# *Broadcasting of Correlations in a Quantum World*


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# Contents

# Few (popular) Impossibilities in the Quantum World

- Cloning of arbitrary quantum states — [W.K. Wootters et al., Nature **266**, 802 (1982)]
  - Broadcasting of arbitrary quantum states — [H. Barnum et al., Phys. Rev. Lett. **99**, 240501 (2000)]
- Deletion of arbitrary quantum states — [A.K. Pati et al., Nature **404**, 164 (2000)]
- Partial erasure of quantum information — [A.K. Pati et al., Phys. Lett. A **359**, 31 (2006)]
- Partial flipping of quantum information — [V. Buzek et al., Phys. Rev. A **60**, R2626 (1999)]

***Cloning*** *and* ***Broadcasting*** *demarcate the boundary between the Classical and Quantum Worlds.*

# No Cloning Theorem

***Noiseless* and *deterministic* cloning of an *arbitrary* and *unknown* quantum state is impossible!**


[W.K. Wootters et al., Nature **266**, 802 (1982)]

**Proofs exists from: Linearity, Unitarity & No-Signaling principles**

**Can we take a modest approach?**

# Modest Approaches to Quantum Cloning

*No-cloning theorem doesn't rule out the possibility of cloning states approximately.*

- Symmetric (B-H type) cloning operations:
  - State independent (universal) optimal cloners → Local / Nonlocal
    [V. Buzek et al., Phys. Rev. A **54**, 1844 (1996)]
    [V. Buzek et al., Phys. Rev. Lett. **81**, 5003 (1998)]
  - State dependent optimal cloners → Local / Nonlocal
    [M. Shukla et al., In Prep.]
    [S. Chatterjee et al., Phys. Rev. A **93**, 042309 (2015)]
- Asymmetric (Pauli) cloning operations:
  - State independent (universal) optimal cloners → Local / Nonlocal
    [I. Ghui et al., Phys. Rev. A **67**, 012323 (2003)]
    [A. Jain et al., arXiv:1702:02123 (2018)]
- Probabilistic Cloners
  [L. M. Duan et al., Phys. Rev. Lett. **80**, 4999 (1998)]

# No Broadcasting Theorem

***Weaker version of the no-cloning theorem***

For a CPTP Map $\mathcal{E}$,

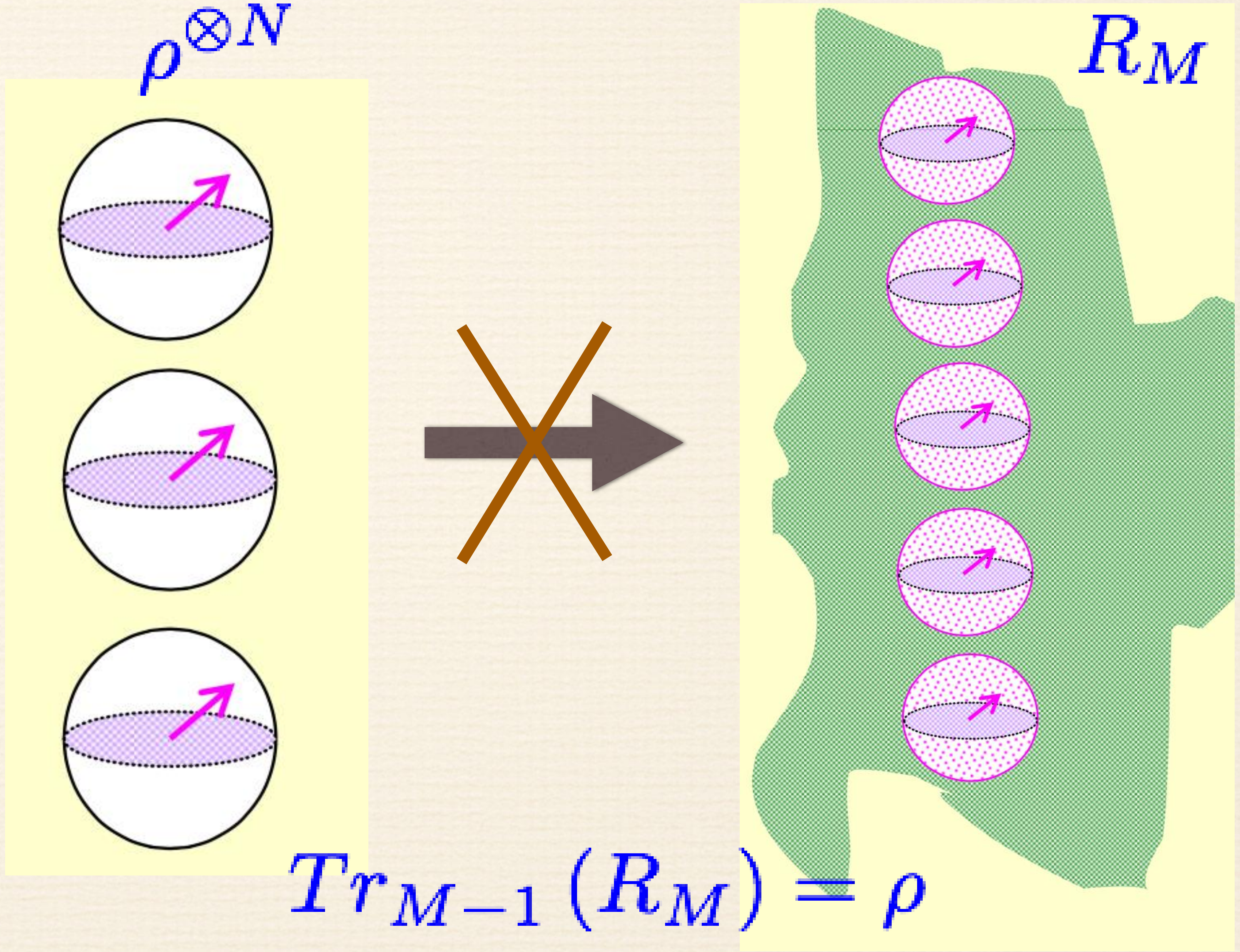
1. **Clonable**: $\varepsilon(\rho) = \rho \otimes \rho$

2. **Broadcastable**: marginals of $\varepsilon(\rho)$ are $\rho$

[Dieks et al., Phys. Lett. A **92**, 271 1982]

<u>For Broadcasting of Correlations:</u>

$$C(TrM\ \_\ _{1}(R_M)) = C(\rho)$$



# Works enforcing the no-broadcasting theorem

- Non-commuting mixed states cannot be broadcast. [Barnum et al., PRL 1996]
- No local broadcasting of quantum correlation. [Piani et al., PRL 2008]
- No uni-local broadcasting of quantum correlation. [Luo et al., Lett. Math. Phys. 2010]
- Equivalence of the no-broadcasting theorems. [Luo et al., PRA 2010]

Similar to "approximate Cloning", Can we take a modest approach in "Broadcasting" as well?

# Our Motivation: Broadcasting of Entanglement

- If, **compression** of quantum entanglement exists: Distillation procedures. [C. H. Bennett et al., Phys. Rev. A **53**, 2046 (1996)]

- Can we **decompress** (broadcast) quantum entanglement?

- "**Broadcasting of entanglement via local cloning**": generation of lesser entangled pairs from a given pair where $C(\rho^{a_1 b_1}) \leq C(\rho^{a_2 b_2}) \ll C(\rho^{ab})$. [V. Buzek et al., Phys. Rev. A **55**, 3327 (1997)]

- Broadcasting of correlations is **approximate** local or nonlocal **copying** of nonlocal correlations.

# Towards Quantum Networks

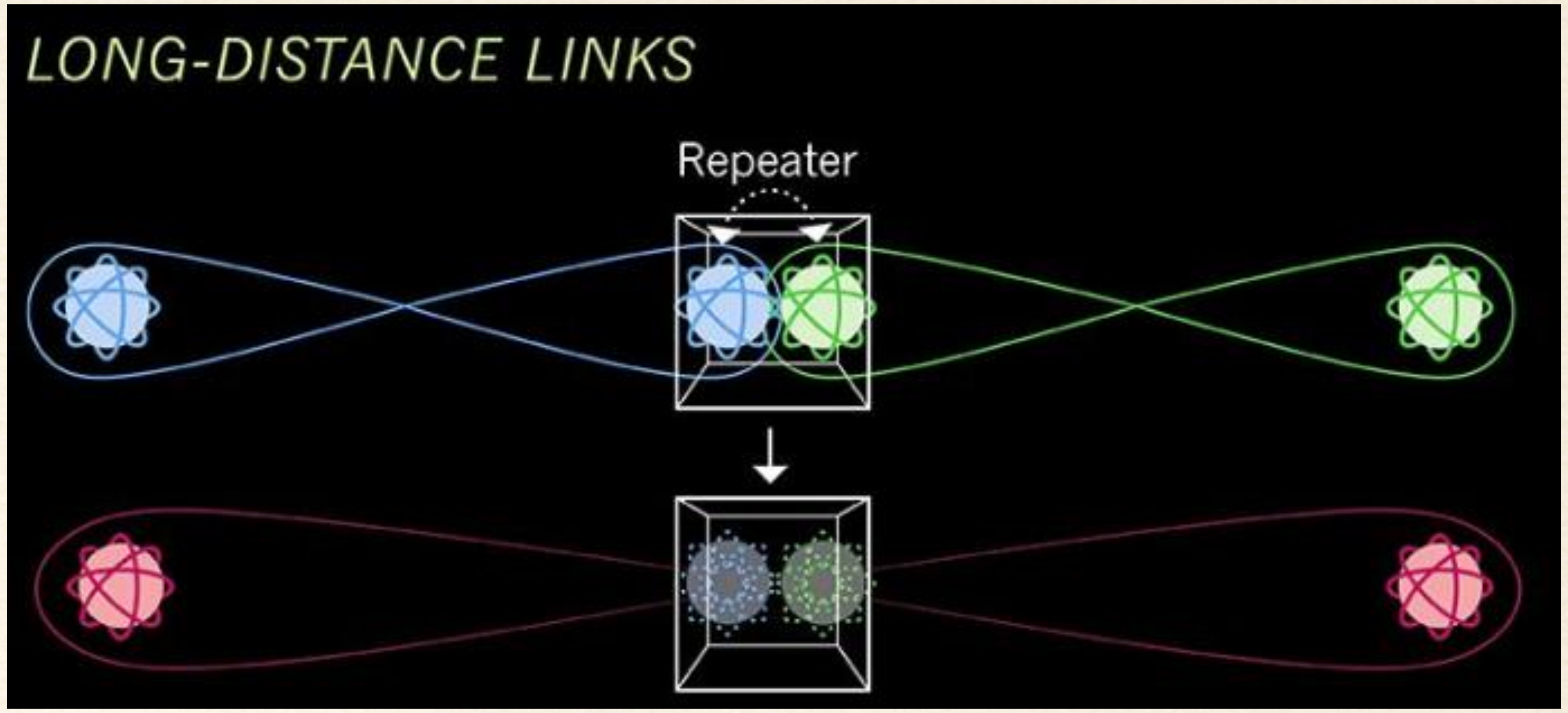
- Quantum Correlations: imperative resources for QIP tasks

- Manufacturing maximally entangled pairs is difficult

- Cloning is less costly operation: unitary

- Distribution of entanglement across various nodes in a network via cloning is of practical importance

- Increase in number of entangled pairs at the cost of reduction in the degree of entanglement




Quantum Internet

Nature 554, 289-292 (2018)

Pic. Courtesy: G. Shafiee, MPL Erlangen

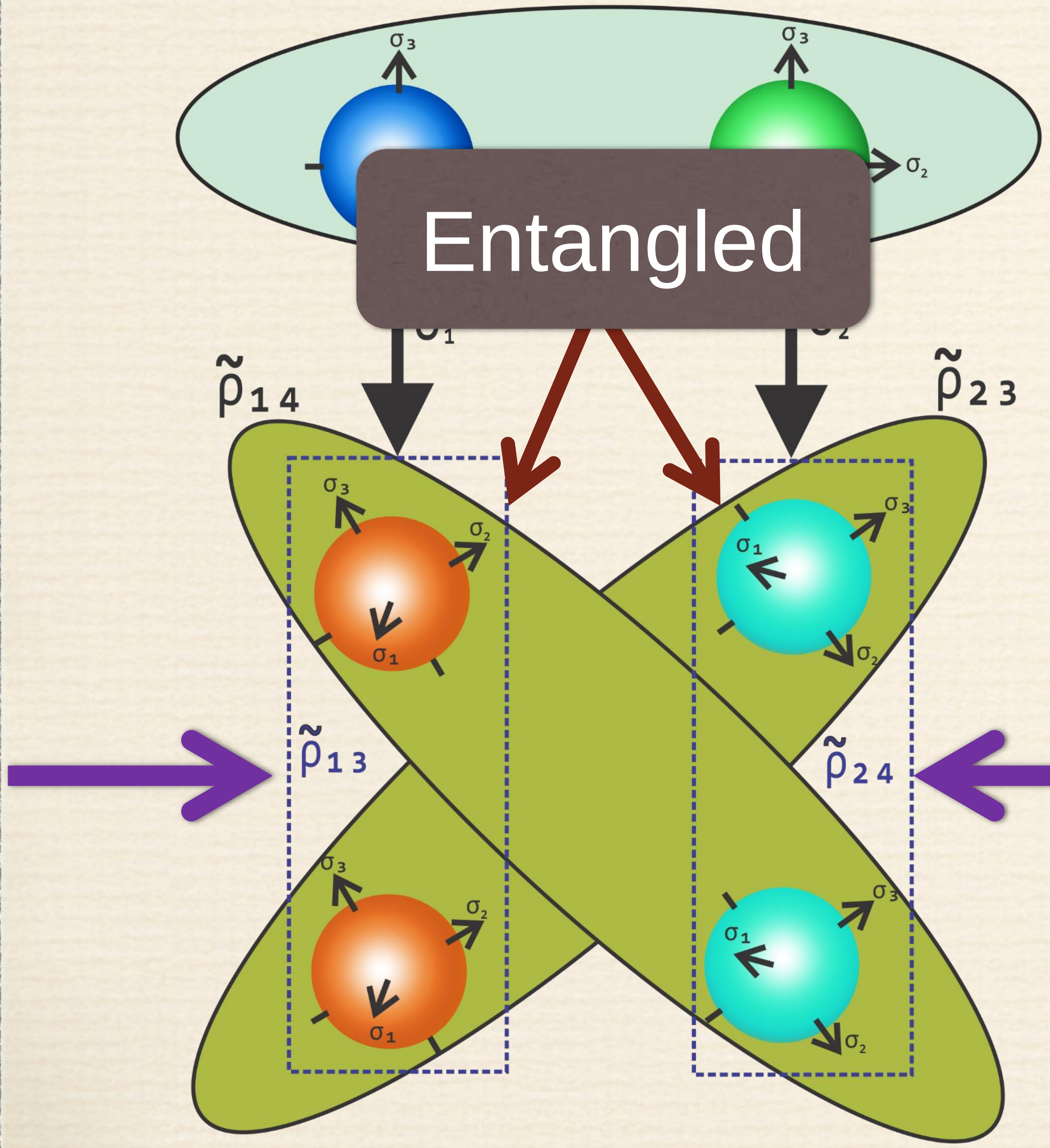
So, using cloning operations on an input entangled pair how can we best decompress (broadcast) entanglement?

# 1. With Symmetric Optimal State Independent (universal) B-H Cloners

# Broadcasting of Entanglement via Symmetric Universal Optimal Cloning operations

**B-H** QCM symmetric **M**-dimensional cloning transformation: (M=$2^m$, m = no. of qubits)

$$U_{bh}\,|\Psi_i\rangle_{a_0}\,|0\rangle_{a_1}\,|X\rangle_x \rightarrow c\,|\Psi_i\rangle_{a_0}\,|\Psi_i\rangle_{a_1}\,|X_{ii}\rangle_x + d\sum_{j\neq i}^{M}\left(|\Psi_i\rangle_{a_0}\,|\Psi_j\rangle_{a_1} + |\Psi_j\rangle_{a_0}\,|\Psi_i\rangle_{a_1}\right)|Y_{ij}\rangle_x$$

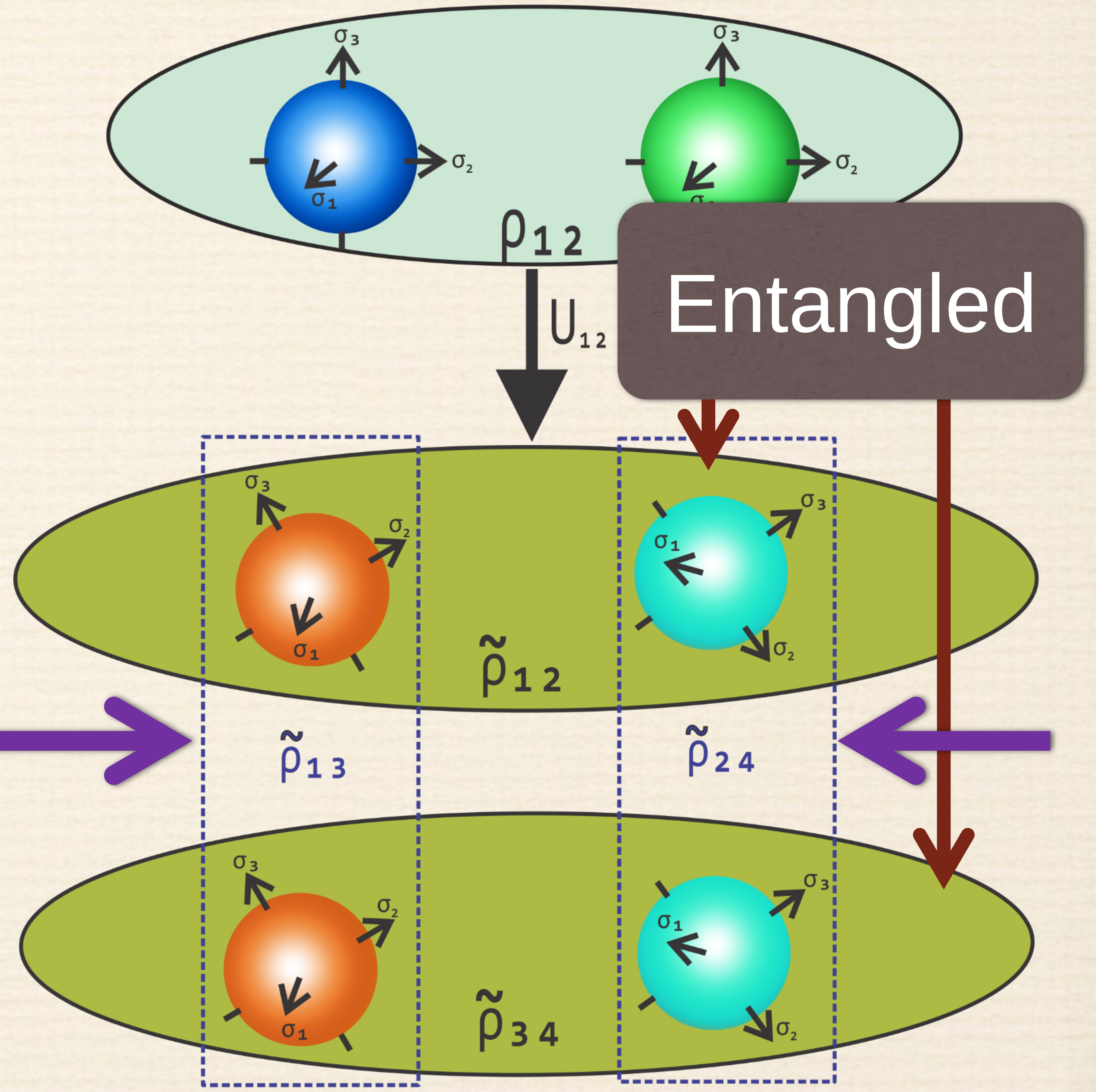



**State Independent Cloners**

$$c^2 = \frac{2}{M+1}$$

$$d^2 = \frac{1}{2(M+1)}$$

Separable




**Optimal Fidelity = $^5/_6$**

**[V. Buzek et al., PRA 1997, PRL 1998]**

**Via Local Cloners (M=2)**

**Via Nonlocal Cloners (M=4)**

# Broadcasting of Entanglement via Symmetric Cloning for any general two qubit input state

"Given a most general state of two qubits as input, we explicitly derive the range of input state parameters for which broadcasting of entanglement will be possible"

Local and nonlocal B-H cloners were used for this purpose

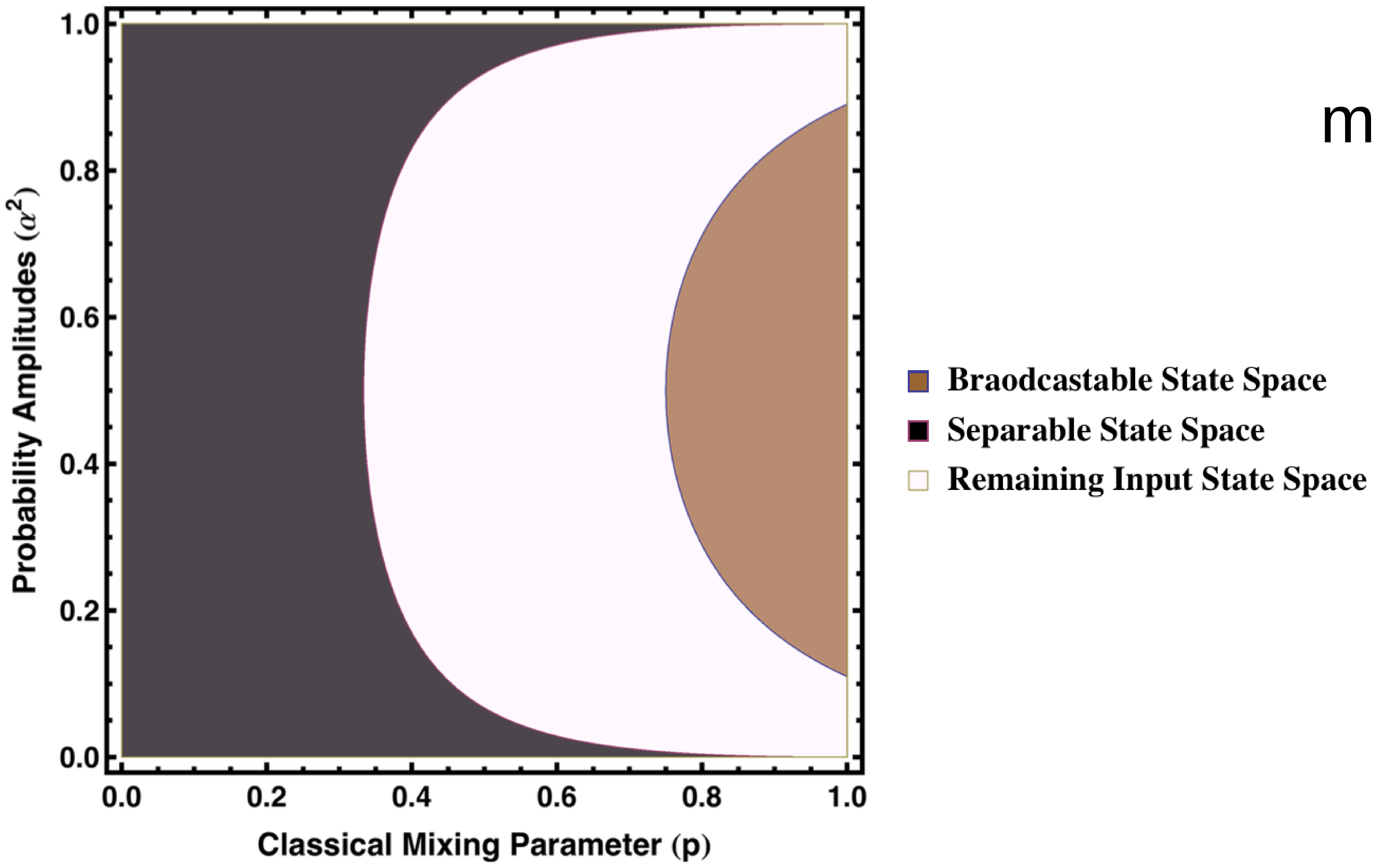
[S. Chatterjee et al., Phys. Rev. A **93**, 042309 (2016)]

Let’s try to visualize the results with certain well-known class of states!

# Exemplifying with Werner-like States via B-H Local Cloners

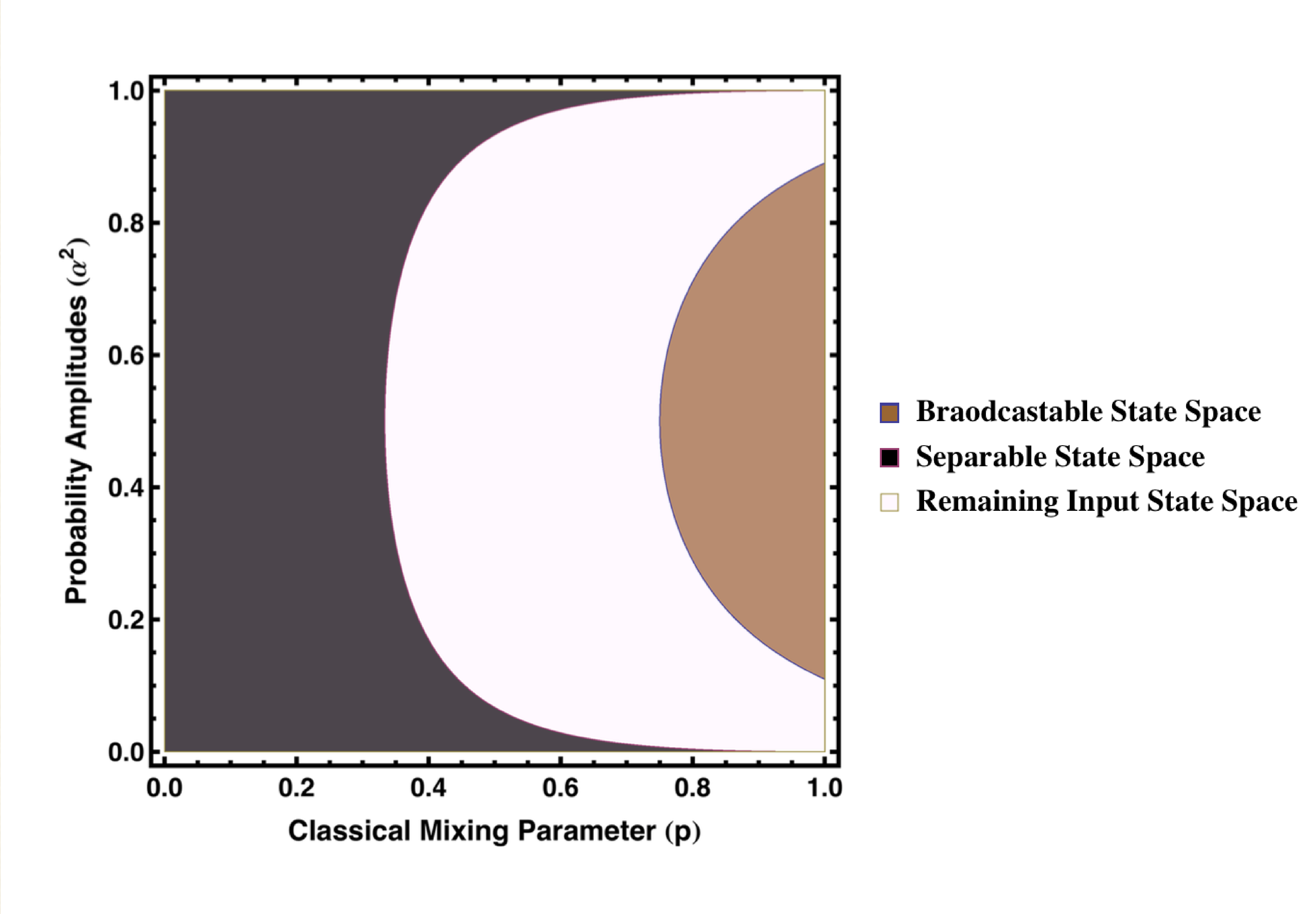
Input State: $\rho_{12}^{w} = \left(\frac{1-p}{4}\right) I \otimes I + p\,|\varphi\rangle\,\langle\varphi|$; $|\varphi\rangle = \alpha\,|00\rangle + \beta\,|11\rangle$

$p$ = classical mixing parameter

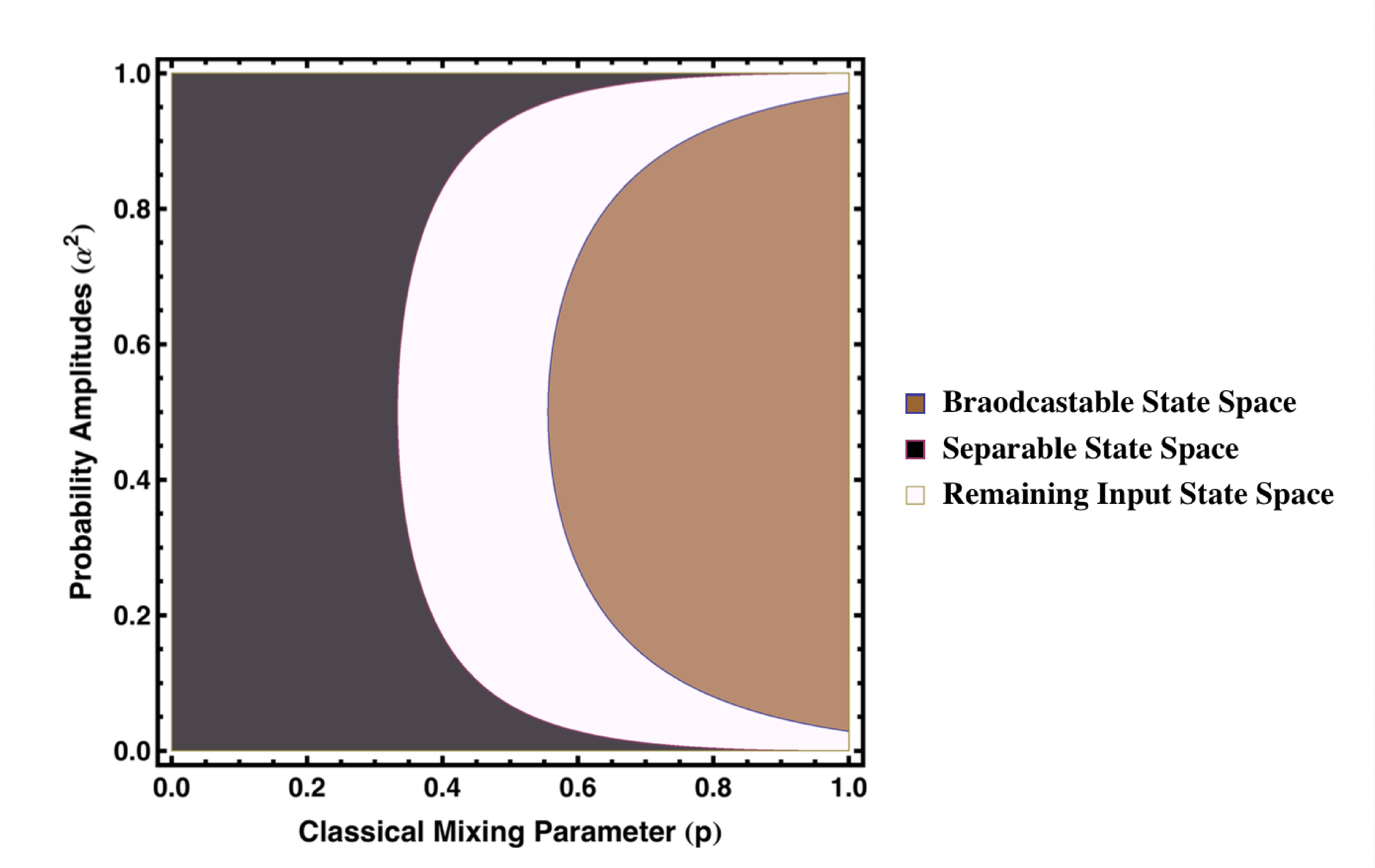



[S. Chatterjee et al., Phys. Rev. A **93**, 042309 (2016)]

# Exemplifying with Werner-like States: Comparison b/w Symmetric Universal Local & Nonlocal Cloning Operations




**With Local Cloner**




**With Nonlocal Cloner**

[S. Chatterjee et al., Phys. Rev. A **93**, 042309 (2016)]

# Exemplifying with Bell-diagonal states via B-H Local Cloning Operations

- Input State:

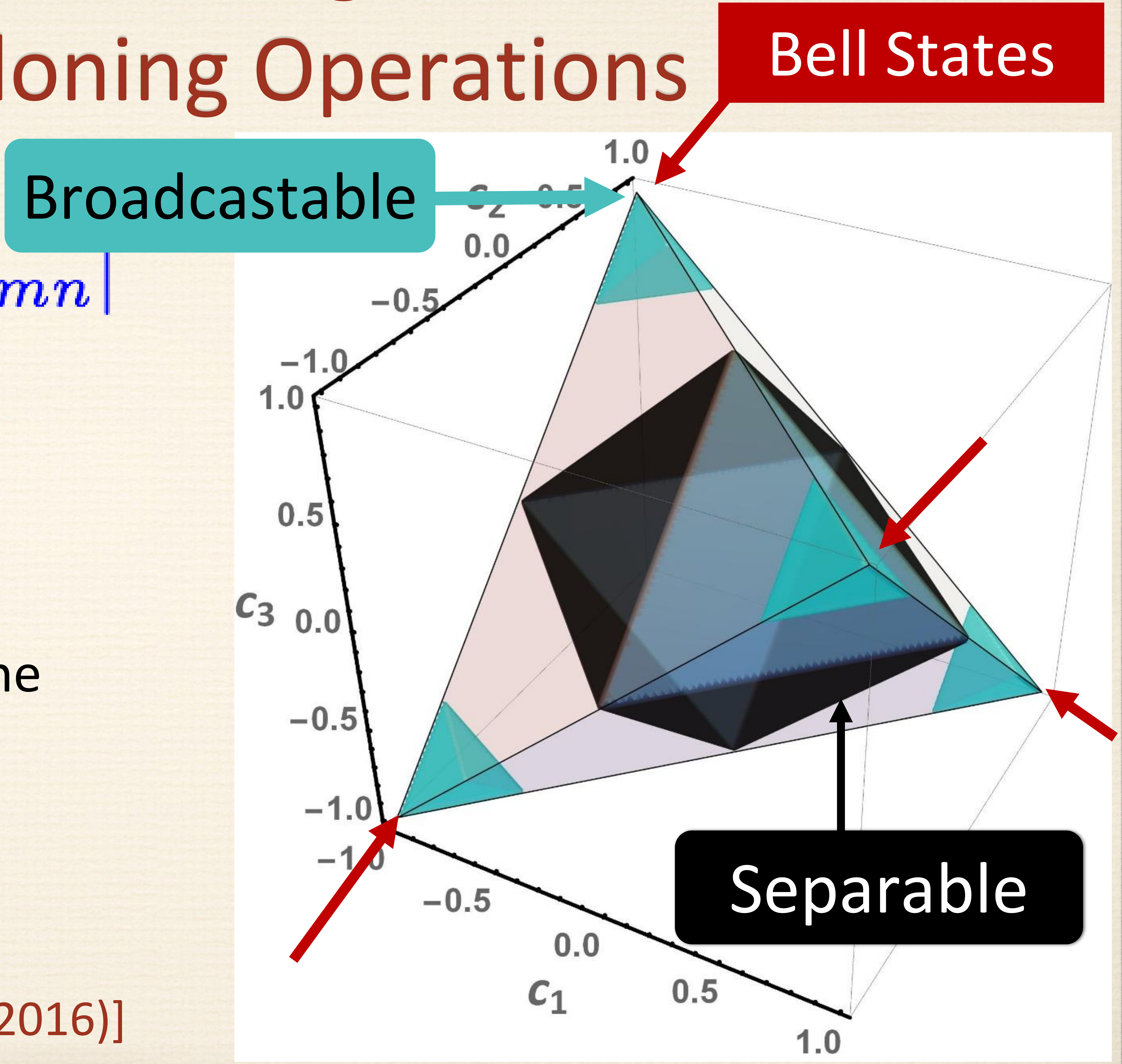
$$\rho_{12}^{b} = \sum_{m,n} \lambda_{mn} |\gamma_{mn}\rangle \langle\gamma_{mn}|$$

$|\gamma_{mn}\rangle$ = Eigenstates or Bell States

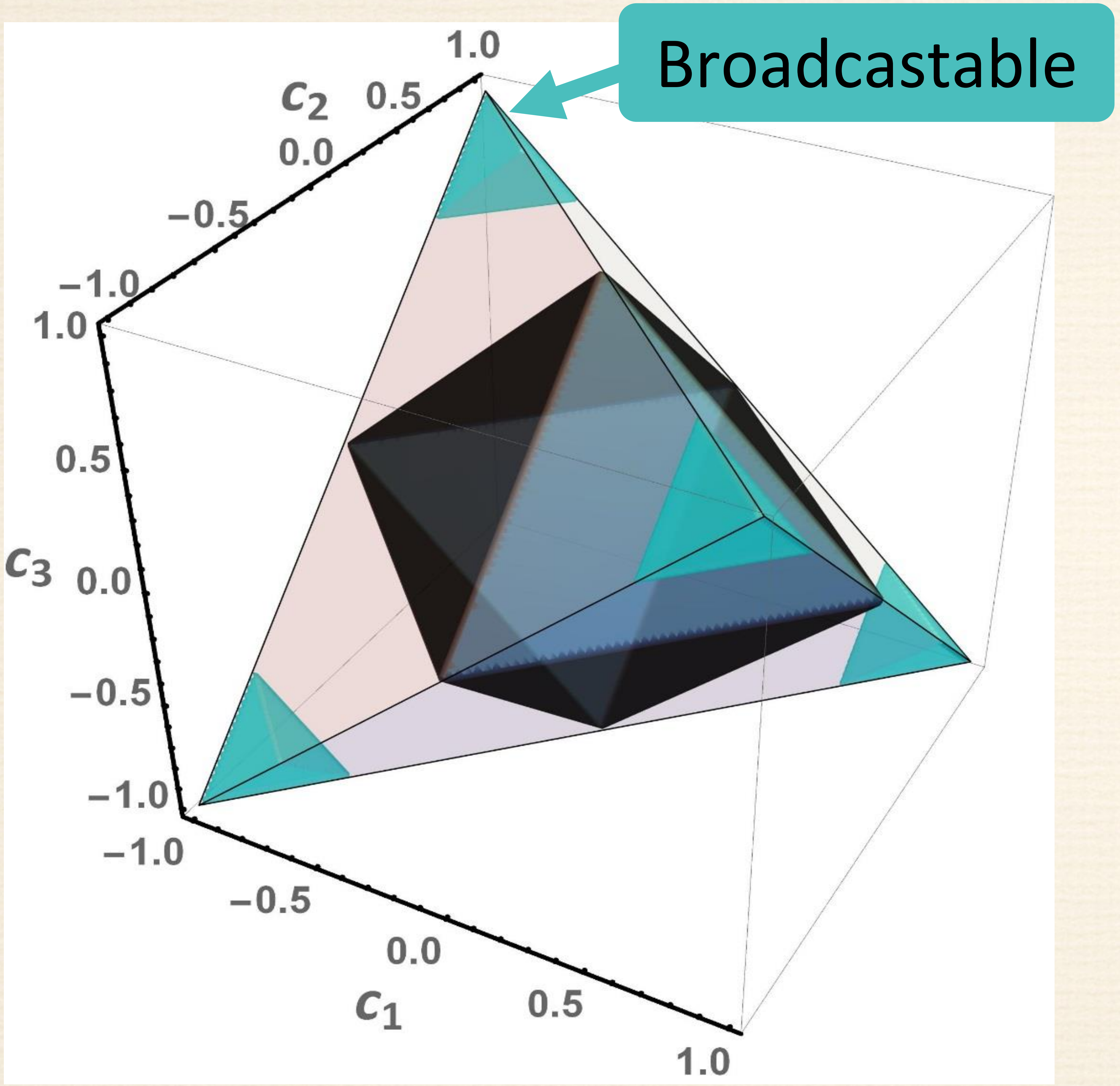
$$\lambda_{mn}(Ci) \geq 0$$

- Here $\mathbf{C_i}$'s, with $\mathbf{i} = \{1,2,3\}$, represent the input state parameters or probability amplitudes.

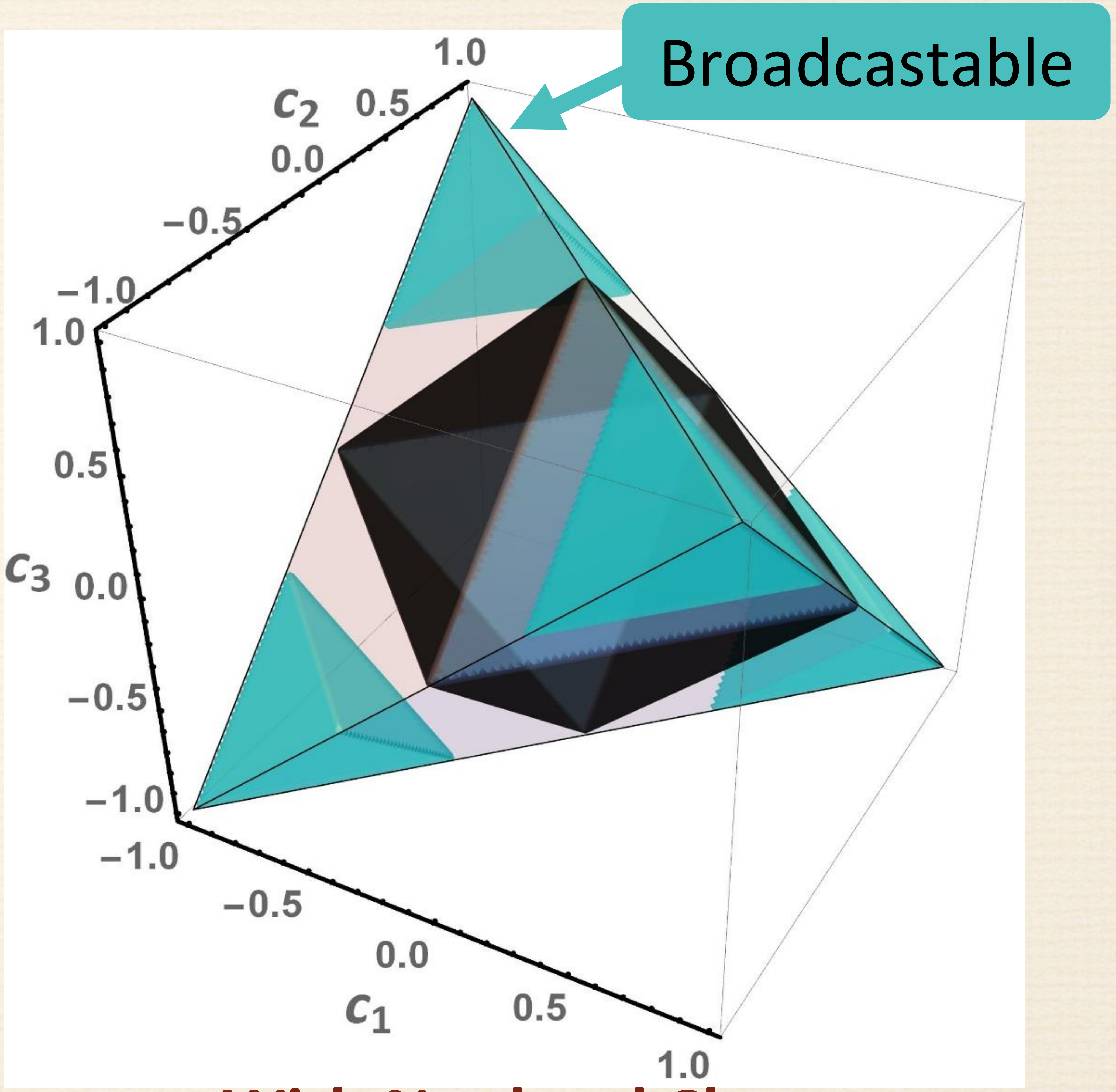
[S. Chatterjee et al., Phys. Rev. A **93**, 042309 (2016)]



# Comparison of Broadcastable zones with Symmetric Cloning operations for Bell-diagonal input states




**With Local Cloner**




**With Nonlocal Cloner**

[S. Chatterjee et al., Phys. Rev. A **93**, 042309 (2016)]

Can asymmetric cloning operations outperform symmetric cloners in the broadcasting entanglement?

# Asymmetric Universal Optimal Cloning Operations

**Pauli QCM are asymmetric universal optimal cloning transformations**

[I. Ghui et al., Phys. Rev. A **67**, 012323 (2003)]

**Local case:**

$$U_a^l|0\rangle|00\rangle = \frac{1}{\sqrt{(1+p^2+q^2)}}(|000\rangle + p|011\rangle + q|101\rangle)$$

$$U_a^l|1\rangle|00\rangle = \frac{1}{\sqrt{(1+p^2+q^2)}}(|111\rangle + p|100\rangle + q|010\rangle)$$

**Nonlocal case:**

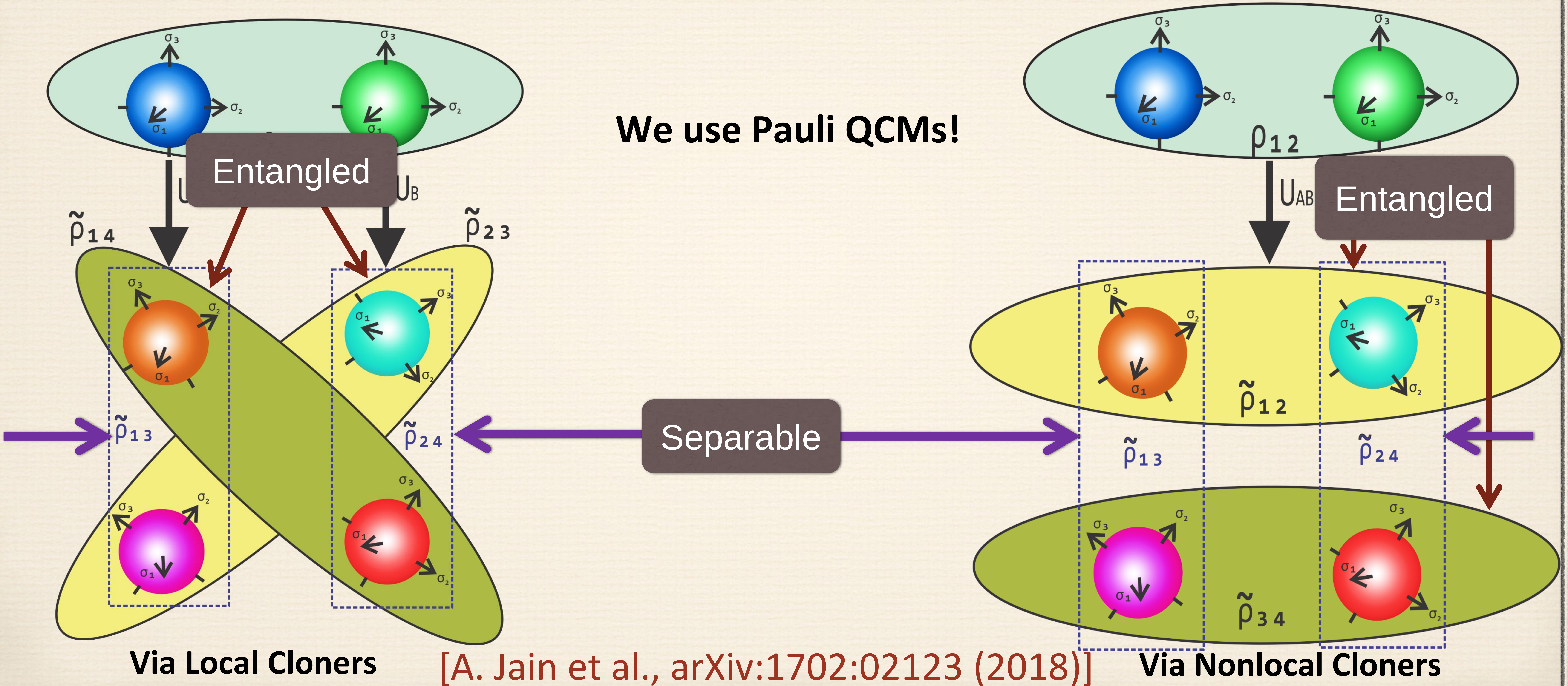
$$U_a^{nl}|j\rangle|00\rangle = \frac{1}{\sqrt{1+3(p^2+q^2)}}\left(|jjj\rangle + p\sum_{r=1}^{3}|j\rangle|\overline{j+r}\rangle|\overline{j+r}\rangle + q\sum_{r=1}^{3}|\overline{j+r}\rangle|j\rangle|\overline{j+r}\rangle\right)$$

$$where,\ j \in \{0,1\}\ and\ \overline{j+r} = (\mathrm{j}+\mathrm{r})\ \%\ 4$$

$$Here, p+q=1\ \&\ p=q=1/2\ \ gives\ symmetric\ cloning\ transformations$$

# 1. With Asymmetric Optimal State Independent (universal) B-H Cloners

# Broadcasting of Entanglement (1 → 2) via Asymmetric Universal Optimal Cloning operations




[A. Jain et al., arXiv:1702:02123 (2018)]

# Broadcasting of Entanglement via Asymmetric Cloning for any general two qubit input state

"Given a most general state of two qubits as input, we explicitly derive the range of input state parameters for which broadcasting of entanglement will be possible"

Local and nonlocal Pauli cloners were used for this purpose

[A. Jain et al., arXiv:1702:02123 (2018)]

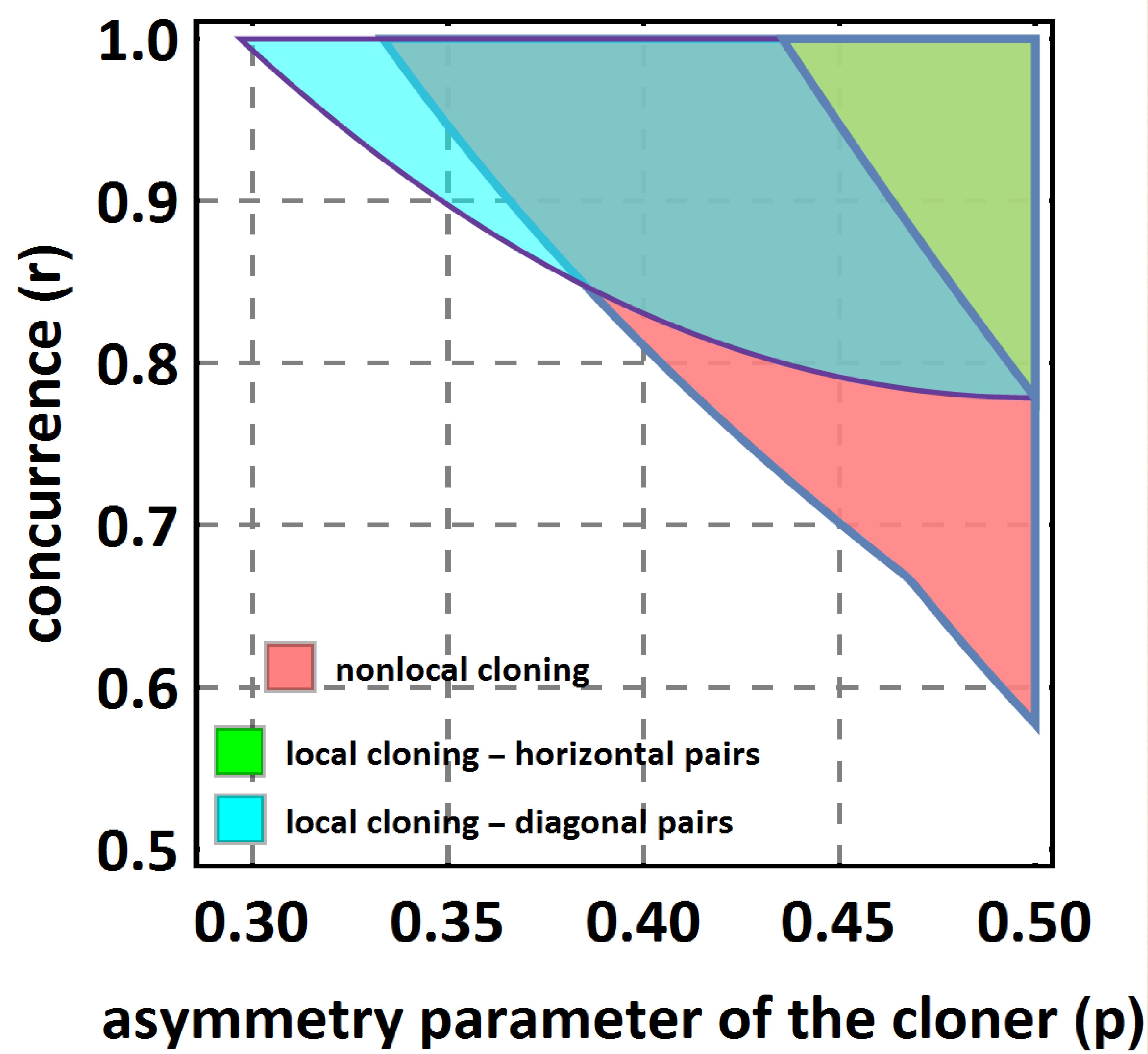
# Exemplifying with MEMS via Asymmetric Optimal Universal Cloners

MEMS: Maximally Entangled Mixed States

Possess maximal entanglement (concurrence squared) for a given degree of mixedness (linear entropy)

$$\rho_{MI} = \frac{r}{2}(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11| + \frac{2(1-r)}{r}|01\rangle\langle 01|), \frac{2}{3} \leq r \leq 1$$

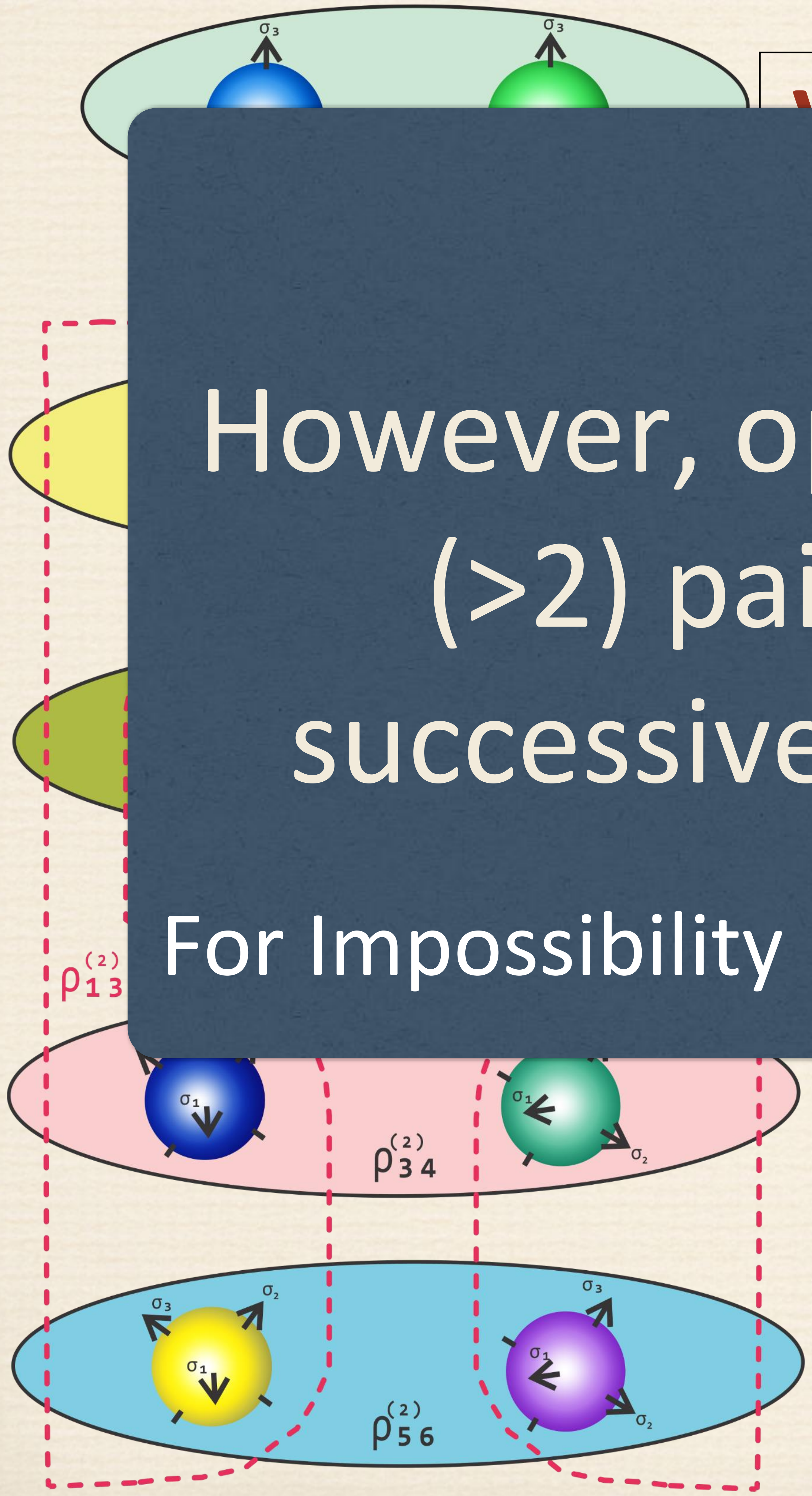
$$\rho_{MII} = \frac{1}{3}(|00\rangle\langle 00| + \frac{3r}{2}|00\rangle\langle 11| + \frac{3r}{2}|11\rangle\langle 00| + |11\rangle\langle 11| + |01\rangle\langle 01|), 0 \leq r \leq \frac{2}{3}$$

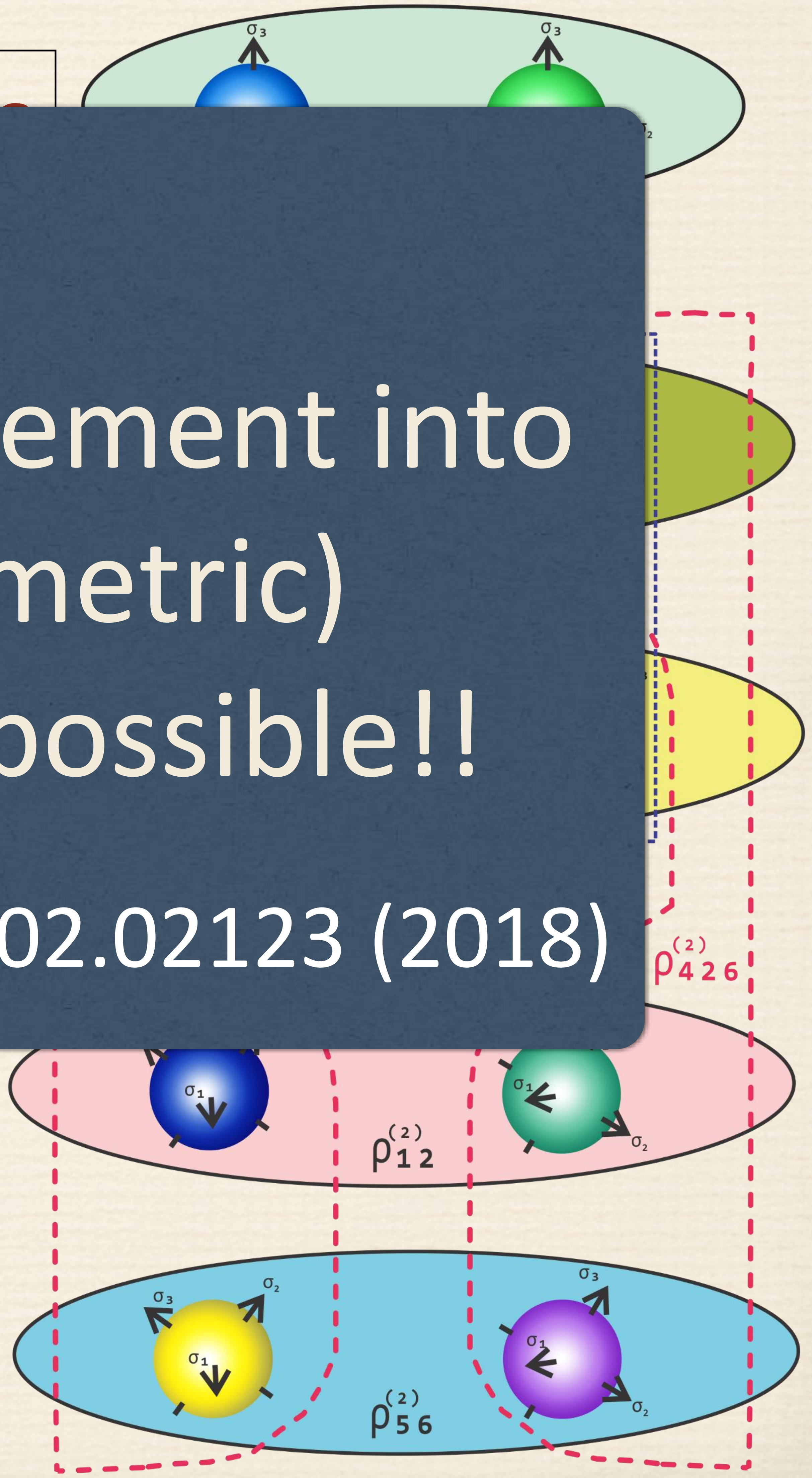



[A. Jain et al., arXiv:1702:02123 (2018)]

Can we optimally broadcast entanglement locally or nonlocally into more than two pairs via cloning operations?

# Broadcasting of Entanglement (1 → 3) via successive Asymmetric Universal Optimal Cloning operations




However, optimal broadcasting of entanglement into (>2) pairs via any (symmetric or asymmetric) successive local cloning operations is impossible!!

For Impossibility Proofs & its generalization, Check arXiv:1702.02123 (2018)

[A. Jain et al., arXiv:1702:02123 (2018)]

# Exemplifying with NMEs for Broadcasting Entanglement into 3 pairs via Successive Asymmetric Nonlocal Cloning

NMEs: Non-Maximally Entangled States

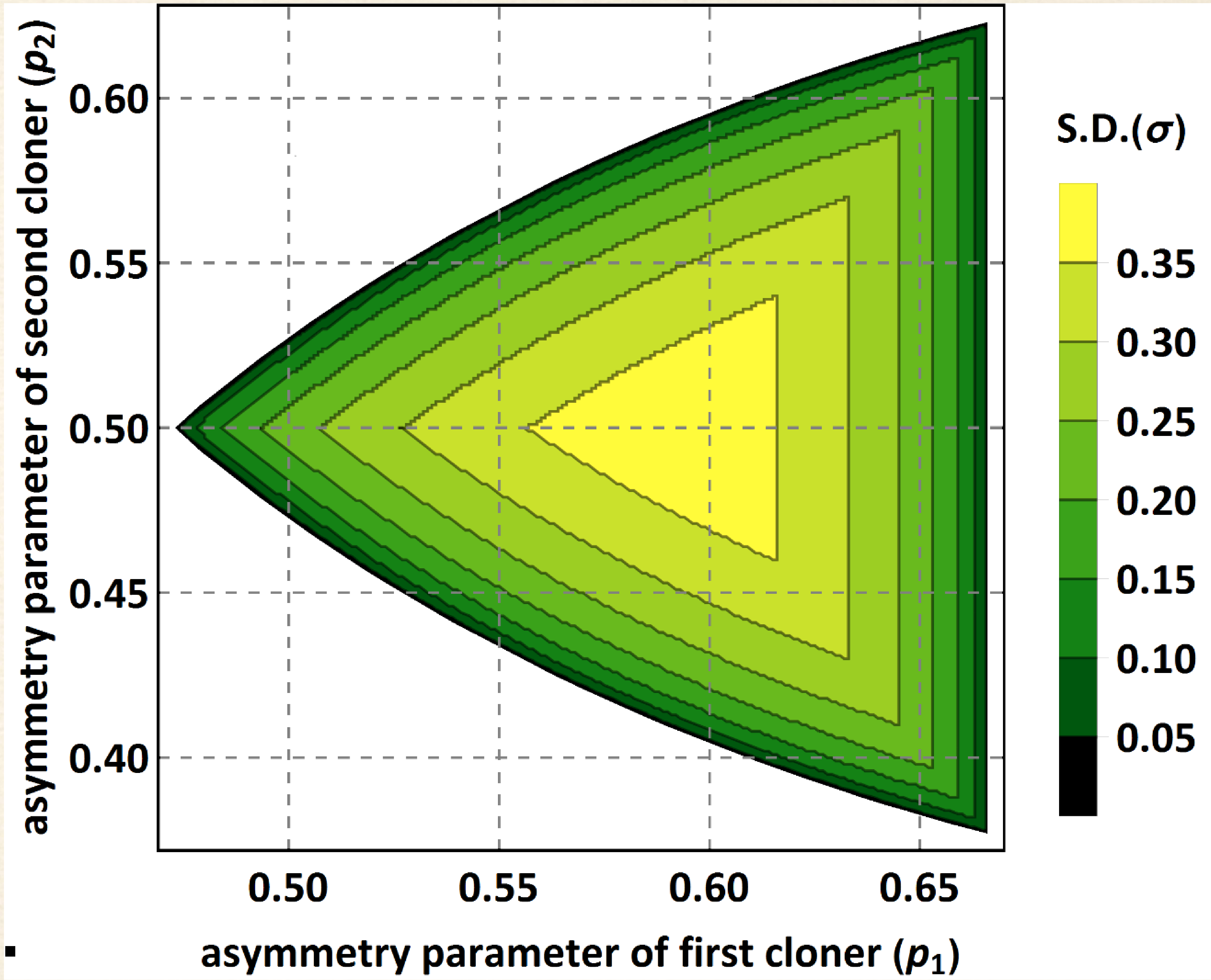
$$|\psi_{12}\rangle = \sqrt{k}|00\rangle + \sqrt{(1-k)}|11\rangle, 0 \le k \le 1$$

Hue depicts the value of standard deviation (SD) i.e. $\sigma$

$$(0.5-\sigma) \le k \le (0.5+\sigma)$$

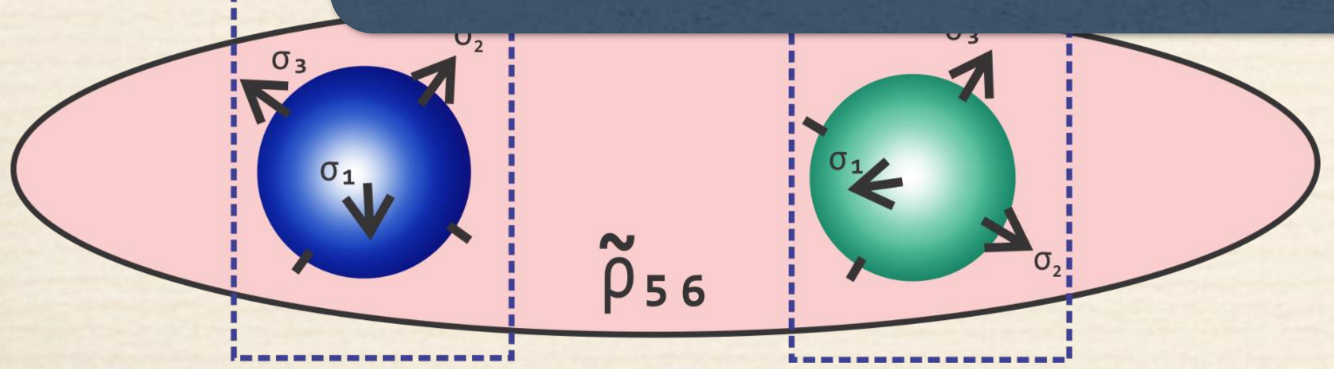
$$0.13 \le k \le 0.87$$

**Interestingly, the maximal range for the allowed values of *k* is not achieved by successive use of two symmetric cloners.**



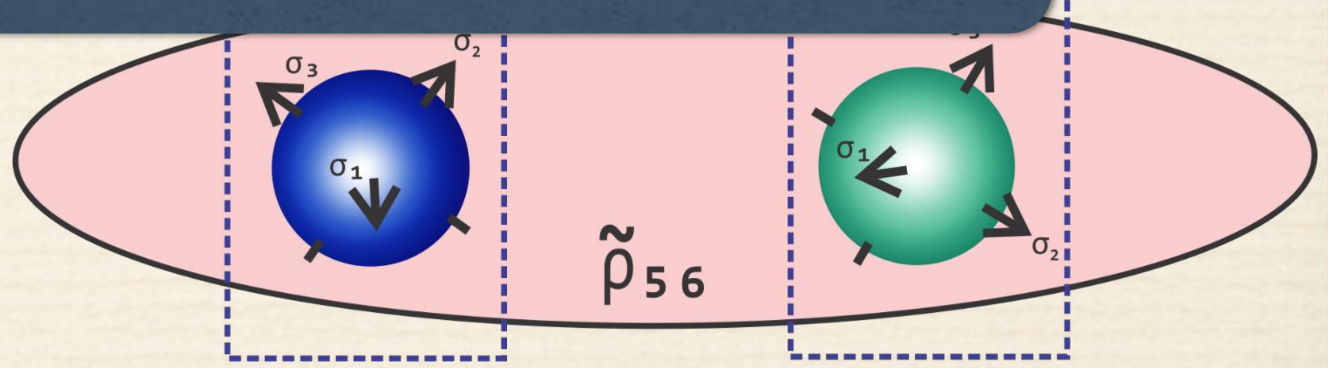

[A. Jain et al., arXiv:1702:02123 (2018)]

# Broadcasting of Entanglement (1 → 3) via direct Asymmetric Universal Optimal Cloning operations



However, optimal broadcasting of entanglement into (>2) pairs via any (symmetric or asymmetric) direct local cloning operations is impossible!!

For Impossibility Proofs & its generalization, Check arXiv:1702.02123 (2018)



[A. Jain et al., arXiv:1702:02123 (2018)]

# Exemplifying with NMEs for Broadcasting Entanglement into 3 pairs via Successive Asymmetric Nonlocal Cloning

NMEs: Non-Maximally Entangled States

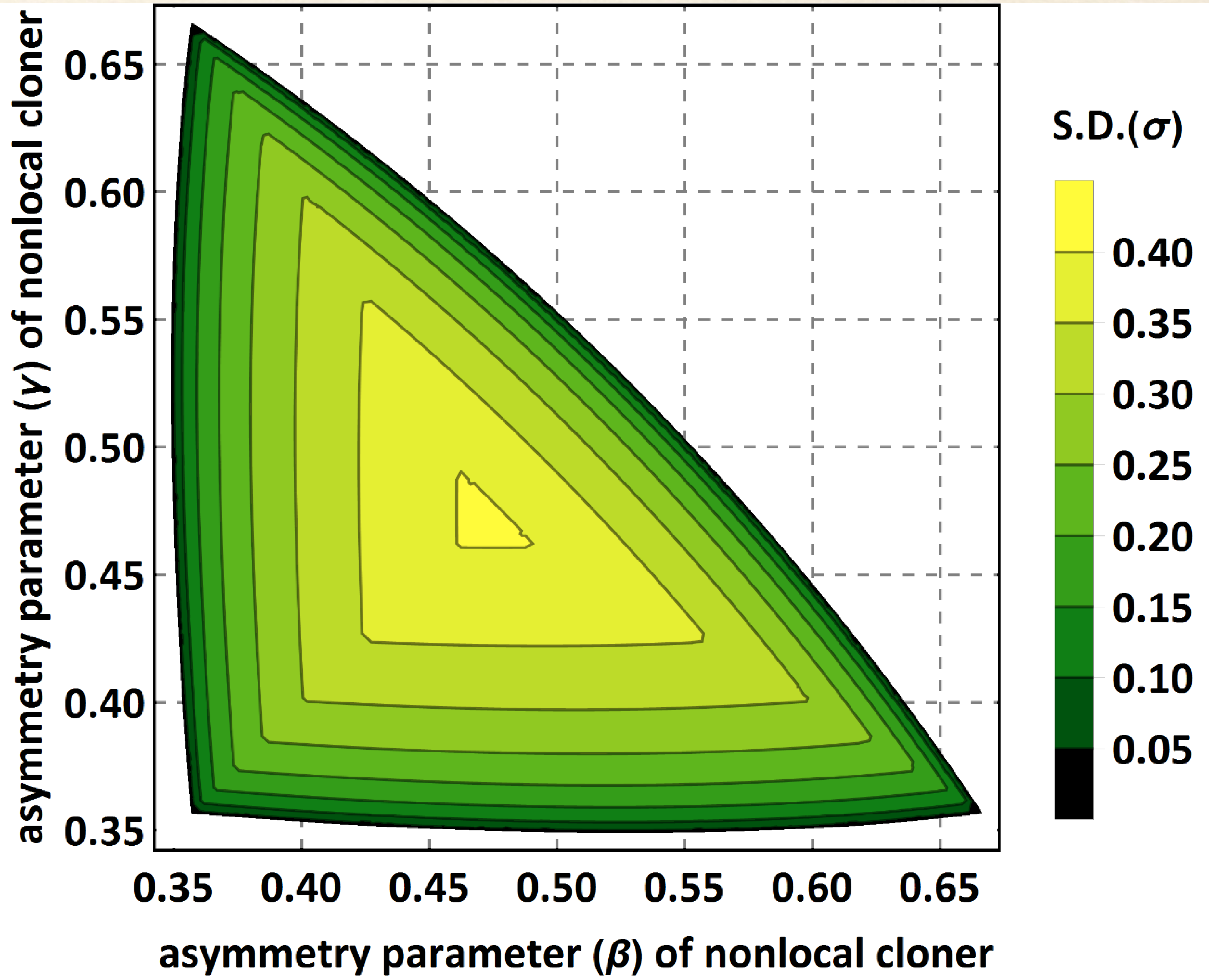
$$|\psi_{12}\rangle = \sqrt{k}|00\rangle + \sqrt{(1-k)}|11\rangle, 0 \leq k \leq 1$$

Hue depicts the value of standard deviation (SD) i.e. $\sigma$

$$(0.5-\sigma) \leq k \leq (0.5+\sigma)$$

$$0.09 \leq k \leq 0.91$$

For broadcasting of entanglement into 3 pairs; 1→3 **direct** cloner does a **better job** than two **successive** 1→ 2 ones.




[A. Jain et al., arXiv:1702:02123 (2018)]

# Comparison of any Local v/s Nonlocal Universal Cloning Operations in Broadcasting of Entanglement

**Observation:** **Nonlocal cloners produce a much wider range for broadcasting of entanglement.**

- **Local cloning: nonlocal outputs get the entanglement as a by-product of the process. Can broadcast only upto two pairs.**

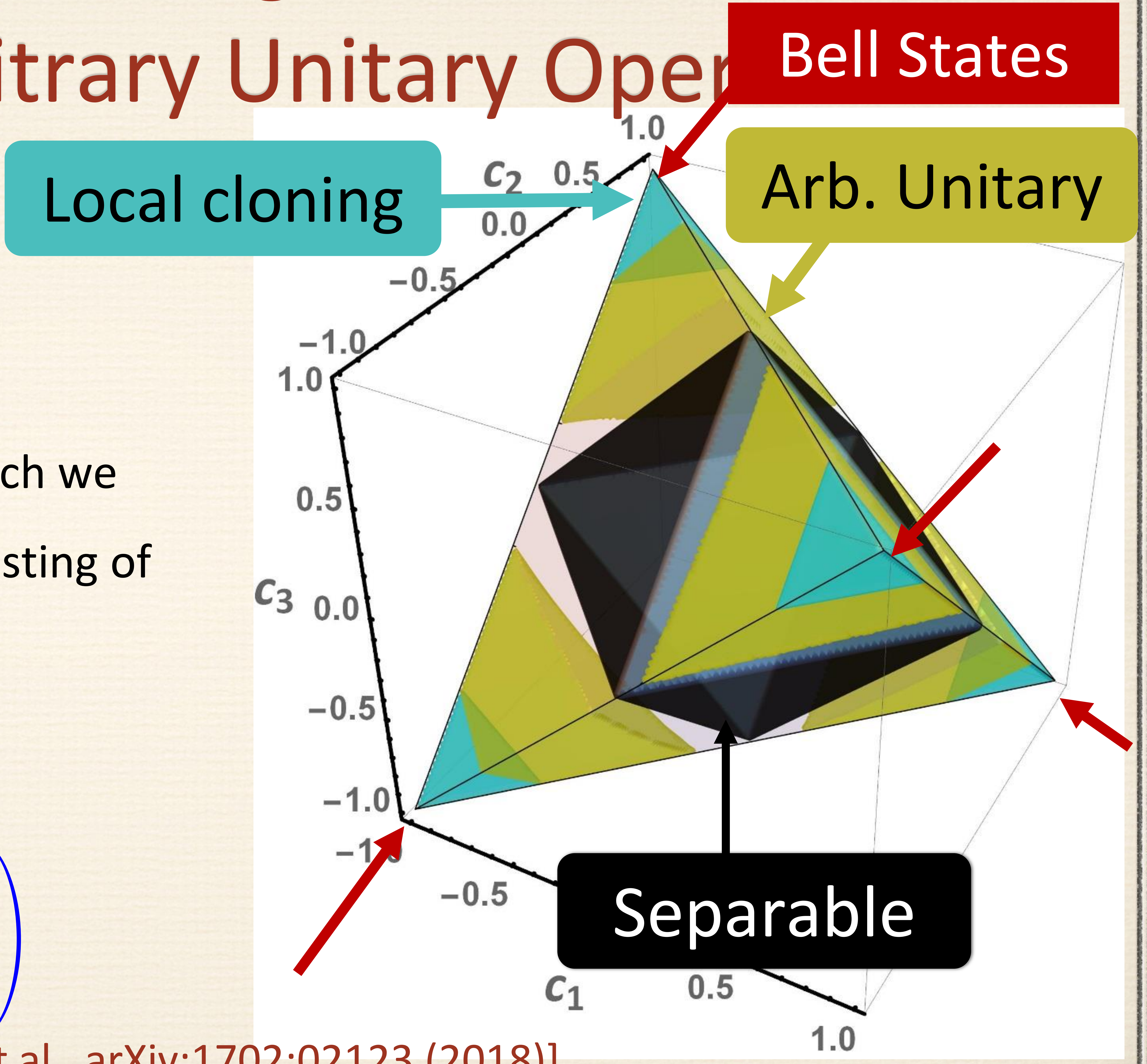
- **Nonlocal cloning: entanglement is directly copied. Can broadcast upto six pairs.**

**Inference:** **Entanglement is much well copied when nonlocal cloners are used.**

[S. Chatterjee et al., Phys. Rev. A **93**, 042309 (2016)]
[S. Bandyopadhyay et. al., Phys. Rev. A 60, 3296 1999]
[A. Jain et al., arXiv:1702:02123 (2018)]

Can we outperform cloning with some other unitary operation to broadcast entanglement?

# Exemplifying with Bell-diagonal states for Broadcasting with Arbitrary Unitary Oper




**$C_i$**'s, with **i** = {1,2,3}, represent the input state parameters or probability amplitudes.

- Existence of arbitrary unitary operations which we can achieve wider range for optimal broadcasting of entanglement.

- Arbitrary (Arb.) local unitary:

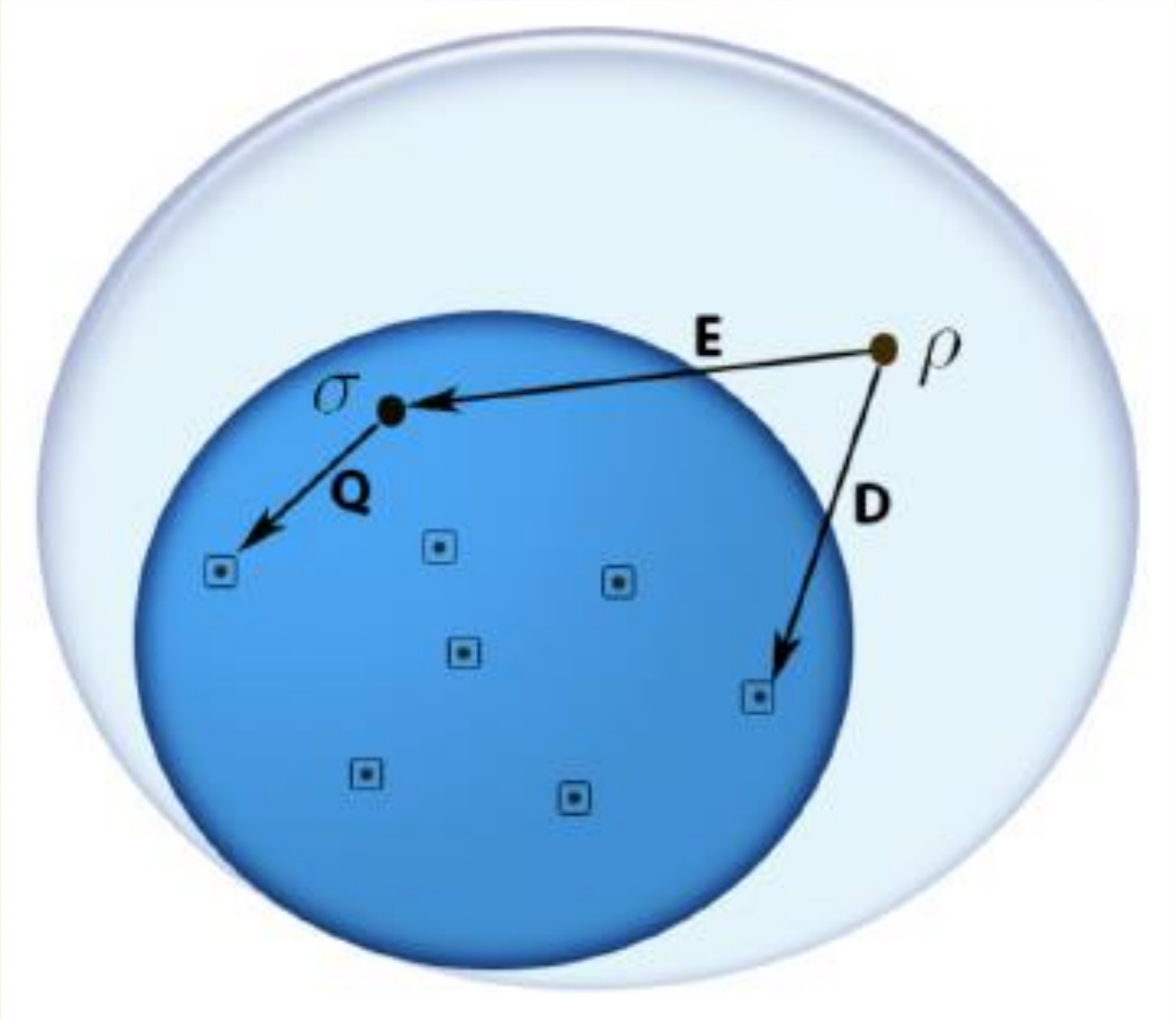
$$U_{bd} = \begin{pmatrix} 0.809 & 0.1816 & -0.4523 & 0.3286 \\ -0.1816 & -0.8273 & -0.4301 & 0.3125 \\ 0.559 & -0.5317 & 0.5148 & -0.3740 \\ 0 & 0 & -0.5878 & -0.8090 \end{pmatrix}$$

[A. Jain et al., arXiv:1702:02123 (2018)]

# Can we broadcast quantum correlations beyond entanglement?

# Non-classical Correlation beyond Entanglement

E = Entanglement




D = Discord

Q = Dissonance

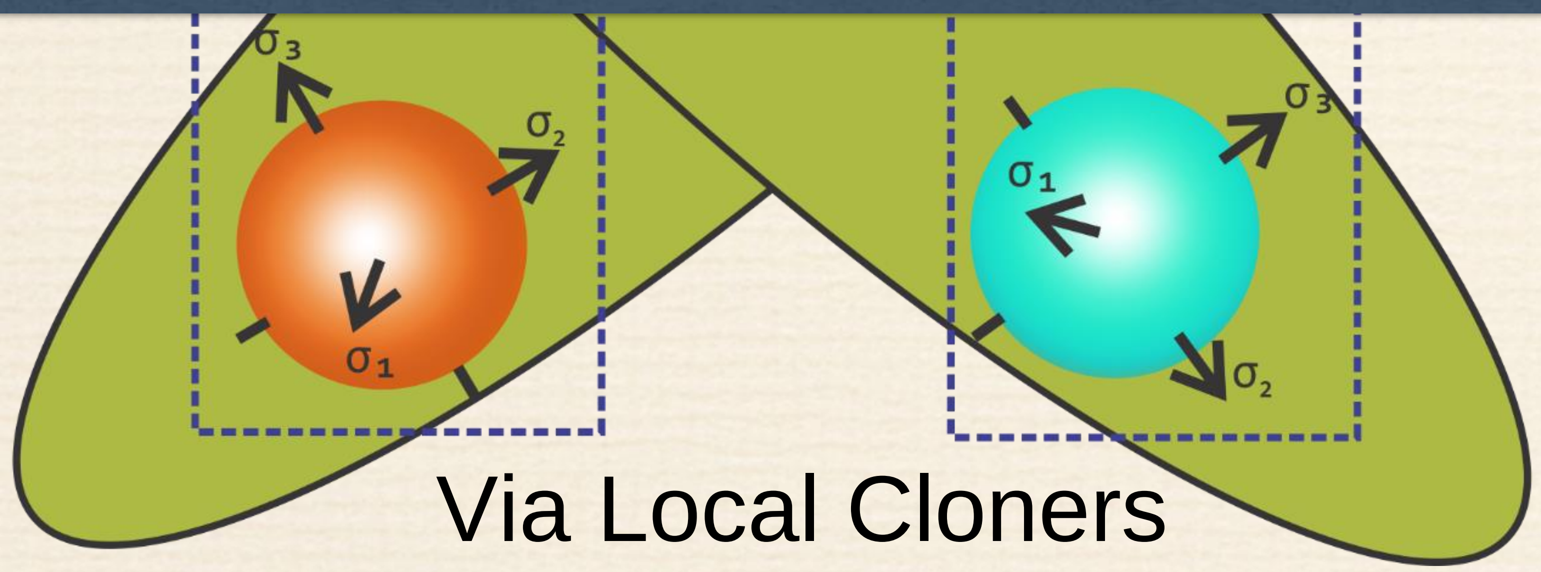
[K. Modi et al., arXiv:1104.1520]

# Optimal Broadcasting of Correlation beyond Entanglement using Symmetric Cloners

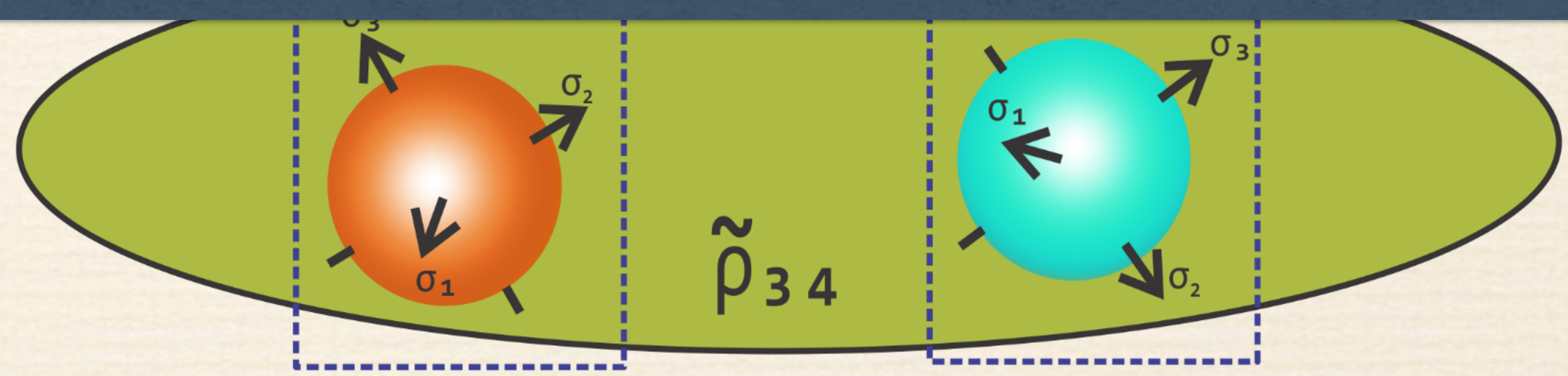
However, optimal broadcasting of correlations beyond entanglement via symmetric cloning is impossible!!

For Impossibility Proofs and its generalization, Check PRA **93**, 042309 (2016)

Disco … d=0




Via Local Cloners
(M=2)




Via Nonlocal Cloners
(M=4)

[Chatterjee et al., Phys. Rev. A **93**, 042309 (2016).]

# Optimal Broadcasting of Correlation beyond Entanglement using Asymmetric Cloners



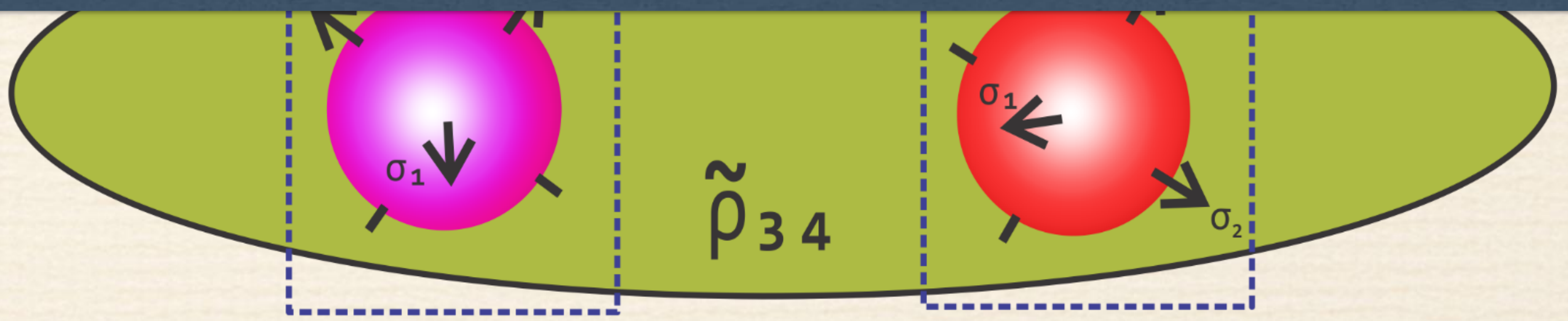
However, optimal broadcasting of correlations beyond entanglement via asymmetric cloning is also impossible!!

For Impossibility Proofs & its generalization, Check arXiv:1702.02123 (2018)

Discord=

0


Via Local Cloners
(M=2)




Via Nonlocal Cloners
(M=4)

[A. Jain et al., arXiv:1702:02123 (2018)]

# Progresses in broadcasting of correlations

## (1 → 2) Broadcasting of entanglement

| Operation | Resource state | Author(s) |
|---|---|---|
| Symmetric cloner | Non-maximally Entangled | Buzek et al., PRA **55**, 3327 (1997) |
| Symmetric cloner | 2-qubit general | Chatterjee et al., PRA **93**, 042309 (2016) |
| Asymmetric cloner | Non-maximally Entangled | Ghiu, PRA **67**, 012323 (2003) |
| Asymmetric cloner | 2-qubit general | Jain et al., arXiv:1702.02123 (2018) |
| Arbitrary unitary | Werner-like & Bell-diagonal state | Jain et al., arXiv:1702.02123 (2018) |

## (1 → 2) Broadcasting of Quantum Discord

| Cloning Operation | Resource state | Author(s) |
|---|---|---|
| Symmetric | 2-qubit general | Chatterjee et al., PRA **93**, 042309 (2016) |
| Asymmetric | 2-qubit general | Jain et al., arXiv:1702.02123 (2018) |

## (1 → 3) Broadcasting of Entanglement

| Operation | Resource state | Author(s) |
|---|---|---|
| 1→ 3 Symmetric | Non-maximally Entangled State | Bandyopadhyay et al., PRA **60**, 3296 (1999) |
| 1→ 3 Asymmetric | Non-maximally Entangled State | Jain et al., arXiv:1702.02123 (2018) |
| 1→ 2 Asymmetric | Non-maximally Entangled State | Jain et al., arXiv:1702.02123 (2018) |

# Other similar works

| Cloning Operation | Resource-type | Author(s) |
|---|---|---|
| Symmetric – state dependent | Entanglement | Shukla et al., In Preparation |
| Symmetric – state independent | Coherence | Sharma et al., PRA **96**, 052319 (2017) |

# Summary

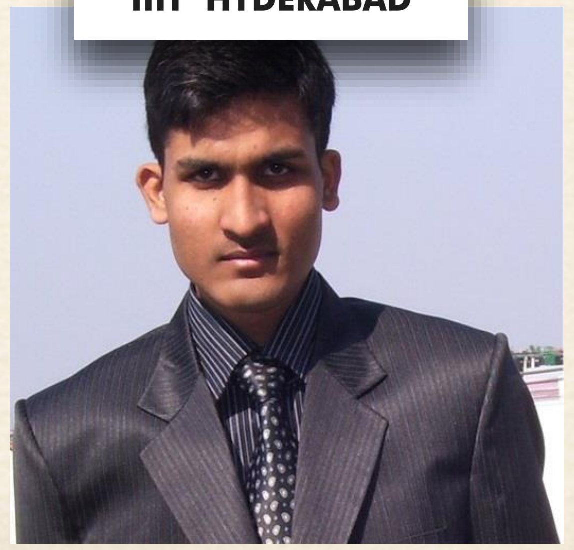
- **Decompression of entanglement using cloning operations is possible.**
- **Nonlocal cloning gives a better range of broadcasting of entanglement than local ones.**
- **For broadcasting of entanglement to more than 2 pairs, asymmetric cloning operations outperform the symmetric ones.**
- **Decompression (broadcasting) of quantum correlations beyond entanglement using cloning operations is impossible!**
- **Suitable arbitrary unitaries can be employed to increase the efficiency of broadcasting process instead of cloning operations.**

# Project Teams

**Broadcasting with universal symmetric cloners**

**Broadcasting with universal asymmetric cloners**

**Broadcasting with state-dependent cloners**



# “THANK YOU”