Total rate for $n \to p$ conversion per neutron

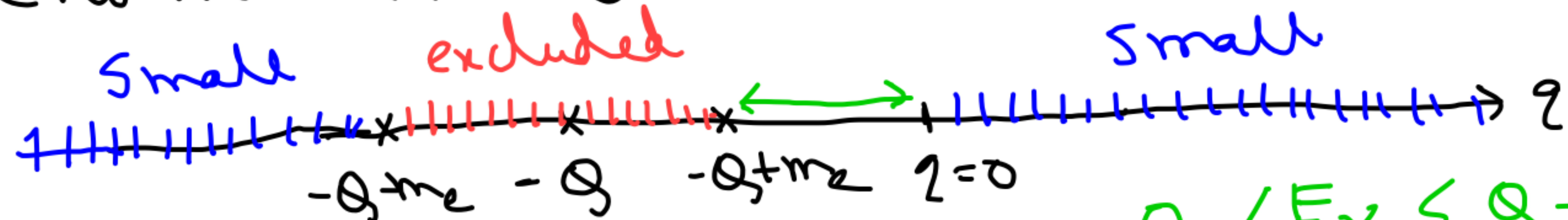
$$R(n\to p) = A \int_{-\infty}^{\infty} dq \left(1 - \frac{m_e^2}{(q+Q)^2}\right)^{1/2} \frac{q^2 (q+Q)^2}{\left(1+e^{\frac{q}{T_\nu}-\frac{\mu_\nu}{T_\nu}}\right)\left(1+e^{-\frac{q+Q}{T}+\frac{\mu_e}{T}}\right)}$$

$$|q+Q| \geq m_e$$

$R(p\to n) \to$ similar formula with the sign of the exponents reversed in the denominator.

① $T \ll Q \sim \text{MeV}, \Rightarrow T_\nu \ll Q$

Denominator for $R(n\to p)$ is large $\left\langle \begin{array}{l} q \gg 0 \\ q+Q \ll 0 \end{array} \right\}$ Unit T



$-Q+m_e \leq q \leq 0$

$0 \leq E_\nu \leq Q - m_e \Rightarrow n \to p + e^- + \bar{\nu}_e$

Ex. Check that for $T \ll Q \sim \text{MeV}$, $R(p \to n)$ is small (Denominator is large for all q).

② Estimate the time/energy at which the rates go below the expansion rate.

Consider $T \gtrsim \text{MeV}$.

$$R(n \to p) = A \int dq \left(1 - \frac{m_e^2}{(q+Q)^2}\right) \frac{q^2 (q+Q)^2}{\left(1 + e^{-\frac{q+Q}{T}}\right)\left(1 + e^{\frac{q}{T}}\right)}$$

$$\sim A \int_{-T}^{T} dq\, q^4 \sim A T^5 \to 10^{-10} (\text{GeV})^{-4} T^5$$

$$= 10^{-10} (\text{GeV}) \left(\frac{T}{\text{GeV}}\right)^5$$

$$\overset{\text{Ex}}{=} (1\,\text{sec})^{-1} (T/\text{MeV})^5$$

$$A \sim (G_{\text{weak}})^2 \to 10^{-5} (\text{GeV})^{-2}$$

$\frac{\dot{a}}{a} \sim T^2 \Rightarrow \quad \frac{1}{\text{sec}} \left(\frac{T}{\text{MeV}}\right)^2$

Recall the neutrino decoupling analysis

$\Rightarrow R(n \to p) \sim \frac{1}{\text{sec}} \left(\frac{T}{\text{MeV}}\right)^5$ falls below the expansion rate when $\frac{T}{\text{MeV}} < 1$

$\Rightarrow T < \text{MeV}$.

Time evolution of n_n and n_p:

$$\frac{d}{dt}\left(n_n a^3\right) = -R(n\to p)\, n_n a^3 + R(p\to n)\, n_p a^3$$

$$\frac{d}{dt}\left((n_n+n_p) a^3\right) = 0$$

Define: $X_n = \dfrac{n_n}{n_n+n_p}$, $1-X_n = \dfrac{n_p}{n_n+n_p}$

$$\Rightarrow \frac{d}{dt} X_n = -R(n\to p)\, X_n + R(p\to n)(1-X_n)$$

Divide by $(n_n+n_p)\, a^3$

T (in K)	t (sec)	X_n
10^{11} K (8.6 MeV)	0	.4626 (stat mech)
3×10^{10} K	.101	.3798
10^{10} K	.998	.2386
3×10^{9} K	12.66	.1654
Nucleosynthesis → 10^{9} K (.086 MeV)	168.1	.1333
3×10^{8} K	1980	.0172
10^{8} K	17780	3.07×10^{-10}

$$\frac{X_n}{1-X_n} \sim e^{-Q/T} \sim e^{-\frac{1}{10}}$$

$$e^{-.01} = .99$$

Step 3: Process of nucleosynthesis.

$p + n \rightleftarrows d + \gamma$, $d + d \rightleftarrows H^3 + p$, $d + d \rightleftarrows He^3 + n$

$d + H^3 \rightleftarrows He^4 + n$, $d + He^3 \rightleftarrows He^4 + p$

em interaction

↓

not as fast as strong interaction.

strong interaction.

fast

For full analysis, define $Y_i = \frac{n_i}{n_B}$

density of ith type nucleus.

$$\frac{dY_i}{dt} = \text{rate of production} - \text{rate of destruction.}$$

cross-sections are known.

We'll use a simplifying assumption that gives reasonably accurate result:

We'll estimate the error

We'll assume that nucleosynthesis begins at some time t_1 when the density of d reaches a critical value.

Once it begins it happens very fast and we can estimate the X_i's by using equilibrium stat. mech. formula.

Two Steps.

a) Estimate the time t, (equivalently the temperature) at which nucleosynthesis happens.

b) Use equilibrium stat mech to compute the Y_i's.

Begin with step (b).

n_i : density of nuclei of type i

Z_i : # of protons inside ith type nucleus

$A_i - Z_i$: # of neutrons inside i-th type "

<u>Conserved charges:</u>

$$(n_p)_{tot} = n_p + \sum_i Z_i n_i$$

$$(n_n)_{tot} = n_n + \sum_i (A_i - Z_i) n_i$$

$$\tilde{\mu}_1 (n_p)_{tot} + \tilde{\mu}_2 (n_n)_{tot} = \mu_p n_p + \mu_n n_n + \sum_i \mu_i n_i$$

$$\Rightarrow \mu_p = \tilde{\mu}_1 , \quad \mu_n = \tilde{\mu}_2 , \quad \mu_i = Z_i \tilde{\mu}_1 + (A_i - Z_i) \tilde{\mu}_2 = Z_i \mu_p + (A_i - Z_i) \mu_n$$

$$\mu_i = Z_i \mu_p + (A_i - Z_i)\mu_n \quad , \qquad Y_i = n_i/n_B, \; Y_p = n_p/n_B$$

All baryons are non-relativistic. $\quad Y_n = n_n/n_B$

$$n_p = 2\, e^{-\frac{m_p - \mu_p}{T}} \left(\frac{m_p T}{2\pi}\right)^{3/2} \qquad T = T(t_i)$$

m_n

$$n_n = 2\, e^{-\frac{m_n - \mu_n}{T}} \left(\frac{m_n T}{2\pi}\right)^{3/2}$$

$A_i m_n$

$$n_i = g_i\, e^{-\frac{M_i - \mu_i}{T}} \left(\frac{M_i T}{2\pi}\right)^{3/2}$$

$$M_i = Z_i m_p + (A_i - Z_i) m_n - B_i$$ $\rightarrow$ binding energy

$$\frac{Y_i}{Y_p^{Z_i} Y_n^{A_i - Z_i}} = n_B^{A_i - 1} \frac{n_i}{n_p^{Z_i} n_n^{A_i - Z_i}} \overset{\text{Ex}}{=} \frac{g_i}{2} A_i^{3/2} e^{B_i/T} \epsilon^{A_i - 1}$$

$$\epsilon = \frac{1}{2} n_B \left(\frac{T m_n}{2\pi}\right)^{-3/2}$$

$$\boxed{Y_i = Y_p^{Z_i}\, Y_n^{A_i - Z_i}\, \frac{g_i}{2}\, A_i^{3/2}\, e^{B_i/T}\, \epsilon^{A_i - 1}}$$

$$\epsilon = \frac{1}{2}\, n_B \left(\frac{T m_p}{2\pi}\right)^{-3/2}$$

$\lambda T = \text{const.}$

$$n_B = \frac{n_B(t_0)\, \lambda(t_0)^3}{\lambda(t_1)^3} = \frac{n_B(t_0)}{T_0^3}\, T(t_1)^3$$

$\lambda(t_0)^3 = 1$

$$n_B = \Omega_B \rho_c = \Omega_B \frac{3 H_0^2}{8\pi G}$$

$\Omega_B \sim .04$

$H_0 = 70\,\text{km/sec/Mpc} \rightarrow 3\times 10^{24}\,\text{cm.} \Rightarrow \epsilon \sim 10^{-15}$

Ex

$$n_p + \sum_i Z_i n_i = (n_p)_{tot} = (1 - X_n(t_1))\, n_B$$

$$\boxed{Y_p + \sum_i Z_i Y_i = 1 - X_n(t_1), \quad Y_n + \sum_i (A_i - Z_i) Y_i = X_n(t_1)}$$