KKLT construction (in more detail).

4-d viewpoint:

Superpotential W: depends on Complex structure moduli and τ but not on Kahler moduli.

Assume that we have a single Kahler modulus ($h_{1,1}=1$)

$\to$ Overall size modulus ρ.

$$\operatorname{Im}\rho = (\mathrm{Vol}\ CY_3)^{2/3} \sim R^4$$

$$\operatorname{Re}\rho : C^{(4)}_{mn\mu\nu} = \underset{\text{Kahler form}}{\omega_{mn}}\ \underset{\text{dualize}}{b_{\mu\nu}}.$$

Equivalently. $\operatorname{Re}\rho = \int_{4\text{-cycle}} C^{(4)}$

unique 4-cycle isomorphic to Kahler form.

Kahler potential:

$$K = -3\ln(-i(\rho-\bar\rho)) - \ln(-i(\tau-\bar\tau))$$
$$- \ln\left(-\frac{i}{2}\int \Omega\wedge\bar\Omega\right)$$

Note: $\mathrm{Im}\,\rho \sim \left(V_{CY_3}\right)^{2/3}$

↳ measured in 10-d Planck unit.

From now on we shall work in 4-d and express all masses in 4-d Planck unit.

ρ will just be a field, with its interpretation remaining unchanged.

$$V = e^K \left(G^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2 \right)$$

$$D_i W = \partial_i W - (\partial_i K) . W$$

$$D_\rho W = \underbrace{\partial_\rho W}_{=0} - \underbrace{(\partial_\rho K)}_{-\frac{3}{(\rho-\bar{\rho})}} . W = \frac{3}{\rho-\bar{\rho}} . W$$

$$G_{\rho\bar{\rho}} = \partial_\rho \partial_{\bar{\rho}} K = -\frac{3}{(\rho-\bar{\rho})^2}$$

$$G^{\rho\bar{\rho}} = -\frac{(\rho-\bar{\rho})^2}{3}$$

$$G^{\rho\bar{\rho}} D_\rho W \overline{D_\rho W} = + \frac{(\rho-\bar{\rho})^2}{3} . \frac{9}{(\rho-\bar{\rho})^2} |W|^2 = + 3|W|^2$$

$\cancel{e^K 3|W|^2 =}$ -

$$V = e^{K} G^{\alpha\bar\beta} D_\alpha W \overline{D_\beta W}$$

run over complex structure moduli and τ.

$D_\alpha W = 0$ minimizes the potential.

→ # of eqs. = # of variables.

→ Typically has discrete set of solutions.

$$D_\rho W = \frac{3}{\rho - \bar\rho} W.$$

If $W_{min} = 0$ $D_\rho W = 0 \Rightarrow$ SUSY is preserved.

If $W_{min} \neq 0$, SUSY is broken.

We'll not assume $W_{min} = 0$. (Special).

$$W_{min} = W_0.$$

Order of masses of complex structure moduli:

$$V = e^{K} G^{\alpha\bar\beta} D_\alpha W \overline{D_{\beta} W}$$

$$\downarrow$$

$$e^{-3\ln(\rho-\bar\rho) + \cdots}$$

ρ independent.

$$\sim \frac{1}{(\rho-\bar\rho)^3}$$

Recall $\rho-\bar\rho \sim V_{CY}^{2/3} = R^4$ in 10-d Planck units.

$$\Rightarrow V \sim \frac{1}{R^{12}}$$

The kinetic term:

$$G_{\alpha\bar\beta}\, \partial_\mu X^\alpha\, \partial_\nu X^\beta\, g^{\mu\nu}_{(4)}$$

$g^{\mu\nu}_{(4)}$: 4-d canonical metric.

$G_{\alpha\bar\beta}$: ρ-independent

$$\partial_\alpha \partial_{\bar\beta} K$$

$\Rightarrow$ The kinetic term does not have any R dependence.

$$V = V_0 + \frac{1}{2}\sum_\alpha a_{\alpha\beta}(\phi^\alpha - \phi^\alpha_0)(\phi^\beta - \phi^\beta_0)$$

$a_{\alpha\beta}$: mass matrix.

$$\Rightarrow M^2 \propto \frac{1}{R^{12}}$$

$$M \propto 1/R^6.$$

R large $\Rightarrow$ M small.

Nevertheless we shall assume that M is large enough so that we can integrate out the fields τ and complex structure moduli and consider the effective action of ρ.

$W = W_0 \ldots$, ρ is unfixed.

~~Now~~ Now we need to include non-perturbative corrections to W.

Euclidean D3-branes wrapped on certain 4-cycles:

$\Delta \propto e^{i c \rho}$ $\rightarrow$ $R^4 \sim$ Volume of Euclidean D3-brane

Note: There ~~will~~ could be additional factors of g_s ~~but we are taking $g_s \sim 1$~~

~~actually somewhat less than 1~~

~~for perturbative control.~~

Let us check.

D3-brane action:

$$\frac{1}{g_s} \times R_s^4$$

$\hookrightarrow R_s$ measured in 10-d string unit.

$$G_{sMN} = e^{\frac{\Phi}{2}} g_{MN}$$

10-d string metric

10-d canonical metric.

$$R_s = e^{\frac{\Phi}{4}} R = g_s^{1/4} R$$

$$\frac{1}{g_s} \times R_s^4 = R^4$$

$\rightarrow$ No factor of R.

$e^{2\pi i \rho}$ (Respects periodicity of ρ).

Gaugino condensation
~~Instantons~~ on N_c coincident D-brane
could also generate similar superpotential

$$e^{2\pi i \rho / N_c}.$$

Assume $\Delta W = A e^{ic\rho}$ some constant.

$$W_{eff} = W_0 + A e^{ic\rho}$$

$$K = -3 \ln(-i(\rho - \bar{\rho}))$$

Look for soln. to

$$D_\rho W = 0 \Rightarrow icAe^{ic\rho} - \frac{3}{\rho - \bar{\rho}}(W_0 + Ae^{ic\rho}) = 0$$

$\rho = i\sigma$
real.

$$\Rightarrow cAe^{-c\sigma} + \frac{3}{2\sigma}(W_0 + Ae^{-c\sigma}) = 0$$

$$W_0 = -Ae^{-c\sigma}\left(1 + \frac{2c\sigma}{3}\right)$$

W_0 small $\Rightarrow$ σ large.

$$V = e^K\left(|D_\rho W_{eff}|^2 - 3|W_{eff}|^2\right)$$

$$= -\frac{1}{6} c^2 A^2 e^{-2c\sigma}\Big|_{\sigma = \sigma_{cr}}$$

$\rightarrow$ AdS vacuum.

Useful to Look at $V(\sigma)$ before extremization:

$$V = \frac{c A e^{-c\sigma}}{2\sigma^2}\left(\frac{1}{3}\sigma c A e^{-c\sigma} + W_0 + A e^{-c\sigma}\right)$$

$(\text{Mass})^2$ of $\sigma \Rightarrow$ second derivative of $V \propto e^{-c\sigma} \sim |W_0|$.

~~Assu~~ Taking W_0 to be small enough (fine tuning) we can ensure that σ mass is small compared to the masses of other moduli.

$\rightarrow$ Will justify integrating them out.

Now we turn to lifting to dS.

Add anti-D3-brane.

$\rightarrow$ Tension $\sim \frac{1}{g_s}\sqrt{\det_4 G^{(10)}}$ $\rightarrow$ 4-d string metric.

$$G^{(10)}_{MN} = e^{\frac{\Phi}{2}} g^{(10)}_{MN}. \qquad e^{\frac{\Phi}{2}} = g_s^{1/2}$$

Upon compactification:

$$\underbrace{V_{CY_3}}_{= R^6} \int \sqrt{\det_4 g^{(10)}} \, R^{(4)}_{g^{(10)}}$$

Canonically normalized 4-d metric

$$\equiv g^{(4)}$$

$$g^{(10)}_{\mu\nu} = \frac{1}{R^6} g^{(4)}_{\mu\nu}$$

will get rid of R^6 factor.

$$\Rightarrow \sqrt{\det_4 G^{(10)}} = \underbrace{\frac{1}{R^{12}}}_{\sim \sigma^3} \det g^{(4)} \left(g_s^2\right).$$

$\Rightarrow$ $\overline{D3}$-brane ~~contributes~~ action:

$$\frac{1}{\sigma^3} \sqrt{\det g^{(4)}} \; \frac{1}{g_s} \cdot g_s^2.$$

$$ds^2_4 = a(y)^2 \, \eta_{\mu\nu} \, dx^\mu dx^\nu$$

warp factor

$$\sqrt{\det g^{(4)}} \propto a_0^4$$

Warp factor

The $\overline{D3}$ brane will sit at the minimum.

There is also a factor of 2.

We are adding a $D3-\overline{D3}$ pair

dissolving into flux.

still contributes energy density.

By adjusting flux we can

adjust a. at the minimum.

Net upshot: Adding $\overline{D3}$

generates a potential:

$$V = \frac{D}{\sigma^3}$$

$D \to$ some constant. (could involve g_s factors, warp factor etc.)

All fixed before σ is stabilized.

$$V = c\,\frac{A e^{-c\sigma}}{2\sigma^2}\left(\frac{1}{3}\sigma c A e^{-c\sigma} + W_0 + A e^{-c\sigma}\right) + \frac{D}{\sigma^3}$$

① Re-extremize.

② Assume that D has been fine-tuned by fluxes so that V_{min} = small

observed cosmological constant.

Shift in σ:

$$\sigma = \sigma_{0} + \Delta\sigma \quad (\text{say})$$

extremizes the potential without D/σ^3 term.

$$\frac{\partial V_0}{\partial \sigma} + \frac{\partial}{\partial \sigma}(\Delta V) = 0$$

$$\Downarrow$$

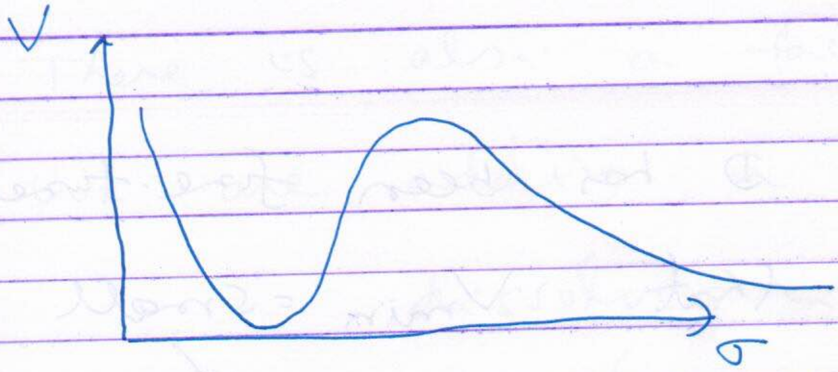
$$\frac{\partial^2 V_0}{\partial \sigma^2}\Delta\sigma + \frac{\partial}{\partial \sigma}(\Delta V) = 0.$$

$$\sim c^2 V_0 \Delta\sigma \qquad \frac{1}{\sigma}(\Delta V_0)$$

If $V_0 + \Delta V_0 \simeq 0$ then $|\Delta V_0| \sim |V_0|$.

$$\Delta\sigma = \frac{1}{c^2\sigma_0} \rightarrow \text{small for large } \sigma_0.$$

Thus the shift in σ due to $\overline{D3}$ brane will be small.



As $\sigma \to \infty$ $V \to 0$

$\Downarrow$ 10-d Minkowski vacuum.

As $\sigma \to 0$, $V \to \infty$

Somewhere around σ_0 there is a metastable vacuum.

→ describes us.

We need to study the stability of the metastable vacuum.