

The inequality:

$$\frac{1}{4} (T_m^m - T_\mu^\mu)^{loc.} - T_3 \rho_3^{loc.} \geq 0.$$

We shall examine to leading order in α' .

p-brane: $\pi_\alpha^\beta = -T_p \delta_\alpha^\beta \delta(x_\perp)$
 tangential \perp directions

$$T_m^m = -T_p (p-3) \delta(x_\perp)$$

$$T_\mu^\mu = -T_p \cdot 4 \cdot \delta(x_\perp)$$

$$(T_m^m - T_\mu^\mu) = T_p (7-p) \delta(x_\perp) > 0 \text{ for } p \leq 7$$

$T_p > 0$

Only these are used

$$\rho_3 \rho_3^{loc.} = 0 \text{ except for } p=3,$$

$$p=3: \frac{1}{4} T_3 \cdot 4 \delta(x_\perp) \mp \overset{D3}{T_3} \delta(x_\perp) \geq 0.$$

$\underbrace{\hspace{1cm}}_{\bar{D3}}$

= 0 for D3.

Inequality is violated for

$$05: T_5 < 0 \quad \overline{03} \quad T_3 < 0$$

So far we have discussed compactification of IIB in presence of localized sources and fluxes in general terms.

~ Requires first solving eqs. for τ and metric in absence of fluxes.

Assume on $\mathcal{H}^{(3)}$, $F^{(2)} \Rightarrow$ no 5-brane source, total # of 3-branes = 0.

Assume ^{localized.} no 3-brane source either.

There can be localized 7-branes/O7-planes.

① Orientifolds ② F-theory.

Type I: IIB/ Ω

↳ world-sheet parity.

$$\Omega: B_{MN} \rightarrow -B_{MN}, \quad C^{(0)} \rightarrow -C^{(0)}, \quad C^{(2)} \rightarrow C^{(2)}, \\ C^{(4)} \rightarrow -C^{(4)}$$

~~Another~~ Another \mathbb{Z}_2 symmetry $(-1)^{F_L}$

→ changes sign of all R-sector states on left.

G_{MN}, B_{MN}, Φ invariant.

$C^{(0)}, C^{(2)}, C^{(4)}$ charges sign

$RR \rightarrow -RR.$

Consider IIB on $S^1 \leftarrow IIA$ on S^2

$G_{\mu\nu}$
even $\leftrightarrow G_{g\mu}$

odd $\leftrightarrow B_{g\mu}$

Ω

$G_{\mu\nu}$
 $B_{g\mu} \rightarrow$ even

$G_{g\mu} \rightarrow$ odd.

$\Omega, \Phi,$

$\downarrow \quad \downarrow$
 $x^9 \rightarrow -x^9$

$B \rightarrow -B, G \rightarrow G, \Phi \rightarrow \Phi$

Confirmed in RR sector.

odd $\leftrightarrow C^{(0)}$

\leftrightarrow

$C^{(1)}$

odd since

$x^9 \rightarrow -x^9$

even $\leftrightarrow C^{(2)}$
 $\mu\nu$

\leftrightarrow

$C^{(3)}$

\rightarrow even since

$C^{(3)} \rightarrow C^{(3)}, x^9 \rightarrow -x^9$

$\leftrightarrow C^{(4)}$
 $m\mu$

\leftrightarrow

$C^{(5)}$

\rightarrow even

etc.

$$\text{II B on } S' \times S' = \text{II A on } \tilde{S}^1 \times S' = \text{II B on } \tilde{S}^1 \times \tilde{S}^1$$

even \leftrightarrow ~~G_{89}~~
 ~~B_{89}~~

~~G_{89}~~
 ~~B_{89}~~

odd ~~even~~ $\leftrightarrow B_{89}$

B_{89}

even $\leftrightarrow G_{9K}$

B_{9K}

even $\leftrightarrow G_{8K}$

~~B_{8K}~~

odd $\leftrightarrow B_{9K}$

G_{9K}

odd $\leftrightarrow B_{8K}$

G_{8K}

$$\tilde{x}^8 \rightarrow -\tilde{x}^8, \quad \tilde{x}^9 \rightarrow -\tilde{x}^9 = \mathbb{Z}_2$$

B_{89} odd $\Rightarrow \Omega$

Even. $\begin{pmatrix} 2 \\ 89 \end{pmatrix}$

\leftrightarrow

$\begin{pmatrix} 0 \\ 89 \end{pmatrix}$

\downarrow

even.

odd under ~~\mathbb{Z}_2~~ $\cdot \mathbb{Z}_2 \cdot \Omega$

Need $(-1)^{F_L} \cdot \Omega \cdot \mathbb{Z}_2$

Ex. check that all other fields

transform correctly if we postulate

that Ω maps to $(-1)^{F_L} \cdot \Omega \cdot \mathbb{Z}_2$

$(-1 | F_L \cdot \Omega$ leaves $\mathbb{C}^{(1)}$ & $\mathbb{C}^{(2)}$ invariant.

$\Rightarrow \tau$ is invariant.

$$\begin{pmatrix} B^{(1)} \\ C^{(1)} \end{pmatrix} \rightarrow - \begin{pmatrix} B^{(1)} \\ C^{(1)} \end{pmatrix}$$

\rightarrow Equivalent to $SL(2, \mathbb{Z})$ trs:

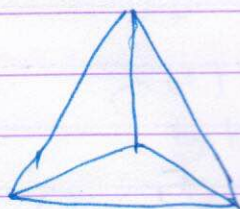
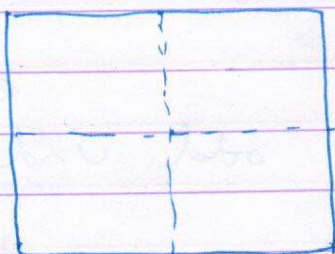
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S_2: (x^8, x^9) \rightarrow (-x^8, -x^9)$$

(x^8, x^9) has period 2π each.

Fixed points: $(0, 0), (0, \pi), (\pi, 0), (\pi, \pi)$.

\mathbb{B}



As we move around a doped circle around a vertex, fields get transformed by $(-1 | F_L \cdot \Omega$

vertices

D-branes: IIB/ Ω has 09-plane
carrying -ve D-brane charge.

\Rightarrow Needs to be cancelled by 16.

D-branes & their images.

\Rightarrow SO(32).

~~Ex~~ In T-dual picture we have

16 D7-branes.

\hookrightarrow along x^0, \dots, x^7

\rightarrow placed at points on the tetrahedron.

Vertices carry -4 units of D7-brane
charge.

Total D-brane charge = 0.

If 4 D-branes are placed at each

vertex we shall have no net

D7-charge even locally.

\rightarrow No source for C^0 , $\tau = \text{constant}$.

Gauge group $(SO(8))^4 \rightarrow$ in IIB/ Ω
we switch on Wilson lines.

What happens when we pull the
D7's away from O7-planes?

Localized sources \Rightarrow ~~τ~~ \approx not constant

First approximation: ignore backreaction
on the metric, treating it as flat.

$$ds^2 = |dz|^2$$

τ -eq: $\partial_z \partial_{\bar{z}} \tau = \rho$. Localized sources.

~~τ~~ $e^{i\theta} \rightarrow e^{i\theta} + 1$ as we go

anti-clockwise around a D7.

$$\Rightarrow \tau = \frac{1}{2\pi\alpha'} \ln |z - z_0| + c$$

Location of D7.

$$\overline{D7} \Rightarrow \tau = \frac{1}{2\pi\alpha'} \ln |\bar{z} - \bar{z}_0| + c$$

For preserving SUSY we only
consider D7.

O7 carries -4 unit of D7 charge

$$\tau = -\frac{4}{2\pi\lambda} \ln(z-z_0) + c.$$

1. $0\eta + 4 D\eta$:

$$\tau = -\frac{4}{2\pi\lambda} \ln(z-z_0) + \sum_{i=1}^4 \frac{1}{2\pi\lambda} \ln(z-z_i) + c_i.$$

Note: As $z \rightarrow z_0$: $\tau \rightarrow \frac{1}{\lambda} \times (-\infty) = i\infty$.

$$\begin{array}{l} \text{Cof.} \\ + i\epsilon \end{array} \quad \begin{array}{l} -\Phi \\ \oplus \\ \Phi \rightarrow -\infty \end{array}$$

weak coupling

As $z \rightarrow z_0$: $\tau \rightarrow -i\infty$

\Downarrow
not sensible

Solution breaks down near the core of 0η .

Later we shall see how to resolve these problems.

Monodromy: $\tau \rightarrow \tau + 1$ around $D\eta$.

$SL(2, \mathbb{Z})$ transform $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

$\tau \rightarrow \tau - 4$ together with $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\Rightarrow SL(2, \mathbb{Z}) \text{ trs } \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 4 \\ 0 & -1 \end{pmatrix}$$

This can be generalised to $\mathbb{C}P^1$ on M to get 4-d compactification
 L_4 ~~could~~ take to be CY_3

Suppose M has a \mathbb{Z}_2 isometry \mathcal{G}_2
with fixed 4-planes.

Near the \mathcal{G}_2 fixed plane we have

① tangential complex coordinates w_1, w_2

② transverse complex coordinate z

$\mathcal{G}_2: z \rightarrow -z, (w_1, w_2) \rightarrow (w_1, w_2)$

Take quotient by $(-1)^{F_L - \Omega} \mathcal{G}_2$

Note: Ω : exchanges two susy trs.

$(-1)^{F_L} \Omega$ - produces at most some

extra sign acting on susy trs

\neq One linear combination survives.

$N=1$ susy in $D=4$