$\mathcal{N}=1$ Supergravity: $K=1$ $(8\pi G=1)$

$$V = e^{K}\left(G^{\alpha\beta} D_\alpha W \overline{D_\beta W} - 3|W|^2\right)$$

$$D_\alpha W = \partial_\alpha W + \partial_\alpha K\, W.$$

At SUSY extremum $D_\alpha W = 0$.

$$\Rightarrow \quad V = -3\, e^{K} |W|^2$$

We shall work in the weak gravity limit.

$$e^{K} \simeq 1 \qquad K = \frac{1}{M_{Pl}^2}(\Phi \bar{\Phi}) + \cdots$$

$$\rightarrow 0 \text{ as } M_{Pl} \rightarrow \infty$$

(This is for simplicity).

$$\Rightarrow \quad V = -3|W|^2$$

$$V_A = -3|W_A|^2, \quad V_B = -3|W_B|^2$$

Next we need to find σ.

the ~~tension~~ energy/area of the domain wall separating vacua A & B.

Weak gravity:

$$V = \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2$$

Assume one scalar:

$$V = \left|\frac{\partial W}{\partial \phi}\right|^2$$

$$\phi = \psi + i\chi$$

Consider the solns where $\chi = 0$

$$V = \left|\frac{\partial W}{\partial \psi}\right|^2$$

Kinetic term $= \partial_\mu \phi \, \partial^\mu \phi^*$

$$= \partial_\mu \psi \, \partial^\mu \psi + \partial_\mu \chi \, \partial^\mu \chi$$

ψ eq: ψ depends on x only

$$-\partial_x^2 \psi + \frac{\partial V}{\partial \psi} = 0 \qquad V = \left|\frac{\partial W}{\partial \psi}\right|^2$$

$$\frac{\partial}{\partial x} \qquad V_A = V_B = 0.$$

(Note: ignore $-3|W|^2$ term for this calculation)

$\rightarrow$ gravity effect

$$\frac{\partial}{\partial x}\left\{\left(\frac{d\psi}{dx}\right)^2 - V(\psi)\right\} = 0.$$

$$\left(\frac{d\psi}{dx}\right)^2 - V(\psi) = 0$$

$$\Rightarrow \frac{d\psi}{dx} = \frac{\partial W}{\partial \psi}$$

since it $\to 0$ as $x \to \pm\infty$.

$$\sigma = \int dx \left\{\left(\frac{d\psi}{dx}\right)^2 + V(\psi)\right\}$$

$$= 2\int \left(\frac{d\psi}{dx}\right)^2 dx$$

$$= 2\int \frac{\partial W}{\partial \psi}\frac{d\psi}{dx}\,dx$$

$$= 2\left(W(\psi_B) - W(\psi_A)\right)$$

$$= 2(W_B - W_A)$$

Minkowski $\to$ AdS

Need $\epsilon < \frac{3}{4}k\sigma^2 = \frac{3}{4}\sigma^2$

$\epsilon = V_A - V_B = 0 - (-3W_B^2) = 3W_B^2$

$\sigma = 2(W_B - W_A) = 2W_B$

r.h.s. $= \frac{3}{4}\sigma^2 = \frac{3}{4}\cdot 4W_B^2 = 3W_B^2$

$\epsilon = \frac{3}{4}\sigma^2 \Rightarrow$ No bubble exists

AdS $\to$ AdS.

Need $\epsilon^2 > 3k\sigma^2\Lambda = 3\sigma^2\Lambda$ (for $k=1$)

$\epsilon = V_A - V_B = 3(W_B^2 - W_A^2)$,

$\sigma = 2(W_B - W_A)$

$W_B^2 - W_A^2 = \frac{\epsilon}{3}$

$W_B = \left(W_A + \frac{\epsilon}{6W_A}\right)$ $\quad \Lambda = 3W_A^2$

r.h.s. $= 3\sigma^2\Lambda = 3 \cdot 4\,(W_B - W_A)^2\, 3W_A^2$

$= 12 \cdot \frac{\epsilon^2}{36 W_A^2} \cdot 3W_A^2 = \epsilon^2$

$\Rightarrow$ l.h.s. = r.h.s.

$\Rightarrow$ No bubble nucleation of finite size is possible.

Can we make jumps from one flux vacuum to another?

→ No if we are looking at SUSY vacuum, but suppose we are looking at a pair of generic non-SUSY vacuum which differ ~~by~~ in their fluxes.

– ~~e~~ Ignore $\overline{D3}$ brane for now, just assume we have a ~~extremum~~ minimum of

$$V = e^{K}\left(D_i W\, \overline{D_j W}\, G^{i\bar{j}} - 3|W|^2\right)$$

which does not satisfy $D_i W = 0$.

Q. Is there a domain wall that separates two such flux vacua?

$$H^{(3)} = \Sigma(m_i A_i + n_i B_i)$$

$$F^{(3)} = \Sigma(p_i A_i + q_i B_i)$$

to $(m_i, n_i, p_i, q_i) \to (m_i', n_i', p_i', q_i')$

$$\int H^{(3)} \wedge F^{(3)} = \int H^{(3)\prime} \wedge F^{(3)\prime}$$

$(\mathcal{H}^{(3)\prime}, F^{(3)\prime})$ | $(\mathcal{H}^{(3)}, F^{(3)})$

Consider an NS-5 wrapped on A_i.

→ produces flux in 1 direction.

⇒ $\int_{B_i} \mathcal{H}^{(3)}$ will differ on two sides of the NS 5-brane.

Simplest example T^6. $x^0, \ldots x^3$ – non-compact, $y^4, \ldots y^9$ – compact.

Suppose we have NS-5 along 0 1 2 4 5 6 direction.

$\perp$ direction: 3 7 8 9.

Flux through asymptotic boundary

$S^0 \times (T^3)_{789}$

$\pm\infty$ of 3 direction.

The flux through $(T^3)_{789}$ at $x^3 \to \infty$ & $x^3 \to -\infty$ will differ by one unit.

Thus ~~so~~ if we take

~~NS5~~ NS-5 wrapped on: $\Sigma(\Delta m_i) B_i - \Sigma(\Delta n_i) A_i$

D5 wrapped on $\Sigma(\Delta p_i) B_i - \Sigma(\Delta q_i) A_i$

This will change flux from

(m_i, n_i, p_i, q_i) to $(m_i + \Delta m_i, n_i + \Delta n_i, p_i + \Delta p_i, q_i + \Delta q_i)$

We have to produce a bubble whose wall is made of a bound state of NS-5 & D5 of this type.

The euclidean ~~sol~~ solution creating this bubble.

$\Rightarrow$ Euclidean (NS5 + D5) instanton

(Reverse process:

Earlier we analytically continued from Euclidean - Lorentzing to study time evolution.

Now we ~~st~~ find the Euclidean soln. from the time evolution.

Radial direction $\Rightarrow$ $\perp$ direction

The tangential directions:

$$\sum (m_i \cdot B_i - n_i \cdot A_i) \quad * \quad (x^2 + y^2 + z^2 + \tau^2 = R^2)$$

R has to be found by equating

the total energy to zero.

(or its AdS or dS counterpart)