Type I on CY_3

→ similar to SO(32) heterotic

at least at the supergravity level.

⇒ Same low energy phenomenology.

We shall now return to type II theories and look for more general compactification with reduced supersymmetry.

To have control, need, at the leading order:

① Unbroken SUSY

② Minkowski 4-d space-time.

Later we have to break SUSY and look for de Sitter soln.

IIB action

$$\frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-\det g}\,\Big[R_g - \frac{1}{2|\tau_2|^2}\, g^{\mu\nu}\partial_\mu\tau\,\partial_\nu\bar{\tau}$$

$$-\frac{1}{2\tau_2}\frac{1}{3!}\, G^{(3)}_{\mu\nu\rho}\,\bar{G}^{(3)}_{\mu'\nu'\rho'}\, g^{\mu\mu'} g^{\nu\nu'} g^{\rho\rho'}$$

$$-\frac{1}{2}\frac{1}{5!}\,\tilde{F}^{(5)}_{\mu\nu\rho\sigma\tau}\,\tilde{F}^{(5)}_{\mu'\nu'\rho'\sigma'\tau'}\, g^{\mu\mu'}\cdots g^{\tau\tau'}\Big]$$

$$-\frac{1}{4\kappa_{10}^2}\int C^{(4)}\wedge \mathcal{H}^{(3)}\wedge F^{(3)}$$

$$\tau = \tau_1 + i\tau_2 = C^{(0)} + i e^{-\Phi}$$

$$G^{(3)} = F^{(3)} - \tau\,\mathcal{H}^{(3)}$$

$$\mathcal{H}^{(3)} = dB^{(2)},\quad F^{(3)} = dC^{(2)}.$$

$$\tilde{F}^{(5)} = dC^{(4)} - \frac{1}{2}C^{(2)}\wedge\mathcal{H}^{(3)} + \frac{1}{2}B^{(2)}\wedge F^{(3)}$$

We look for a general class of solutions with the ansatz:

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}\,dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}(y)\,dy^m dy^n$$

convention

$$\tau = \tau(y)$$

y^m: coordinates along compact direction

$4 \le m \le 9$

x^μ : coordinates along 4-d space-time

$0 \leq \mu \leq 3$.

$\mathcal{H}^{(3)}, F^{(3)}, \widetilde{F}^{(5)}$ can be non zero but preserves 4-d Lorentz invariance.

$\Rightarrow \mathcal{H}^{(3)}_{mnp}(y), F^{(3)}_{mnp}(y)$

$$\widetilde{F}^{(5)} = (1+*) S(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$$

manifest self-duality.

1-form along compact direction

$$d\widetilde{F}^{(5)} = F^{(3)} \wedge \mathcal{H}^{(3)}$$

$\Downarrow$

has only components along compact direction.

$$d\widetilde{F}^{(5)} = \underbrace{dS(y) \wedge dx^0 \wedge \cdots dx^3}$$

$$+ d * (A(y) \wedge dx^0 \wedge \cdots dx^3)$$

$\Downarrow$

Must vanish.

5-form in compact directi

$\Rightarrow dS(y) = 0 \Rightarrow S(y) = d\alpha$ (assuming $\mathcal{H}^{(1)}(M) = 0$)

$$\tilde{F}^{(5)} = (1+*)\, d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$$

We shall also add localized sources of various fields along compact directions.

e.g. a 3-brane along ~~non-compact~~ non-compact space-time, sitting at a given point on the compact space M.

5-brane along $x^0, \dots x^3$ wrapping a 2-cycle on M.

7-brane along $x^0, \dots x^3$ wrapping a 4-cycle on M. etc.

$\Rightarrow$ preserves $(3+1)$ dim. Poincare invariance.

Note: Most of the discussion also valid when $(3+1)$-d space is AdS or dS.

$\to$ Use isometries of AdS & dS instead of Poincare invariance.

Metric equation:

$$\frac{\delta S}{\delta g_{MN}} = 0 \qquad M, N = 0, \cdots 9$$

$$\Rightarrow \quad R_{MN} - \frac{1}{2} R\, g_{MN} = K_{10}^2\, T_{MN}.$$

T_{MN}: stress tensor due to various fields other than gravity + due to localized sources.

$$T_{MN} = -\frac{2}{\sqrt{-\det g}} \frac{\delta S'}{\delta g_{MN}} + T^{loc}_{MN}$$

S': The action of IIB supergravity minus the Einstein-Hilbert term

Taking trace:

$$R = -\frac{1}{4} K_{10}^2\, T^M{}_M.$$

$$\Rightarrow \quad R_{MN} = K_{10}^2 \left(T_{MN} - \frac{1}{8} T^P{}_P\, g_{MN} \right)$$

Ex. For IIB supergravity:

$$T_{\mu\nu} = \frac{1}{2\kappa_{10}^2} g_{\mu\nu} \Big[-\frac{1}{2\tau_2^2} \partial_m \tau \, \partial^m \bar{\tau} - \frac{1}{2} e^{-8A} \partial_m \alpha \, \partial^m \alpha - \frac{1}{12\tau_2} G_{mnp} \bar{G}^{mnp} \Big] + T^{loc}_{\mu\nu}.$$

(Indices raised & lowered by $\tilde{g}^{mn}$, $\tilde{g}_{mn}$)

$$T_{mn} = \frac{1}{2\kappa_{10}^2} g_{mn} \Big[-\frac{1}{2\tau_2^2} \partial_p \tau \, \partial^p \bar{\tau} + \frac{1}{2} e^{-8A} \partial_p \alpha \, \partial^p \alpha - \frac{1}{12\tau_2} G^{(3)}_{pqr} \bar{G}^{(3)\,pqr} \Big]$$

$$+ \frac{1}{\kappa_{10}^2} \Big[\frac{1}{2\tau_2^2} \partial_{\{m} \tau \, \partial_{n\}} \bar{\tau} + \frac{1}{4\tau_2} G_{pq\{m} \bar{G}^{pq}{}_{n\}} - \frac{1}{2} e^{-8A} \partial_m \alpha \, \partial_n \alpha \Big] + T^{loc}_{mn}.$$

Strategy: Calculate $R_{\mu\nu}$ & R_{mn} from the metric ansatz & substitute into e.o.m.

$$R_{\mu\nu} = \kappa_{10}^2 \left(T_{\mu\nu} - \frac{1}{8} T^M{}_M \, g_{\mu\nu}\right)$$

$$\parallel$$

$$-\eta_{\mu\nu} e^{4A} \tilde{\nabla}^2 A$$

$$\Rightarrow \tilde{\nabla}^2 A = e^{-2A} \frac{1}{48\tau_2} G_{mnp} \bar{G}^{mnp} + \frac{1}{4} e^{-10A} \partial_m \alpha \, \partial^m \alpha + \frac{1}{8} \kappa_{10}^2 e^{-2A} \left(T^m{}_m - T^\mu{}_\mu\right)_{loc.}$$

$$\Rightarrow \tilde{\nabla}^2 e^{4A} = e^{2A} \frac{1}{12\tau_2} G_{mnp} \bar{G}^{mnp} + e^{-6A} \partial_m \alpha \, \partial^m \alpha + e^{-6A} \partial_m (e^{4A}) \partial^m (e^{4A}) + \frac{1}{12} \kappa_{10}^2 e^{2A} \left(T^m{}_m - T^\mu{}_\mu\right)_{loc.}$$

Note: Indices are still raised and lower by g^{mn}, g_{mn} except in $\tilde{\nabla}$.

$$\int d^6 y \sqrt{-\det \tilde{g}} \; \tilde{\nabla}^2 e^{4A} = 0$$

$$\Rightarrow 0 = \int d^6 y \sqrt{-\det \tilde{g}} \Big[e^{2A} \frac{1}{12\tau_2} G_{mnp} \bar{G}^{mnp} + e^{-6A} \partial_m \alpha \, \partial^m \alpha + e^{-6A} \partial_m (e^{4A}) \partial^m (e^{4A}) + \frac{1}{12} \kappa_{10}^2 e^{2A} \left(T^m{}_m - T^\mu{}_\mu\right)_{loc.} \Big]$$

$\Rightarrow$ In absence of $(T_{MN})_{loc.}$, $G_{mnp} = 0$, $A = \text{Constant}$, $\alpha = \text{constant}$.