$Y_i = \frac{n_i}{n_B}$ → Number density of i-th type nucleus d, He^3, H^3, He^4, n, p

Systematic procedure:

$\frac{dY_i}{dt}$ = rate of creation − rate of destruction

rate of creation → processes in which i appears in final state

rate of destruction → processes in which i appears in initial state

claim: Values of Y_i can be obtained by assuming that the system reaches equilibrium at $T \sim 10^9 K$. → to be justified later.

We got

$$Y_i = \frac{g_i}{2} \; Y_n^{A_i - Z_i} \; Y_p^{Z_i} \; (A_i)^{3/2} \; \epsilon^{A_i - 1} \; e^{B_i/T}$$

B_i → binding energy

$$\epsilon = \frac{1}{2} n_B \left(\frac{T m_p}{2\pi}\right)^{-3/2} = 10^{-15} \times \left(\frac{T}{10^9 K}\right)^{3/2}$$

$10^9 K$ → .086 MeV

n_B ↓

$$\frac{1}{\lambda^3} \; \frac{\Omega_B \rho_c}{m_p} = \left(\frac{T}{T_0}\right)^3 \frac{\Omega_B}{m_p} \; \frac{3 H_0^2}{8\pi G}$$

$H_0 = 100 h$ Km/sec/Mpc

Mpc ↪ 3.086×10^{24} cm

h ↓ .7

$\Omega_B = .04$

$$\epsilon = 1.46 \times 10^{-12} \times \left(\frac{T}{10^{10} K}\right)^{3/2} \Omega_B h^2$$

$$Y_n + Y_d + Y_{He^3} + 2 Y_{H^3} + 2 Y_{He^4} = X_n$$

$$Y_p + Y_d + 2 Y_{He^3} + Y_{He^3} + 2 Y_{He^4} = 1 - X_n$$

Actual numbers at $T = 10^9$ K. | $Y_i \leq 1$

Ex.

$$Y_{He^4} = Y_n^2 Y_p^2 10^{97}$$ → Either Y_n or Y_p or both must be $< 10^{-24}$

$Y_n \sim 10^{-48}$

$$Y_{He^3} = Y_n Y_p^2 10^9$$

$$Y_{H^3} = Y_n^2 Y_p 10^{12}$$

$$Y_d = Y_n Y_p 10^{-4}$$

} small $< 10^{-12}$

$$Y_n + 2Y_{He^4} = X_n, \quad Y_p + 2Y_{He^4} = 1 - X_n$$

$$X_n < \tfrac{1}{2} \Rightarrow 1 - X_n > X_n \Rightarrow Y_p > Y_n \Rightarrow Y_n < 10^{-24}$$

$Y_{He^4} = \frac{X_n}{2}$, $\quad Y_p = 1 - 2X_n \sim 1$

$$Y_{He^4} = \frac{X_n}{2}, \quad Y_p = 1-2X_n.$$

$$\Rightarrow \frac{\text{Helium}}{\text{Hydrogen}} = \frac{Y_{He^4}}{Y_p} = \frac{X_n}{2(1-2X_n)} \quad \text{by No. of nuclei}$$

$$\frac{\rho_{He}}{\rho_H} \rightarrow \text{mass density} = \frac{4\,Y_{He^4}}{Y_p} = \frac{2X_n}{1-2X_n} \sim \frac{.27}{.73}$$

At $T = 10^9$ K, $X_n = .133$

Remaining task: Show that equilibrium is established at 10^9 K.

$p + n \rightleftarrows d + \gamma$ — EM

$d + d \rightleftarrows He^3 + n$, $d + d \rightleftarrows H^3 + p$

$d + He^3 \rightleftarrows He^4 + p$, $d + H^3 \rightleftarrows He^4 + n$ — strong

Rate per neutron

$$\sigma_d v \, n_p \longrightarrow \sim n_B \sim \left(\frac{T}{T_0}\right)^3 \Omega_B \frac{3H_0^2}{8\pi G} \quad 1 - X_n = X_p$$

$\sigma_d v$: $4.55 \times 10^{-20}\, cm^3/s$

Ex. $\sigma_d v \, n_p \simeq 2.52 \times 10^4 \sec^{-1} \times \left(\frac{T}{10^{10} K}\right)^3 X_p \Omega_B h^2$

$$\frac{\dot{\lambda}}{\lambda} = \sqrt{\frac{8\pi G}{3}\rho} \longrightarrow \frac{1}{2} a_B T^4 \left(2 + \frac{7}{8} \times 6 \times \left(\frac{T_\nu}{T}\right)^4\right)$$

Ex. $\frac{\sigma_d v n_p}{\dot{\lambda}/\lambda} \sim 10^5 \frac{T}{10^{10}K}(1-X_n)\,\Omega_B h^2$

($\Omega_B \sim .04$, $h \sim .7$)

large for $T > 10^9$ K.

Conclusion: For $T \gtrsim 10^9$ K, p, n, d, γ system is in equilibrium.

Assume for now that the heavier nuclei are not produced appreciably at this time (to be seen later).

We use a formula similar to what we had earlier to determine the density of d.

$$\tilde{Y}_d = \frac{3}{2}\,\tilde{Y}_n \tilde{Y}_p \, 2\sqrt{2} \, \epsilon \, e^{B_d/T}$$

($3/2 = g_d$)

$$Y_p + \tilde{Y}_d = 1 - X_n, \quad \tilde{Y}_n + \tilde{Y}_d = X_n$$

Around 10^9 K, $\epsilon \sim 10^{-15}$, $e^{B_d/T} \sim 10^{11}$

$$\Rightarrow \tilde{Y}_d = \frac{3}{2}\,\tilde{Y}_n \tilde{Y}_p \, 2\sqrt{2} \, 10^{-15} \times 10^{11} \rightarrow \text{small.}$$

Suppose $T \sim .75 \times 10^9$ K. $e^{B_d/T} \simeq 10^{11 \times \frac{4}{3}} \sim 10^{15}$

$d + d \to H^3 + p \qquad \sigma v \simeq 1.8 \times 10^{-17}\ cm^3/s$

$d + d \to He^3 + p \qquad \sigma v \simeq 1.6 \times 10^{-17}\ cm^3/s$

Total rate for d, $\sigma v \tilde{n}_d$, $\tilde{n}_d = \tilde{Y}_d \times n_B$

Ex. $$\frac{\sigma v n_d}{\dot{\lambda}/\lambda} \sim 10 \left(\frac{T}{10^9 K}\right)^{5/2} \frac{\exp\left(25.87 \times \frac{10^9 K}{T}\right)}{\exp(25.87)}$$

$B_d = 25.87 \times .086\ MeV \sim 2\ MeV.$

Ratio ~ 10 at $T = 10^9 K$, but drops sharply for larger T and rises sharply at lower T.

A more detailed analysis shows that nucleosynthesis happens between 10^9 K and $.95 \times 10^9$ K.

Uncertainty in T: $\pm 10\%$

$t \propto T^2 \Rightarrow$ Uncertainly in $t \sim \pm 20\%$

t at 10^9 K $= 168$ sec.

Uncertainly in t . $(168 \pm 168 \times .2)$ sec.

880 sec mean life of n

$$\frac{dX_n}{dt} = -\frac{1}{880\,\text{sec}} X_n \Rightarrow \Delta X_n = \pm X_n \times \frac{168 \times .2}{880}$$

.04

$\Delta X_n = .04 X_n$

$Y_{He} = 2X_n \Rightarrow$ Error in determination of Y_{He} by this argument is about 4%.

What about the other nuclei?

Equilibrium values: at 10^9 K, since $Y_p \sim 1$, $Y_n \sim 10^{-48}$

$Y_{He^4} \sim Y_n^2 Y_p^2 \, 10^{97} \rightarrow 10^{-40}$

$Y_{He^3} \sim Y_n Y_p^2 \, 10^{9} \rightarrow 10^{-84}$

$Y_{H^3} \sim Y_n^2 Y_p \, 10^{12}$, $Y_d \sim Y_n Y_p \, 10^{-4} \sim 10^{-52}$

In actual practice the number densities of d, H^3, He^3 are larger.

If the only reaction involving d had been

$$d + d \rightleftarrows He^3 + n$$
$$H^3 + p$$

the the reaction rate per d would be

$$\sigma_d v \, n_d.$$

Relavant quantity is $\dfrac{\sigma_d v n_d}{\dot{a}/a}$

The system goes out of equilibrium when $\sigma_d v n_d / |\dot{a}/a|$ falls below 1. $\Rightarrow$ determines n_d.

This process leaves us with some n_d, n_{H^3} and n_{He^3} besides p, He^4 which are the main components.

H^3 decays to $He^3 + \cdots$ (lifetime ~ 12 years)

What we observe ~~today~~ as He^3 abundance is actually the sum of H^3 and He^3 abundance during nucleosynthesis.

These numbers do not include production of these elements in stars which happen much later.