Fermions : 16-component Majorana in $D=10$

Under $SO(6) \times SO(3,1) \subset SO(9,1)$

$$(4, 2_L) + (\bar{4}, 2_R) \leftarrow 16$$

Under $SU(3) \subset SO(6)$

$$4 \to 3+1 \qquad \bar{4} \to \bar{3}+1.$$

$\Rightarrow$ Fermion in 4 rep. of $SU(4)$
$\equiv \psi^r \to \psi^r E_r{}^m g_{m\bar{n}}$
$\equiv$ (Anti-holomorphic vector

+ scalar) as far as coupling

to background metric is concerned.

$\psi_{\bar{i}} \to$ left-handed from 4-d viewpoint

$\psi_{i} \to$ right handed from 4-d viewpoint

$\Leftrightarrow$ same as $A_{\bar{i}}$, A_i etc.

Explains why $A_{\bar{i}}$ is part of
(left-)
chiral multiplet and A_i is part

of anti-chiral multiplet.

Breaking E_6:

Suppose we have a discrete $\mathbb{Z}_n$ group under which M_{CY_3} is invariant.

Take quotient of M_{CY} by the group.

$\Leftrightarrow$ Keep only those fields which are invariant under $\mathbb{Z}_n$.

$\chi \to \chi/n$.

$\to$ reduces # of generations - antigenerations.

We can also quotient by $\mathbb{Z}_n$ with a twist.

① Identify a $\tilde{\mathbb{Z}}_n$ subgroup of $E_6 \times E_8$.

② Given an element of $\mathbb{Z}_n$ acting on M, ~~[illegible]~~ keep only fields which are invariant under simultaneous action of $\mathbb{Z}_n$ and $\tilde{\mathbb{Z}}_n$

= Swiching on Wilson line along the cycle generated by $\mathbb{Z}_n$ quotient.

Now only the subgroup of E_6 invariant under $\tilde{\mathbb{Z}}_n$ will ~~be~~ survive as a gauge group.

Example: Taken a E_6 gauge field in 4-d. ~~&~~ Regarded as a 10-d field it is constant along M_{CY_3}.

$\Rightarrow$ $\mathbb{Z}_n$ acts trivially.

$\Rightarrow$ Only $\tilde{\mathbb{Z}}_n$ invariant massless gauge fields will ~~[illegible]~~ survive.

~~@~~ This way we can break E_6 to a lower dim. group.

Note: since $\mathbb{Z}_n$ is abelian, it can be embedded in the Cartan subgroup.

$\Rightarrow$ The Cartan generators survive.

$\Rightarrow$ rank ~~=~~ remains 6.
(Can be reduced by giving up spin connection = gauge connection.)

e.g. $E_6 \to SU(3) \times SU(2) \times U(1) \times U(1) \times U(1)$

$\to$ possible.

Computation of $h_{2,1}$.

$$\sum_{i=1}^{5} z_i^5 = 0 \quad \text{in} \quad CP^4$$

# of deformations of the homogeneous polynomial

− # of linear coordinate redefs.

$$c = \frac{(1+J)^5}{1+5J} = (1+5J+10J^2+10J^3)(1-5J+25J^2-125J^3)$$

Coefficient of J^3:

$\cancel{-64+80-40+10=6}$

$-125+125-50+10 = -40$

$\chi = 5 \times (-40) = -200$

↓
degree (product of degrees)

$h_{1,1} - h_{2,1} = -100 \qquad h_{1,1} = 1 \qquad h_{2,1} = 101$