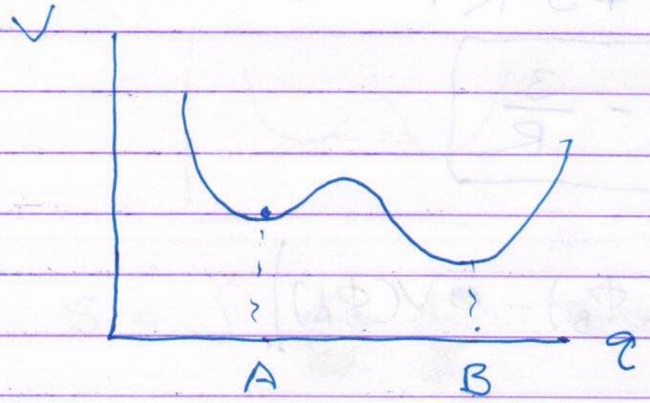
Decay of false vacuum

Generalities: Particle in ~~the~~ a local minimum of potential.



Classically the particle will sit at A.

Quantum mechanically, it will have tunneling probability to B.

→ Can be calculated using WKB.

Quantum field theory: ϕ is a field.

False vacuum : $\phi(x) = A$ for all A.

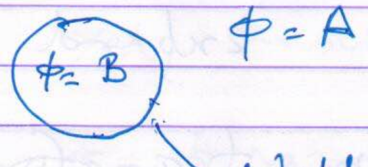
How does it decay to the true vacuum

$\phi(x) = B$.

Changing $\phi(x)$ from A to B everywhere at once requires passing through an energy barrier.

This is not the way it happens.

Create a small bubble in space at



Wall has some thickness along which ϕ changes from B to A.

$$E_{bubble} - E_{False} = (V_B - V_A)\cdot\frac{4}{3}\pi r^3 + 4\pi r^2 \sigma$$

σ energy/area of the wall.

For $r > \dfrac{3\sigma}{V_A - V_B}$

$$E_{bubble} < E_{false}.$$

Such a bubble ~~evolve~~ will expand classically transferring the potential excess energy density to K.E. of the wall.

The wall will accelerate and soon reach ~~the~~ almost the speed of light

~~If M: mass scale of $V(\phi)$ then the time to reach a speed almost class close~~ As it expand it consumes the false vacuum transforming it to true vacuum B.

(Gravity will make things worse)
~ later.

- Role of quantum mechanics: Provide tunnelling probability to create a bubble of radius

$$R = \frac{3\sigma}{V_A - V_B}$$

$\rightsquigarrow$ Finite energy barrier & hence finite transition probability. (to be computed)

Is the standard model unstable?

Does the Higgs potential has a minimum below the ~~current~~ one we are living in?

We do not yet know (requires better measurement of parameters - top quark pole mass).

But this question may not be relevant.

It is hard to imagine that SM will hold till Planck mass.

~~→ take ex~~ After all we have to explain dark matter, ~~?~~ neutrino mass etc

→ expects new fields.

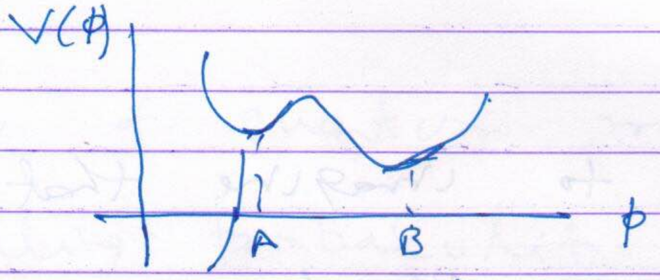
- Some may be scalars.

Could we be at the false vacuum of ~~the~~ their potential?

We do not know.

— Highly UV sensitive theory.

Given a perfectly sensible, stable theory, we could add a new scalar of very heavy mass which could destabilize the vacuum.



We are here.

As long as $V''(\phi)$ at A is large the scalar will have large mass & undetectable.

Nevertheless it could induce vacuum decay rate large enough to affect us.

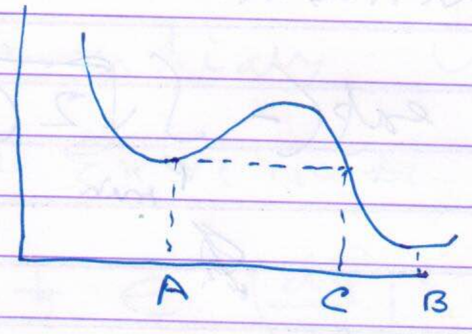
We need a top down approach that tells us precisely what fields we have & what are their couplings.

Unfortunately ~~a~~ string theory has not yet given such ~~a~~ information.

Nevertheless we could at least take a given KKLT model and test how stable it is.

→ Need to learn how to compute vacuum decay probability in QFT.

<u>Particle (1-d)</u>



WKB ⇒ $\Gamma = K \exp\left(-2\int_A^C \sqrt{2(E-V(q))}\, dq\right)$

Constant since we are computing probability.

Particle : Multi-dim. $\vec{q}$: coordinate.

$\vec{q}_A$: Local minimum of $V(\vec{q})$

Σ : The subspace of $\vec{q}$ space in which $V(\vec{q}) = V(\vec{q}_A)$ $\vec{q} \neq \vec{q}_A$.

$\vec{q}_1$: Some point on Σ

① Draw a path P from $\vec{q}_A$ to $\vec{q}_1$.

② Calculate $2\displaystyle\int_P^{\hat{q}} \sqrt{2(E - V(\vec{q}(s)))}\left|\frac{d\vec{q}}{ds}\right| ds$

parameter labelling the path P

③ Find P and $\vec{q}_1 \in \Sigma$ for which this is minimum.

$$\Gamma = K \exp\left(-\int_{P_{min}} \sqrt{2(E - V(\vec{q}))}\left|\frac{d\vec{q}}{ds}\right| ds\right)$$

The particle tunnels along the easiest direction.

Minimization:

① w.r.t. path for fixed $\vec{q}_1$

② w.r.t. $\vec{q}_1$ for $\vec{q}_1 \in \Sigma$

→ can be done later.

(If not we'll get an upper bound on Γ)

Alternative action:

$$\cancel{S = 2\int_P \left\{ \left[e(s) \left(\frac{1}{2} \dot{\vec{q}}^2 + V(\vec{q}) - V(\vec{q}_A) \right] + e(s)^{-1} \right\} ds}$$

$$S = \int_P \left[\frac{1}{2} e(s)^{-1} \left| \frac{d\vec{q}}{ds} \right|^2 + e(s) \left(V(\vec{q}) - V(\vec{q}_A) \right) \right] ds.$$

$e(s)$ auxiliary variable.

~~Minimize~~ Extremize w.r.t. $e(s)$ & $\vec{q}(s)$

$e(s)$: $\quad -\frac{1}{2} e^{-2} \left| \frac{d\vec{q}}{ds} \right|^2 + V(\vec{q}) - V(\vec{q}_A) = 0$

$$e = \left\{ \sqrt{2 (V(\vec{q}) - V(\vec{q}_A))} \right\}^{-1} \left| \frac{d\vec{q}}{ds} \right|$$

Substitute:

$$S = \int ds \left(\frac{1}{2} \sqrt{2(V(\vec{q}) - V(\vec{q}_0))} \left| \frac{d\vec{q}}{ds} \right| \times 2 \right)$$

$$= \int ds \sqrt{V(\vec{q}(s)) - V(\vec{q}_0)} \left| \frac{d\vec{q}}{ds} \right|.$$

$\rightarrow$ same problem as before.

But now we note reparametrization invariance.

$e: \sqrt{g_{ss}}$

Choose gauge $e = 1$.

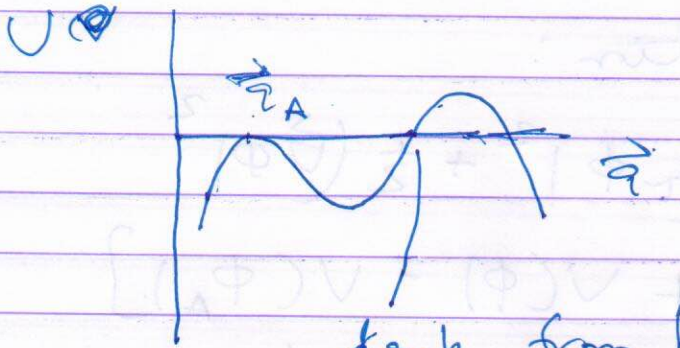
$$S = \int_P \left[\left(\frac{d\vec{q}}{ds} \right)^2 + V(\vec{q}(s)) - V(\vec{q}_A) \right]$$

$$\frac{d^2 q^i}{ds^2} - \frac{\partial V}{\partial q^i} = 0.$$

Constraint: $\left(\frac{d\vec{q}}{ds} \right)^2 - V(\vec{q}) + V(\vec{q}_A) = 0$

consistent with e.o.m.

A zero energy particle moving in a potential $-V(\vec{q}) + V(\vec{q}_A) = U$



drop from here. & Let it reach $\vec{q}_A$

$\Gamma \propto \exp(-S|_{\text{on the soln.}}$

Slightly different interpretation.

① Begin at $\vec{q}_A$ (very close to $\vec{q}_A$)

② Drop it along some direction (initial velocity direction)

③ Let it reach the other end & come back.

→ Bounce.

$\Gamma \propto \exp($− Total Euclidean action of the bounce solution$)$.

We shall now apply this to QFT

$\to$ a theory of ∞ # of particles.

Euclidean action:

$$S = \int \left[\frac{1}{2}(\partial_\tau \phi)^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + V(\phi) - V(\phi_A)\right]$$

False vacuum energy.

Find the bounce soln. & evaluate the action.

$$(\partial_\tau^2 + \vec{\nabla}^2)\phi - V'(\phi) = 0$$

Constraint:

$$\int d^3x \left(\frac{1}{2}(\partial_\tau\phi)^2 - \frac{1}{2}(\vec{\nabla}\phi)^2 - V(\phi) + V(\phi_A)\right) = 0.$$

As long as we choose this at $\tau = -\infty$, it will continue to be satisfied (check)