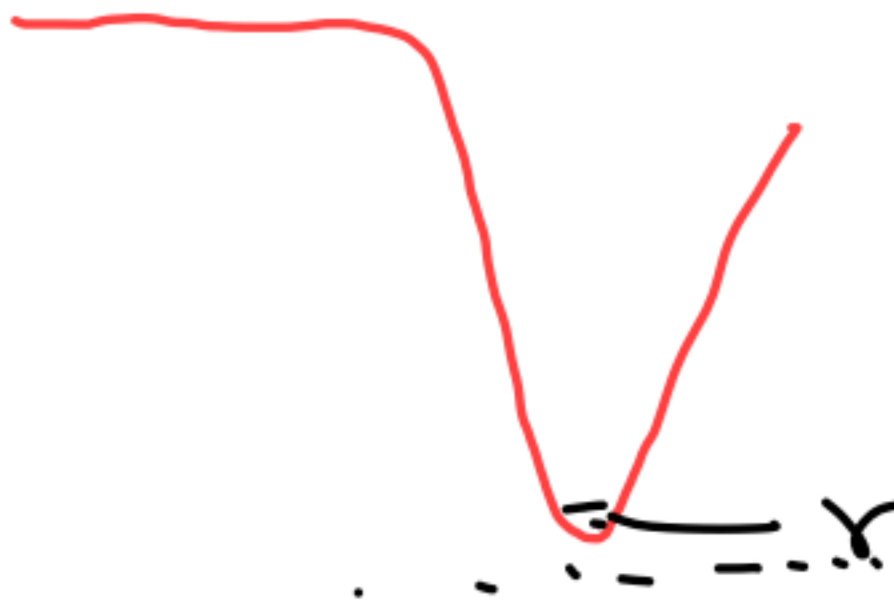
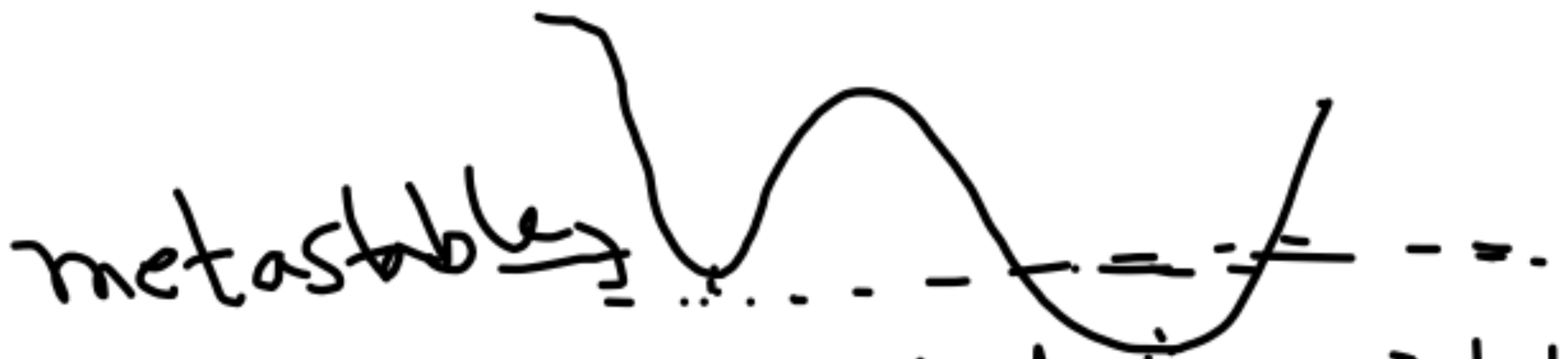
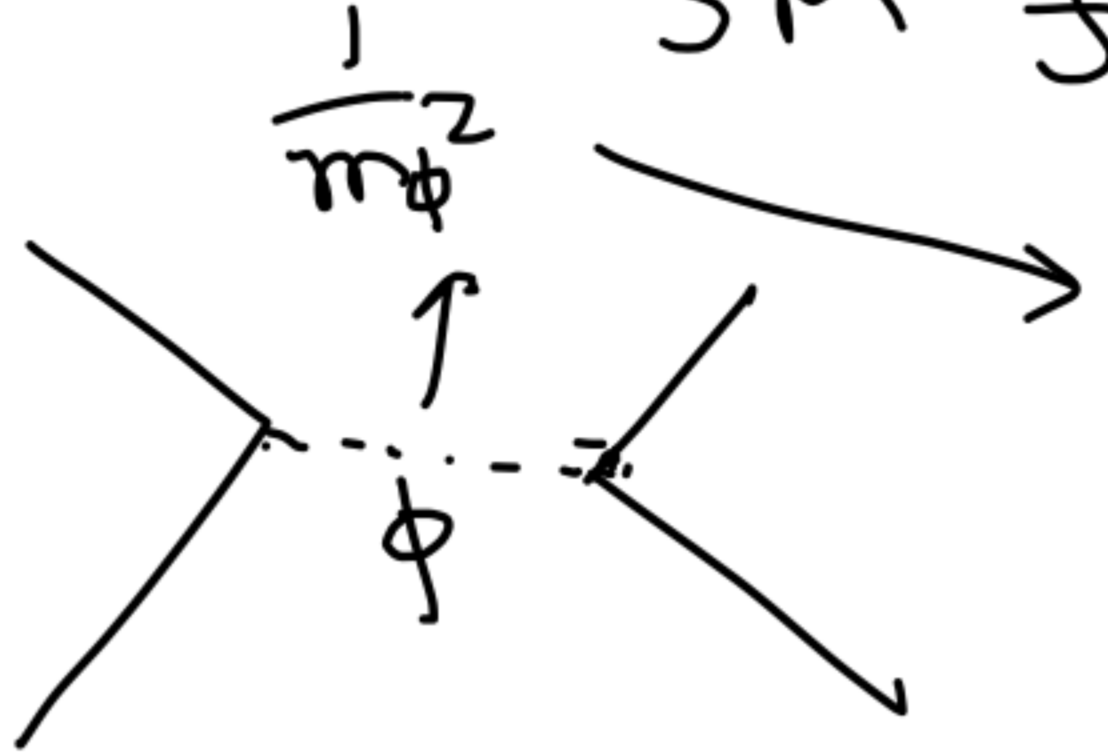
<u>String Landscape.</u>





massive field with interaction with SM fields.

$\phi\bar{\Psi}\Psi$



not observable today.

$\Lambda = V(\phi_1, \dots \phi_n)$ $\quad S = -\int V(\phi)\sqrt{-\det g}\, d^4x$

$V(\phi)\Big|_{\phi_{min}}$ always acts a cosmological const

Ex. Suppose that for $\lambda < \lambda_I$, the energy density of the universe was stored in a fluid of eq. of state $p = w\rho$

Show that we can solve the horizon problem if $w < -\frac{1}{3}$.

Taking λ_* (beginning of inflation) to be sufficiently small we can get the reqd. contribution to r_H.

Flatness problem:

$$\left(\frac{\dot{\lambda}}{\lambda}\right)^2 = -\frac{k}{a_0^2\lambda^2} + \frac{8\pi G}{3}\rho \rightarrow \left(\frac{\rho_{m0}}{\lambda^3} + \frac{\rho_{r0}}{\lambda^4} + \rho_{\Lambda 0}\right)$$

$$\frac{k}{a_0^2\lambda^2} \ll \frac{8\pi G}{3}\rho$$

$$\left.\frac{k}{a_0^2\lambda^2} \Big/ \left(\frac{8\pi G}{3}\rho_r\right)\right|_{\substack{\text{today} \\ \lambda=1}} \ll \frac{\rho}{\rho_r} \sim 10^4$$

$$\frac{k}{a_0^2\lambda^2} \Big/ \left(\frac{8\pi G}{3}\underbrace{\rho_r}_{=\frac{\rho_{r0}}{\lambda^2}}\right) = \left(\frac{k}{a_0^2} \Big/ \frac{8\pi G}{3}\rho_{r0}\right)\lambda^2 \ll 10^4\lambda^2$$

$$\left|\frac{k}{a_0^2 \lambda^2}\right| \Big/ \left|\frac{8\pi G}{3}\rho_r\right| << 10^4 \lambda^2$$

$\underline{k_B T \sim 10^{+16}\ \text{GeV}.}$ $\qquad \lambda \sim \dfrac{10^{-4}\,\text{ev}}{10^{16} \times 10^{9}} \sim 10^{-29}$

$$\frac{k}{a_0^2 \lambda^2} \Big/ \left(\frac{8\pi G}{3}\rho_r\right) << 10^{-58+4} \sim 10^{-54}$$

$$\left(\frac{\dot{\lambda}}{\lambda}\right)^2 = -\frac{k}{a_0^2 \lambda^2} + \frac{8\pi G}{3}\rho_r$$

$\ni$ curvature was "very small"

$\downarrow$

much smaller than the second term.

$\Rightarrow$ Flatness problem.

$\lambda(t) = \tilde{c}\, t \qquad ds^2 = -dt^2 + \tilde{c}^2 t^2 \left(\frac{dr^2}{1+r^2} + r^2 d\Omega^2 \right|$

Solution of the flatness problem via inflation:

Suppose that the beginning of inflation $\lambda = \lambda_*$, the two terms were of the same order.

$$\frac{k}{a_0^2 \lambda_*^2} \sim \frac{8\pi G}{3} \rho$$

During inflation, ρ remains constant.

$\frac{k}{a_0^2 \lambda^2}$ changes. End of inflation we get $\frac{k}{a_0^2 \lambda_f^2}$

End of inflation:

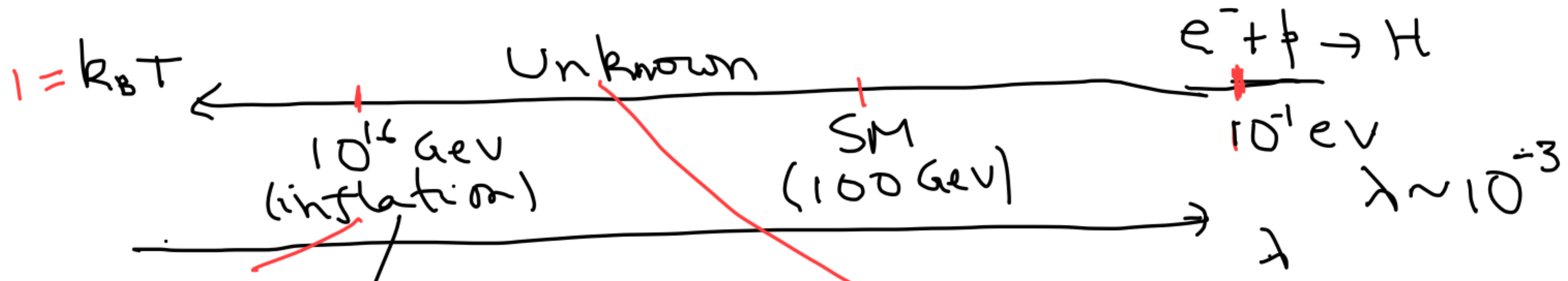
$$\frac{k}{a_0^2 \lambda_I^2} = \frac{k}{a_0^2 \lambda_*^2} \left(\frac{\lambda_*}{\lambda_I}\right)^2 \sim \left(\frac{8\pi G}{3}\rho_I\right) \times \left(\frac{\lambda_*}{\lambda_I}\right)^2$$

$$\frac{\lambda_*}{\lambda_I} \sim e^{-N}. \qquad N \text{ large} > 60-70$$

\# of e-folding.

$\Rightarrow$ Inflation provides an explanation of the flatness.

Broad picture



erases everything

Speculative
10^{16} is just a guess
$> 10^2$ GeV is all we know.

Q. Is it possible to make any prediction without knowing this part?

Ans. It is possible because thermal equilibrium erases most past information except a few details.

One example of the data that is not erased is the excess of matter over anti-matter.

SM cannot produce this and cannot erase this.

Inflation erases this.

This excess must have been produced between inflation and $k_B T \sim 100\,\text{GeV}$.

$\rightarrow$ will be studied later.

For now we have ~~take~~ this as an initial condition at $100\,\text{GeV}$.

Main tool in our study: Statistical mechanics.

Review some of the main results in statistical mechanics.

Consider a system of N particles in a volume V, carrying total energy between E & $E+\delta E$ (microcanonical ensemble)

$\Omega(E,N,V)\,\delta E$: # of quantum states of the system

$$S(E,N,V) = \ln \Omega(E,N,V)$$

→ entropy

In the usual statistical system, S is an extensive quantity.

$$S(\lambda E, \lambda N, \lambda V) = \lambda S(E,N,V)$$

Some defn:

$$s = \frac{S}{V} \Rightarrow \boxed{s = f\left(\frac{E}{V}, \frac{N}{V}\right).} \Rightarrow s = f(\rho, n)$$

Set $\lambda = \frac{1}{V}$ $\Rightarrow$ $S\left(\frac{E}{V}, \frac{N}{V}, 1\right) = \underbrace{\frac{1}{V} S(E,N,V)}_{= s}$

$$\rho = \frac{E}{V}, \quad n = \frac{N}{V}$$

$$\frac{1}{T} = \beta = \frac{\partial S}{\partial E}, \quad \mu = -T\frac{\partial S}{\partial N},$$

$$p = T\frac{\partial S}{\partial V}$$

$$S = Vs = V f(e,n), \quad N = Vn, \quad E = Ve,$$

$$\frac{1}{T} = \frac{\partial S}{\partial E}, \quad -\frac{\mu}{T} = \frac{\partial S}{\partial N}, \quad \frac{p}{T} = \frac{\partial S}{\partial V}$$

Consider arbitrary infinitesimal variation of V, E and N.

$$\delta S = \frac{\partial S}{\partial V}\delta V + \frac{\partial S}{\partial E}\delta E + \frac{\partial S}{\partial N}\delta N = \frac{p}{T}\delta V + \frac{1}{T}\delta E - \frac{\mu}{T}\delta N$$

$$\delta E = \delta V e + V\delta e, \quad \delta N = \delta V n + V\delta n, \quad \delta S = \delta V s + V\delta s.$$

substitute & divide the eq. by V.

Ex.
$$\underset{\frac{\partial s}{\partial e}\delta e + \frac{\partial s}{\partial n}\delta n}{\delta s \overset{\shortparallel}{}} = \frac{\delta V}{V}\Big\{\underbrace{-s + \frac{1}{T}(e - \mu n + p)}_{0 =}\Big\} + \frac{1}{T}\delta e - \frac{\mu}{T}\delta n$$

Final eqs:

$$\frac{\partial s}{\partial e} = \frac{1}{T}$$ $\rightarrow$ gives T in terms of e, n (2)

$$\frac{\partial s}{\partial n} = -\frac{\mu}{T}$$ $\rightarrow$ μ in terms of e, n (3)

$$s = \frac{p+e-\mu n}{T}$$ $\rightarrow$ p in terms of e, n. (4)

$s = f(e, n)$ (1)

$\hookrightarrow$ determined from counting of states.

We assumed that the no. of particles is conserved.

If N is not conserved, then we need to sum over N while doing statistical average.

$$\langle O \rangle = \frac{\sum_N \Omega(E,N,V)\, O}{\sum_N \Omega(E,N,V)}$$

In the thermodynamic limit, this is dominated by the value of N at which Ω is maximized.

$S = \ln \Omega$

Ω is maximized at $\frac{\partial S}{\partial N} = 0 \Rightarrow \mu = 0$.

We can still use the results of the microcanonical ensemble, except that we set $\mu = 0$ & determine n by solving $\mu = 0$ eq.

Intensive variables.

Instead of two independent variables ρ, n we have only one independent variable ρ.
n is determined by demanding $\mu = 0$.

The actual calculation of Ω is difficult for quantum statistics.

$\rightarrow$ Use grand canonical ensemble.

$$\mathcal{Q}(\mu, \beta, V) = \underbrace{\sum_{\alpha}}_{\text{sum over all quantum states}} e^{-\beta \underbrace{E_\alpha}_{\text{energy}} + \mu\beta \underbrace{N_\alpha}_{\text{\# of particles in the states}}}$$

$$= \sum_{N'} \int dE' \, e^{-\beta E' + \mu\beta N'} \, \underbrace{\Omega(E', N', V)}_{= e^{S(E', N', V)}}$$

$\neq$ Maximize the exponent w.r.t. E', N'

$$\frac{\partial}{\partial E'}\left(-\beta E' + S + \mu\beta N'\right) = 0 \quad \Rightarrow \quad \boxed{\beta = \frac{\partial S}{\partial E}}$$

$$\frac{\partial}{\partial N'}\left(-\beta E' + S + \mu\beta N'\right) = 0 \quad \Rightarrow \quad \boxed{\beta\mu = \frac{\partial S}{\partial N}}$$

Same as in microcanonical case.

E, N : the values of E', N' at the maximum.

$$\ln Q = -\beta E + \beta\mu N + S(E,V,N)$$

$$\frac{1}{V}\ln Q = -\beta e + \beta\mu n + s$$

$$= \beta p \left(= \frac{p}{T}\right).$$

$$\Rightarrow s = \frac{e - \mu n + p}{T}$$

Knowing Q gives $p(\beta,\mu)$..

$$\frac{\partial}{\partial\beta}\ln Q = -E\,, \quad \frac{\partial}{\partial\mu}\ln Q = \beta \cdot N$$

$$\Rightarrow \quad \rho = \frac{E}{V} = -\frac{1}{V}\frac{\partial}{\partial\beta}\ln Q = -\frac{\partial}{\partial\beta}(\beta p) = -p + T\frac{\partial p}{\partial T}$$

$$n = \frac{N}{V} = \frac{1}{\beta}\frac{\partial}{\partial\mu}\left(\frac{1}{V}\ln Q\right) = \frac{1}{\beta}\frac{\partial}{\partial\mu}(\beta p)$$

$$= \frac{\partial p}{\partial\mu}$$

$p = g(\mu, T) \longrightarrow$ from calculation of Q.

$$s = \frac{1}{T}(\rho - \mu n + p)$$