Nucleosynthesis: Happens at T in the range:

$$0.1\,\text{MeV} \lesssim T \lesssim \text{MeV}.$$

First step: Find time dependence of T, T_ν and λ in this range.

equations ① $\left(\frac{\dot{\lambda}}{\lambda}\right)^2 = \frac{8\pi G}{3}\rho$.

② $\frac{d}{dt}(s\lambda^3) = 0$ ③ $T_F \lambda_F = T_\nu \lambda$

$$s = \frac{4}{3} a_B T^3 + \frac{1}{2\pi^2}(2+2)\int_0^\infty k^2\, dk$$

$\frac{\pi^2}{15}$ '' e^- e^+

$$\times \left[\ln\left(1 + e^{-\sqrt{k^2+m^2}/T}\right) + \frac{1}{T}\sqrt{k^2+m^2}\, \frac{e^{-\sqrt{k^2+m^2}/T}}{1+e^{-\sqrt{k^2+m^2}/T}} \right]$$

p/T e/T

(Used $s = \frac{p + e - \overset{=0}{\mu n}}{T}$

$u = \frac{k}{T}$, $x = \frac{m}{T}$

$$\frac{4}{3} a_B T^3 + \frac{2}{\pi^2} T^3 \int_0^\infty u^2\, du \left[\ln\left(1 + e^{-\sqrt{u^2+x^2}}\right) + \sqrt{u^2+x^2}\; e^{-\sqrt{u^2+x^2}} \Big/ \left(1 + e^{-\sqrt{u^2+x^2}}\right) \right]$$

$$= \frac{4}{3} a_B T^3 f_s(x)$$

$$s = \frac{4}{3} a_B T^3 f_s(x) \,, \qquad x = \frac{m}{T}$$

$$f_s(x) \xrightarrow[x\to\infty]{} 1$$

$$\xrightarrow[x\to 0]{} \frac{11}{4} \left(1 + \frac{7}{8} + \frac{7}{8}\right)$$

$$s\lambda^3 = \text{constant} \Rightarrow T^3 \lambda^3 f_s(x) = \text{constant}$$

$$\Rightarrow x^{-3} \lambda^3 f_s(x) = \text{const.} = \frac{11}{4} x_F^{-3} \lambda_F^3$$

$$T_\nu = \frac{T_F \lambda_F}{\lambda} \,, \quad \frac{T_\nu}{T} = \frac{T_F \lambda_F}{T \lambda} = \frac{\lambda_F}{\lambda} \frac{x}{x_F} = \left(\frac{4}{11} f_s(x)\right)^{1/3}$$

Now we compute ρ

$\nu_e, \bar{\nu}_e$ $\quad$ $\nu_\mu, \bar{\nu}_\mu$ $\quad$ $\nu_\tau, \bar{\nu}_\tau$

$$\rho = a_B T^4 + \frac{1}{2} a_B T_\nu^4 \, \frac{7}{8} \, (2+2+2)$$

e^- $\quad$ e^+

$$+ \frac{1}{2\pi^2}(2+2) \int_0^\infty k^2 \, dk \, \sqrt{k^2+m^2} \; \frac{e^{-\sqrt{k^2+m^2}/T}}{1+e^{-\sqrt{k^2+m^2}/T}}$$

$x = \frac{m}{T}, \; u = \frac{k}{T}$

$$= a_B T^4 + \frac{21}{8} a_B T^4 \left(\frac{4}{11} f_s(x)\right)^{4/3}$$

$$+ \frac{2}{\pi^2} T^4 \int_0^\infty u^2 \, du \, \sqrt{u^2+x^2} \; \frac{e^{-\sqrt{u^2+x^2}}}{1+e^{-\sqrt{u^2+x^2}}}$$

$$= a_B T^4 f_e(x)$$

$$① \quad \lambda^3 x^{-3} f_S(x) = \text{constant.}$$

$$② \quad T_\gamma = T\left(\frac{4}{11} f_S(x)\right)^{1/3}$$

$$③ \quad \rho = a_B T^4 f_\rho(x).$$

Substitute into

$$\left(\frac{\dot{\lambda}}{\lambda}\right)^2 = \frac{8\pi G}{3}\rho$$

$$\frac{d}{dt}(\ln ①) \Rightarrow 3\frac{\dot{\lambda}}{\lambda} - 3\frac{\dot{x}}{x} + \frac{f_S'(x)}{f_S(x)}\dot{x} = 0$$

$$\Rightarrow \frac{\dot{\lambda}}{\lambda} = \frac{\dot{x}}{x}\left(1 - \frac{1}{3} x \frac{f_S'(x)}{f_S(x)}\right)$$

Eliminate $\frac{\dot{\lambda}}{\lambda}$ & rewrite Friedmann eq. as

$$\left(\frac{\dot{x}}{x}\right)^2 \left(1 - \frac{x}{3}\,\frac{f_s'(x)}{f_s(x)}\right)^2 = \frac{8\pi G}{3}\, a_B\, \frac{m^4}{x^4}\, f_e(x)$$

$$\Rightarrow \underset{\substack{\parallel \\ \frac{dx}{dt}}}{\dot{x}} = \left(1 - \frac{x}{3}\,\frac{f_s'(x)}{f_s(x)}\right)^{-1} \left(\frac{8\pi G}{3} a_B\right)^{1/2} m^2 x^{-1} \left(f_e(x)\right)^{1/2}$$

$$\Rightarrow t = \int_0^x d\tilde{x} \left(1 - \frac{\tilde{x}}{3}\,\frac{f_s'(\tilde{x})}{f_s(\tilde{x})}\right) \left(\frac{8\pi G}{3} a_B\right)^{-1/2} m^{-2}\, \tilde{x}\, f_e(\tilde{x})^{-1/2}$$

B.C. As $t \to 0$, $x \to 0$

T (Mev/K)	T/T_ν	t (Sec)
10^{11} K (8.6 Mev)	1	$\simeq 0$
3×10^{10} K	1.001	.101
10^{10} K (.86 MeV)	1.008	.998
3×10^{9} K	1.08	12.66
10^{9} K (.086 Mev)	1.345	168
3×10^{8} K	$1.401 = \left(\frac{11}{4}\right)^{1/3}$	1980
10^{8} K (.0086 Mev)	1.401	1.78×10^{4} K.

← BBN.

Neutron half life ~ 880 Secs.

Note: If we had an extra decoupled particle (which might have decoupled at higher temp.) then it will contribute to $\rho \Rightarrow f_\rho(x)$ will be larger.

$\Rightarrow$ Nucleosynthesis will begin earlier.

$\Rightarrow$ More n at the time of nucleosynthesis.

$\Rightarrow$ Larger He/H ratio.

$\Rightarrow$ Has been used to put bounds on number of extra light particles at the time of nucleosynthesis.

We now turn to the next step.
Study n_p/n_n ratio during this time
(ignoring nuclei formation).
n_p/n_n ratio can change only by
weak interaction
(EM and strong interaction conserves
n_p and n_n separately).

Processes that are relevant:

$n \rightleftarrows p + e^- + \bar{\nu}_e$, $n + e^+ \rightleftarrows p + \bar{\nu}_e$, $n + \nu_e \rightleftarrows p + e^-$

$\mu, \tau, \nu_\mu, \nu_\tau$ do not play any role.

μ^-, τ^- are both heavy.

$\Rightarrow$ # densities have become small by $T \sim \text{MeV}$.

$\Rightarrow$ ignore them in our analysis and focus on $n, p, e^\pm, \gamma, \nu_e, \bar{\nu}_e$ system.

n_n, n_p are much smaller than n_γ.

$\Rightarrow$ n, p cannot affect the $e^+, e^-, \gamma, \nu_e, \bar{\nu}_e$ system.

Converse is not true.

$\Rightarrow$ We can regard n, p system to be in thermal & chemical equilibrium with $e^\pm, \gamma, \nu_e, \bar{\nu}_e$ system at $T \gtrsim 1$ MeV.

For this problem we cannot ignore the chemical potentials any more.

$$n_n = \frac{1}{2\pi^2} \times 2 \times \int_0^\infty k^2 \, dk \; \frac{e^{-\left(\sqrt{k^2+m_n^2}-\mu_n\right)/T}}{1+e^{-\left(\sqrt{k^2+m_n^2}-\mu_n\right)/T}}$$

$T \sim \text{MeV} \ll m_n \sim 10^3 \text{ MeV}$.

$$n_n \simeq e^{\mu_n/T} \times 2\, e^{-m_n/T} \left(\frac{m_n T}{2\pi}\right)^{3/2}, \quad n_\gamma \propto T^3$$

$$\frac{n_n}{n_\gamma} \sim e^{\mu_n/T} \underbrace{e^{-m_n/T}}_{e^{-1000}} \cdot \left(\frac{m_n}{2\pi T}\right)^{3/2} \sim 10^{-9} \sim e^{-20}$$

$$\Rightarrow \frac{\mu_n}{T} \simeq \frac{m_n}{T} - 20 \sim 980.$$

$$\frac{n_n}{n_\gamma} = e^{\mu_n/T}\, e^{-m_n/T}\left(\frac{m_n}{2\pi T}\right)^{3/2} \sim 10^{-9}$$

$\Rightarrow \frac{\mu_n}{T}$ cannot be taken to be small if $e^{-\frac{m_n}{T}}\left(\frac{m_n}{2\pi T}\right)^{3/2} \ll 10^{-9}$

$$\Rightarrow \frac{m_n}{T} \sim 20$$

$\Rightarrow$ For $T < \frac{m_n}{20}$, $\frac{\mu_n}{T}$ becomes appreciable.

$\Rightarrow$ Same logic $\Rightarrow$ $\frac{\mu_e}{T}$ becomes appreciable at $T \sim \frac{m_e}{20} \sim .02$ MeV.

Similar logic tells us that $\frac{\mu_{\nu_e}}{T}$ can also be taken to be small.

Hidden assumption:

$$\frac{n_{\nu_e} - n_{\bar{\nu}_e}}{n_\gamma} \sim 10^{-9} \rightarrow$$ Has not been measured experimentally.

Later we'll argue theoretically, using physics at 100 GeV, that this is the case.

Conclusion:

At T between 1 MeV and .1 MeV, we have appreciable values of $\frac{\mu_n}{T}, \frac{\mu_p}{T}$

but we can set

$$\frac{\mu_{e^-}}{T} \simeq 0, \quad \frac{\mu_{\nu_e}}{T} \simeq 0, \quad \mu_\gamma = 0$$

n: # of collisions with $\nu, e^\pm, \gamma$ is >1 per . $\boxed{T > 1\text{ MeV}}$