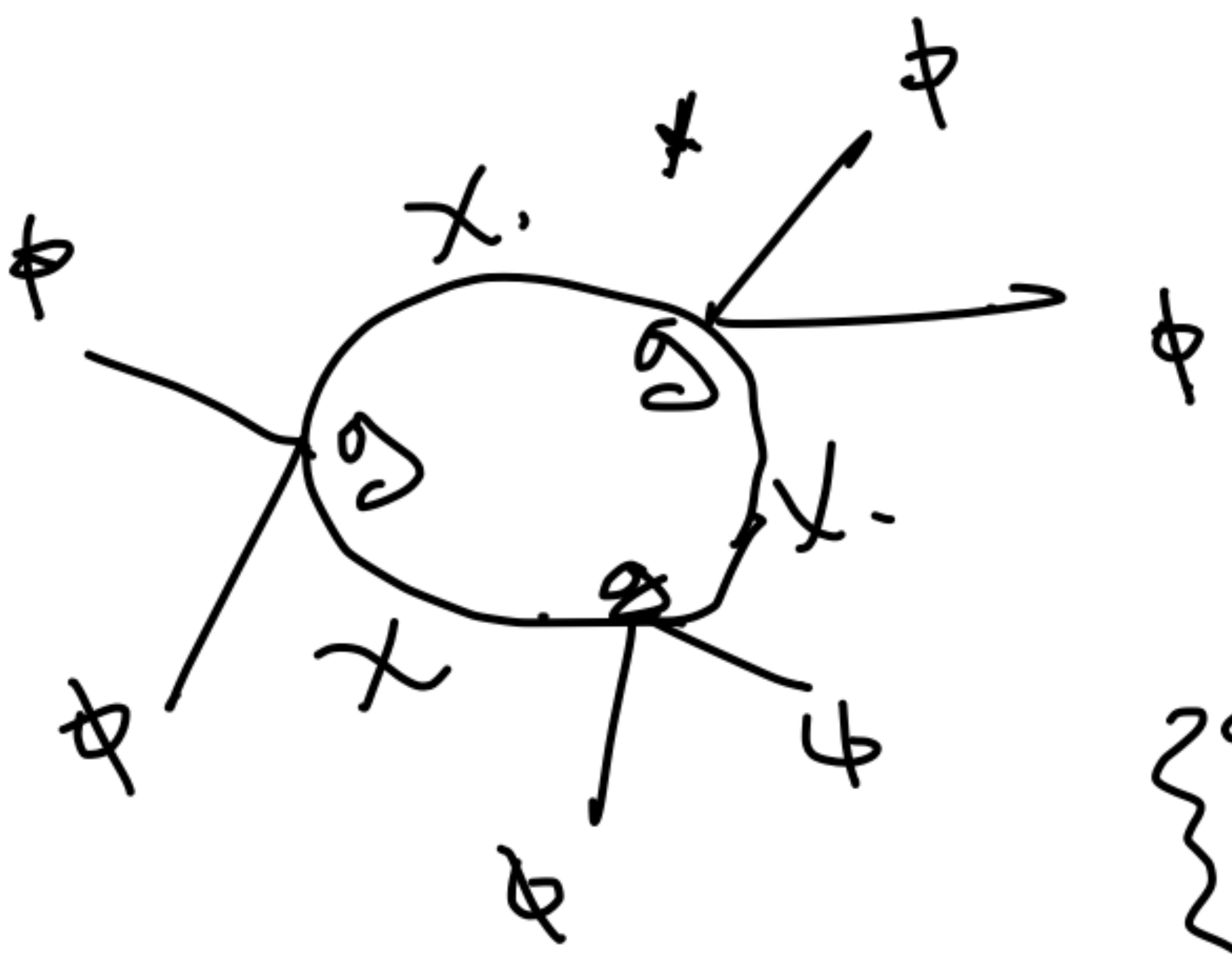
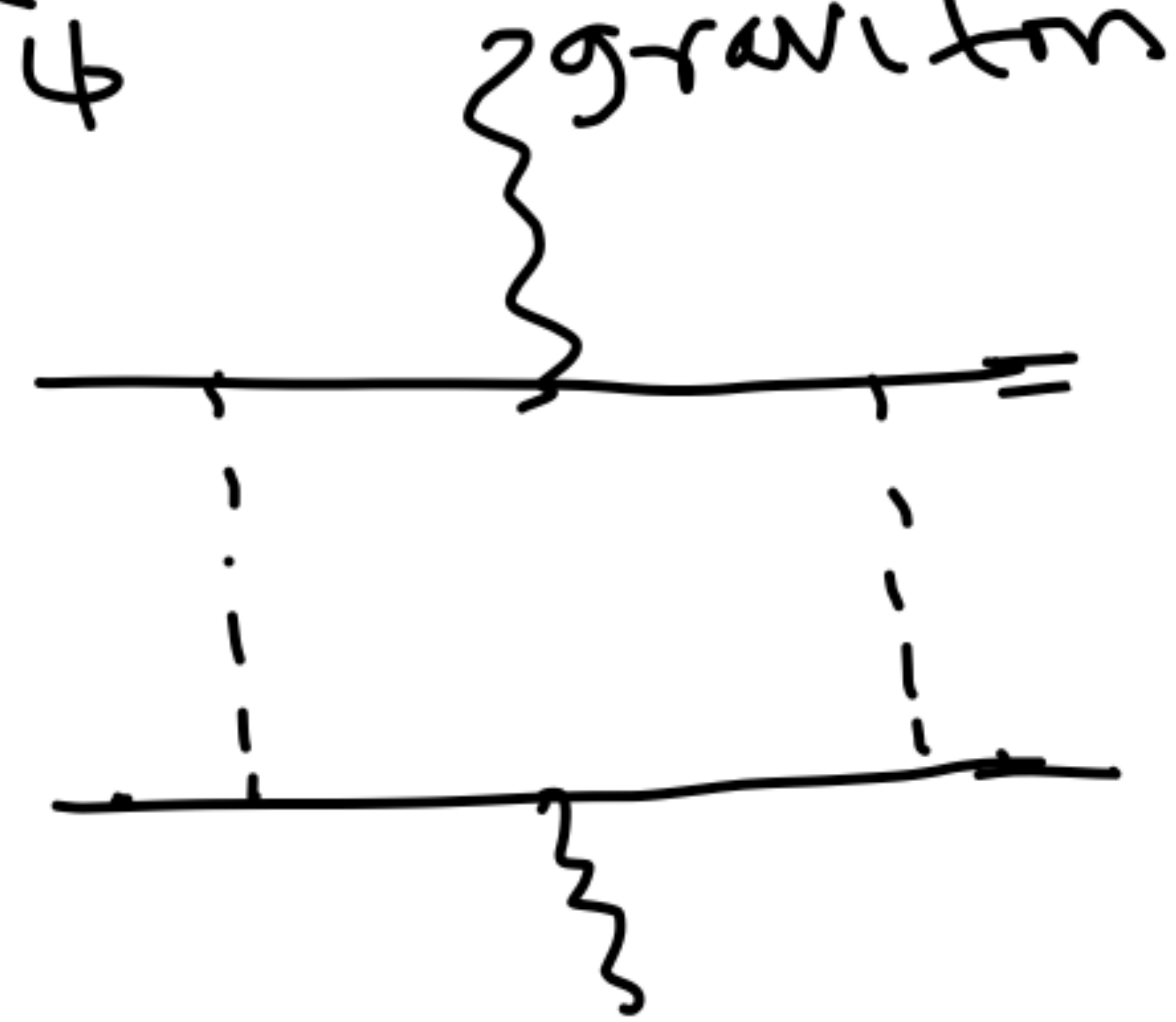


$$\frac{\mathcal{H}^2}{\omega^2}$$

$$\frac{\phi^6}{M_{pl}^2}$$

$$\frac{\phi^6}{m^2}$$

$$\sim \frac{\phi^6}{M_\chi^2}$$



$$= \;\Rightarrow\; \partial_\mu \to D_\mu$$

$$g\, R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \phi^2$$

So far we studied perturbation during inflation

Quantum fluctuation $\rightarrow$ classical fluctuation after the horizon exit.

Next step: Evolve them forward in time using Einstein's eq. (classical)

Inflation: Provides the initial condition at the horizon exit.

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

We shall use general form and not just the form during inflation.

Decompose $T_{\mu\nu}$ into background + fluctuation

Define $T_\mu{}^\nu = g_{\mu\rho} T^{\rho\nu}$ — full $g_{\mu\nu}$, $T^{\rho\nu}$.

$$T_0{}^0 = \underbrace{\bar{T}_0{}^0}_{= -\bar{\rho}} + \underbrace{\tilde{T}_0{}^0}_{= -\tilde{\rho}}, \quad T_i{}^0 = q_i, \quad \bar{T}_i{}^0 = 0, \quad T_i{}^j = \underbrace{\bar{T}_i{}^j}_{= \bar{p}\,\delta_{ij}} + \underbrace{\tilde{T}_i{}^j}_{= \tilde{p}\,\delta_{ij} + \Sigma_{ij}}, \quad \eta^{ij}\Sigma_{ij} = 0$$

$$T_0{}^0 = -(\bar{\rho}+\tilde{\rho}), \quad T_i{}^0 = q_i, \quad T_i{}^j = (\bar{p}+\tilde{p})\delta_{ij} + \Sigma_{ij}$$

$$\Sigma_{ij}\delta_{ij} = 0$$

$T_0{}^i$ is not independent.

Proof: $T_{\mu\nu} \equiv T_{\nu\mu}$

$$\Rightarrow g_{\nu\sigma} T_\mu{}^\sigma = g_{\mu\sigma} T_\nu{}^\sigma \Rightarrow (\bar{g}_{\nu\sigma}+\tilde{g}_{\nu\sigma})(\bar{T}_\mu{}^\sigma + \tilde{T}_\mu{}^\sigma) = \mu \leftrightarrow \nu$$

Expand this to first order in perturbation:

$\mu = i, \ \nu = 0$

$$\bar{g}_{00}\tilde{T}_i{}^0 + \tilde{g}_{0j}\bar{T}_i{}^j = \bar{g}_{ik}\tilde{T}_0{}^k + \tilde{g}_{i0}\bar{T}_0{}^0$$

$$\bar{g}_{00}\tilde{T}_i{}^0 + \tilde{g}_{0k}\bar{T}_i{}^k = \bar{g}_{ik}\tilde{T}_0{}^k + \tilde{g}_{i0}\bar{T}_0{}^0$$

$$ds^2 = \cdots + 2\lambda(t)\, B_i\, dx^i\, dt.$$

$$\Rightarrow \quad \tilde{g}_{0k} = \lambda B_k.$$

$$-\ q_i + \lambda B_k \bar{p}\, \delta_i{}^k = \lambda^2 \delta_{ik}\tilde{T}_0{}^k + \lambda B_i(-\bar{\rho})$$

$$\Rightarrow \quad \boxed{\tilde{T}_0{}^i = \lambda^{-1} B_i(\bar{p}+\bar{\rho}) - \lambda^{-2} q_i}$$

Ex. $T_{ij} = T_{ij} \quad \Rightarrow \quad \Sigma_{ij} = \Sigma_{ji}$

$\tilde{\rho}, \tilde{p}, q_i, \Sigma_{ij} \Rightarrow$ Perturbations in $T_{\mu\nu}$

$$q_i = \partial_i \tilde{q} + Q_i \,, \quad \partial_i Q_i = 0.$$

$$\Sigma_{ij} = \partial_i \Sigma_j + \partial_j \Sigma_i + \left(\partial_i \partial_j \tilde{\Sigma} - \frac{1}{3}\delta_{ij} \partial^2 \tilde{\Sigma}\right) + \Sigma^T_{ij}$$

$$\partial_i \Sigma_i = 0 \,, \quad \partial_i \Sigma^T_{ij} = 0 \,, \quad \delta_{ij} \Sigma^T_{ij} = 0$$

Scalar perturbations of $T_{\mu\nu}$: $\tilde{p}, \tilde{\rho}, \tilde{q}, \tilde{\Sigma}$

Recall that metric fluctuation has 4 scalar modes : Φ, Ψ, E, B | Two gauge tr. $\xi^0, \xi^i = \partial_i \xi$

Gauge trs: (coordinate trs)

$$x^\mu \to y^\mu = x^\mu - \xi^\mu(\vec{x},t), \quad x^\mu = y^\mu + \xi^\mu(\vec{y},t)$$

$$T_\mu{}^\nu(x) \to T_\rho{}^\sigma(y) \frac{\partial y^\rho}{\partial x^\mu} \frac{\partial x^\nu}{\partial y^\sigma}$$

$$= \left(T_\rho{}^\sigma(x) - \xi^\tau \partial_\tau T_\rho{}^\sigma(x)\right)\left(\delta_\mu{}^\rho - \partial_\mu \xi^\rho\right)\left(\delta_\sigma{}^\nu + \partial_\sigma \xi^\nu\right)$$

$T_\rho{}^\sigma(x) = \bar{T}_\rho{}^\sigma(x)$

$$= \bar{T}_\mu{}^\nu - \xi^\tau \partial_\tau \bar{T}_\mu{}^\nu - \bar{T}_\rho{}^\nu \partial_\mu \xi^\rho + \bar{T}_\mu{}^\sigma \partial_\sigma \xi^\nu$$

$$\delta \tilde{T}_\mu{}^\nu = -\xi^\tau \partial_\tau \bar{T}_\mu{}^\nu - \bar{T}_\rho{}^\nu \partial_\mu \xi^\rho + \bar{T}_\mu{}^\sigma \partial_\sigma \xi^\nu$$

$$\delta \tilde{T}_0{}^0 = -\xi^0 \partial_t \underbrace{\bar{T}_0{}^0}_{=-\bar{\rho}} - \bar{T}_0{}^0 \cancel{\partial_0 \xi^0} + \bar{T}_0{}^0 \cancel{\partial_0 \xi^0}$$

$-\delta\tilde{\rho}$ #

$$\boxed{\delta\tilde{\rho} = -\xi^0 \partial_t \bar{\rho}}$$

$$\delta \tilde{T}_i{}^0 = 0 - \bar{T}_0{}^0 \partial_i \xi^0 + \bar{T}_i{}^k \partial_k \xi^0 = (\bar{\rho} + \bar{p})\partial_i \xi^0$$

$= q_i$

$$\Rightarrow \boxed{\delta q_i = (\bar{\rho} + \bar{p}) \partial_i \xi^0} \qquad \boxed{\begin{array}{l} \delta\tilde{p} = -\xi^0 \partial_t \tilde{p} \\ \delta\Sigma_{ij} = 0 \end{array}}$$

$$\delta \tilde{T}_i{}^j = -\xi^0 \partial_0 \bar{p}\, \delta_i^j - \bar{T}_k{}^j \cancel{\partial_i \xi^k} + \bar{T}_i{}^k \cancel{\partial_k \xi^j}$$

$\bar{T}_k{}^j \partial_i \xi^k = -\bar{p}\, \partial_i \xi^j$, $\quad \bar{T}_i{}^k \partial_k \xi^j = \bar{p}\, \partial_i \xi^j$

$\delta \tilde{T}_i{}^j = \delta\tilde{p}\, \delta_{ij} + \delta\Sigma_{ij}$

$$\boxed{\delta\tilde{\rho} = -\xi^0\partial_t\bar{\rho}\,,\quad \delta\tilde{p} = -\xi^0\partial_t\bar{p}}\,,\quad \delta q_i = (\bar{\rho}+\bar{p})\partial_i\xi^0$$

$$\delta\Sigma_{ij} = 0 \qquad q_i = \partial_i\tilde{q} + Q_i \qquad \xi^i = \partial_i\xi$$

$$\Rightarrow\ \boxed{\delta\tilde{q} = (\bar{\rho}+\bar{p})\xi^0}\,,\quad \delta Q_i = 0\,,\quad \boxed{\delta\tilde{\Sigma} = 0} \quad +\xi^i$$

Recall trs. of metric scalar d.o.f Φ, Ψ, E, B

$$\boxed{\delta\Phi = -\partial_t\xi^0\,,\quad \delta B = \lambda^{-1}\xi^0 - \lambda\partial_t\xi\,,\quad \delta E = -\xi\,,\quad \delta\Psi = \mathcal{H}\xi^0}$$

$\boxed{\Phi_B, \Psi_B}$ gauge invariant

$$\boxed{-\zeta = \Psi + \frac{\mathcal{H}}{\partial_t\bar{\rho}}\tilde{\rho}} \qquad \boxed{\mathcal{R} = \Psi - \frac{\mathcal{H}}{\bar{\rho}+\bar{p}}\tilde{q}} \qquad \boxed{\tilde{\Sigma}} \qquad \boxed{p_e = \tilde{p} - \frac{\partial_t\bar{p}}{\partial_t\bar{\rho}}\tilde{\rho}}$$

6 scalar variables: $\Phi_B, \Psi_B, R, S, \rho_e, \tilde{\Sigma}$.

Einsteins eq: $K_{\mu\nu} = 0$, $K_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - 8\pi G T_{\mu\nu}$

How many scalar equations

$K_{00} \to$ Scalar.

$K_{i0} = \partial_i K + L_i$, $\partial_i L_i = 0$

$K \to$ scalar.

$K_{ij} = \delta_{ij} M + \partial_i \partial_j P + (\partial_i V_j + \partial_j V_i) + V^T_{ij}$

$\partial_i V_i = 0$, $\partial_i V^T_{ij} = 0$

$V^T_{ii} = 0$

$$\boxed{\begin{array}{l} \text{4 scalar eqs.} \\ K_{00} = 0,\ K = 0 \\ M = 0,\ P = 0 \end{array}}$$

eq. of state $\rightarrow$ 1 eq. $p = f(\rho)$

The other equation: Assume perfect fluid for of $T_{\mu\nu}$.

$T_{\mu\nu} = p \cdot g_{\mu\nu} + (p+\rho) U_\mu U_\nu$ for some choice of p, ρ, U_μ, with $g_{\mu\nu} U^\mu U^\nu = -1$

Physically this means that in the choice of local inertial frame in which $U^\mu = (1,0,0,0)$ we have $T_{00} = \rho$, $T_{ij} = p \delta_{ij}$, $T_{0i} = 0$.