Axionic dark matter:

Axion: Low mass scalar field with "approximate" shift symmetry $\chi \to \chi + \text{constant}$.

$$S = \int d^4x \sqrt{-\det g} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} m^2 \chi^2 \right) + \cdots$$

m is small.

Inflation: Spatial inhomogenities in χ get smoothened out $\Rightarrow \vec{\nabla} \chi \cong 0$.

Ex. Eqs. of motion of χ in FRW metric:

$$\ddot{\chi} + 3H\dot{\chi} + m^2\chi = 0 \qquad H = \frac{\dot{\lambda}(t)}{\lambda(t)}$$

(Assumed $\vec{\nabla}^2\chi \simeq 0$)

$\rightarrow$ damped harmonic oscillator.

Early universe: H is large. $H \gg m$.

$\rightarrow$ ignore the $m^2\chi$ term.

Soln. χ = constant $\rightarrow$ could depend on the region of the universe we are in.

As the universe expands $H(t)$ falls.

When $H \ll m$, then we can ignore the $3H\dot{x}$ term.

$$\ddot{x} + m^2 x = 0 \Rightarrow x = x_0 \cos(mt + A)$$

constant.

Ex. $\rho = \frac{1}{2}\dot{x}^2 + \frac{1}{2}m^2 x^2, \quad p = \frac{1}{2}\dot{x}^2 - \frac{1}{2}m^2 x^2$

$$\langle p \rangle = \frac{1}{2}\langle \dot{x}^2 \rangle - \frac{1}{2}m^2 \langle x^2 \rangle = 0.$$

over a period

$$\langle \rho \rangle = \frac{1}{2}\langle \dot{x}^2 \rangle + \frac{1}{2}m^2 \langle x^2 \rangle = \frac{1}{2}m^2 x_0^2 \neq 0.$$

We have $T_{\mu\nu}$ for which $p = 0$, $\rho \neq 0$

$\rightarrow$ acts as non-relativistic matter.

— Axionic dark matter.

Particle interpretation:

In QFT, in momentum space we have a harmonic oscillator for each momentum.

$$\omega = \sqrt{\vec{k}^2 + m^2}$$

classical oscillation of a harmonic oscillator.

$\rightarrow$ coherent state (superposition of highly excited states)

In QFT, highly excited state of h.o.

→ large number of particles.

Since χ is uniform ($|\vec{\nabla}\chi| \simeq 0$), only

$\vec{k}=0$ modes are excited.

→ These have nearly 0 momentum.

Axionic dark matter: Large no. of light particles of almost 0 momentum.

responsible for why it represents non-relativistic matter.

$\chi \bar{\psi} \cancel{\psi}$ $\qquad \partial_\mu \chi \; (\cdots) \to k_\mu$

The axion field begins to roll when

$H \sim m$. $\qquad m^{-1}$. $\quad H = \frac{\dot{\lambda}}{\lambda} = \sqrt{\frac{8\pi G}{3}\frac{1}{2} a_B T^4 N}$.

$$\pi^2$$

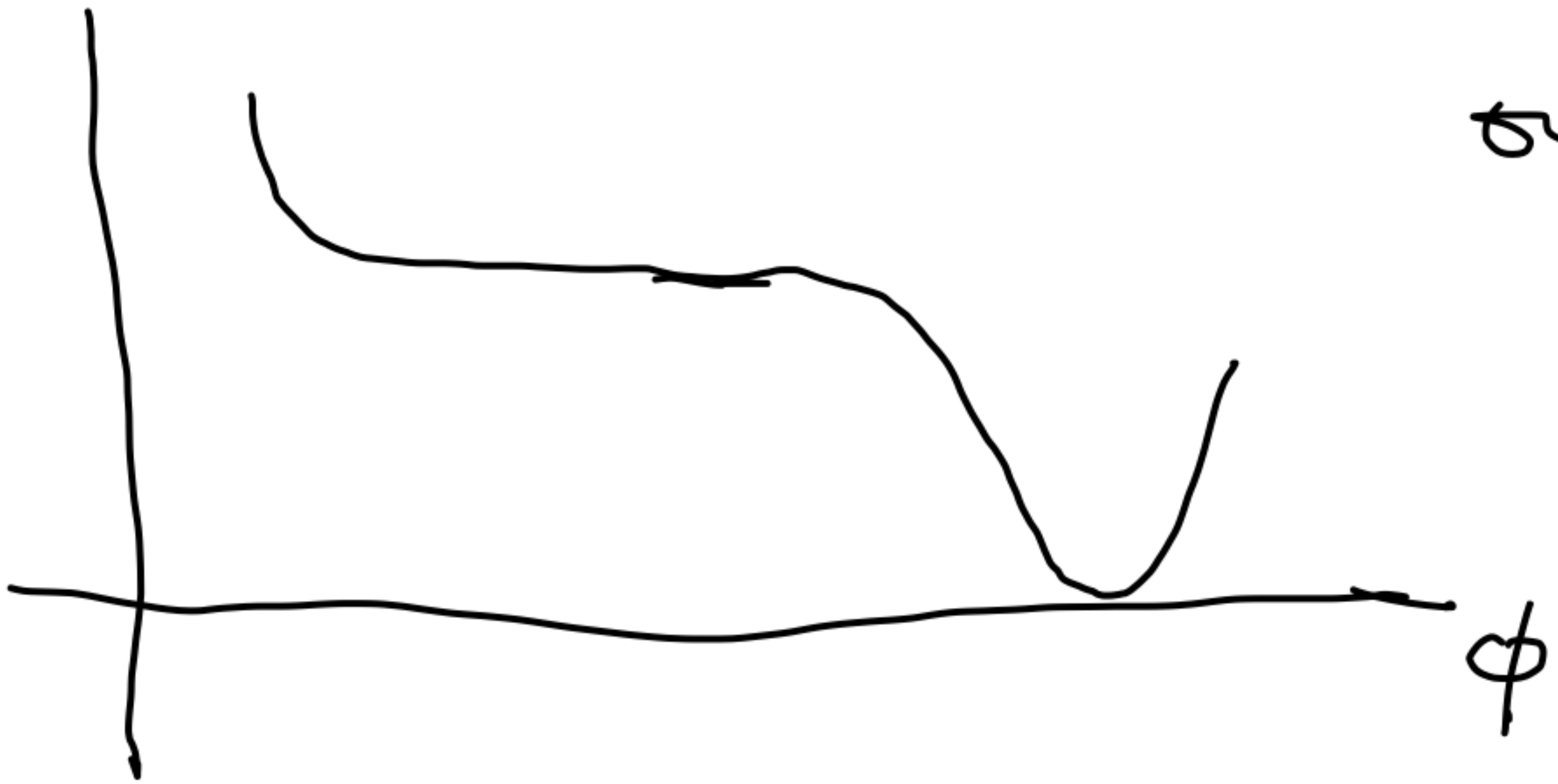
$$=$$

$$m = T^2 \sqrt{\frac{8\pi G}{3}\frac{1}{2} a_B N} = T^2 / M_{pl} \times \text{const.}$$

$$T \sim \sqrt{m M_{pl}} \gg m$$

$\boxed{\text{arXiv: 1610.08297}}$

Inflation



$V(\phi) \simeq$ constant over some range of ϕ.

$\rightarrow$ the universe expands exponentially

$\rightsquigarrow$ solves some puzzles like horizon problem and flatness problem.

Slow roll inflation:

$$S = \int \sqrt{-\det g} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right)$$

Spatial inhomogeneities in ϕ get smoothened out quickly as inflation progresses.

$\rightsquigarrow$ We'll set $\vec{\nabla}\phi = 0$ in subsequent analysis.

Eq. of motion: $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p = \frac{1}{2}\dot{\phi}^2 - V(\phi).$$

$$H(t) = \frac{\dot{\lambda}}{\lambda} = \sqrt{\frac{8\pi G}{3}\rho} = \sqrt{\frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)}$$

Slow roll conditions

Time required for λ to double is $\sim H^{-1}$

For nearly exponential expansion, H should not change much during this time.

$\dot{H} H^{-1} < H$ $\quad \epsilon = -\dot{H}/H^2 \rightarrow$ first slow roll parameter.

2nd slow roll parameter

$$\eta = -\frac{\ddot{\phi}}{\dot{\phi}} H^{-1}$$

Small $|\eta| \Rightarrow |\ddot{\phi} H^{-1}| << |\dot{\phi}|$

$\Rightarrow$ Change in $\dot{\phi}$ over time H^{-1} should be small compared to $\dot{\phi}$.

$\rightarrow$ Not necessary, but is a useful condition to impose for simplifying the analysis.

We'll try to translate these into conditions on $V(\phi)$.

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

$$\frac{d}{dt}: \quad 2H\dot{H} = \frac{8\pi G}{3}\left(\ddot{\phi}\dot{\phi} + V'(\phi)\dot{\phi}\right) = -8\pi G\dot{\phi}^2 H$$

$$\epsilon = -\frac{\dot{H}}{H^2} = 4\pi G\frac{\dot{\phi}^2}{H^2} = \frac{3\dot{\phi}^2}{\dot{\phi}^2 + 2V(\phi)}$$

Small $\epsilon \Rightarrow \boxed{\dot{\phi}^2 \ll V(\phi)}$

$$\eta = -\frac{\ddot{\phi}}{\dot{\phi} H} = \frac{3H\dot{\phi} + V'(\phi)}{\dot{\phi} H}$$

$$\Rightarrow \boxed{3H\dot{\phi} + V'(\phi) \text{ should be small.}}$$

$$\dot{\phi} \simeq -\frac{V'(\phi)}{3H}$$

$$\dot{\phi}^2 \ll V(\phi) \Rightarrow \{V'(\phi)\}^2 \ll 9H^2 V(\phi)$$

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \simeq \frac{8\pi G}{3} V(\phi)$$

$$\boxed{(V'(\phi))^2 \ll 24\pi G\, V(\phi)^2}$$

$$\boxed{|V''| \ll 8\pi G V(\phi)}$$

$$\dot{\phi} \simeq -\frac{V'(\phi)}{3H}$$

$$\ddot{\phi} \simeq -\frac{V''}{3H}\dot{\phi} + \frac{V'(\phi)}{3H^2}\dot{H}$$

$$\eta = -\frac{\ddot{\phi}}{\dot{\phi} H} = \frac{V''}{3H^2} - \frac{V'\dot{H}}{3H^3\dot{\phi}}$$

$$\frac{\dot{H}}{H^2} \times \frac{V'}{3H\dot{\phi}}, \qquad \frac{V'}{3H\dot{\phi}} \simeq -1$$

small

$$|\eta| \ll 1$$

$$\Rightarrow |V''| \ll 3H^2 = 8\pi G V(\phi)$$

$$\boxed{\begin{aligned} V'(\phi)^2 &\ll 24\pi G\, V(\phi)^2 \\ |V''(\phi)| &\ll 8\pi G\, V(\phi) \end{aligned}} \rightarrow \text{slow roll condition}$$

Other conditions:

1. If inflation lasts from t_i to t_f, then $a(t_f)/a(t_i)$ should be sufficiently large so as to solve the horizon and the flatness problem.

If inflation happened at 10^{16} GeV,

then we need $\ln \frac{\lambda(t_f)}{\lambda(t_i)} > 60$.

During inflation: | Slow roll:

$$3H\dot{\phi} \simeq -V'(\phi)$$

$$\frac{\dot{\lambda}}{\lambda} = H$$

$$\phi_i = \phi(t_i), \quad \phi_f = \phi(t_f)$$

$$= H^2\left(-\frac{3\dot{\phi}}{V'(\phi)}\right).$$

$$\frac{d}{dt}\ln\lambda = -\frac{1}{V'(\phi)} \frac{8\pi G}{\cancel{3}} V(\phi) \cdot \cancel{3}\frac{d\phi}{dt}.$$

$$\Rightarrow \ln\frac{\lambda_f}{\lambda_i} = -\int_{\phi_i}^{\phi_f} \frac{8\pi G}{V'(\phi)} V(\phi)\, d\phi$$

$$\boxed{-8\pi G \int_{\phi_i}^{\phi_f} \frac{V(\phi)}{V'(\phi)}\, d\phi > 60}$$

Final condition:

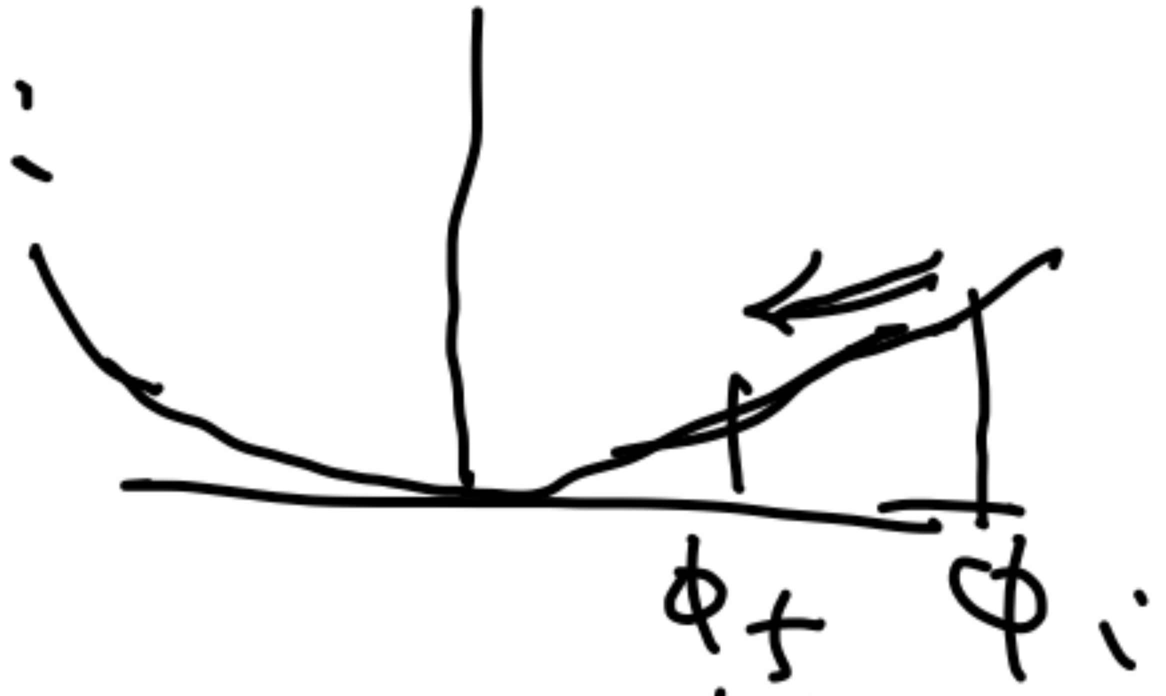
Quantum gravity effects should be small:

$$\Rightarrow \boxed{V(\phi) \ll M_{Pl}^4}$$

$\Rightarrow$ space-time curvature during inflation is small in Planck units.

Ex. Check that all these four conditions can be satisfied by a simple class of potentials for appropriate choice of ϕ_i, ϕ_f:

$$V(\phi) = g\,\phi^{\alpha} \qquad \alpha \geq 2.$$



g needs to be sufficiently small.

- Includes a free scalar field with low mass.