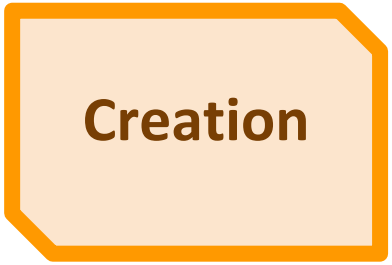
Magic in quantum systems

Chiranjib Mukhopadhyay



Rubbing pieces of paper
Lighting a candle

Dancing frog
See things in the dark

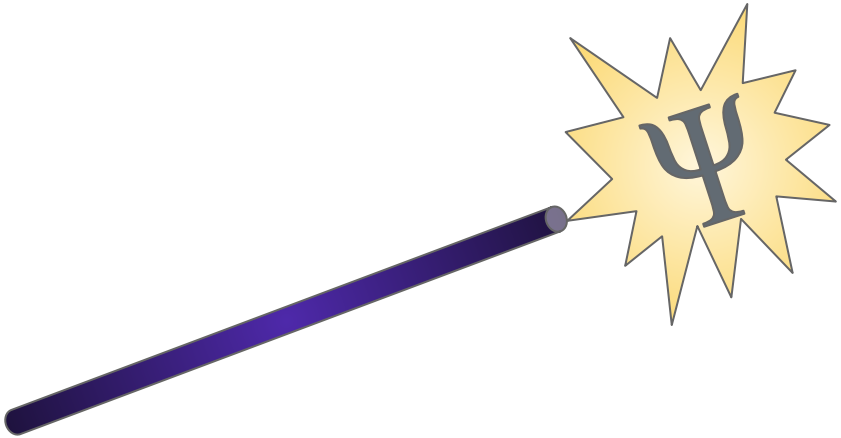
Electromagnetic Theory

Trinity of Resources

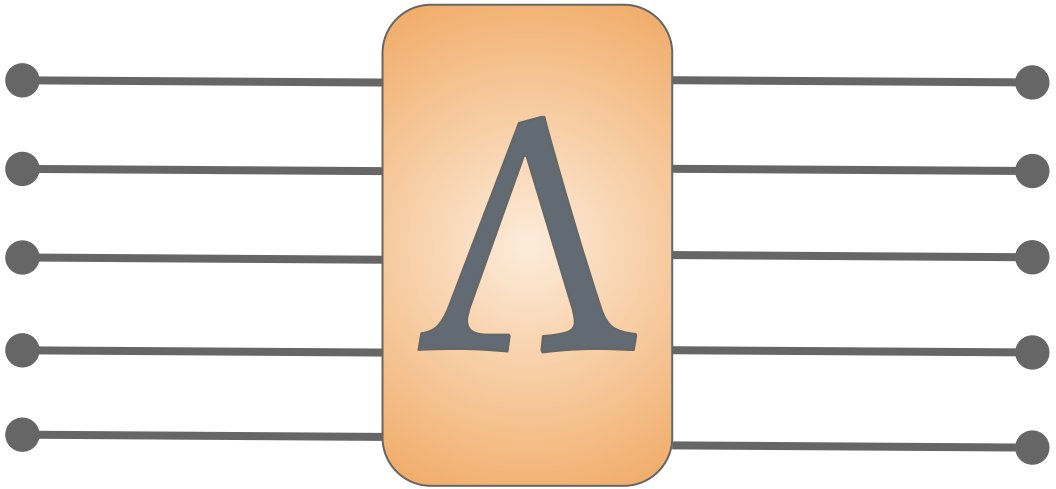
- What is magic ?
- Link with other resources
- How to create magic ?
- Conclusions

Magic (n.)

An extraordinary power or influence seemingly from a supernatural source.

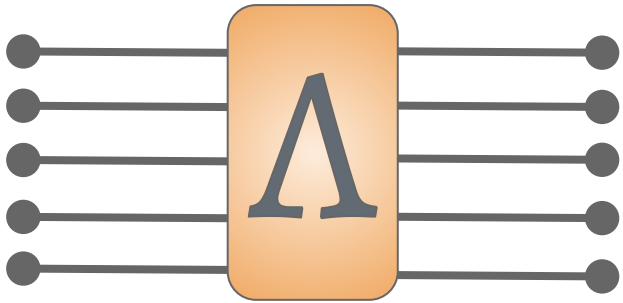


What is ‘quantum’ about quantum computers ?



What kind of circuits can you efficiently simulate on a classical computer ?

What is ‘quantum’ about quantum computers ?



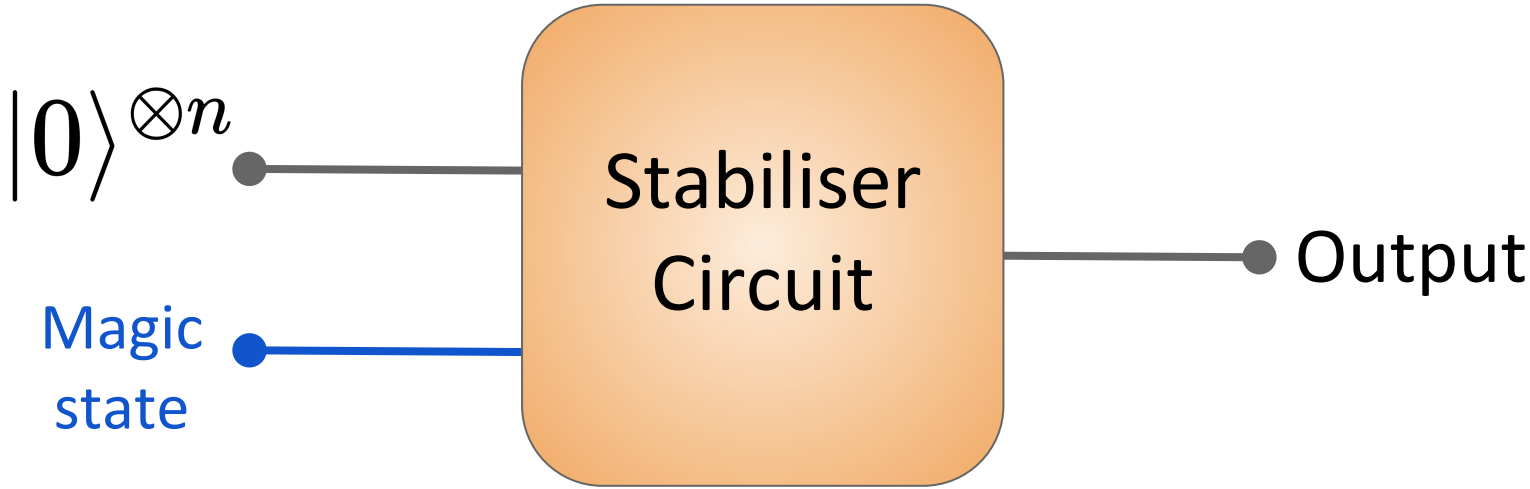
Entanglement does not imply quantum computation

Gottesman Knill (1998)

Circuits solely consisting of normalizers to the (generalized) Pauli group are classically simulable.

- CNOT
- Hadamard
- Phase gate

Quantum computing by state injection



Magic states are the resource for quantum computing

Resource Theory

Resource

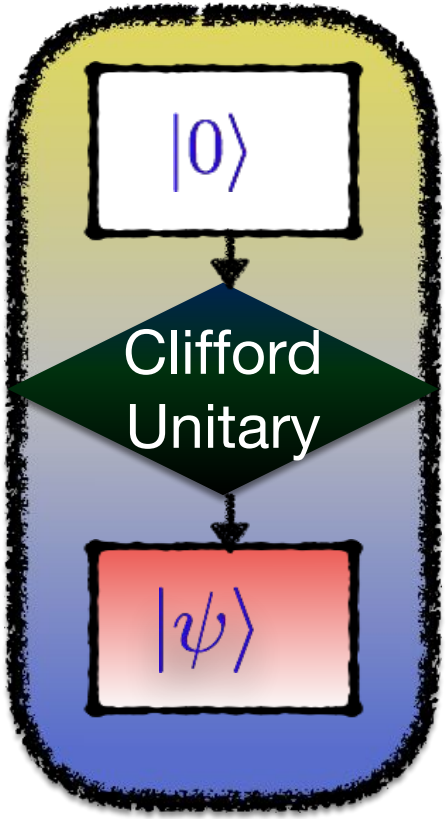
Resource Theory

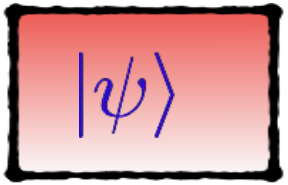
Free State

Resource Theory

Free operations

What kind of PURE ancilla states are NOT helpful ?





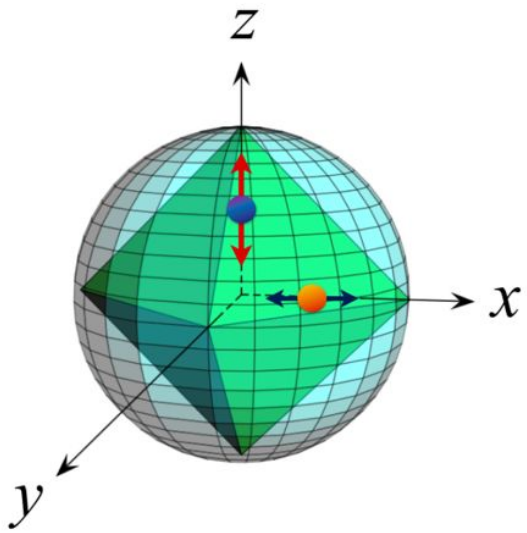
This family is useless

Pure Stabiliser states

Free states : convex Polytope with d(d+1) vertices in prime power dimensions.

Resource theory of magic states

Free states



Free operations

- ★ Clifford unitaries
- ★ Measurement in computational basis
- ★ Composition with free ancilla
- ★ Partial Trace
- ★ Classical randomness

Monotones

$$M(\rho) = \min_{\sigma \in S} S(\rho||\sigma)$$

Relative Entropy of Magic

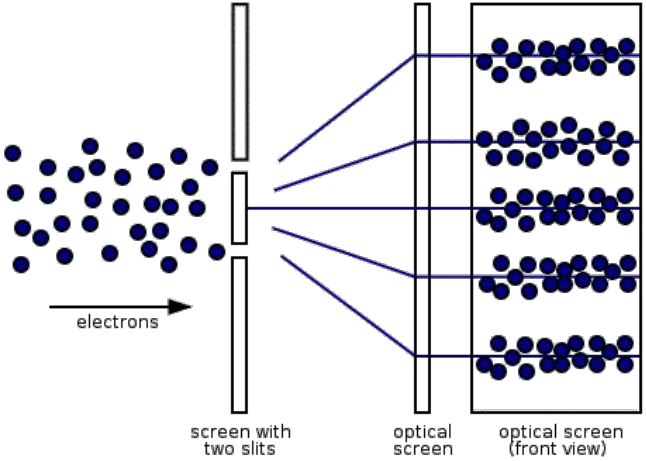
$$M(\rho) = \min_{\sigma \in S} \left[\xi : \frac{\rho + \xi\sigma}{1+\xi} \in S \right]$$

Robustness of Magic

Difficult to calculate

Other Quantum Resource Theories and Magic

- What is magic ?
- Link with other resources
- How to create magic ?
- Conclusions



"Only mystery in quantum mechanics"
superposition a.k.a. coherence

CM, S. Sazim, A.K. Pati - *J Phys A* 51 414006(2018).

Resource theory of magic states : links with other resources

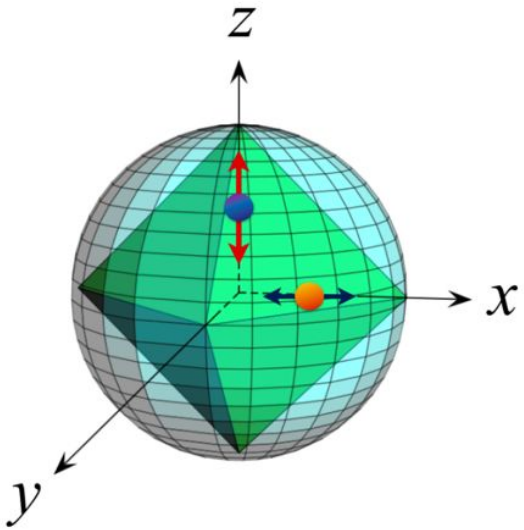
- Pure Gaussian state if and only if Wigner function is a genuine probability distribution.

Resource theory of magic states : links with other resources

- Pure Gaussian state if and only if Wigner function is a genuine probability distribution.

- non-Gaussianity is a resource.

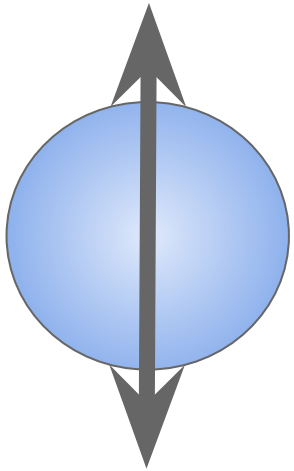
Resource theory of magic states : links with other resources



- Pure odd-dimensional stabilizer state if and only if discrete Wigner function is a genuine probability distribution.
- Keyword : odd dimensions.
- Set of Gaussian states isn't convex.

Entanglement distillation possible using stabilizer codes.

Resource theory of coherence



Resource Theory of Coherence

- Diagonal states Δ
- Incoherent Operations

$$\Lambda = \{ K_i \} : \; K_i \, \Delta \, K_i^{\dagger} \in \Delta \; \forall i$$

Question : Is there any link with the resource theory of magic ?

Coherence makes quantum systems magical

(Contractive) distance

$$R_d(\rho) = \min_{\sigma \in Q} D[\rho, \sigma]$$

Resource

Free State

Result - I

$$M_d\left[\Lambda_{IC}(\rho)\right] \leq C_d[\rho]$$

Result - II

$$\sup_{\Lambda_{IC}} M_d\left[\Lambda_{IC}(\rho)\right]$$

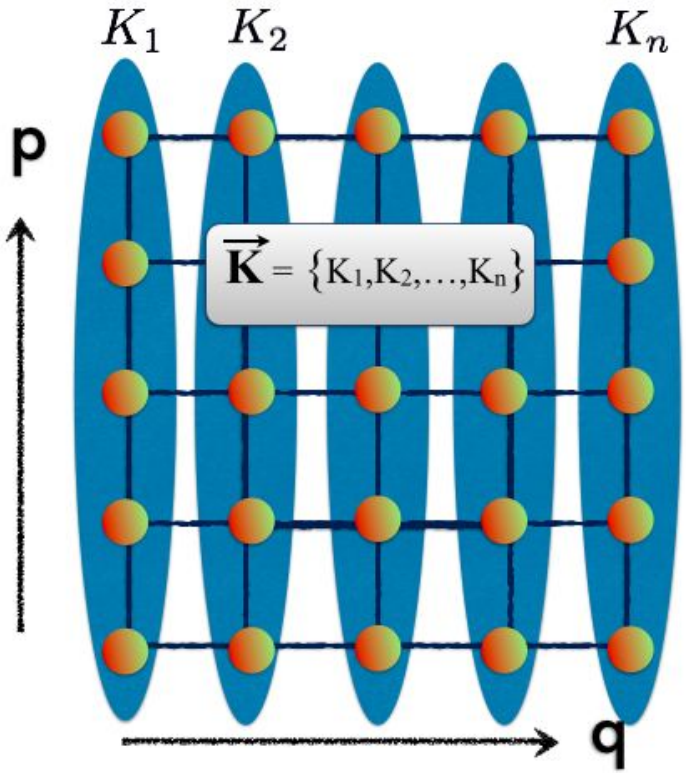
Coherence monotone

Coherence in a system is equal to the maximal amount of magic you can generate using incoherent operations.

Similar result w/ Entanglement

Streltsov, Singh, Dhar, Bera, Adesso PRL 115 020403 (2015)

Monotones using discrete Wigner function

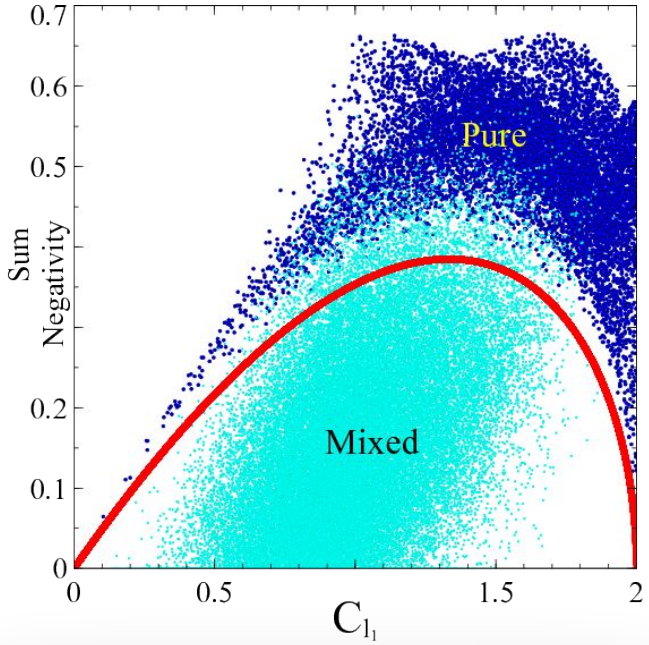


Magic monotone

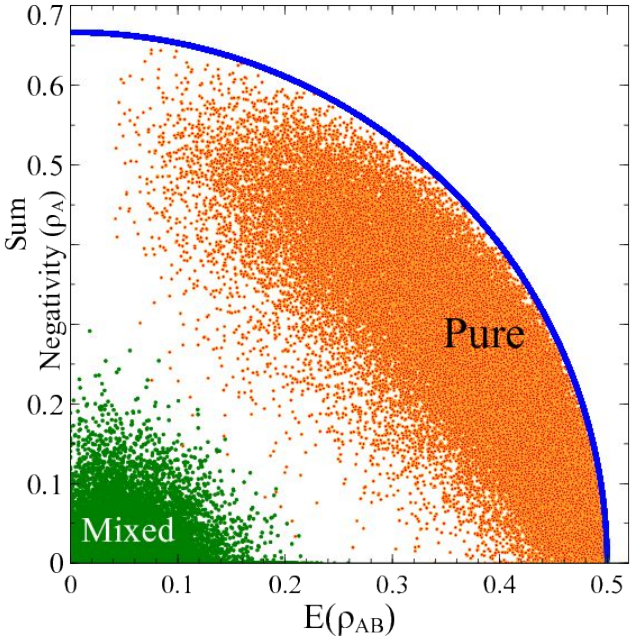
Sum of negative components of (discrete) Wigner

Coherence monotone

$$C_w[\rho] = \min_{\sigma \in \mathcal{I}, \lambda \geq 0} ||\vec{K}_\rho - \lambda \vec{K}_\sigma||$$



$$M_{SN}[|\psi\rangle] \geq \frac{C_{l_1}[|\psi\rangle]}{2}\sqrt{1-\frac{C_{l_1}[|\psi\rangle]}{2}}$$



$$16E_{AB}^2 + 9M_{SN_A}^2 < 4$$

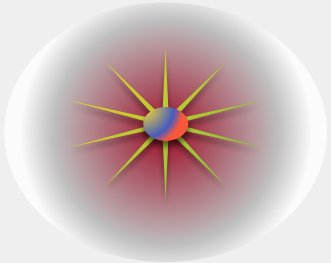
Quantitative links with coherence in a qutrit, and entanglement in a qubit-qutrit system

- What is magic ?
- Link with other resources
- How to create magic ?
- Conclusions

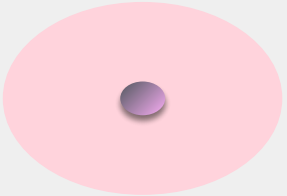
Go to Hogwarts !!!

CM - *Phys Rev A* 98 012102 (2018).

Heat Bath

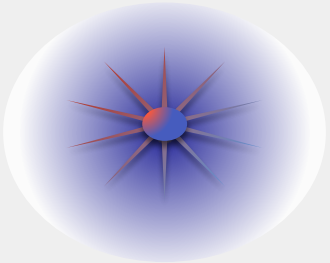


Energy exchange with environment



$$\tau = \frac{1}{Z}|0\rangle\langle 0| + \frac{e^{-\beta\Delta_E}}{Z}|1\rangle\langle 1|$$

Angular Momentum Bath



X-component of angular momentum exchange with environment

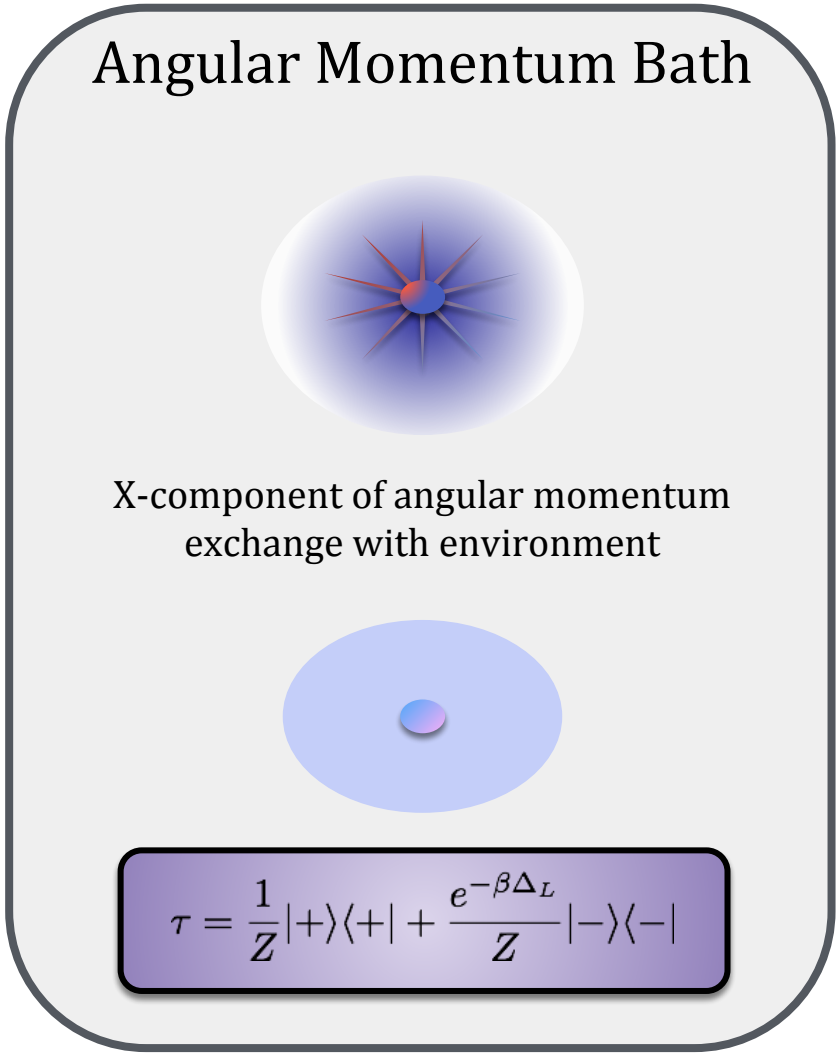
$$\tau = \frac{1}{Z}|+\rangle\langle +| + \frac{e^{-\beta\Delta_L}}{Z}|-\rangle\langle -|$$

Landauer erasure with no energy cost

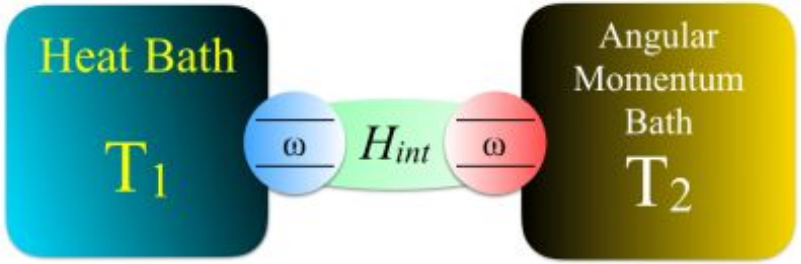
Barnett Vaccaro Entropy (2013)

Otto cycle engines and Carnot efficiency

Wright *et al* Phys Rev A (2018)



Design of the autonomous machine



Goal

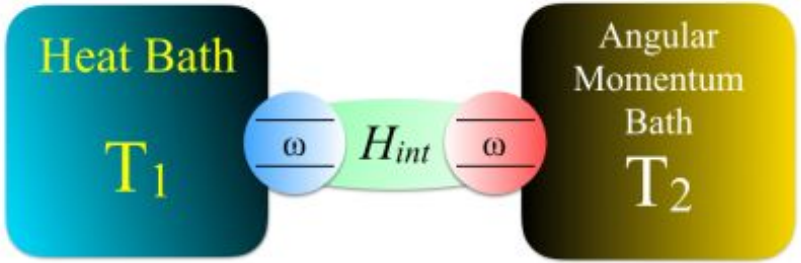
Generate magic in the blue qubit

$$H = H_1 + H_2 + H_{int}$$

$$\frac{1}{2}\omega_1\sigma_z \qquad \frac{1}{2}\omega_2\sigma_z$$

$$g\left(|01\rangle\langle 10| + h.c.\right)$$

Design of the autonomous machine



Goal

Generate magic in the blue qubit

$$\frac{d\rho_{12}(t)}{dt} = -i[H, \rho_{12}] + \sum_i p_i \left[\tau_i \otimes Tr_i \rho_{12}(t) - \rho_{12}(t)\right]$$

Simplified Master Equation

Warm up

Steady state coherence of thermal qubit

Steady state coherence of thermal qubit

$$C_{l_1} = \frac{4gp_2}{\sqrt{(1+4p_1^2)(1+4p_2^2)}} \left| \tanh\left(\frac{1}{2T_1}\right) \tanh\left(\frac{1}{2T_2}\right) \right| + \mathcal{O}(g^2)$$

Steady state coherence of thermal qubit

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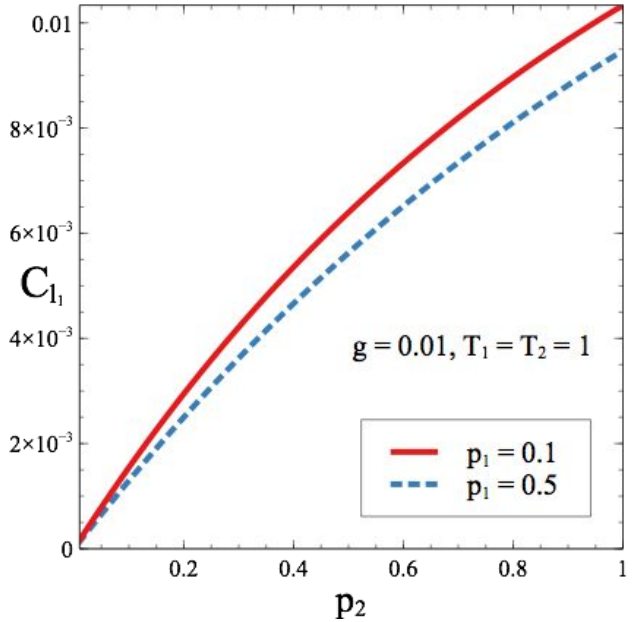

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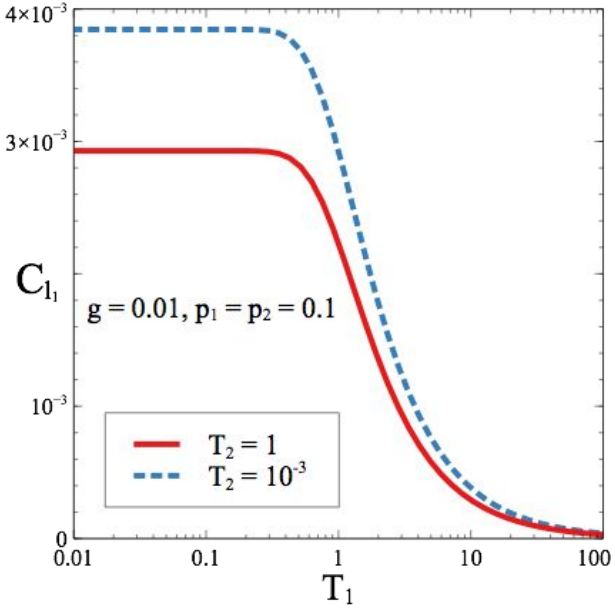
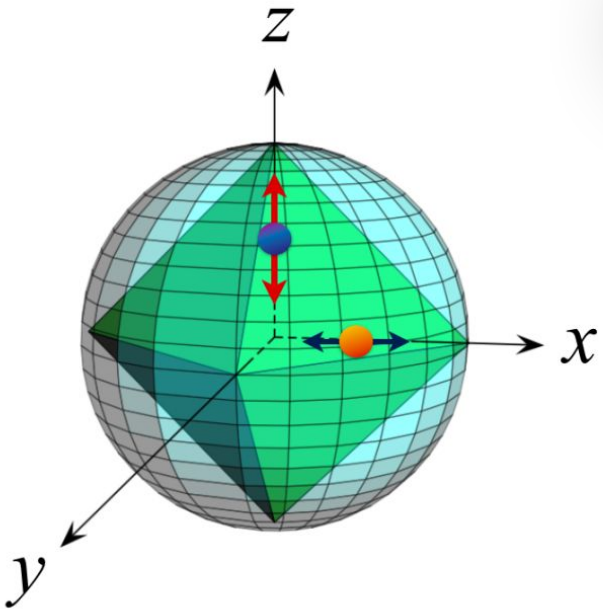


Magic creation in the thermal qubit

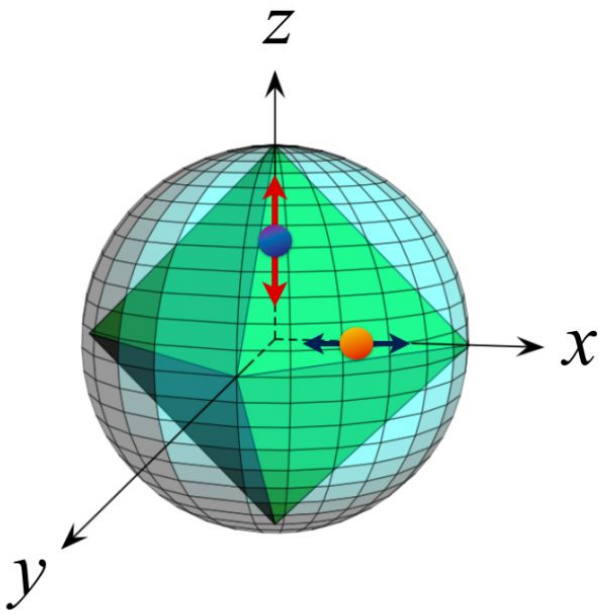
Magic creation in the thermal qubit



Stabilizer polytope

$$-1 \leq x \pm y \pm z \leq 1$$

Magic creation in the thermal qubit

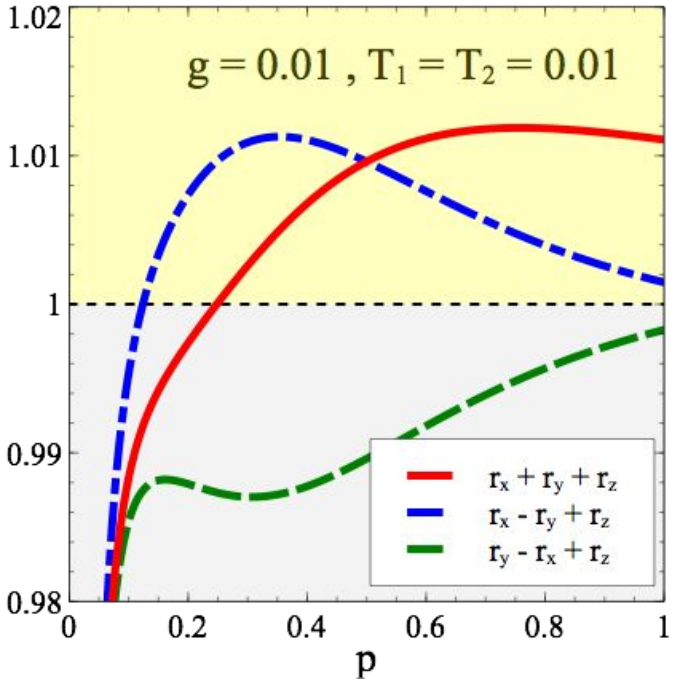


Stabilizer polytope

$$-1 \leq x \pm y \pm z \leq 1$$

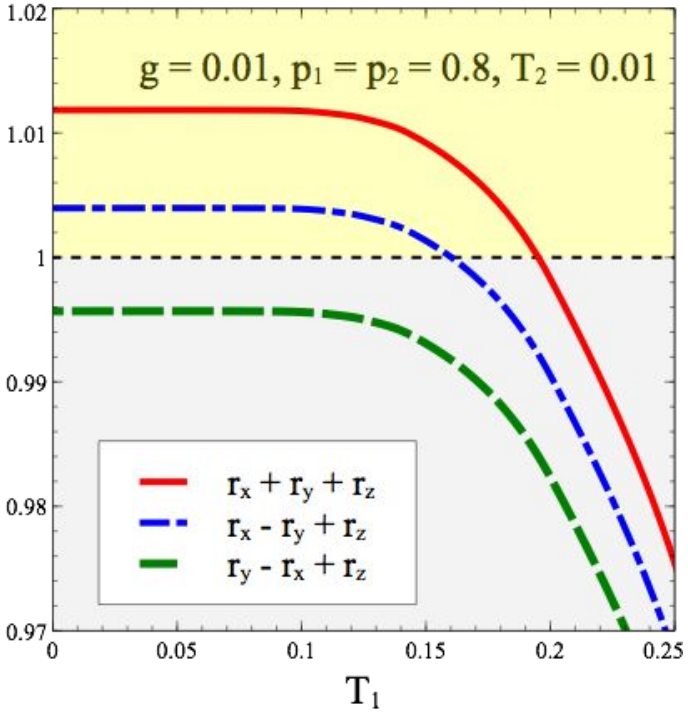
Simply dipping in spin-bath won't help

Exact solution for steady state of thermal qubit



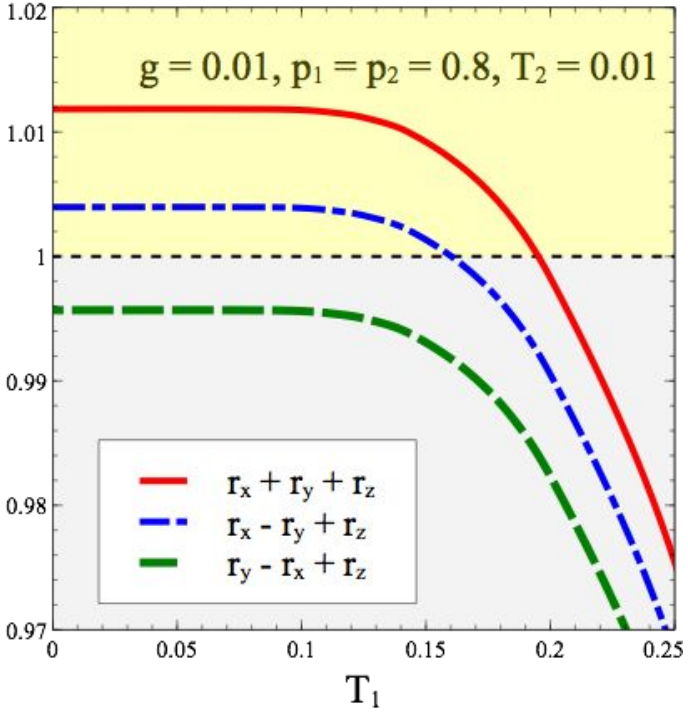
Magic Created !

Exact solution for steady state of thermal qubit



But only if the heat bath is Cold enough….

Exact solution for steady state of thermal qubit



But only if the heat bath is Cold enough….

A critical temperature must exist …

Perturbative analysis of critical temperature

Low T_2 limit

$$f_1 = \frac{p(4p^2+4p-1)}{(1+4p^2)^2}$$

$$f_2 = \frac{1+6p^2+24p^4}{p^2(1+4p^2)^2}$$

$$g_1 = \frac{p(1+4p-4p^2)}{(1+4p^2)^2}$$

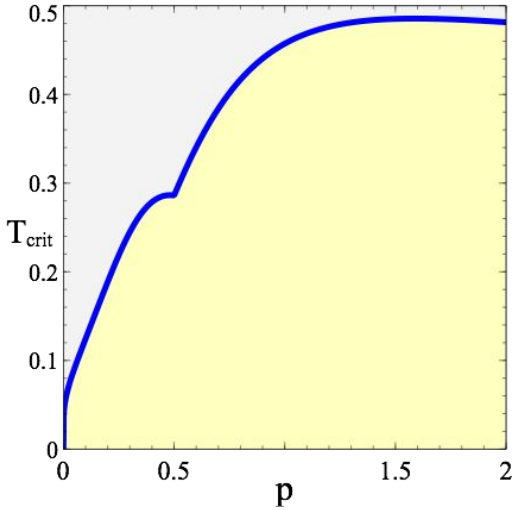
$$h_1 = \frac{p(4p^2-4p-1)}{(1+4p^2)^2}$$

$$T^1_{\text{crit}} = \frac{1}{\ln\left(1+\frac{f_2}{2f_1^2}\right)}, T^2_{\text{crit}} = \frac{1}{\ln\left(1+\frac{f_2}{2g_1^2}\right)}, T^3_{\text{crit}} = \frac{1}{\ln\left(1+\frac{f_2}{2h_1^2}\right)}$$

Critical temperature is the maximum of these three

Perturbative analysis of critical temperature

Low T_2 limit



Bath helps…

Perturbative analysis of critical temperature

High T_2 limit

$$F_1 = \frac{p(4p^2+4p-1)}{T_2(1+4p^2)^2}$$

$$F_2 = 1/p^2$$

$$G_1 = \frac{p(1+4p-4p^2)}{T_2(1+4p^2)^2}$$

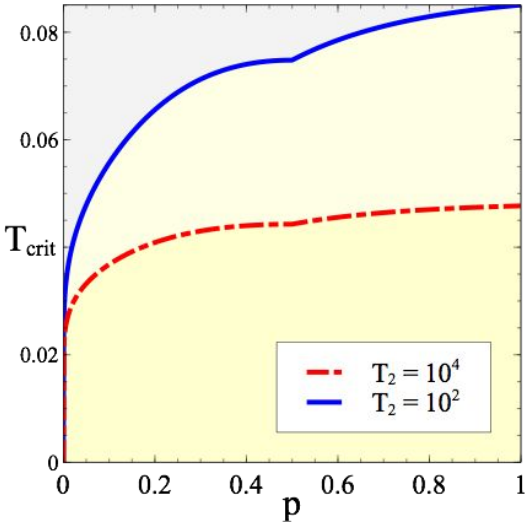
$$H_1 = \frac{p(4p^2-4p-1)}{T_2(1+4p^2)^2}$$

Critical temperature

$$T_{\text{crit}} = \max\left[\frac{1}{\ln\left(1+\frac{F_2}{F_1^2}\right)}, \frac{1}{\ln\left(1+\frac{F_2}{G_1^2}\right)}, \frac{1}{\ln\left(1+\frac{F_2}{H_1^2}\right)}\right]$$

Perturbative analysis of critical temperature

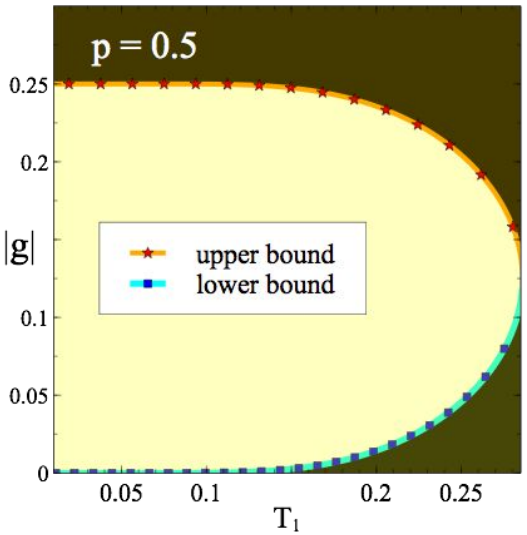
High T_2 limit



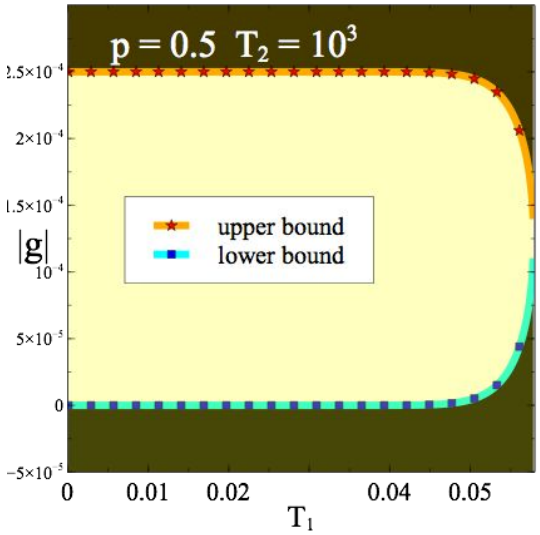
Less unpolarised spin bath helps…

Even below critical temperature, $\boldsymbol{g}$ must lie in some parameter region

Low T_2 limit

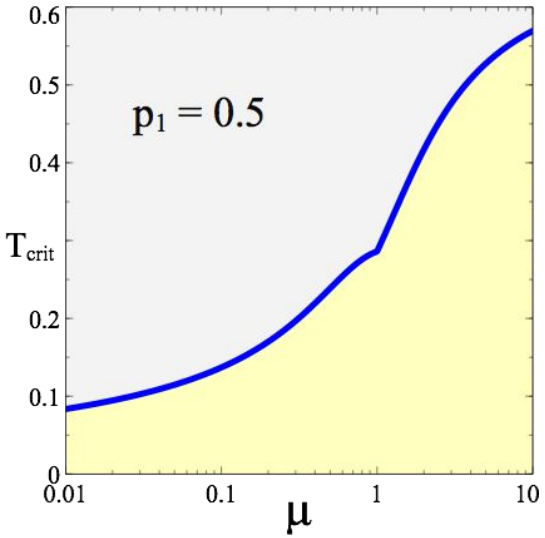


High T_2 limit

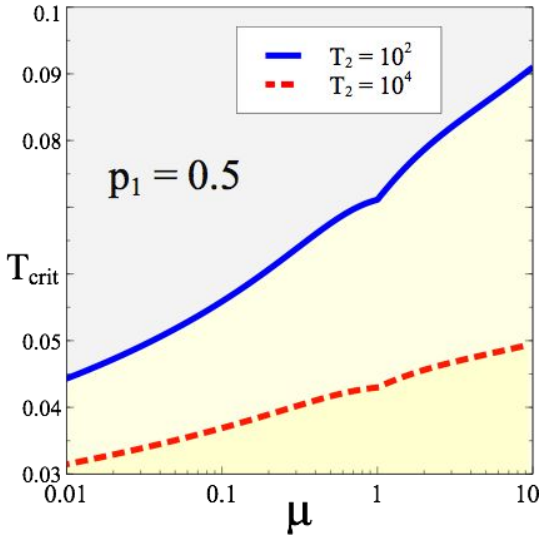


Asymmetry between reset probabilities also helps

Low T_2 limit



High T_2 limit



- What is magic ?
- Link with other resources
- How to create magic ?
- Conclusions

➔ **Resource Theories of Stabiliser computation and Coherence are intimately connected.**

Q : What about connections with other resource theories ?

➔ **You can create magic in qubits using autonomous thermodynamic machines.**

Q: Can you distill ?

➔ **Magic as a witness of Physical changes ?**

Hopefully… ongoing work

Thank you