})} \leq 1.39952$$

$$0.1123 \leq I_{6,3}^{(\mathrm{SIC})} \leq 1.48175$$

$$\frac{3}{20} \leq I_{7,3}^{(\mathrm{SIC})} \leq \frac{3}{2}$$

$$\frac{3}{8} \leq I_{8,3}^{(\mathrm{SIC})} \leq \frac{3}{2}.$$

$$\frac{3}{4} \leq I_{9,3}^{(\mathrm{SIC})} \leq \frac{3}{2}.$$

QST can be applied

Remarks. SICs vs. Capability of Entanglement Detection

Entanglement vs. properties of SICs

arXiv.org > quant-ph > arXiv:1803.02708

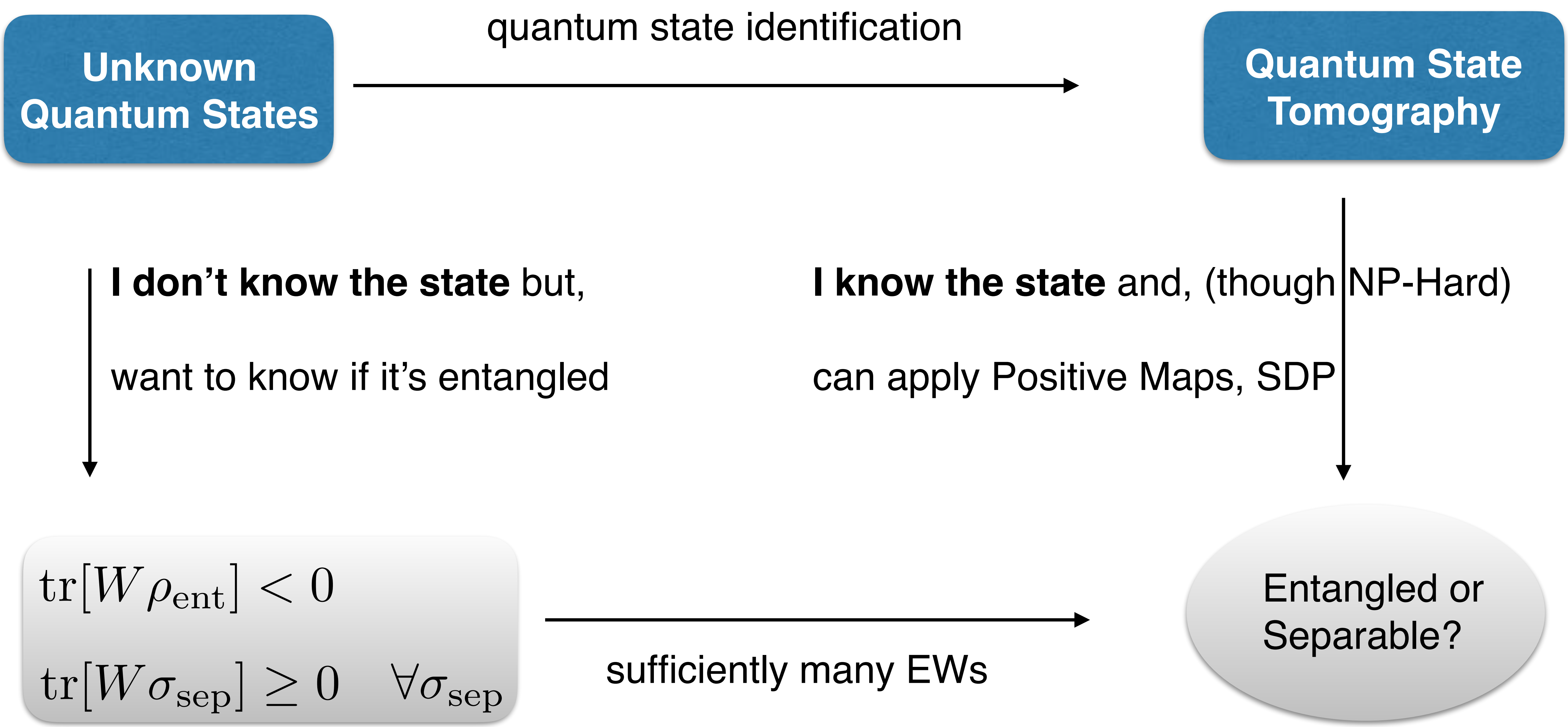
Linking Entanglement Detection and State Tomography via Quantum 2-Designs

Joonwoo Bae, Beatrix C. Hiesmayr, Daniel McNulty

(Submitted on 7 Mar 2018)

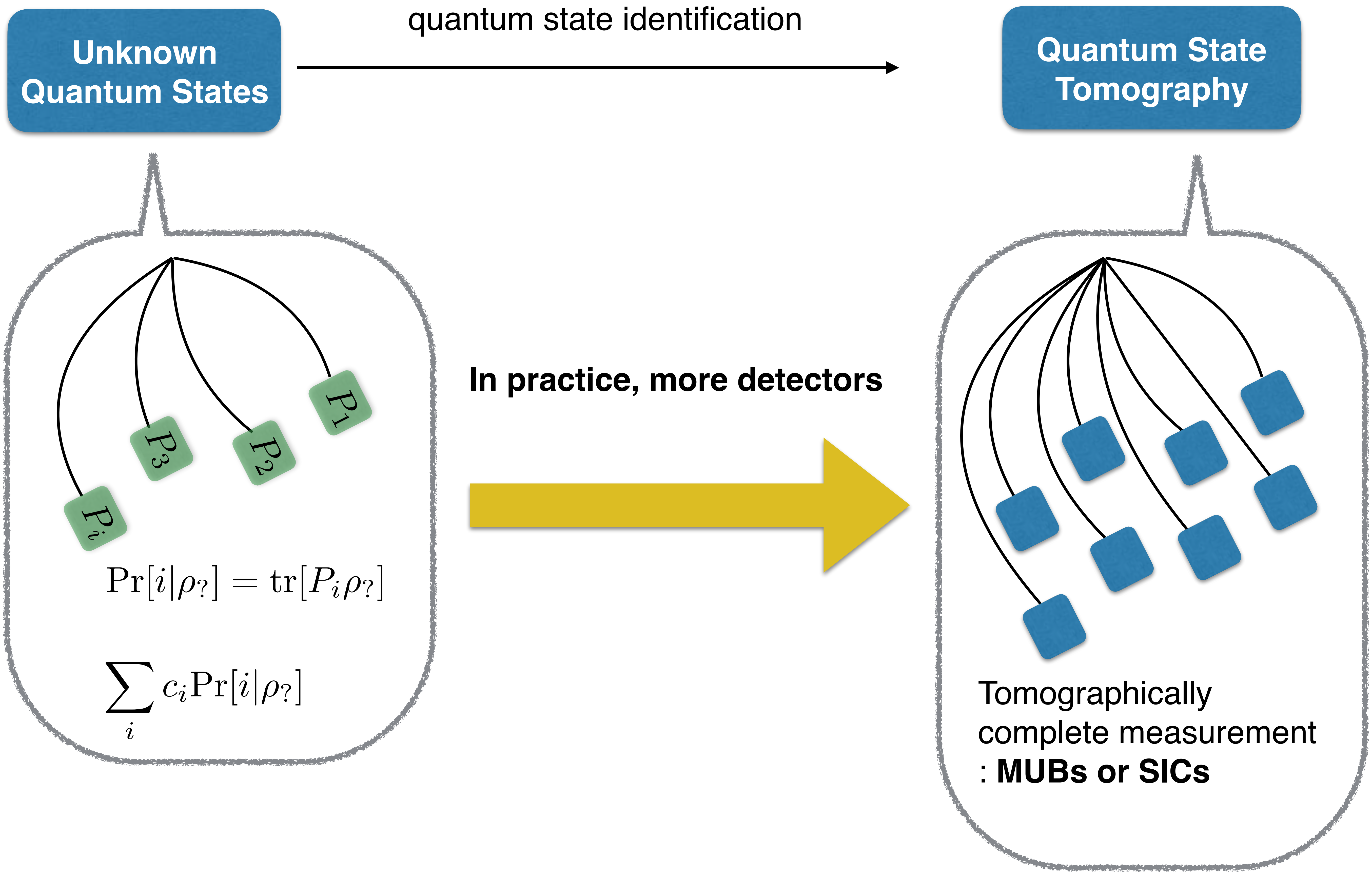
We present an experimentally feasible and efficient method for detecting entangled states with measurements that extend naturally to a tomographically complete set. Our detection criterion is based on measurements from subsets of a quantum 2-design, e.g., mutually unbiased bases or symmetric informationally complete states, and has several advantages over standard entanglement witnesses. First, as more detectors in the measurement are applied, there is a higher chance of witnessing a larger set of entangled states, in such a way that the measurement setting converges to a complete setup for quantum state tomography. Secondly, our method is twice as effective as standard witnesses in the sense that both upper and lower bounds can be derived. Thirdly, the scheme can be readily applied to measurement-device-independent scenarios.

General Picture of Entanglement Detection



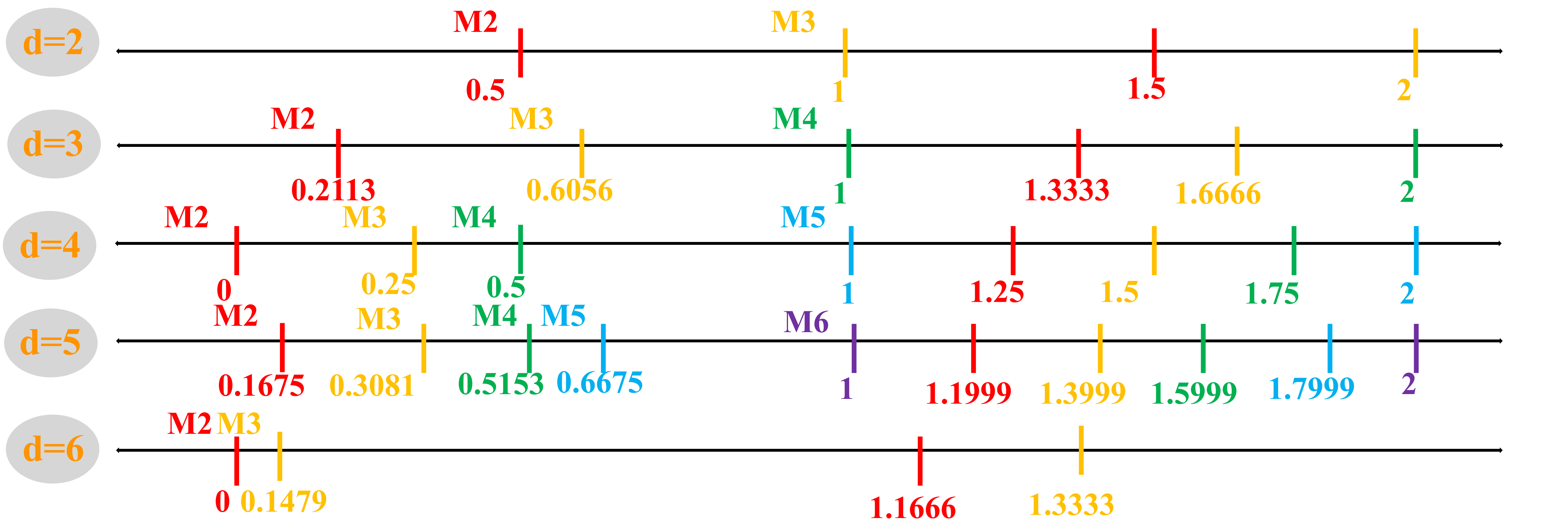
Advantage of EWs: Entanglement of unknown states can be detected !

General Picture of Entanglement Detection

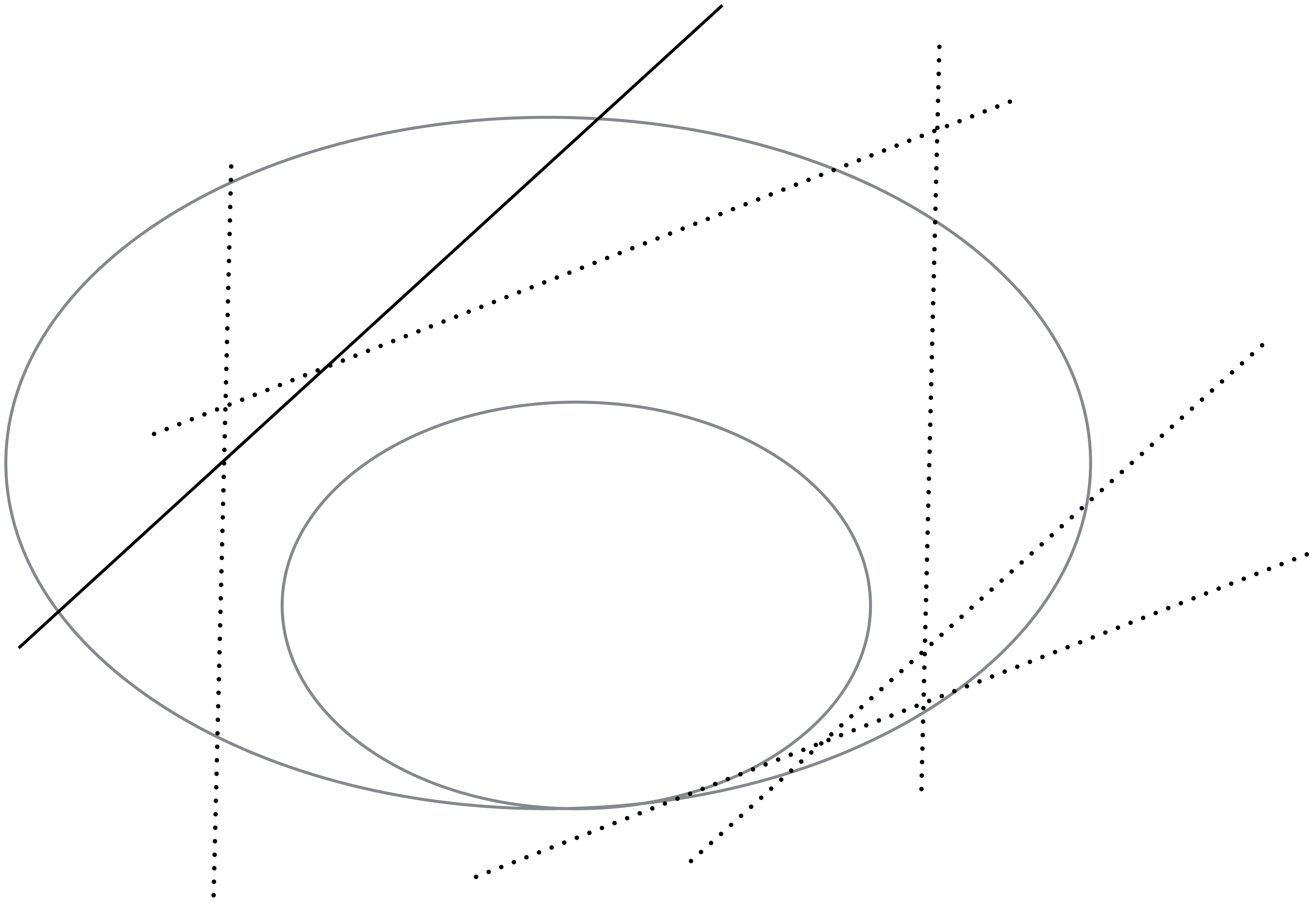


Result 4 : 3 MUBs in d=6 cannot detect all entangled states

$$(d+1)\ MUBs \qquad\qquad (d+1)\ MUBs$$



To Be Continued … with higher-order SPAs



Introduction: Entanglement Witnesses (EWs)

Main Question: Entanglement Detection vs. Quantum State Tomography

Our contribution : EWs are more useful than we thought

Theoretical parts: Many hyperplanes can be generated

Experimental proposal: Entanglement detection can be tested many times

Discussions: I no longer need Positive Maps to construct EWs.

Applications : MUBs, the conjecture in d=6, etc.

On-going directions and questions

I learned that **EWs are more useful than I thought**