

$$Y_i = \frac{n_i}{n_B} \rightarrow \text{Number density of } i\text{-th type nucleus } d, \text{He}^3, \text{H}^3, \text{He}^4, n, p$$

Systematic procedure:

$$\frac{dY_i}{dt} = \text{rate of creation} - \text{rate of destruction}$$

processes in which i appears in final state \downarrow processes in which i appears in initial state \uparrow

claim: values of Y_i can be obtained by assuming that the system reaches equilibrium at $T \sim 10^9 \text{ K}$. \rightarrow to be justified later.

W r Jot

$$Y_i = \frac{g_i}{2} Y_n^{A_i - Z_i} Y_p^{Z_i} (A_i)^{3/2} \leftarrow A_i^{-1} \quad \leftarrow B_i/T \text{ binding energy}$$

$$\leftarrow = \frac{1}{2} n_B \left(\frac{T m_p}{2\pi} \right)^{-3/2} = 10^{-15} \times \left(\frac{T}{10^9 \text{ K}} \right)^{3/2}$$

$$\chi^2_3 \frac{\Omega_B \rho_c}{m_p} = \left(\frac{T}{T_0} \right)^3 \frac{\Omega_B}{m_p} \frac{3 H_0^2}{8\pi G} \rightarrow .086 \text{ MeV}$$

$$H_0 = 100 h \text{ Km/sec/Mpc} \rightarrow 3.086 \times 10^{24} \text{ Cm}$$

$$\rightarrow \Omega_B = .04 \times \left(\frac{T}{10^{10} \text{ K}} \right)^{3/2} \Omega_B h^2$$

$$E = 1.46 \times 10^{-12}$$

$$\left[\begin{aligned} Y_n + Y_d + Y_{\text{He}^3} + 2 Y_{\text{H}^3} + 2 Y_{\text{He}^4} \\ = X_n \\ Y_p + Y_d + 2 Y_{\text{He}^3} + Y_{\text{He}^3} + 2 Y_{\text{He}^4} \\ = 1 - X_n \end{aligned} \right]$$

Actual numbers at $T = 10^9 \text{ K}$. | $Y_i \leq 1$

ΣX {

$$Y_{\text{He}^4} = Y_n^2 Y_p^2 10^{99}$$

$$Y_{\text{He}^3} = Y_n Y_p^2 10^9$$

$$Y_{\text{H}^3} = Y_n^2 Y_p 10^{12}$$

$$Y_d = Y_n Y_p 10^{-4}$$

} → Either Y_n or Y_p or both must be $< 10^{-24}$
 Small $< 10^{-12}$

$Y_n \sim 10^{-48}$

$$Y_n + 2Y_{\text{He}^4} = X_n$$

$$Y_p + 2Y_{\text{He}^4} = 1 - X_n$$

$$X_n < \frac{1}{2} \Rightarrow 1 - X_n > X_n$$

→ $Y_{\text{He}^4} = \frac{X_n}{2}$

$$\Rightarrow Y_p > Y_n \Rightarrow Y_n < 10^{-24}$$

→ $Y_p = 1 - 2X_n \sim 1$

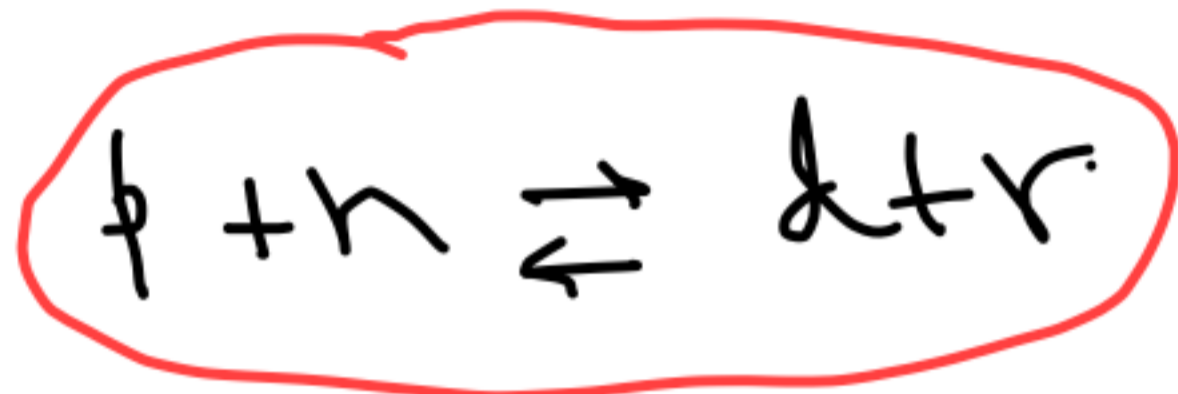
$$Y_{\text{He}^4} = \frac{X_n}{2}, \quad Y_p = 1 - 2X_n.$$

$$\Rightarrow \frac{\text{Helium}}{\text{Hydrogen}} = \frac{Y_{\text{He}^4}}{Y_p} = \frac{X_n}{2(1-2X_n)} \quad \text{by No. of nuclei}$$

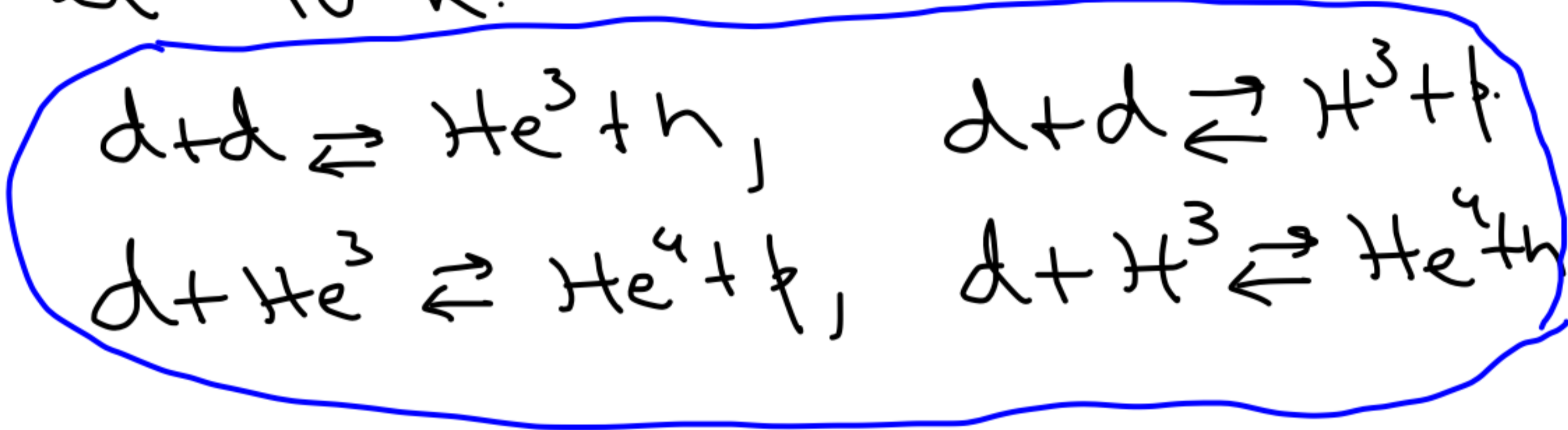
$$\frac{\rho_{\text{He}} \rightarrow \text{mass density}}{\rho_{\text{H}}} = \frac{4 Y_{\text{He}^4}}{Y_p} = \frac{2X_n}{1-2X_n} \sim \frac{.27}{.73}$$

$$\text{At } T = 10^9 \text{ K, } X_n = .133$$

Remaining task: Show that equilibrium is established at 10^9 K.



EM



Strong

Rate per neutron

$$\begin{aligned}
 \sigma_d \ll n_p &\rightarrow \sim n_B \sim \left(\frac{T}{T_0}\right)^3 & \Omega_B &= \frac{3H_0^2}{8\pi G} (1 - X_n) \\
 4.55 \times 10^{-20} \text{ cm}^3/\text{s} & & & \\
 \text{Ex. } \sigma_d \ll n_p &\approx 2.52 \times 10^4 \text{ sec}^{-1} \times \left(\frac{T}{10^{10} \text{ K}}\right)^3 & & = X_p \Omega_B h^2 \\
 \lambda &= \sqrt{\frac{8\pi G \rho}{3}} & & \rightarrow \frac{1}{2} \Omega_B T^4 \left(2 + \frac{7}{8} \times 6 \times \left(\frac{T}{10^{10} \text{ K}}\right)^4\right)
 \end{aligned}$$

$$\text{Ex. } \frac{\sigma_{\alpha} \rho n_p}{j/\lambda} \approx 10^5 \frac{T}{10^{10} \text{ K}} (1 - X_n) \Omega_B h^2$$

$\begin{matrix} 1 \\ -0.4 \end{matrix} \quad \begin{matrix} 1 \\ 1.7 \end{matrix}$

large for $T \gg 10^9 \text{ K}$.

Conclusion: For $T \gtrsim 10^9 \text{ K}$, p, n, d, r system is in equilibrium.

Assume for now that the heaviest nuclei are not produced appreciably at this time (to be seen later).

We use a formula similar to what we had earlier to determine the density of d .

$$\tilde{y}_d = \frac{3}{2} \tilde{y}_n \tilde{y}_p \cdot 2\sqrt{2} \in \rho^{B_d/T}$$

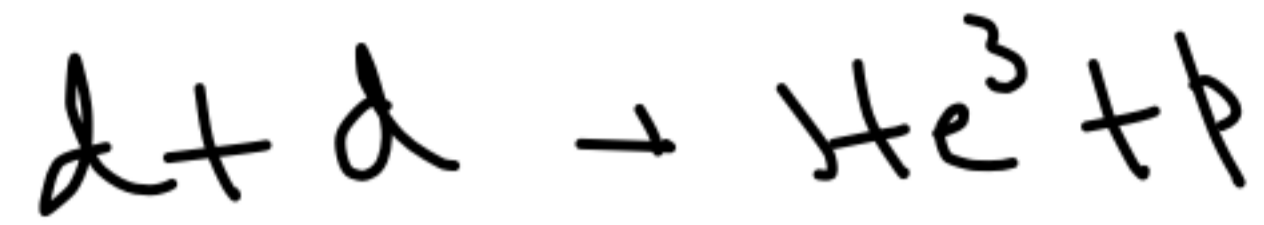
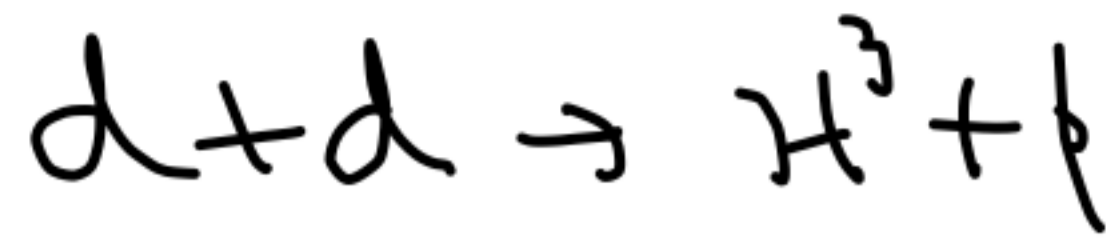
$$\tilde{y}_p + \tilde{y}_d = 1 - X_n, \quad \tilde{y}_n + \tilde{y}_d = X_n$$

Around 10^9 K , $\rho \sim 10^{-15}$, $\rho^{B_d/T} \sim 10^{11}$

$$\Rightarrow \tilde{y}_d = \frac{3}{2} \tilde{y}_n \tilde{y}_p \cdot 2\sqrt{2} \cdot 10^{-15} \times 10^{11} \rightarrow \text{Small.}$$

Suppose $T \sim 0.75 \times 10^9 \text{ K}$

$$\rho^{B_d/T} \approx 10^{11} \times \frac{4}{3} \sim 10^{15}$$



$$\sigma_U \approx 1.8 \times 10^{-17} \text{ cm}^3/\text{s}$$

$$\sigma_U \approx 1.6 \times 10^{-17} \text{ cm}^3/\text{s}$$

Total rate for d , $\sigma_U \bar{n}_d = \bar{\gamma}_d \times n_B$

$$\text{Ex. } \frac{\sigma_U n_d}{\lambda/\lambda} \approx 10 \left(\frac{T}{10^9 \text{ K}} \right)^{5/2} \exp \left(\frac{25.87 \times 10^9 \text{ K}}{T} \right)$$

$$B_\lambda = 25.87 \times 0.086 \text{ MeV} \approx 2 \text{ MeV.}$$

Ratio ≈ 10 at $T = 10^9 \text{ K}$, but drops sharply for larger T and rises sharply at lower T .

A more detailed analysis shows that nucleosynthesis happens between 10^9 K and $.95 \times 10^9$ K.

Uncertainty in $\sigma: \pm 10\%$

$t \propto \sigma^2 \Rightarrow$ Uncertainty in $t \sim \pm 20\%$

t at 10^9 K = 168 sec.

Uncertainty in t = $(168 \pm 168 \times .2)$ sec.

$$\frac{dX_n}{dt} = -\frac{1}{880 \text{ sec}} X_n$$

$$\Rightarrow \Delta X_n = \pm X_n \times \frac{168 \times .2}{880} \text{ mean life of } n \text{ } \leftarrow .04$$

$$\Delta X_n = .04 X_n$$

$Y_{He} = 2X_n \Rightarrow$ Error in determination of Y_{He} by this argument is about 4%.

What about the other nuclei?

Equilibrium values: at $10^9 K$ since $Y_p \sim 1$, $Y_n \sim 10^{-48}$

$$Y_{He^4} \sim Y_n^2 Y_p^2$$

$$Y_{He^3} \sim Y_n Y_p^2$$

$$Y_{H^3} \sim Y_n^2 Y_p$$

$$10^{-97}$$

$$10^{-97}$$

$$10^{-97}$$

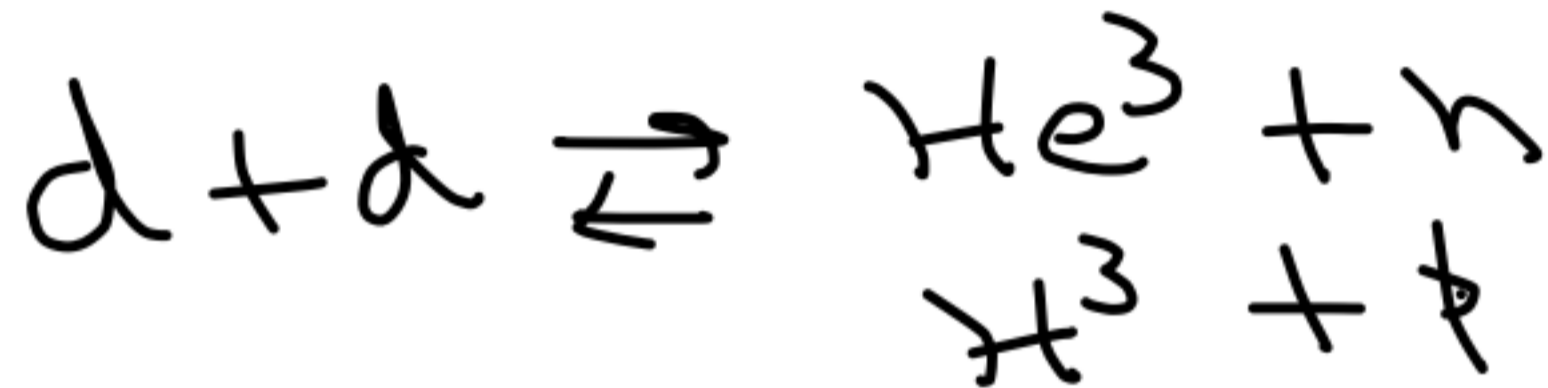
$$\rightarrow 10^{-40}$$

$$\rightarrow 10^{-84}$$

$$Y_2 \sim Y_n Y_p 10^{-4} \sim 10^{-52}$$

In actual practice the number densities of d , H^3 , He^3 are larger.

If the only reaction involving d had been



the reaction rate per d would be

$$\sigma_d v n_d$$

Relevant quantity is $\frac{\sigma_d v n_d}{\lambda/\lambda}$

The system goes out of equilibrium when $\sigma_d v n_d / (\lambda/\lambda) < 1 \Rightarrow$ determines n_d .

This process leaves us with some n , n_{H^3} and n_{He^3} besides p , He^4 which are the main components.

H^3 decays to $He^3 + \dots$ (lifetime ~ 12 years)

What we observe today as He^3 abundance is actually the sum of H^3 and He^3 abundance during nucleosynthesis.

These numbers do not include production of these elements in stars which happen much later: