Comments:

① C-map should work even after inclusion of α' corrections but not after inclusion of string loop corrections.

$$\text{IIB on } CY_3 \times S^1 = \text{IIA on } CY_3 \times S^1$$

We compute the moduli spaces on both sides by dimensional reduction and compare.

→ goes through even after inclusion of α' correction on both sides.

Quantum Correction: Momentum/Winding modes propagating in the loop can correct the effective action/metric ~~[illegible]~~ on ~~[illegible]~~ both sides.

⇒ Classical dimensional reduction not valid.

② Geometry of the manifold spanned by vector multiplet scalars is called special Kahler geometry.

$\rightsquigarrow$ a Kahler manifold with special properties.

Need "holomorphic gauge" to see this structure.

Recall the Einstein-Hilbert term:

$$-\frac{i}{2}\left(Z^I \bar{F}_I - \bar{Z}^I F_I\right) R + \cdots$$

D-gauge: $-i\left(Z^I \bar{F}_I - \bar{Z}^I F_I\right) = 1$

"Holomorphic gauge": Set some holomorphic homogeneous fn of Z^I's to 1.

e.g. $Z^0 = 1$ & absorb $-\frac{i}{2}\left(Z^I \bar{F}_I - \bar{Z}^I F_I\right)$ by metric rescaling

Equivalently, define $t^A = \frac{Z^A}{Z^0}$, $A = 1, \cdots, n_V$

\# of vector multiplets.

Regard Z^I's as fixed fn. of t^A

n_v+1 of them $\qquad$ n_v of them.

One gauge condition.

$$e^{-K} \equiv -i(Z^I \bar{F}_I - \bar{Z}^I F_I)$$

Then the metric for t^A's are:

$$G_{AB} = \partial_{t_A} \partial_{\bar{t}_B} K$$

e.g. $Z^0 = 1$ gauge, $t^A = \frac{Z^A}{Z^0}$

Ex. $G_{AB} = \partial_{Z_A} \partial_{\bar{Z}_B} K \Big|_{Z^0 = \bar{Z}^0 = 1}$

~~Another gauge: $Z^A \to f(\vec{t}) Z^A$~~

Different holomorphic gauge choice.

$$\Rightarrow K \to K + f(t_A) + \overline{f(\vec{t})}$$

$$(Z^I \to g(\vec{t}) Z^I, \quad F_I \to g(\vec{t}) F_I)$$

K: Kahler potential.

e.g. for IIA on CY_3:

$$F = -\frac{1}{6} d_{ABC} \frac{Z^A Z^B Z^C}{Z^0}$$

$$e^{-K} = -\frac{i}{} \left[Z^A \bar{F}_A - \bar{Z}^A F_A + Z^0 \bar{F}_0 - \bar{Z}^0 F_0 \right]$$

$$= -\frac{i}{} \left(-\frac{1}{6}\right) d_{ABC} \left[\frac{Z^A \bar{Z}^B \bar{Z}^C}{\bar{Z}^0} + \frac{\bar{Z}^A Z^B \bar{Z}^C}{\bar{Z}^0} \right.$$

$$\left. + \frac{\bar{Z}^A \bar{Z}^B Z^C}{\bar{Z}^0} - \frac{Z^0 \bar{Z}^A \bar{Z}^B \bar{Z}^C}{\bar{Z}^{0\,2}} - c.c. \right]$$

$$= \frac{i}{6} d_{ABC} Z_0 \bar{Z}_0 \left[t^A \bar{t}^B \bar{t}^C + \bar{t}^A t^B \bar{t}^C \right.$$

$$+ \bar{t}^A \bar{t}^B t^C - \bar{t}^A \bar{t}^B \bar{t}^C - \bar{t}^A t^B t^C - t^A \bar{t}^B t^C$$

$$\left. - t^A t^B \bar{t}^C + \bar{t}^A t^B t^C \right]$$

$$= \frac{i}{6} d_{ABC} Z_0 \bar{Z}_0 \left[(t^A - \bar{t}^A)(t^B - \bar{t}^B)(t^C - \bar{t}^C) \right]$$

$$= \frac{1}{6} d_{ABC} Z_0 \bar{Z}_0 \, \mathrm{Im}(t^A)\, \mathrm{Im}(t^B)\, \mathrm{Im}(t^C),$$

$$K = - \ln\left(\mathrm{Im}(t^A)\, \mathrm{Im}(t^B)\, \mathrm{Im}(t^C)\right).$$

$B + iJ = \sum_A t^A \omega_A$
↳ basis of 2-forms.

$t^A \leftrightarrow$ metric information only.

Shift-symmetry $B \to B +$ const. preserved.

Add to F: $-i\overset{c}{}(Z^0)^2$
↓
Constant.

$F_0 \to F_0 + 2\overset{c}{}Z^0$

$e^{-K} = \frac{1}{6} d_{ABC} Z_0 \bar{Z}_0 \; \mathrm{Im}(t^A)\,\mathrm{Im}(t^B)\,\mathrm{Im}(t^C)$

$-i\{Z^0(2i\underset{c}{}\bar{Z}^0) - \bar{Z}_0(-2i\underset{c}{}Z^0)\}$

$= 4c Z^0 \bar{Z}^0$ $\propto$ Volume of CY_3 $(\mathcal{V})$.

$e^{-K} = \{\frac{1}{6} d_{ABC} \mathrm{Im}(t^A)\,\mathrm{Im}(t^B)\,\mathrm{Im}(t^C)$
$+ 4c\} Z^0 \bar{Z}^0$ | $\mathcal{V} \propto \int J \wedge J \wedge J$

$t^A \sim$ area. $\Rightarrow$ Correction $\sim \frac{c}{\mathrm{Vol}(CY_3)}$

$c \sim (\alpha')^3 \Rightarrow$ effect of $(\alpha')^3 R^4$ term

c is computable.

Higher derivative correction:

$$\mathcal{V} \to \mathcal{V} - \frac{1}{2}\,\zeta(3)\,\frac{\chi(X)}{2\cdot(2\pi)^3}$$ → Euler numbers of CY_3.

No further perturbative world-sheet corrections.

There are world-sheet instanton corrections $\sim e^{-a\mathcal{V}}$ → constant.

<u>Heterotic string theory on CY_3.</u>

- Space-time SUSY $\mathcal{N}=1$.

Multiplets in low energy theory:

① Gravity : Graviton + Gravitino

② Vector : Gauge fields + Gaugino.

③ Chiral : Complex scalar + ~~Majorana~~ Weyl fermion (left-handed)

④ Anti-chiral : Complex scalar + Weyl fermion (right-handed)
conjugate to chiral.

Action involving Scalars + Metric:

$\rightsquigarrow$ Controlled by two functions:

Kahler potential : $K(\Phi, \bar{\Phi})$

Superpotential : $W(\Phi)$

$$S = \int d^4x \sqrt{-\det g}\Big[\tfrac{1}{2} R$$

$$- G_{i\bar{j}}(\Phi, \bar{\Phi})\, \partial_\mu \Phi^i \partial^\mu \bar{\Phi}^{\bar{j}} - V_F(\Phi, \bar{\Phi})$$

$$- V_D(\Phi, \bar{\Phi})\Big]$$

$$G_{i\bar{j}} = \partial_i \partial_{\bar{j}} K$$

$$V_F = e^{K}\left(G^{i\bar{j}} D_i W\, D_{\bar{j}} \bar{W} - 3|W|^2\right)$$

$$D_i W \equiv \partial_i W + W \partial_i K$$

$$V_D = \frac{1}{2} \sum_a e_a^2 \left(\phi^*, T^a \phi\right)^2$$

Usual D-term potential.

Heterotic on CY_3:

Massless fields:

Metric $\to$ Complex structure: $h_{1,2}$ complex.

Kahler $\to$ $h_{1,1}$ real.

2-form $\to$ $h_{1,1}$ real.

Φ, Dual of $B_{\mu\nu}$ $\to$ 1. complex.

What is their action?

Consider IIA on CY_3.

Metric + 2-form $\Rightarrow$ Vector multiplet scalars.

$\to$ Identical to what we have in Heterotic.

(Same sugra action in 10-d)

$$\Rightarrow K_{\text{kahler}} = -\ln\left(v - \tfrac{1}{2}\zeta(3)\frac{\chi(X)}{2(2\pi)^3} + O(e^{-av})\right)$$

Note: complex structure moduli ~~is~~ in this theory is part of hypermultiplet which also includes RR fields.

not present in heterotic.

Consider IIB on CY_3.

Complex structure moduli

→ $h_{1,2}$ complex

→ ~~part~~ vector multiplet scalars.

The effective action of these should coincide with the effective action of complex structure moduli of vector multiples.

$$K_{com} = -\ln\left(-\frac{i}{2}\left(Z^I \bar{F}_I - \bar{Z}^I F_I\right)\right)$$

Recall: $Z^I = \int_{A^I} \Omega \qquad F_I = \int_{B^I} \Omega$

3-form $\Rightarrow$ dual to 3 cycles $\Rightarrow$ dual to 3-cycles

$\Rightarrow$ 3 forms are isomorphic to 3-cycles.

$$A^I \leftrightarrow \alpha^I$$

$$B_I \leftrightarrow \beta_I$$

$$A^I \cap A^J = 0 \Rightarrow \int_{A^I} \alpha^J = 0.$$

$$A^I \cap B_J = \delta^I{}_J \Rightarrow \int_{A^I} \beta_J = \delta^I{}_J$$

$$B_I \cap B_J = 0 \Rightarrow \int_{B_I} \beta_J = 0$$

$$B_I \cap A^J = -\delta_I{}^J \Rightarrow \int_{B_I} \alpha^J = -\delta_I{}^J.$$

$$\int \alpha^I \wedge \alpha^J = A^I \cap A^J = 0$$

$$\int \alpha^I \wedge \beta_J = A^I \cap B_J = \delta^I{}_J$$

$$\int \beta_I \wedge \beta_J = B_I \cap B_J = 0$$

$$\int \beta_J \wedge \alpha^I = B_J \cap A^I = -\delta_J{}^I$$

$$\Omega = Z^I \beta_I - F_I \alpha^I$$

$$\int_{A^I} \Omega = Z^J \int_{A^I} \beta_J = Z^I$$

$$\int_{B_I} \Omega = -F_J \int_{B_I} \alpha^J = F_I$$

$$\int \Omega \wedge \bar{\Omega} = (Z^I \beta_I - F_I \alpha^I) \wedge (\bar{Z}^J \beta_J - \bar{F}_J \alpha^J)$$

$$= + Z^I \bar{F}_I - \bar{Z}^J F_J$$

$$= - (\bar{Z}^J F_J - Z^I \bar{F}_I)$$

$$K_{com} = - \ln \left(-\frac{i}{2} \int \Omega \wedge \bar{\Omega} \right)$$

We are left with axion-dilaton modulus:

$$S = a + i e^{-2\Phi}$$

dual to $B_{\mu\nu}$

$$K_S = - \ln \left(-i (S - \bar{S}) \right)$$

from explicit calculation.

Potential: Moduli are uncharged

$\Rightarrow V_D = 0.$

Moduli cannot have potential $\Rightarrow V_F = 0$

$\Rightarrow W = 0.$