Witnessing bipartite entanglement sequentially by multiple observers

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HRI

Witnessing entanglement sequentially: Maximally entangled states are not special

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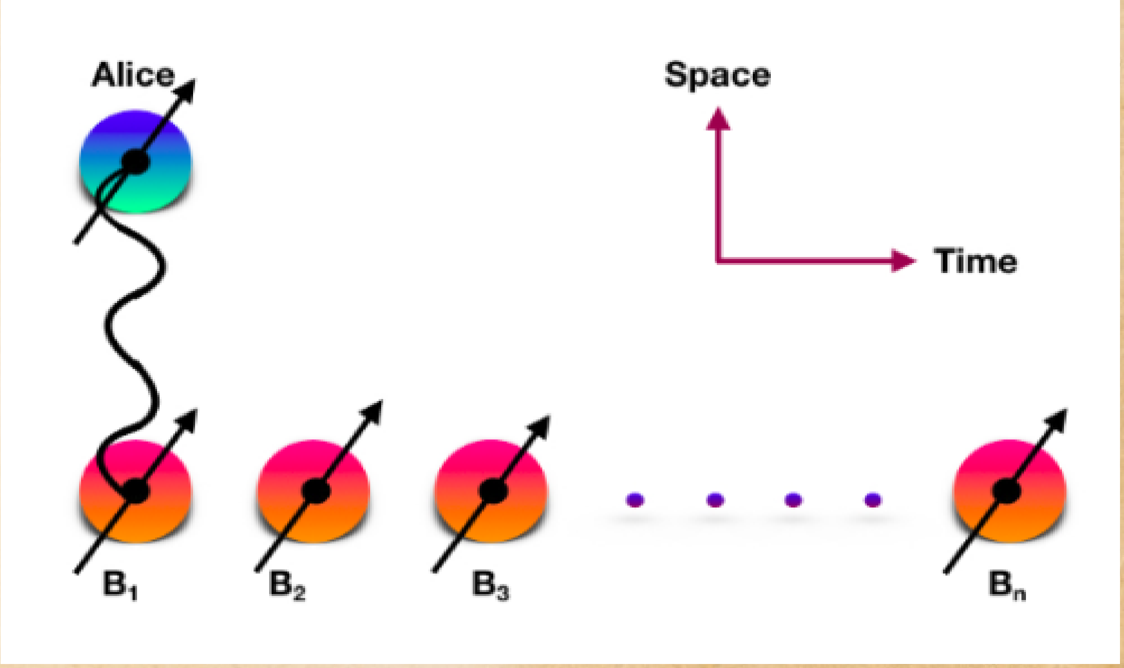
Witnessing bipartite entanglement sequentially by multiple observers



Witnessing bipartite entanglement sequentially by multiple observers

- Entanglement theoretic
- Measurement formalism

Tribute

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Quantum entanglement

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Entanglement theoretic

- Best possible knowledge of an entire system is not contained in the best possible knowledge of its subparts.
- Characterization, detection, manipulation and quantification of entanglement.

Quantification of entanglement

- Relative entropy of entanglement
- Geometric measure of entanglement
- Log negativity
- Distillable entanglement
- Entanglement cost

Detection of entanglement

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Method for Direct Detection of Quantum Entanglement

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(Received 7 February 2002; published 30 August 2002)

Structural physical approximation of a nonphysical map

1. The map can be implemented by applying selected products of unitary (Pauli) transformations with the pre-scribed probabilities.

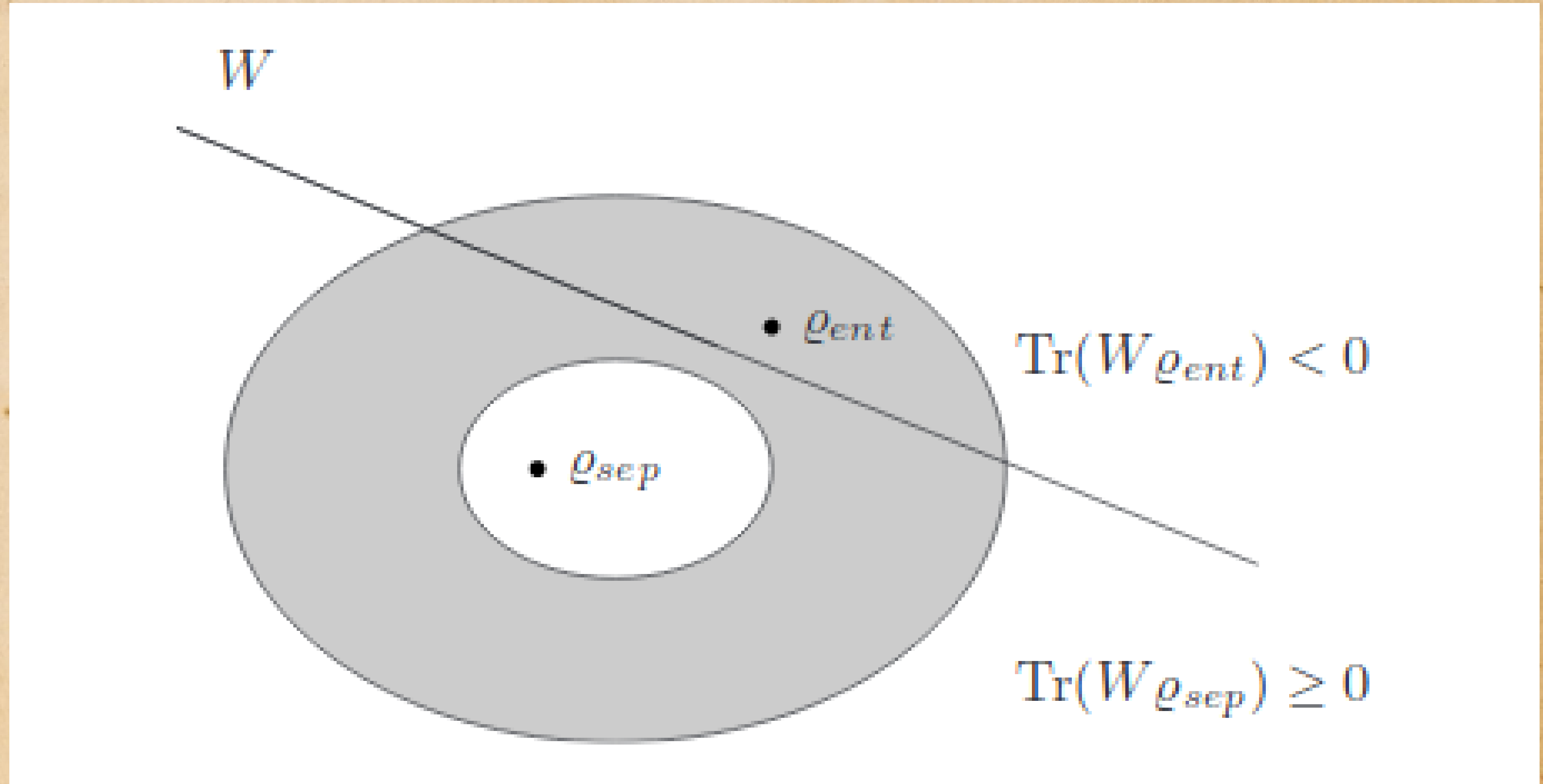
2. Have to measure lowest eigenvalue of the transformed state.

- Measuring quantum entanglement without prior state estimation, P. Horodecki, Phys. Rev. Lett. 90, 167901 (2003).

Entanglement witness

- Quantum cryptography
- Statistical physics
- Quantum optics
- Condensed matter nanophysics
- Bound entanglement
- Hidden nonlocality

Entanglement witness



JOURNAL OF MODERN OPTICS, 2003, VOL. 50, NO. 6–7, 1079–1102 Taylor & Francis
Taylor & Francis Group

Experimental detection of entanglement via witness operators and local measurements

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$$\varrho(p, d) := p|\psi\rangle\langle\psi| + (1-p)\sigma,$$

$$\left\|\sigma - \tfrac{1}{4}\mathbb{1}\right\| \leqslant d.$$

If the Schmidt decomposition of $|\psi\rangle$ is $|\psi\rangle = a|01\rangle + b|10\rangle$ with $a, b \geqslant 0$,

eigenvector corresponding to the minimal eigenvalue λ_- is given by

$$|\phi_-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle),$$

and thus the witness W_0 is given by

$$W_0 = |\phi_-\rangle\langle\phi_-|^{T_B} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Decomposition of witness

$$W = \sum_{i=1}^{k} c_i |e_i\rangle\langle e_i| \otimes |f_i\rangle\langle f_i|.$$

$$\begin{aligned}|\psi\rangle\langle\psi|^{T_B} &= \alpha^2|z^+z^+\rangle\langle z^+z^+| + \beta^2|z^-z^-\rangle\langle z^-z^-| + \alpha\beta(|x^+x^+\rangle\langle x^+x^+| \\ &\quad + |x^-x^-\rangle\langle x^-x^-| - |y^+y^-\rangle\langle y^+y^-| - |y^-y^+\rangle\langle y^-y^+|) \\ &= \tfrac{1}{4}(1\otimes 1 + \sigma_z\otimes\sigma_z + (\alpha^2-\beta^2)(\sigma_z\otimes 1 + 1\otimes\sigma_z) \\ &\quad + 2\alpha\beta(\sigma_x\otimes\sigma_x + \sigma_y\otimes\sigma_y)).\end{aligned}$$

Measurement formalism

$$|\Psi\rangle \otimes |\varphi(q)\rangle \longrightarrow \sum_a \langle a|\Psi\rangle \cdot |a\rangle \otimes |\varphi(q - g_0 a)\rangle.$$

$H(t) = g(t)A \otimes p$

$\int g(t)dt = g_0$

In a strong measurement the pointer's initial state is narrower than the distance between the eigenvalues, i.e., $\langle\varphi(q-a)|\varphi(q-a')\rangle = \delta_{aa'}$; hence, reading the pointer's position provides full information of the measured physical quantity and collapses the system into the corresponding eigenstate of the observable.

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Multiple Observers Can Share the Nonlocality of Half of an Entangled Pair by Using Optimal Weak Measurements

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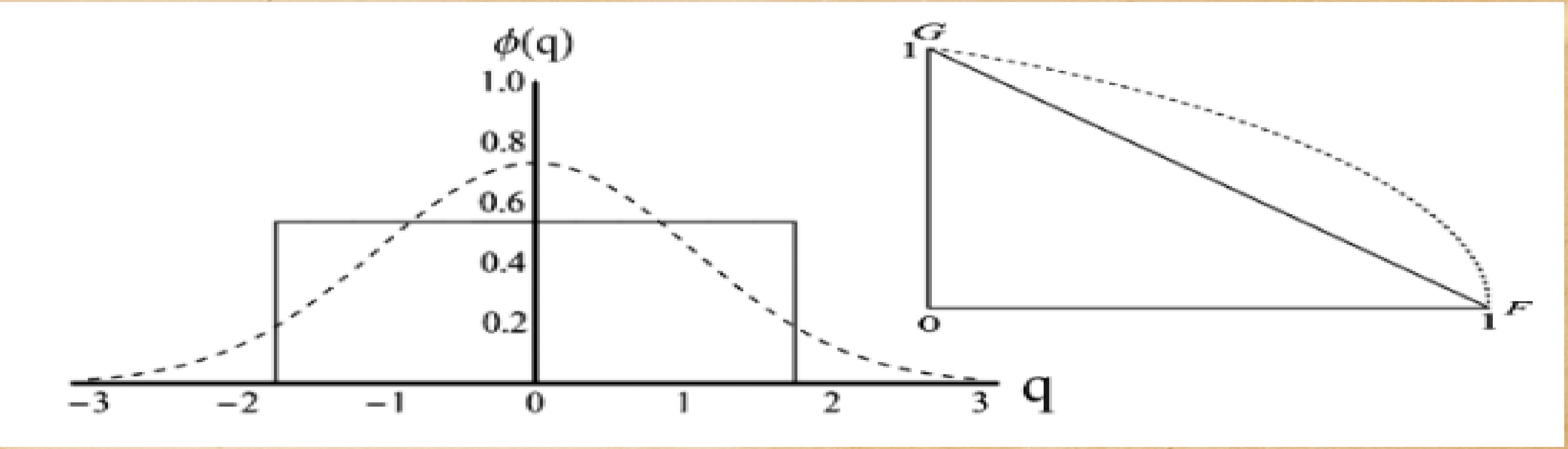
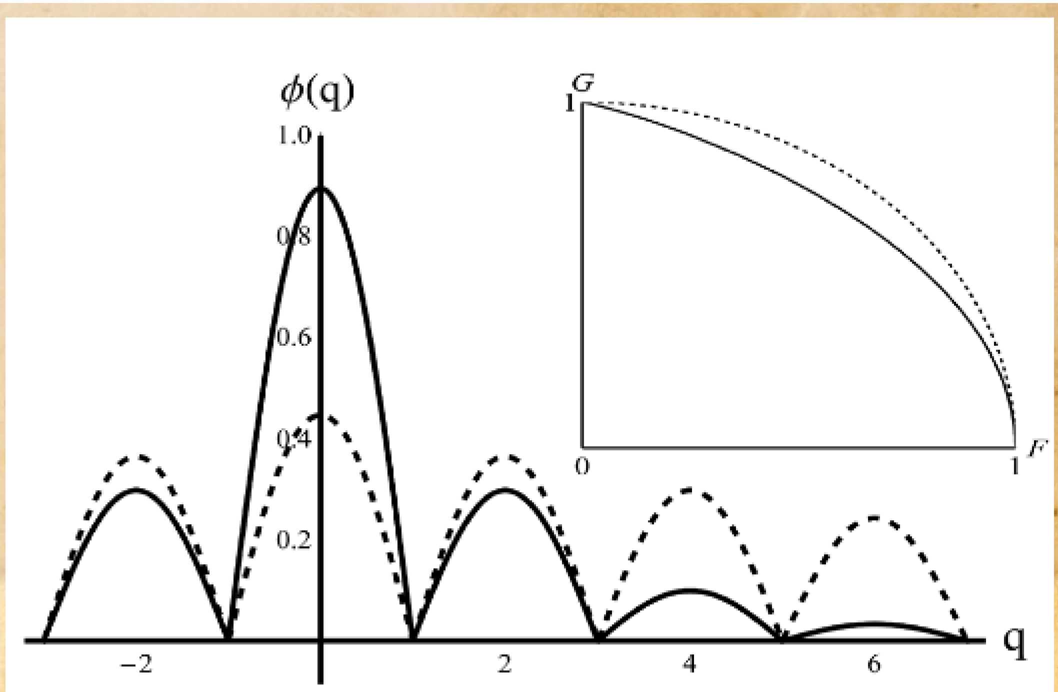


ϕ(q)
1.0
0.8
0.6
0.4
0.2
−3 −2 −1 0 1 2 3
q
G
1
0
1
F



ϕ(q)
1.0
0.8
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0.2
−2 2 4 6
q
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1
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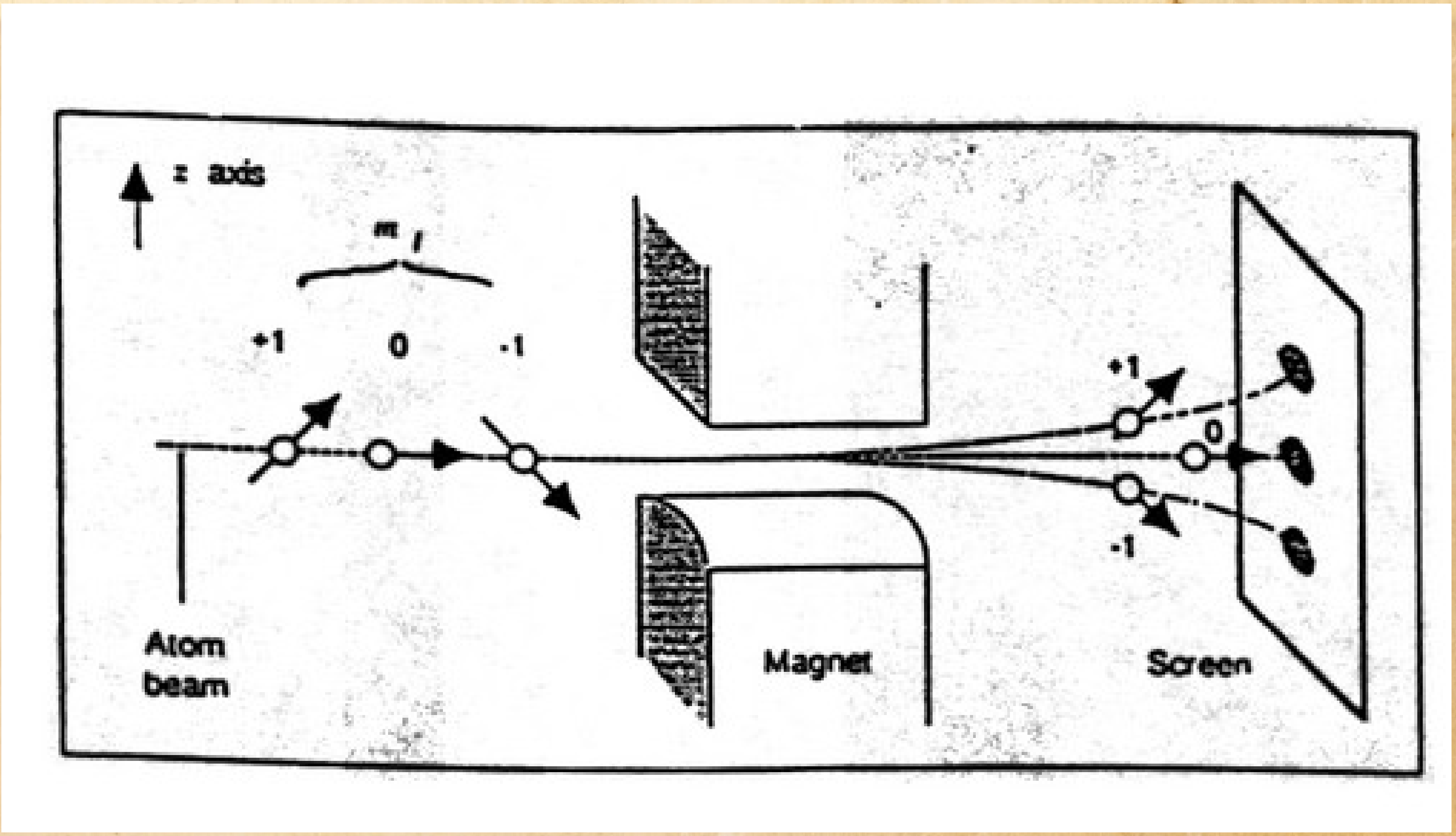
Stern Gerlach apparatus



Unsharp POVM

PHYSICAL REVIEW D VOLUME 33, NUMBER 8 15 APRIL 1986

Unsharp reality and joint measurements for spin observables

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(Received 21 October 1985)

$$E^{\lambda}_{\pm|\hat{n}} = \lambda P^{\pm}_{\hat{n}} + \frac{1-\lambda}{2}\mathbb{I}.$$

$$\rho \to \frac{1}{\tilde{p}}\sqrt{E^{\lambda}_{\pm|\hat{n}}}\rho\sqrt{E^{\lambda}_{\pm|\hat{n}}},$$

$$\tilde{p} = \mathrm{Tr}\left(\sqrt{E^{\lambda}_{\pm|\hat{n}}}\rho\sqrt{E^{\lambda}_{\pm|\hat{n}}}\right)$$

Modification of witness operator

$$\mathrm{Tr}\Big(\rho(P^{i}_{\hat{n}} \otimes E^{\lambda}_{j|\hat{m}})\Big)$$

$$\begin{aligned}\langle\sigma_{\hat{n}} \otimes \sigma^{\lambda}_{\hat{m}}\rangle &\equiv \mathrm{Tr}\Big[(P^{+}_{\hat{n}} - P^{-}_{\hat{n}}) \otimes (E^{\lambda}_{+|\hat{m}} - E^{\lambda}_{-|\hat{m}})\rho\Big] \\ &= \mathrm{Tr}\Big[(P^{+}_{\hat{n}} - P^{-}_{\hat{n}}) \otimes \lambda(P^{+}_{\hat{m}} - P^{-}_{\hat{m}})\rho\Big] \\ &= \lambda\langle\sigma_{\hat{n}} \otimes \sigma_{\hat{m}}\rangle.\end{aligned}$$

$$W^{\lambda}_{0} = \frac{1}{4}\Big(\mathbb{I} \otimes \mathbb{I} + \sigma_z \otimes \lambda\sigma_z - \sigma_x \otimes \lambda\sigma_x - \sigma_y \otimes \lambda\sigma_y\Big)$$

$$\mathrm{Tr}\left[|\psi^+\rangle\langle\psi^+|W^{\lambda_1}_0\right] = \frac{1}{4}(1 - 3\lambda_1).$$

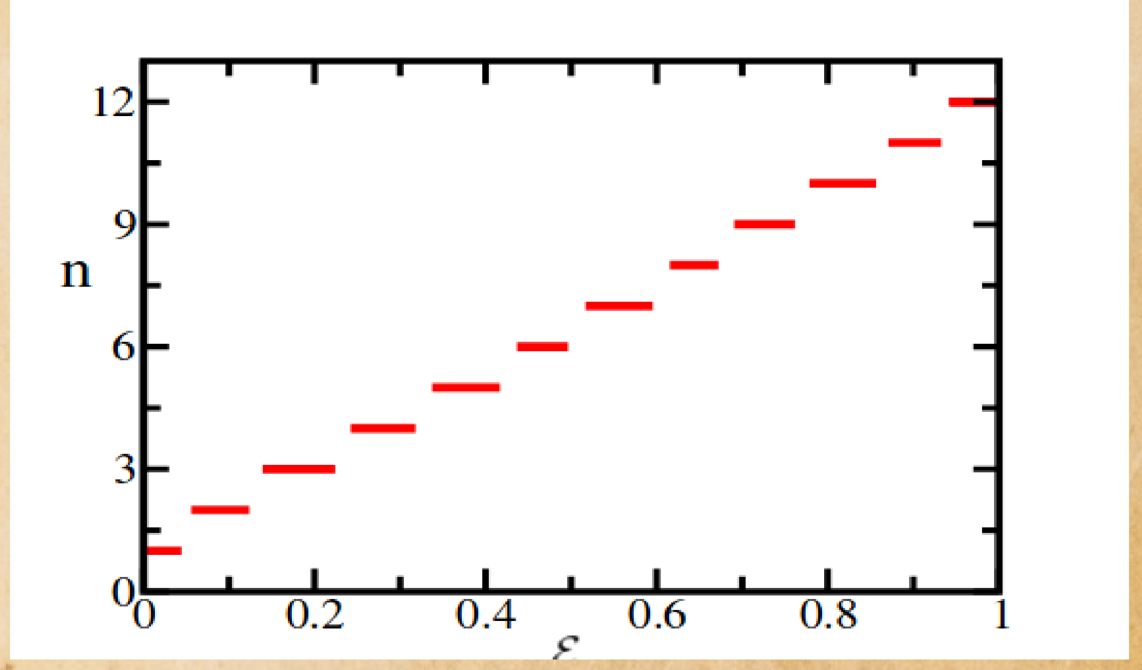
$$|\psi^+\rangle\langle\psi^+| \to \rho^{\lambda_1}_1 = \frac{1}{3}\sum_{i,\hat{n}} \sqrt{E^{\lambda_1}_{i|\hat{n}}}|\psi^+\rangle\langle\psi^+|\sqrt{E^{\lambda_1}_{i|\hat{n}}}$$

$$\rho^{\lambda_1}_1 = \frac{1}{4}\Big[p\rho_{\psi^+} + (1-p)\mathbb{I} \otimes \mathbb{I}\Big]$$

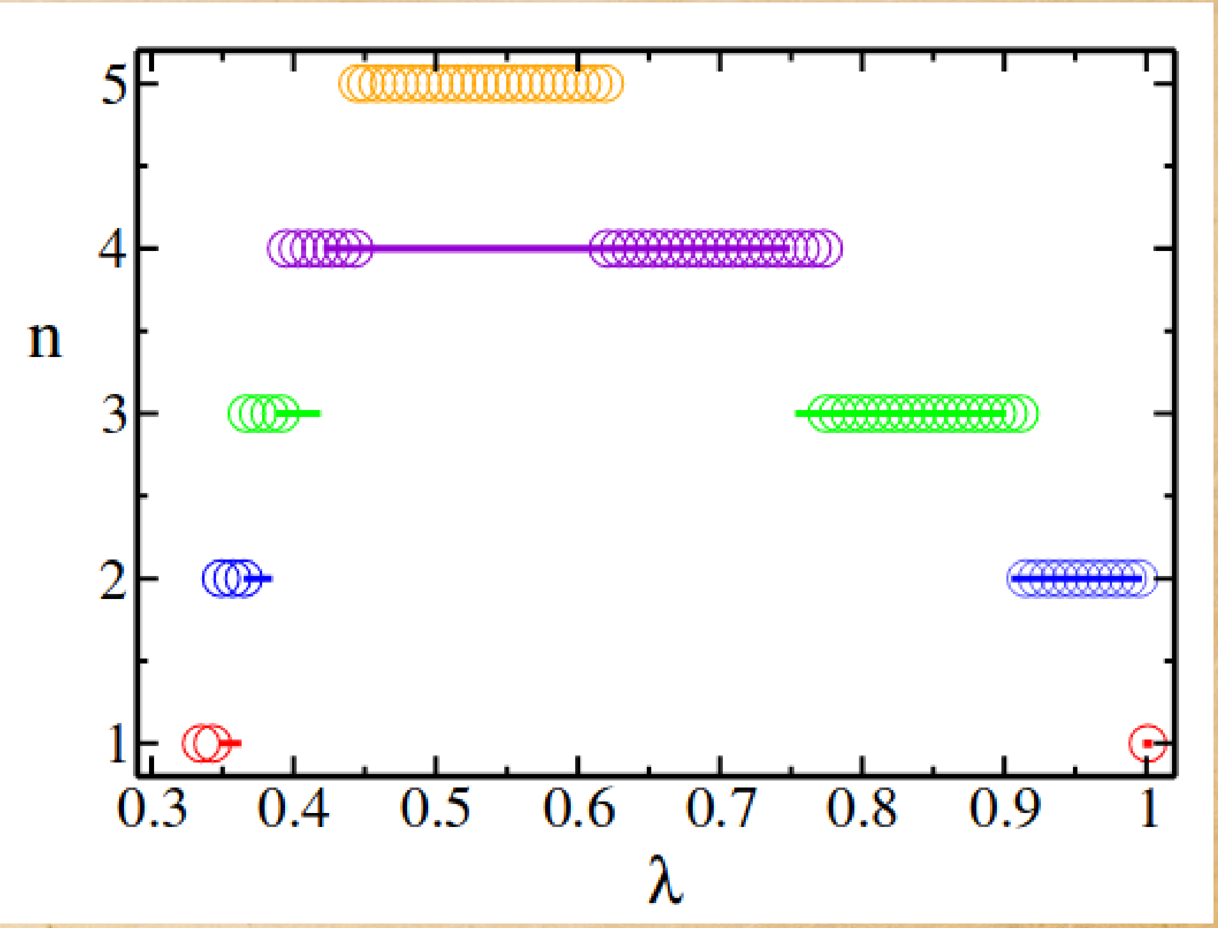
$$\mathrm{Tr}[W^{\lambda_2}_0 \rho^{\lambda_1}_1] = -\frac{1}{4}\Big[1 - (1 + 2\sqrt{1-\lambda_1^2})\lambda_2\Big].$$

Now if $\lambda_1 = 1/3$ in the first stage, then to detect entanglement in the second stage, the sharpness parameter λ_2, of B_2, must be greater than 0.3465 (correct up to four

$$E_n = \frac{1}{4}\left[3^{n-1} - \frac{1}{3^{n-1}}(1+4ab)\lambda_n \Pi_{i=1}^{n-1}(1+2\sqrt{1-\lambda_i^2})\right]$$



Equal sharpness parameter



Summary

- Witnessing entanglement through imprecise apparatus.
- Limit of observers sharing bipartite entanglement.
- An operational measure of entanglement.
- Coarse gained measure of entanglement.
- A less entangled state also upto maximally entangled state.