Copy of this file can be found at:

http://www.mri.ernet.in/∼sen/asian12.pdf
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Slightly older version can be found at the dropbox site containing the lectures at the school

Review article:

I. Mandal, A.S., arXiv:1008.3801

Extremal Black Hole Entropy

Ashoke Sen

Harish-Chandra Research Institute, Allahabad, India

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Lecture 1

Introduction

A black hole is a classical solution in general theory of relativity with special properties.

It is surrounded by an event horizon which acts as a one way membrane.

Nothing can escape from inside the event horizon to the outside.

Thus in classical general theory of relativity a black hole behaves as a perfect black body at zero temperature and is an infinite sink of entropy.

It has been known since the work of Bekenstein, Hawking and others that in quantum theory a black hole behaves as a thermodynamic system with finite temperature, entropy etc.

$$S_{BH} = \frac{A}{4\,G_N}$$

Bekenstein, Hawking

A: Area of the event horizon

G_N: Newton's gravitational constant

Our units: $\hbar = c = k_B = 1$

For ordinary objects the entropy of a system has a microscopic interpretation.

We fix the macroscopic parameters (e.g. total electric charge, energy etc.) and count the number of quantum states – known as microstates – each of which has the same charge, energy etc.

d_{micro}: number of such microstates

Define microscopic (statistical) entropy:

$$S_{micro} = \ln d_{micro}$$

Question: Does the entropy of a black hole have a similar statistical interpretation?

The best tests involve a class of supersymmetric extremal black holes in string theory, also known as BPS states.

Strategy:

1. Identify a supersymmetric black hole carrying a certain set of electric charges $\{Q_i\}$ and magnetic charges $\{P_i\}$ and calculate its entropy $S_{BH}(Q, P)$ using the Bekenstein-Hawking formula.

2. Identify the supersymmetric quantum states in string theory carrying the same set of charges and calculate the number $d_{micro}(Q, P)$ of these states.

3. Compare $S_{micro} \equiv \ln d_{micro}(Q, P)$ with $S_{BH}(Q, P)$.

For these one indeed finds a match:

$$\mathbf{A/4G_N} = \ln \mathbf{d_{micro}}$$

Strominger, Vafa, ...

However this agreement also opens up new questions.

1. The computation of the entropy on the black hole side is valid when gravity is sufficiently strong so that the horizon radius is much larger than the compton wavelength.

The microscopic computation is valid in the opposite limit.

How can we compare the two?

Suggested remedy: Use supersymmetric index $\sim \mathrm{Tr}(-1)^F$

Protected from quantum corrections and is easier to compute on the microscopic side.

Is it reasonable to compare this with black hole entropy which counts $\mathrm{Tr}(1)$?

2. Both $A/4G_N$ and d_{micro} are computed in the large charge approximation.

On the black hole side this is needed to keep the curvature at the horizon small so that we can use classical Bekenstein-Hawking formula.

On the microscopic side the large charge approximation is needed so that we can use some asymptotic formula for estimating $\ln d_{micro}$.

Does the agreement between the microscopic and the macroscopic results hold beyond the large charge limit?

– need tools for more accurate computation of entropy on both sides.

3. On the microscopic side we can compute the entropy in different ensembles, *e.g.* grand canonical, canonical, microcanonical etc.

They all agree in the large charge limit, but differ from each other for finite charges.

Which of these entropies should we compare with the black hole entropy?

4. Do black holes carry more information than just the total number of states?

Example 1: Can we tell if most of the black holes are bosonic or fermionic, i.e. is $\mathrm{Tr}(-1)^F$ positive or negative?

Example 2: Suppose the theory has a discrete $\mathbb{Z}_N$ symmetry generated by g.

Can the black holes tell us the answer for $\{\mathrm{Tr}(-1)^F g\}$?

$\Leftrightarrow$ distribution of $\mathbb{Z}_N$ quantum numbers among the microstates.

Why do we want to study these questions?

On the black hole side addressing these questions invariably leads us to the study of quantum gravity corrections to the black hole entropy.

Thus successfully addressing these questions will require understanding the rules for quantizing gravity.

Testing the gravity prediction against microscopic prediction will enable us to test whatever tools we use to study quatum gravity in black hole background.

We can then try to apply the same tools to more general situations possibly going beyond supersymmetric black holes.

Some exact microscopic results in D=4

Exact microscopic results are known for

1. Type II on T^6,

2. Heterotic on T^6 or equivalently type II on K3 $\times$ T^2,

3. Some special orbifolds of the above theories with 16 unbroken supersymmetries

– known as CHL models

Chaudhuri, Hockney, Lykken

The role of index

The microscopic analysis is always done in a region of the moduli space where gravity is weak and hence the states do not form a black hole.

In order to be able to compare it with the results from the black hole side we must focus on quantities which do not change as we change the coupling from small to large value.

– needs appropriate supersymmetric index.

The appropriate index in D=4 is the helicity trace index.

Bachas, Kiritsis

Suppose we have a BPS state that breaks 4n supersymmetries.

$\rightarrow$ there will be 4n fermion zero modes (goldstino) on the world-line of the state.

Consider a pair of fermion zero modes ψ_0, $\psi_0^\dagger$ satisfying

$$\{\psi_0, \psi_0^\dagger\} = 1$$

If $|0\rangle$ is the state annihilated by ψ_0 then

$$|0\rangle, \qquad \psi_0^\dagger|0\rangle$$

give a degenerate pair of states with $J_3 = \pm 1/4$ and hence

$$(-1)^F = (-1)^{2J_3} = (-1)^{\pm 1/2} = \pm i$$

Thus

$$Tr(-1)^F = 0, \qquad Tr(-1)^F(2J_3) = i$$

Lesson: Quantization of the fermion zero modes produces Bose-Fermi degenerate states and make $\mathrm{Tr}(-1)^F$ vanish.

Remedy: Define

$$B_{2n} = \frac{1}{(2n)!}\mathrm{Tr}(-1)^F(2J_3)^{2n} = \frac{1}{(2n)!}\mathrm{Tr}(-1)^{2J_3}(2J_3)^{2n}$$

Since there are 2n pairs of zero modes,

$$\begin{aligned} B_{2n} &= \frac{1}{(2n)!}\mathrm{Tr}_{rest}\mathrm{Tr}_{zero}(-1)^{2J_3^{(1)}+\cdots 2J_3^{(2n)}+2J_3^{rest}} \\ &\quad \times \left(2J_3^{(1)}+\cdots 2J_3^{(2n)}+2J_3^{rest}\right)^{2n} \\ &= \mathrm{Tr}_{rest}\mathrm{Tr}_{zero}(-1)^{2J_3^{(1)}+\cdots 2J_3^{(2n)}+2J_3^{rest}} \times 2J_3^{(1)} \times \cdots 2J_3^{(2n)} \\ &= (i)^{2n} \times \mathrm{Tr}_{rest}(-1)^{2J_3^{rest}} \end{aligned}$$

$$B_{2n} = (i)^{2n} \times Tr_{rest}(-1)^{2J_3^{rest}}$$

Thus B_{2n} effectively counts $(-1)^n Tr_{rest}(-1)^F$, with the trace taken over modes other than the 4n fermion zero modes associated with broken supersymmetry.

Note: B_{2n} does not receive any contribution from non-BPS states which break more than 4n supersymmetries and hence have more than 4n fermion zero modes.

Due to this property B_{2n} is protected from quantum corrections.

Examples

Type II on T^6 has 32 supersymmetries.

1/8 BPS black holes break 28 of the supersymmetries.

Thus the relevant index is B_{14}.

Heterotic on T^6 (or type II on $K3 \times T^2$) has 16 supersymmetries.

1/4 BPS black hole breaks 12 supersymmetries.

Thus the relevant index is B_6.

Type II on T^6

This theory has 12 NSNS sector gauge fields and 16 RR sector gauge fields.

Consider a dyon carrying NSNS sector charges.

– characterized by 12 dimensional electric and magnetic charge vectors Q and P.

Q and P transform as vectors under the T-duality group $SO(6,6;\mathbb{Z})$

$Q^2, P^2, Q\cdot P$: T-duality invariant inner products.

$$Q^2 = 2\sum_{i=1}^{6} n_i w_i, \quad P^2 = 2\sum_{i=1}^{6} N_i W_i, \quad Q\cdot P = \sum_{i=1}^{6}(n_i N_i + w_i W_i)$$

n_i, w_i: (momentum, winding) along i-th circle

N_i, W_i: (KK monopole, H-monopole) charge along i-th circle

Define $\Delta = Q^2P^2 - (Q\cdot P)^2$

– invariant also under S-duality group

Restrict to states satisfying $\gcd\{Q_iP_j - Q_jP_i\} = 1$

Dabholkar, Gaiotto, Nampuri

Then

$$B_{14} = (-1)^{Q\cdot P} \sum_{s|Q^2/2, P^2/2, Q\cdot P} s\,\widehat{c}(\Delta/s^2)$$

where $\widehat{c}(u)$ is defined through

$$-\vartheta_1(z|\tau)^2\,\eta(\tau)^{-6} \equiv \sum_{k,l} \widehat{c}(4k - l^2)\, e^{2\pi i(k\tau + lz)}$$

Shih, Strominger, Yin

ϑ_1: Jacobi theta function η: Dedekind eta function

$$\widehat{c}(-1) = 1,\ \widehat{c}(0) = -2,\ \widehat{c}(3) = 8,\ \widehat{c}(4) = -12,\ \widehat{c}(7) = 39$$

$$\widehat{c}(8) = -56,\ \widehat{c}(11) = 152,\ \widehat{c}(12) = -208, \cdots$$

B_{14} is <u>negative</u> and for large charges we have

$$\log[-B_{14}] = \pi\sqrt{\Delta} - 2\ln\Delta + \cdots$$

Although we have stated the results for black holes carrying only NSNS sector charges, it also covers many other black holes carrying purely RR charges or both NSNS and RR charges, since U-duality symmetry relates many of these black holes.

$$B_{14} < 0, \quad \log[-B_{14}] = \pi\sqrt{\Delta} - 2\ln\Delta + \cdots$$

Bekenstein-Hawking entropy S_{BH} of a black hole carrying the same charges is given by

$$\pi\sqrt{\Delta}$$

1. Why is there an agreement between ln $|B_{14}|$ and S_{BH} at the leading order?

2. Can we reproduce the subleading $-2\ln\Delta$ correction from the black hole side?

3. Can we explain why B_{14} is negative from the black hole side?

Heterotic string theory on T^6

This theory has 28 U(1) gauge fields.

Thus a generic charged state is characterized by 28 dimensional electric charge vector Q and magnetic charge vector P.

The theory has T-duality symmetry $O(6, 22; \mathbb{Z})$ under which Q and P transform as vectors.

This allows us to define T-duality invariant bilinears in the charges:

$$Q^2, \qquad P^2, \qquad Q \cdot P$$

More general class of $\mathcal{N}=4$ supersymmetric string theories can be constructed by taking orbifolds of heterotic string theory on T^6.

– CHL models

Chaudhuri, Hockney, Lykken

These theories have $(r+6)$ U(1) gauge fields for different values of r.

Thus Q and P are (r+6) dimensional vectors.

We can again construct $O(r,6)$ invariant bilinears

$$Q^2, \qquad P^2, \qquad Q\cdot P$$

In each of these theories, the index $B_6(Q,P)$ has been computed for a wide class of charge vectors (Q,P).

In each case the result is expressed as Fourier expansion coefficients of some well known functions $Z(\rho,\sigma,v)$, called Siegel modular forms:

$$B_6 = (-1)^{Q.P} \int d\rho \int d\sigma \int dv\, e^{-\pi i(\rho Q^2 + \sigma P^2 + 2v Q\cdot P)} Z(\rho,\sigma,v)$$

$Z(\rho,\sigma,v)$: explicitly known in each of the examples, and transform as modular forms of certain weights under subgroups of $Sp(2,\mathbb{Z})$.

Dijkgraaf, Verlinde, Verlinde; Shih, Strominger, Yin; David, Jatkar, A.S.; Dabholkar, Gaiotto, Nampuri;

S. Banerjee, Srivastava, A.S.; Dabholkar, Gomes, Murthy; Govindarajan, Gopala Krishna; $\cdots$

Some microscopic results for $-B_6$ in heterotic on T^6 (Fourier coefficients of a Siegel modular form)

$(Q^2, P^2)\backslash Q.P$	-2	2	3	4	5	6	7
(2,2)	-209304	648	327	0	0	0	0
(2,4)	-2023536	50064	8376	-648	0	0	0
(2,6)	-15493728	1127472	130329	-15600	972	0	0
(4,4)	-16620544	3859456	561576	12800	3272	0	0
(4,6)	-53249700	110910300	18458000	1127472	85176	-6404	0
(6,6)	2857656828	4173501828	920577636	110910300	8533821	153900	26622
(2,10)	-510032208	185738352	16844421	-2023536	315255	-31104	1620

It is also possible to find the systematic expansion of B_6 for large charges.

In each case we find $\underline{B_6 < 0}$ in large charge limit.

$$\ln|B_6| = \pi\sqrt{Q^2P^2-(Q.P)^2} + f\left(\frac{Q.P}{P^2}, \frac{\sqrt{Q^2P^2-(Q.P)^2}}{P^2}\right) + \mathcal{O}(\text{charge}^{-2})$$

f: a known function.

Cardoso, de Wit, Kappeli, Mohaupt; David, Jatkar, A.S.

For example, for heterotic string theory compactified on a six dimensional torus,

$$f(\tau_1,\tau_2) = 12\ln\tau_2 + 24\ln\eta(\tau_1+i\tau_2) + 24\ln\eta(-\tau_1+i\tau_2)$$

η: Dedekind function

$$\ln|\mathbf{B_6}| = \pi\sqrt{\mathbf{Q^2P^2 - (Q.P)^2}} + \mathbf{f}\left(\frac{\mathbf{Q.P}}{\mathbf{P^2}}, \frac{\sqrt{\mathbf{Q^2P^2 - (Q.P)^2}}}{\mathbf{P^2}}\right) + \mathcal{O}(\mathbf{charge}^{-2})$$

Bekenstein-Hawking entropy S_{BH} of a black hole carrying the same charges is given by

$$\pi\sqrt{\mathbf{Q^2P^2 - (Q.P)^2}}$$

1. Why is there an agreement between microscopic index and $\exp[S_{BH}]$ at the leading order?

2. Can we calculate the subleading corrections on the black hole side?

3. Can we explain why $B_6 < 0$ for large charges from the black hole side?

4. Can we explain why B_6 does not have definite sign for finite charges?

Lecture 2

Review of main results

Exact microscopic results for helicity trace index exist in type II string theory on T^6, heterotic string theory on T^6 and four dimensional CHL models with 16 unbroken supersymmetries.

Relevent index: B_{14} for type IIA on T^6 and B_6 for heterotic on T^6 and CHL models.

Important results on B_{14}:

1. $B_{14} < 0$

2. In the large charge limit

$$\log[-B_{14}] = \pi\sqrt{\Delta} - 2\ln\Delta + \cdots$$

$$\Delta \equiv Q^2 P^2 - (Q\cdot P)^2$$

Bekenstein-Hawking entropy S_{BH} of a black hole carrying the same charges is given by

$$\pi\sqrt{\Delta}$$

Important results on B_6:

$B_6 < 0$ in the large charge limit but for finite charges B_6 can be either positive or negative.

In the large charge limit

$$\ln |B_6| = \pi\sqrt{Q^2P^2 - (Q.P)^2} + f\left(\frac{Q.P}{P^2}, \frac{\sqrt{Q^2P^2 - (Q.P)^2}}{P^2}\right) + \mathcal{O}(\text{charge}^{-2})$$

f: a known function

Bekenstein-Hawking entropy S_{BH} of a black hole carrying the same charges is given by

$$\pi\sqrt{Q^2P^2 - (Q.P)^2}$$

There are closely related results in 4+1 non-compact dimensions e.g. in type II on T^5, type II on $K3 \times S^1$ and their orbifolds.

Maldacena, Moore, Strominger; Dijkgraaf, Moore, Verlinde, Verlinde; Jatkar, David, A.S.

Twisted index

On special subspaces of the parameter space of the $\mathcal{N}=8$ and $\mathcal{N}=4$ supersymmetric string theories in (3+1) dimensions, the theory develops $\mathbb{Z}_N$ discrete symmetry generated by an element g which commutes with 16 supersymmetries.

Example: For heteroric on T^6 we can have N=2,3,4,5,6,7,8

On these special subspaces we can define the twisted index:

$$B_6^g = \frac{1}{6!} Tr \left[(-1)^{2h} (2h)^6 g \right]$$

Like B_6, this index is also protected.

In each case we can calculate the twisted index B_6^g, and find that the result is again given by Fourier integrals of modular forms of subgroups of $Sp(2,\mathbb{Z})$.

$$B_6^g = (-1)^{Q.P} \int d\rho \int d\sigma \int dv\, e^{-\pi i(\rho Q^2 + \sigma P^2 + 2v Q\cdot P)} Z_g(\rho, \sigma, v)$$

Z_g are known functions.

Furthermore for large charges we find

$$B_6^g = \exp[\pi\sqrt{Q^2P^2 - (Q\cdot P)^2}/N + \cdots]$$

Can we explain this behaviour of B_6^g from the black hole side?

Macroscopic analysis

Goal:

1. Develop tools for computing the entropy / index of extremal black holes beyond the large charge limit.

2. Apply it to black holes carrying the same charges for which we have computed the microscopic index.

3. Compare the macroscopic results with the microscopic results.

4. Repeat the analysis for g-twisted index.

Computation of macroscopic degeneracy.

To leading order it is given by $\exp[S_{BH}(Q)]$.

Our goal will be to study corrections to this formula.

In string theory the Bekenstein-Hawking formula receives two types of corrections:

1. Higher derivative (α') corrections in classical string theory.
2. Quantum (g_s) corrections.

Of these the α' corrections are captured by Wald's modification of the Bekenstein-Hawking formula.

What about quantum corrections?

Since the metric and the dilaton at the horizon are fixed by the charges, both the higher derivative corrections and string loop corrections are controlled by appropriate combination of the charges.

α' and g_s expansion $\Rightarrow$ an expansion in inverse power of charges.

Example: Consider a black hole in type II string compactification carrying only RR charges, each of order Λ for some large number Λ.

For such a black hole $g_s \sim \Lambda^{-1}$ at the horizon.

Thus string loop expansion gives an expansion for the entropy in inverse powers of Λ.

Proof:

Classical action $\mathcal{S}(\phi, \psi_{\text{NSNS}}, \psi_{\text{RR}})$ satisfies:

$$\mathcal{S}(\phi - \ln\Lambda, \psi_{\text{NSNS}}, \Lambda\psi_{\text{RR}}) = \Lambda^2 \mathcal{S}(\phi, \psi_{\text{NSNS}}, \psi_{\text{RR}})$$

ϕ: Dilaton field, $\psi_{\text{NSNS}}, \psi_{\text{RR}}$**: NSNS and RR fields**

Thus given any classical solution we can get another solution by scaling RR fields by Λ and e^ϕ by $1/\Lambda$.

scales RR charges by Λ and g_s by $1/\Lambda$.

Thus the correction to the entropy of order Λ^{-2n+2} comes at the n-loop order.

Tree level: Λ^2, One loop: Λ^0, $\ln\Lambda$, etc.

How can we calculate these quantum corrections to the entropy?

Strategy: Use euclidean path integral formulation and make use of the presence of AdS_2 in the near horizon geometry.

Example: Reissner-Nordstrom solution in $D = 4$

$$
\begin{aligned}
ds^2 &= -(1-\rho_+/\rho)(1-\rho_-/\rho)d\tau^2 \\
&\quad + \frac{d\rho^2}{(1-\rho_+/\rho)(1-\rho_-/\rho)} \\
&\quad + \rho^2(d\theta^2 + \sin^2\theta d\phi^2)
\end{aligned}
$$

Define

$$
2\lambda = \rho_+ - \rho_-, \quad t = \frac{\lambda\tau}{\rho_+^2}, \quad r = \frac{2\rho - \rho_+ - \rho_-}{2\lambda}
$$

and take $\lambda \to 0$ limit keeping r, t fixed.

$$
ds^2 = \rho_+^2\left[-(r^2-1)dt^2 + \frac{dr^2}{r^2-1}\right] + \rho_+^2(d\theta^2 + \sin^2\theta d\phi^2)
$$

$AdS_2 \quad \times \quad S^2$

This feature holds for all known extremal black hole solutions.

Postulate: Any extremal black hole has an AdS_2 factor / $SO(2,1)$ isometry in the near horizon geometry.

– partially proved

Kunduri, Lucietti, Reall; Figueras, Kunduri, Lucietti, Rangamani

The full near horizon geometry takes the form $AdS_2 \times K$

K: some compact space that includes the S^2 factor.

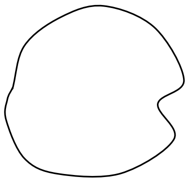
Presence of the AdS_2 factor allows us to apply the rules of AdS/CFT correspondence.

1. Consider the euclidean AdS_2 metric:

$$
\begin{aligned}
ds^2 &= a^2\left((r^2-1)d\theta^2 + \frac{dr^2}{r^2-1}\right), \quad 1 \le r < \infty,\ \theta \equiv \theta + 2\pi \\
&= a^2(\sinh^2\eta\, d\theta^2 + d\eta^2), \quad r \equiv \cosh\eta, \quad 0 \le \eta < \infty
\end{aligned}
$$

Regularize the infinite volume of AdS_2 by putting a cut-off $r \le r_0 f(\theta)$ for some smooth periodic function $f(\theta)$.

This makes the AdS_2 boundary have a finite length L.



2. Define the partition function:

$$Z_{AdS_2 \times K} = \int D\varphi \, \exp[-\text{Action}]$$

φ: set of all string fields

Boundary condition: Asymptotically the field configuration should approach the classical near horizon geometry of the black hole.

By AdS_2/CFT_1 correspondence:

$$Z_{AdS_2 \times K} = Z_{CFT_1}$$

CFT_1: dual (0+1) dimensional CFT obtained by taking the infrared limit of the quantum mechanical system underlying the black hole microstates.

3. Note on boundary condition:

Near the boundary of AdS_2, the θ indepndent solution to the Maxwell's equation has the form:

$$A_r = 0, \quad A_\theta = C_1 + C_2 r$$

C_1 (chemical potential) represents normalizable mode

C_2 (electric charge) represents non-normalizable mode

$\rightarrow$ the path integral must be carried out keeping C_2 (charge) fixed and integrating over C_1 (chemical potential).

Two consequences:

(a) The AdS_2 path integral computes the CFT_1 partition function in the microcanonical ensemble where all charges are fixed.

(b) This also forces us to to include a Gibbons-Hawking type boundary term in the path integral

$$\exp[-i q_k \oint_{\partial(AdS_2)} d\theta \, A_\theta^{(k)}]$$

$A_\mu^{(k)}$: gauge fields on AdS_2.

q_k: associated electric charge

4.

$$Z_{AdS_2\times K} = Z_{CFT_1} = Tr(e^{-LH}) = d_{hor}\, e^{-L\,E_0}$$

H: Hamiltonian of dual CFT_1 at the boundary of AdS_2.

(d_{hor}, E_0): (degeneracy, energy) of the states of CFT_1.

5. Thus we can define d_{hor} by expressing $Z_{AdS_2\times K}$ as

$$Z_{AdS_2\times K} = e^{CL} \times d_{hor} \quad \text{as } L \to \infty$$

C: A constant

d_{hor}: 'finite part' of $Z_{AdS_2\times K}$.

We identify $(\ln d_{hor})$ as the quantum corrected black hole entropy S_{macro}

Classical limit

$$
\begin{aligned}
Z_{AdS_2\times K} &= \exp[-\text{Classical Action} - i q_k \oint d\theta\, A_\theta^{(k)}] \\
&= \exp\left[-\int_1^{r_0} dr \int_0^{2\pi} d\theta [\sqrt{\det g}\,\mathcal{L}_E + i q_k\, F_{r\theta}^{(k)}]\right]
\end{aligned}
$$

$\mathcal{L}_E$: Euclidean Lagrangian density integrated over K.

Now in the near horizon geometry:

$$
\sqrt{\det g} = a^2, \quad \mathcal{L}_E = \text{constant}, \quad F_{r\theta}^{(k)} = -i e_k
$$

Thus

$$
Z_{AdS_2\times K} = \exp\left[-(a^2\mathcal{L}_E + q_k e_k)\int_1^{r_0} dr \int_0^{2\pi} d\theta\right]
$$

$$\int_1^{r_0} dr \int_0^{2\pi} d\theta = 2\pi(r_0 - 1)$$

Length of the boundary of AdS_2 is

$$L = \int_0^{2\pi} \sqrt{g_{\theta\theta}}\, d\theta = 2\pi a \sqrt{r_0^2 - 1} = 2\pi r_0 a + \mathcal{O}(1/r_0)$$

Thus

$$\int_1^{r_0} dr \int_0^{2\pi} d\theta = L/a - 2\pi + \mathcal{O}(L^{-1})$$

$$\begin{aligned} Z_{AdS_2 \times K} &= \exp\left[-(a^2 \mathcal{L}_E + q_k e_k) \int_1^{r_0} dr \int_0^{2\pi} d\theta\right] \\ &= \exp\left[-(a^2 \mathcal{L}_E + q_k e_k)(L/a - 2\pi)\right] \end{aligned}$$

$$\Rightarrow d_{hor} = \exp[2\pi(a^2 \mathcal{L}_E + q_k e_k)] = \exp[S_{wald}]$$

We shall now try to compute quantum corrections to $Z_{AdS_2 \times K}$ and compare them with the microscopic results.

However we need to address several issues first.

1. Microscopic results are for the index but the black hole entropy is related to degeneracy.

We must find a way to relate the two.

A.S.; Dabholkar, Gomis, Murthy, A.S.

2. On the macroscopic side there may be additional modes living outside the horizon – known as hair modes – which contribute to degeneracy / index.

– supersymmetric deformations of the black hole solution with support outside the horizon.

N. Banerjee, Mandal, A.S.; Jatkar, A.S., Srivastava

3. Besides single centered black holes we may also have multi-centered black hole solutons carrying the same total charges.

Denef; · · ·

We need to include their contribution as well.

Example of hair modes:

The fermion zero modes associated with the broken supersymmetry generators are always part of the hair modes.

Proof: Take a black hole solution and deform it by infinitesimal local supersymmetry transformation with parameter $\epsilon(\mathbf{x})$ such that

$$\epsilon(\mathbf{x}) \to \epsilon_0 \quad \textbf{as } \mathbf{x} \to \infty$$

$$\epsilon(\mathbf{x}) = 0 \quad \textbf{for } |\mathbf{x}| < R_0 \textbf{ for some } R_0$$

1. Deformations have support outside the sphere of radius R_0.

2. This is not a pure gauge deformation if ϵ_0 is not the asymptotic value of a Killing spinor.

Lecture 3

Algorithm for computing quantum black hole entropy

1. Regularize infinte volume of AdS_2 by putting a cut-off $r \leq r_0$ so that the boundary has a finite length L.

2. Calculate

$$Z_{AdS_2 \times K} = \int D\varphi \exp[-\text{Action} - iq_k \oint_{\partial(AdS_2)} d\theta\, A_\theta^{(k)}]$$

3. Define d_{hor} through:

$$Z_{AdS_2 \times K} = e^{CL}\, d_{hor} \quad \text{as } L \to \infty$$

4. Identify $\ln d_{hor}$ as the quantum corrected entropy.

Issues to be addressed

1. Microscopic results are for the index but d_{hor} computes degeneracy.

We must find a way to relate the two.

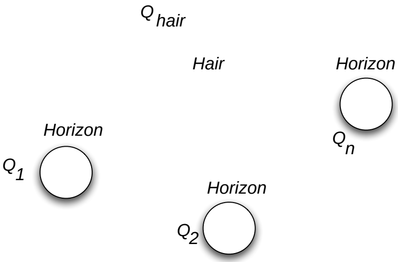
2. On the macroscopic side there may be additional modes living outside the horizon – known as hair modes – which contribute to degeneracy / index.

e.g. the zero modes associated with broken supersymmetries are hair modes.

3. Besides single centered black holes we may also have multi-centered black hole solutons carrying the same total charges.

We need to include their contribution as well.

To address these three issues we begin with a general multi-black hole configuration:



Q_i denotes both electric and magnetic charges of the i-th black hole.

We shall denote the degeneracy associated with the horizon degrees of freedom by d_{hor} and those associated with the hair degrees of freedom by d_{hair}.

d_{hair} can be calculated by explicitly identifying and quantizing the hair modes.

The total degeneracy:

$$\sum_{k} \sum_{\substack{\{Q_i\}, Q_{hair} \\ \sum_{i=1}^{k} Q_i + Q_{hair} = Q}} \left\{ \prod_{i=1}^{k} d_{hor}(Q_i) \right\} d_{hair}(Q_{hair}; \{Q_i\})$$

Now let us compute B_{2n} for the same configuration.

$$B_{2n} = \frac{1}{2n!} Tr(-1)^{2h}(2h)^{2n} = \frac{1}{2n!} Tr(-1)^{h_{hor}+h_{hair}}(2h_{hor} + 2h_{hair})^{2n}, \quad h \equiv J_3$$

For black hole with four unbroken supersymmetries:

SUSY + $SL(2,R)$ isometry of AdS_2 $\to$ $SU(1,1;2)$ supergroup

– symmetry group of the near horizon geometry.

$$SU(1,1;2) \supset SU(2)$$

$\to$ horizon must be spherically symmetric.

Furthermore since the black hole is in the microcanonical ensemble,

spherical symmetry $\to$ zero angular momentum

$\to h_{hor} = 0$.

$$B_{2n} = \frac{1}{2n!}\,\mathrm{Tr}(-1)^{2h}(2h)^{2n} = \frac{1}{2n!}\,\mathrm{Tr}(-1)^{h_{hor}+h_{hair}}(2h_{hor} + 2h_{hair})^{2n}$$

$$h_{hor} = 0$$

Thus

$$B_{2n} = \frac{1}{2n!}\mathrm{Tr}(-1)^{h_{hair}}(2h_{hair})^{2n}$$

$$\| \qquad \mathrm{Tr} \Rightarrow \mathrm{Tr}_{hor}\mathrm{Tr}_{hair}$$

$$\sum_{k} \sum_{\substack{\{Q_i\}, Q_{hair} \\ \sum_{i=1}^{k} Q_i + Q_{hair} = Q}} \left\{ \prod_{i=1}^{k} d_{hor}(Q_i) \right\} B_{2n;hair}(Q_{hair}; \{Q_i\})$$

Let us for now focus on the contribution from single centered black holes (k=1).

Often for single centered black holes the only hair modes are the fermion zero modes.

In this case $Q_{hair} = 0$.

To compute $B_{2n;hair}$ we note that quantization of each pair of fermion zero modes produces states with $h = \pm 1/4$ and hence $Tr(-1)^{2h}(2h) = i$.

Thus 2n pairs of fermion zero modes will gives

$$B_{2n;hair} = (i)^{2n} = (-1)^n$$

Thus

$$B_{2n}(Q) = (-1)^n d_{hor}(Q)$$

$$B_{2n}(Q) = (-1)^n d_{hor}(Q)$$

– explains why we can compare the microscopic index with the macroscopic entropy, and also predicts that

A.S.; Dabholkar, Gomes, Murthy, A.S.

$$B_6 < 0, \qquad B_{14} < 0$$

provided we can ignore the effect of

1. multi-centered black holes,

2. hair modes of single centered black holes other than the fermion zero modes,

The hair modes of single centered black holes are quite restrictive, and known hair modes in D=4 carry positive $B_{2n;hair}$.

Thus they do not change the sign of B_{2n}.

For type II on T^6 the multi-centered black holes do not contribute to B_{14} for $\Delta > 0$. A.S.

– predicts $B_{14} < 0$ for $\Delta > 0$

– in perfect agreement with the explicit microscopic results.

In $\mathcal{N} = 4$ supersymmetric string theories multi-centered black holes contribute to the index but their contribution is exponentially suppressed in the large charge limit.

A.S.; Dabholkar, Guica, Murthy, Nampuri

Thus the previous argument predicts $B_6 < 0$ in the large charge limit, in agreement with the microscopic results.

What about for finite charges?

Some microscopic results for $-B_6$ in heterotic on T^6 (Fourier coefficients of a Siegel modular form)

$(Q^2, P^2) \backslash Q.P$	-2	2	3	4	5	6	7
(2,2)	-209304	648	327	0	0	0	0
(2,4)	-2023536	50064	8376	-648	0	0	0
(2,6)	-15493728	1127472	130329	-15600	972	0	0
(4,4)	-16620544	3859456	561576	12800	3272	0	0
(4,6)	-53249700	110910300	18458000	1127472	85176	-6404	0
(6,6)	2857656828	4173501828	920577636	110910300	8533821	153900	26622
(2,10)	-510032208	185738352	16844421	-2023536	315255	-31104	1620

Red entries: Negative index

Blue entries: $\Delta \equiv Q^2P^2 - (Q.P)^2 < 0$ and hence no single centered black holes

Strategy: Calculate the contribution to the index from multi-centered black holes and subtract from the above result.

$(Q^2, P^2)\backslash Q.P$	-2	2	3	4	5	6	7
(2,2)	648	648	0	0	0	0	0
(2,4)	50064	50064	0	0	0	0	0
(2,6)	1127472	1127472	25353	0	0	0	0
(4,4)	3859456	3859456	561576	12800	0	0	0
(4,6)	110910300	110910300	18458000	1127472	0	0	0
(6,6)	4173501828	4173501828	920577636	110910300	8533821	153900	0
(2,10)	185738352	185738352	16844421	16491600	0	0	0

No more negative index or $\Delta < 0$ states.

Similar results hold for other $\mathcal{N} = 4$ supersymmetric CHL models.

The above results illustrate the power of black holes to explain features of black hole microstates beyond the leading Bekenstein-Hawking entropy.

Proving these positivity relations for all $(\mathbf{Q}^2, \mathbf{P}^2, \mathbf{Q.P})$ remains a challenging problem for the mathematicians and reflects some non-trivial properties of the Siegel modular forms.

Partial progress by Bringmann and Murthy

We shall now try to derive more quantitative predictions about microstates from the black hole side.

This will be done by comparing the asymptotic expansions of entropy / log |index| in the large charge limit.

In this limit the contribution from multicentered black holes as well as the hair modes are exponentially suppressed and so we can directly compare d_{hor} with $|B_{2n}|$.

Logarithmic corrections to the black hole entropy

– corrections of order $\ln\Lambda$ if all charges scale as Λ

– arise from one loop contribution to the path integral from massless fields.

Final results:

S. Banerjee, Gupta, Mandal, A.S.; Ferrara, Marrani; A.S.

The theory	scaling of charges	logarithmic contribution	microscopic
$\mathcal{N}=4$ with n_V matter	$Q_i \sim \Lambda, \quad A_H \sim \Lambda^2$	0	$\surd$
$\mathcal{N}=8$	$Q_i \sim \Lambda, A_H \sim \Lambda^2$	$-8\ln\Lambda$	$\surd$
$\mathcal{N}=2$ with n_V vector and n_H hyper	$Q_i \sim \Lambda, \quad A_H \sim \Lambda^2$	$\frac{1}{6}(23+n_H-n_V)\ln\Lambda$	?*
$\mathcal{N}=6$	$Q_i \sim \Lambda, \quad A_H \sim \Lambda^2$	$-4\ln\Lambda$	?
$\mathcal{N}=5$	$Q_i \sim \Lambda, \quad A_H \sim \Lambda^2$	$-2\ln\Lambda$	?
$\mathcal{N}=3$ with n_V matter	$Q_i \sim \Lambda, \quad A_H \sim \Lambda^2$	$2\ln\Lambda$	?
BMPV in type IIB on $T^5/\mathbb{Z}_N$ or $K3\times S^1/\mathbb{Z}_N$ with n_V vectors	$Q_1, Q_5, n \sim \Lambda$; $J \sim \Lambda^{3/2}, \quad A_H \sim \Lambda^{3/2}$	$-\frac{1}{4}(n_V-3)\ln\Lambda$	$\surd$
BMPV in type IIB on $T^5/\mathbb{Z}_N$ or $K3\times S^1/\mathbb{Z}_N$ with n_V vectors	$Q_1, Q_5, n \sim \Lambda$; $J=0, \quad A_H \sim \Lambda^{3/2}$	$-\frac{1}{4}(n_V+3)\ln\Lambda$	$\surd$

*: various proposals exist but no definite result

Ooguri, Strominger, Vafa; Cardoso, de Wit, Kappeli, Mohaupt; Denef, Moore; David; Cardoso, de Wit, Mahapatra

General procedure

Supersymmetric black holes have some moduli fields which are not fixed at the horizon (e.g. hypermultiplet fields in the N=2 theories).

Utilizing these flat directions we can take all moduli of order 1 at the horizon and the only large number will be the ratio of the horizon size a to Planck length.

Then calculate the one loop determinant of massless fields in the $AdS_2 \times K$ background and collect terms of order $\ln a$ in the entropy.

The integration over the zero modes need to be done separately.

Lecture 4

Logarithmic corrections to black hole entropy

1. Consider an extremal black hole in D=4 with horizon size a

$$ds^2 = a^2\left(\frac{dr^2}{r^2-1} + (r^2-1)d\theta^2 + d\psi^2 + \sin^2\psi d\phi^2\right) + ds^2_{compact}$$

2. Evaluate the one loop contribution to $Z_{AdS_2\times K}$ from massless fields.

3. Identify contribution to $\ln Z_{AdS_2\times K}$ proportional to $\ln a$.

Some details of the computation

Let $\{\psi_r\}$ denote the set of fluctuating massless fields around the near horizon background.

Let the eigenfunctions of the kinetic operator $\mathcal{K}$ be:

$$\psi_r = f_r^{(n)}(x), \qquad x \in AdS_2 \times S^2$$

with eigenvalue κ_n.

$$\mathcal{K} f^{(n)} = \kappa_n f^{(n)}$$

$$\int d^4x \sqrt{g} \sum_r f_r^{(n)}(x) f_r^{(m)}(x) = \delta_{mn}$$

$$\mathcal{K} f^{(n)} = \kappa_n f^{(n)}, \quad \int d^4x \sqrt{g} \sum_r f_r^{(n)}(x) f_r^{(m)}(x) = \delta_{mn}$$

Heat kernel sans zero modes:

$$K'(x, x', s) \equiv \sum_{n,r}{}' e^{-\kappa_n s} f_r^{(n)}(x) f_r^{(n)}(x')$$

One loop correction to $\ln Z$ from non-zero modes:

$$\Delta \ln Z = -\frac{1}{2} \ln \det{}' \mathcal{K} = -\frac{1}{2} \sum_n{}' \ln \kappa_n = \frac{1}{2} \int_\epsilon^\infty \frac{ds}{s} \sum_n{}' e^{-\kappa_n s}$$

ϵ: a string scale UV cut-off.

$$\Delta \ln Z = \frac{1}{2} \int_\epsilon^\infty \frac{ds}{s} \sum_n{}' e^{-\kappa_n s} = \frac{1}{2} \int_\epsilon^\infty \frac{ds}{s} \int d^4x \sqrt{\det g}\, K'(x, x; s)$$

$$\Delta \ln Z = \frac{1}{2} \int_{\epsilon}^{\infty} \frac{ds}{s} \int d^4x \sqrt{\det g}\, K'(x, x; s)$$

Homogeneity of $AdS_2 \times S^2$

$\Rightarrow K'(x, x; s)$ is independent of x.

$$\int d^4x \sqrt{\det g} = 4\pi a^2 \times 2\pi a^2 (r_0 - 1) \simeq 8\pi^2 a^4 (\frac{L}{2\pi a} - 1)$$

Drop the part proportional to L.

One loop correcton to entropy from non-zero modes:

$$-4\pi^2 a^4 \int_{\epsilon}^{\infty} \frac{ds}{s} K'(x, x; s)$$

$$K'(x,x',s) = \sum_{n,r}{}' e^{-\kappa_n s} f_r^{(n)}(x) f_r^{(n)}(x')$$

Since the eigenvalues κ_n are proportional to a^{-2}, and $f_r^{(n)}(x) \propto a^{-2}$, $a^4\, K'(x,x;s)$ is a function of $\bar{s} = s/a^2$.

One loop correcton to entropy from non-zero modes:

$$-4\pi^2 a^4 \int_{\epsilon}^{\infty} \frac{ds}{s} K'(x,x;s) = -4\pi^2 a^4 \int_{\epsilon/a^2}^{\infty} \frac{d\bar{s}}{\bar{s}} K'(x,x;s)$$

The logarithmic correction $\propto \ln a$ comes from the $\mathcal{O}(\bar{s}^0)$ term in the small $\bar{s}$ expansion of $K'(x,x;s)$.

If C_0 denotes the s-independent term in the small s expansion of $a^4 K'(x,x;s)$ then

$$\Delta \ln Z = -4\pi^2\, C_0 \ln a^2$$

Note: $\kappa_n = 0$ modes must be removed.

Zero mode contribution:

The path integral over the fields is defined with the standard general coordinate invariant measure, *e.g.* for gauge fields:

$$\int [DA_\mu] \exp\left[- \int d^4x \sqrt{\det g}\, g^{\mu\nu} A_\mu A_\nu \right] = 1$$

SInce $\sqrt{\det g}\, g^{\mu\nu} \sim a^2$ this shows that $[aA_\mu]$ has a independent measure.

Zero modes of A_μ are of the form $\partial_\mu \Lambda$ with Λ not vanishing at ∞.

Changing variables from aA_μ to $\Lambda \Rightarrow$ 'a' per zero mode

Net contribution to $Z_{AdS_2 \times K}$ from gauge field zero modes is a^{N_z} where N_z is the number of zero modes.

Computation of N_z:

Let

$$A_\mu(x) = h_\mu^{(k)}(x), \qquad k = 1, 2, \cdots$$

be the zero mode wave functions Camporesi, Higuchi

$$N_z = \sum_k 1 = \int d^4x \sqrt{\det g}\, g^{\mu\nu} \sum_k h_\mu^{(k)}(x) h_\nu^{(k)}(x)$$

$c_z \equiv a^4 g^{\mu\nu} \sum_k h_\mu^{(k)}(x) h_\nu^{(k)}(x)$ **is independent of x and a after summing over k.**

$$\begin{aligned} N_z &= c_z\, a^{-4} \int d^4x \sqrt{\det g} = 8\pi^2 c_z (r_0 - 1) \\ &= 8\pi^2 c_z \left(\frac{L}{2\pi a} - 1 + \mathcal{O}(L^{-1}) \right) \end{aligned}$$

$$N_z = 8\pi^2 c_z \left(\frac{L}{2\pi a} - 1 + \mathcal{O}(L^{-1}) \right)$$

$\Rightarrow$ gauge field zero mode contribution to $Z_{AdS_2 \times K}$:

$$a^{N_z} = \exp\left[8\pi^2 c_z \ln a \left(\frac{L}{2\pi a} - 1 + \mathcal{O}(L^{-1}) \right) \right]$$

Comparing with $Z_{AdS_2 \times K} = d_{hor}\, e^{-E_0 L}$ we get the logarithmic contribution to $\ln d_{hor}$ from the zero modes:

$$-8\pi^2 c_z \ln a$$

Contributions from other zero modes can be found similarly.

One loop correction due to massive string loops

Integrating out massive string modes gives a local one loop correction to the effective action.

The contribution of this term to $\ln d_{hor}$ is identical to the correction to the Wald entropy due to this local correction to the effective action.

Caution: Only some special one loop correction to the effective Lagrangian is known and we can make further progress by assuming that only these terms contribute to the entropy at this order.

Consider the CHL models obtained by $\mathbb{Z}_N$ orbifold of type IIB on K3 $\times$ $S^1 \times \widetilde{S}^1$.

At tree level there are no corrections at the four derivative level, but at one loop these theories get corrections proportional to the Gauss-Bonnet term in the 1PI action. Harvey, Moore; Gregori, Kiritsis, Kounnas, Obers, Petropoulos, Pioline

$$\sqrt{-\det g}\,\Delta\mathcal{L}_E$$

$$= -\psi(\tau_1, \tau_2)\,\sqrt{-\det g}\,\left\{R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right\}$$

$\tau = \tau_1 + i\tau_2$: modulus of the torus ($S^1 \times \widetilde{S}^1$).

ψ: a known function dependent on the theory

This contributes $8\pi^2 a^4 \Delta\mathcal{L}_E$ to the Wald entropy to first order.

Result for the Wald entropy

$$\pi\sqrt{\mathbf{Q}^2\mathbf{P}^2 - (\mathbf{Q}.\mathbf{P})^2}$$
$$-64\pi^2\psi\left(\frac{\mathbf{Q}.\mathbf{P}}{\mathbf{P}^2}, \sqrt{\frac{\mathbf{Q}^2\mathbf{P}^2 - (\mathbf{Q}.\mathbf{P})^2}{\mathbf{P}^2}}\right) + \mathcal{O}\left(\frac{1}{\mathbf{Q}^2, \mathbf{P}^2, \mathbf{Q}.\mathbf{P}}\right)$$

– agrees exactly with the result for $\ln|B_6(Q, P)|$ calculated in the microscopic theory to order charge^0.

Cardoso, de Wit, Kappeli, Mohaupt; David, Jatkar, A.S.

Twisted index

Suppose we want to compute the index

$$B_6^g = \frac{1}{6!}\, Tr\left[(-1)^{2h}\,(2h)^6\, g\right]$$

g: some $\mathbb{Z}_N$ symmetry generator.

After separating out the contribution from the hair degrees of freedom, and using $h_{hor} = 0$, we see that the relevant quantity associated with the horizon is

$$-Tr_{hor}(g)$$

What macroscopic computation should we carry out?

By following the logic of AdS/CFT correspondence we find that we need to again compute the partition function on AdS_2, but this time with a g twisted boundary condition on the fields under $\theta \to \theta + 2\pi$.

Other than this the asymptotic boundary condition must be identical to that of the original near horizon geometry since the charges have not changed

The 'finite part' of this partition function gives us $Tr_{hor}(g)$.

Recall AdS_2 metric:

$$ds^2 = a^2\left[(r^2-1)d\theta^2 + \frac{dr^2}{r^2-1}\right] = v\left[\sinh^2\eta\, d\theta^2 + d\eta^2\right]$$

The circle at infinity, parametrized by θ, is contractible at the origin $r = 1$.

Thus a g twist under $\theta \to \theta + 2\pi$ is not admissible.

$\to$ the $AdS_2 \times S^2$ geometry is not a valid saddle point of the path integral.

Question: Are there other saddle points which could contribute to the path integral?

Constraints:

1. It must have the same asymptotic geometry as the $AdS_2 \times S^2$ geometry.

2. It must have a g twist under $\theta \to \theta + 2\pi$.

3. It must preserve sufficient amount of supersymmetries so that integration over the fermion zero modes do not make the integral vanish.

Beasley, Gaiotto, Guica, Huang, Strominger, Yin; N. Banerjee, S. Banerjee, Gupta, Mandal, A.S.

There are indeed such saddle points in the path integral, constructed as follows.

1. Take the original near horizon geometry of the black hole.

2. Take a $\mathbb{Z}_N$ orbifold of this background with $\mathbb{Z}_N$ generated by simultaneous action of

a) $\theta \to \theta + 2\pi/N$

a) $\phi \to \phi + 2\pi/N$ (needed for preserving SUSY)

c) g.

N. Banerjee, Jatkar, A.S.; A.S.; Pioline, Murthy

To see that this satisfies the required boundary condition we make a rescaling:

$$\theta \to \theta/\mathbf{N}, \qquad \mathbf{r} \to \mathbf{N}\,\mathbf{r}$$

The metric takes the form:

$$\mathbf{a}^2 \left((\mathbf{r}^2 - \mathbf{N}^{-2})\mathbf{d}\theta^2 + \frac{\mathbf{dr}^2}{\mathbf{r}^2 - \mathbf{N}^{-2}} \right)$$

Orbifold action: $\theta \to \theta + 2\pi$, $\phi \to \phi + 2\pi/N$, g

The g transformation provides us with the correct boundary condition.

The ϕ shift can be regarded as a Wilson line, and hence is an allowed fluctuation in AdS_2.

The classical action associated with this saddle point, after removing the divergent part proportional to the length of the boundary, is S_{wald}/N.

Thus the leading contribution to the twisted partition function B_6^g from this saddle point is

$$Z_g^{finite} = \exp\left[S_{wald}/N\right]$$

This is exactly what we have found in the microscopic analysis of the twisted index.

Localization

N. Banerjee, S. Banerjee, Gupta, Mandal, A.S; Dabholkar, Gomes, Murthy

Presence of supersymmetry often allows one to restrict the path integral over a finite dimensional subspace which is invariant under a subset of the supersymmetries.

Nekrasov; Pestun; Drukker, Marino,Putrov; $\cdots$

Can we do this for $Z_{AdS_2\times K}$?

The path integral over massless fields in four dimensional $\mathcal{N} \geq 2$ supersymmetric theories involve:

1. **Integration over vector multiplets**

2. **Integration over gravity multiplet**

3. **Integration over hypermultiplets**

4. **Integration over gravitino multiplets (for $\mathcal{N} > 2$)**

So far the integration over the vector multiplets have been localized over a finite dimensional subspace.

Dabholkar, Gomes, Murthy

Conclusion

Quantum gravity in the near horizon geometry contains detailed information about not only the total number of microstates. but also finer details *e.g.* the $\mathbb{Z}_N$ quantum numbers carried by the microstates, the sign of the index etc..

Thus at least for extremal black holes there seems to be an exact duality between

Gravity description $\Leftrightarrow$ Microscopic description

General lesson

Euclidean quantum gravity can be trusted beyond the classical approximation.

Even without the detailed knowledge of ultraviolet completion of the theory we can use this to extract properties of the theory which must hold for all consistent UV completion.

Example: Logarithmic correction to black hole entropy

A proposed UV completion that fails to reproduce either the leading classical result or the subleading logarithmic corrections, must not be a consistent UV completion of gravity.

Inspired by the success we can try to extend the Euclidean gravity techniques to non-supersymmetric black holes.

Example: An extremal Kerr black hole in D=4 has logarithmic correction:

$$\frac{16}{45} \ln A_H$$

Can Kerr/CFT correspondence explain this microscopically?