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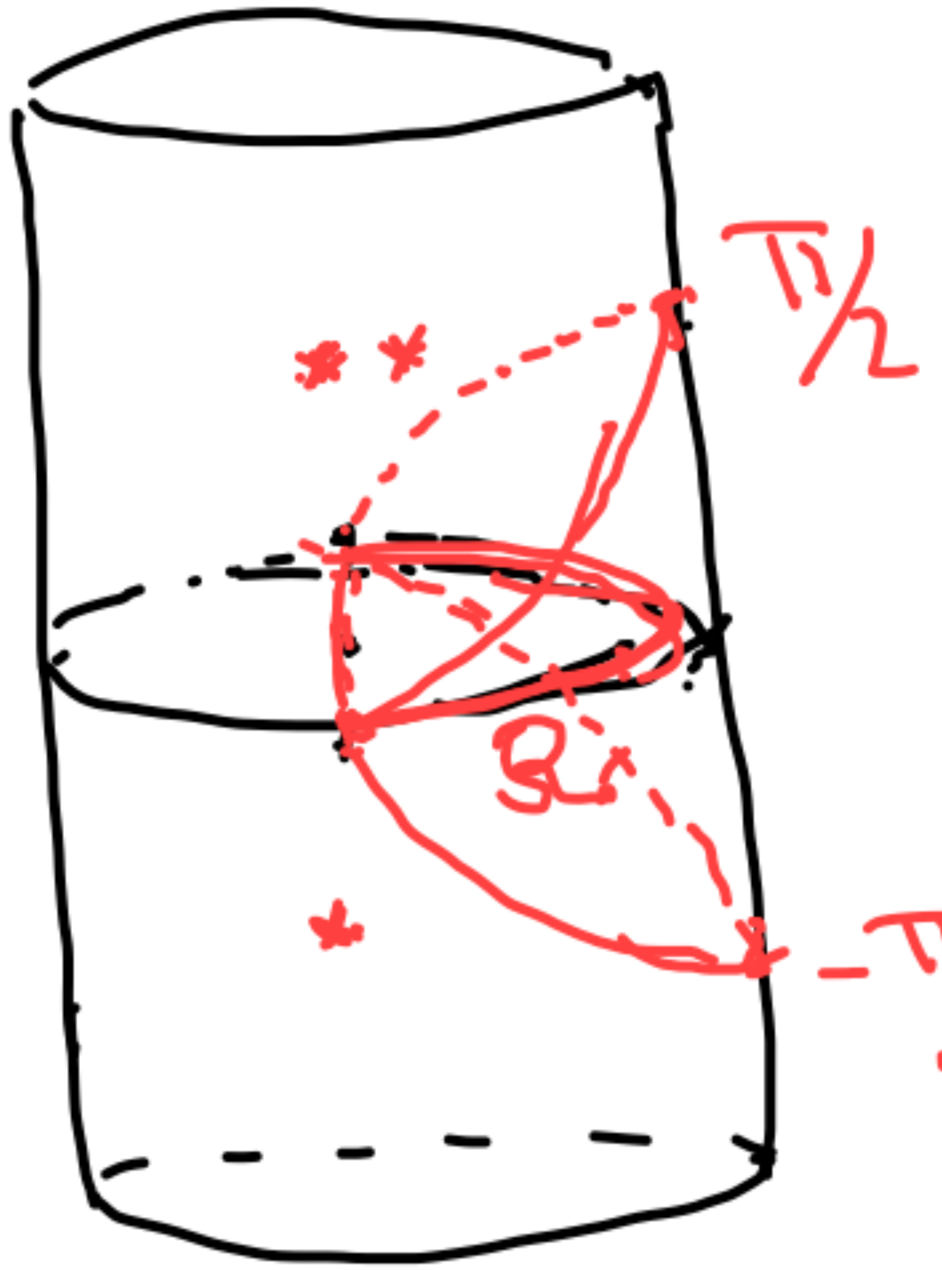
Rindler	Global
$T_1 = \rho \cosh \chi$	$T_1 = \sqrt{r^2 + R^2} \cos t$
$T_2 = \sqrt{\rho^2 - R^2} \sinh \tau$	$T_2 = \sqrt{r^2 + R^2} \sin t$
$X_d = \sqrt{\rho^2 - R^2} \cosh \tau$	$\vec{X}^2 = r^2 \to S^{d-1}$
$\underbrace{X_1^2 + \cdots + X_{d-1}^2}_{S^{d-2}} = \rho^2 \sinh^2 \chi$	$t = 0 \Rightarrow T_2 = 0 \Rightarrow \tau = 0$
$\rho > R, \; -\infty < \chi, \tau < \infty$	For $\lvert t \rvert > 0$, less than half of S^{d-1} is covered.
$T_1 > R, \quad X_d \geq \lvert T_2 \rvert \geq 0$ $\Downarrow$ $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$	For $t \to \pm \frac{\pi}{2}$, $\lvert T_2 \rvert \to \sqrt{r^2 + R^2}$ we must take $r \to \infty$
For $t = 0$, half of S^{d-1} is covered.	$X_d \to r \Rightarrow$ a point on S^{d-1}.

$d=2$ for simplicity.



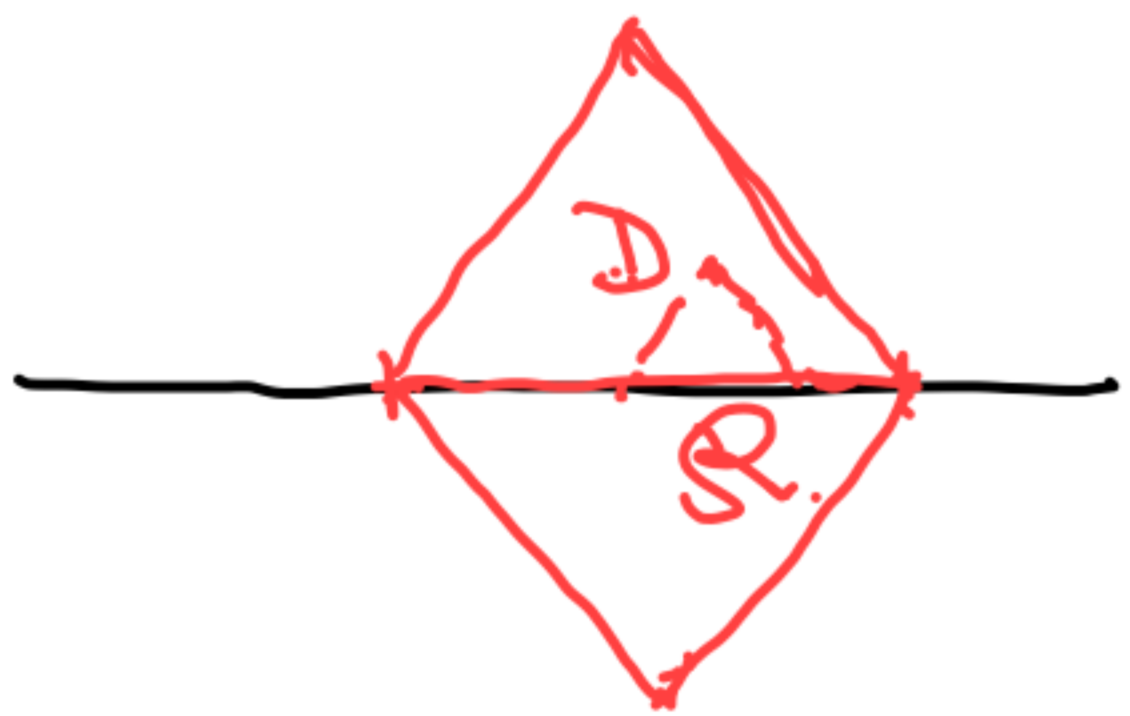
$\uparrow t$

$\mathcal{R}$: Hemisphere on the boundary at $t=0$.

Given $\mathcal{R}$ there is a geometric algorithm to construct the region of global AdS_{d+1} covered by the Rindler coordinates.

① On the boundary find the collection of all points such that any time-like or null curve passing through the point intersect $\mathcal{R}$ either in the past or in the future.

→ Boundary domain of dependence D of $\mathcal{R}$



$t=0$ slice of the boundary.

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(2) Find the causal past of D in the bulk
— collection of points in the bulk
that can send signal to some point
in D.

(3) Find the causal future of D.

(4) Causal wedge of $\mathcal{R}$ is the intersection
of the causal past and the causal future
of D $\Rightarrow$ Gives the region of the bulk
covered by the Rindler coordinates.

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Note: Once $\mathcal{R}$ is given, the rest of the construction is purely geometric ~ do not need explicit coordinate system.

<u>Next task</u>: Show that local bulk operators in the causal wedge of $\mathcal{R}$ can be constructed in terms of boundary operators on $\mathcal{R}$ at $t=0$.

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$$ds^2 = -(\rho^2 - R^2)\, d\tau^2 + \frac{R^2\, d\rho^2}{\rho^2 - R^2} + \rho^2\, dH_{d-1}^2$$

$$\Downarrow$$

$$dx^2 + \sinh^2 x\, d\Omega_{d-2}^2$$

$\uparrow$ S^{d-2}

$\vec{\alpha}$: coordinates on H_{d-1}

$\llcorner$ $(x,$ angles of $S^{d-2})$

Scalar field in Rindler coordinates.

$$(\Box - m^2)\phi = 0.$$

Basis functions:

$$f_{\omega\vec{\lambda}}(\rho, \tau, \vec{\alpha}) = \psi_{\omega\vec{\lambda}}(\rho)\, Y_{\vec{\lambda}}(\vec{\alpha})\, e^{-i\omega\tau}$$

$Y_{\vec{\lambda}}(\vec{\alpha})$: Eigenfn--s of the Laplacian on H_{d-1}

$\hookrightarrow$ partly discrete, partly continuous. | $\psi_{\omega\vec{\lambda}} \sim \rho^{-\Delta}$ as $\rho \to \infty$

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$$\phi(\rho,\tau,\vec{\alpha}) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi} \sum\!\!\!\!\!\!\int d\vec{\lambda}\; f_{\omega\vec{\lambda}}(\rho,\tau,\vec{\alpha})\; \tilde{\phi}_{\omega\vec{\lambda}}$$

$$\tilde{\phi}_{\omega\vec{\lambda}} \to a_{\omega\vec{\lambda}} \text{ for } \omega>0, \quad a^{\dagger}_{-\omega\vec{\lambda}} \text{ for } \omega<0$$

$$(f_{\omega\vec{\lambda}}, f_{\omega'\vec{\lambda}'}) = 2\pi\,\delta(\omega-\omega')\,\delta_{\vec{\lambda}\vec{\lambda}'}$$

$$f_{\omega\vec{\lambda}} = e^{-i\omega\tau}\,\psi_{\omega\vec{\lambda}}(\rho)\,Y_{\vec{\lambda}}(\vec{\alpha})$$

$\vec{\alpha} = x$, angles.

$$\int d^{d-1}\alpha\; Y_{\vec{\lambda}}(\vec{\alpha})^{*}\, Y_{\vec{\lambda}'}(\vec{\alpha}) = \delta_{\vec{\lambda}\vec{\lambda}'}.$$

$\Rightarrow$ Fixes normalization of $\psi_{\omega\vec{\lambda}}$.

$$\psi_{\omega\vec{\lambda}} \xrightarrow[\rho\to\infty]{} e^{-\Delta} N_{\omega\vec{\lambda}} \Rightarrow \text{defines } N_{\omega\vec{\lambda}} \ldots \text{\# number}$$

Boundary operator:

$$O(\tau,\vec{\alpha}) = \lim_{\rho\to\infty} \rho^{\Delta}\, \phi(\rho,\tau,\vec{\alpha})$$

$$= \lim_{\rho\to\infty} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sum\!\!\!\!\!\!\!\int d\vec{\lambda}\;\; \rho^{\Delta} e^{-i\omega\tau}\, \psi_{\omega\vec{\lambda}}(\rho)\, Y_{\vec{\lambda}}(\alpha)\, \tilde{\phi}_{\omega\vec{\lambda}}$$

$$= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sum\!\!\!\!\!\!\!\int d\vec{\lambda}\;\, e^{-i\omega\tau}\, N_{\omega\vec{\lambda}}\, Y_{\vec{\lambda}}(\alpha)\, \tilde{\phi}_{\omega\vec{\lambda}}$$

$$\tilde{\phi}_{\omega\vec{\lambda}} = \frac{1}{N_{\omega\vec{\lambda}}} \int_{-\infty}^{\infty} d\tau'\, e^{i\omega\tau'} \int d^{d-1}\alpha'\, Y_{\vec{\lambda}}(\vec{\alpha}')^{*}\, O(\tau',\vec{\alpha}')$$

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$$\phi(\rho,\tau,\vec{\alpha}) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sum\!\!\!\!\!\!\!\int d\lambda \; f_{\omega\vec{\lambda}}(\rho,\tau,\vec{\alpha})\, \tilde{\phi}_{\omega\vec{\lambda}}$$

$$= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sum\!\!\!\!\!\!\!\int d\lambda \; f_{\omega\vec{\lambda}}(\rho,\tau,\vec{\alpha}) \int d\tau' \, d^{d-1}\alpha'$$

$$\frac{1}{N_{\omega\vec{\lambda}}} \, e^{i\omega\tau'} \, Y_{\vec{\lambda}}(\vec{\alpha}')^{*} \; G(\tau',\vec{\alpha}')$$

$$\phi(x) = \int d\tau' \, d^{d-1}\alpha' \; \tilde{K}(x,x') \; G(x')$$

$$\rho,\tau,\vec{\alpha} \qquad\qquad \tau',\vec{\alpha}'$$

$$\tilde{K}(x,x') = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sum\!\!\!\!\!\!\!\int d\vec{\lambda} \; f_{\omega\vec{\lambda}}(\underbrace{\rho,\tau,\vec{\alpha}}_{x}) \; \underbrace{e^{i\omega\tau'} Y_{\vec{\lambda}}(\vec{\alpha}')^{*}}_{x'}$$

$\int dx'$ runs over D

Using local boundary Hamiltonian we can express this in terms of O on R:

$\Rightarrow$ If x is in the causal wedge of R then $\phi(x)$ can be expressed in terms of boundary operators on R.

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Make $SO(d,2)$ trs. that leaves $t=0$ condition unchanged.

Focus on $d=2$ case for simplicity.

$$T_1 = \rho \cosh \chi$$

$$T_2 = \sqrt{\rho^2 - R^2}\, \sinh \tau$$

$$X_2 = \sqrt{\rho^2 - R^2}\, \cosh \tau$$

$$X_1^2 + X_2^2 = \rho^2 \sinh^2 \chi$$

$$T_1 = \sqrt{r^2 + R^2}\, \cos t$$

$$T_2 = \sqrt{r^2 + R^2}\, \sin t$$

$$X_1 = r \cos \theta$$

$$X_2 = r \sin \theta$$

$X_2 \geq 0 \qquad 0 \leq \theta \leq \pi$

$t=0 \Rightarrow T_2 = 0 \Rightarrow \tau = 0$

$SO(2,1)$ acting on T_1, X_1, X_2 can be used.

At $t=0$, $\tau=0$

$$T_1 = \rho\cosh\chi = \sqrt{r^2+R^2}$$

$$X_2 = \sqrt{\rho^2-R^2} = r\sin\theta$$

$$T_1' = \cosh\alpha\, T_1 + \sinh\alpha\, X_2$$

$$X_2' = \cosh\alpha\, X_2 + \sinh\alpha\, T_1$$

$\theta = 0$ or π

$$\Rightarrow \sin\theta' = \frac{\sinh\alpha}{\cosh\alpha}$$

$$\theta' = \sin^{-1}\tanh\alpha, \quad \pi - \sin^{-1}\tanh\alpha$$

$$\alpha\to\infty, \quad \theta' = \frac{\pi}{2}, \frac{\pi}{2}$$

$$\sqrt{r'^2+R^2} = \cosh\alpha\sqrt{r^2+R^2} + \sinh\alpha\, r\sin\theta$$

$$r'\sin\theta' = \cosh\alpha\, r\sin\theta + \sinh\alpha\sqrt{r^2+R^2}$$

$$r\to\infty \Rightarrow \sin\theta' = \frac{\cosh\alpha\sin\theta + \sinh\alpha}{\cosh\alpha + \sinh\alpha\sin\theta}$$

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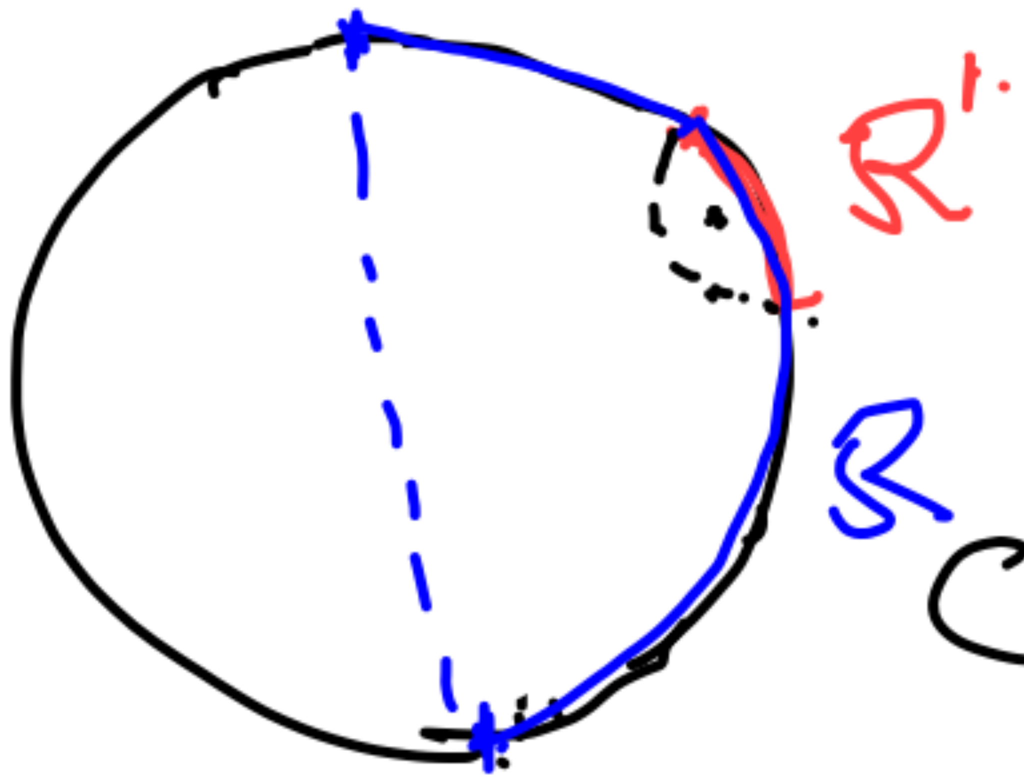


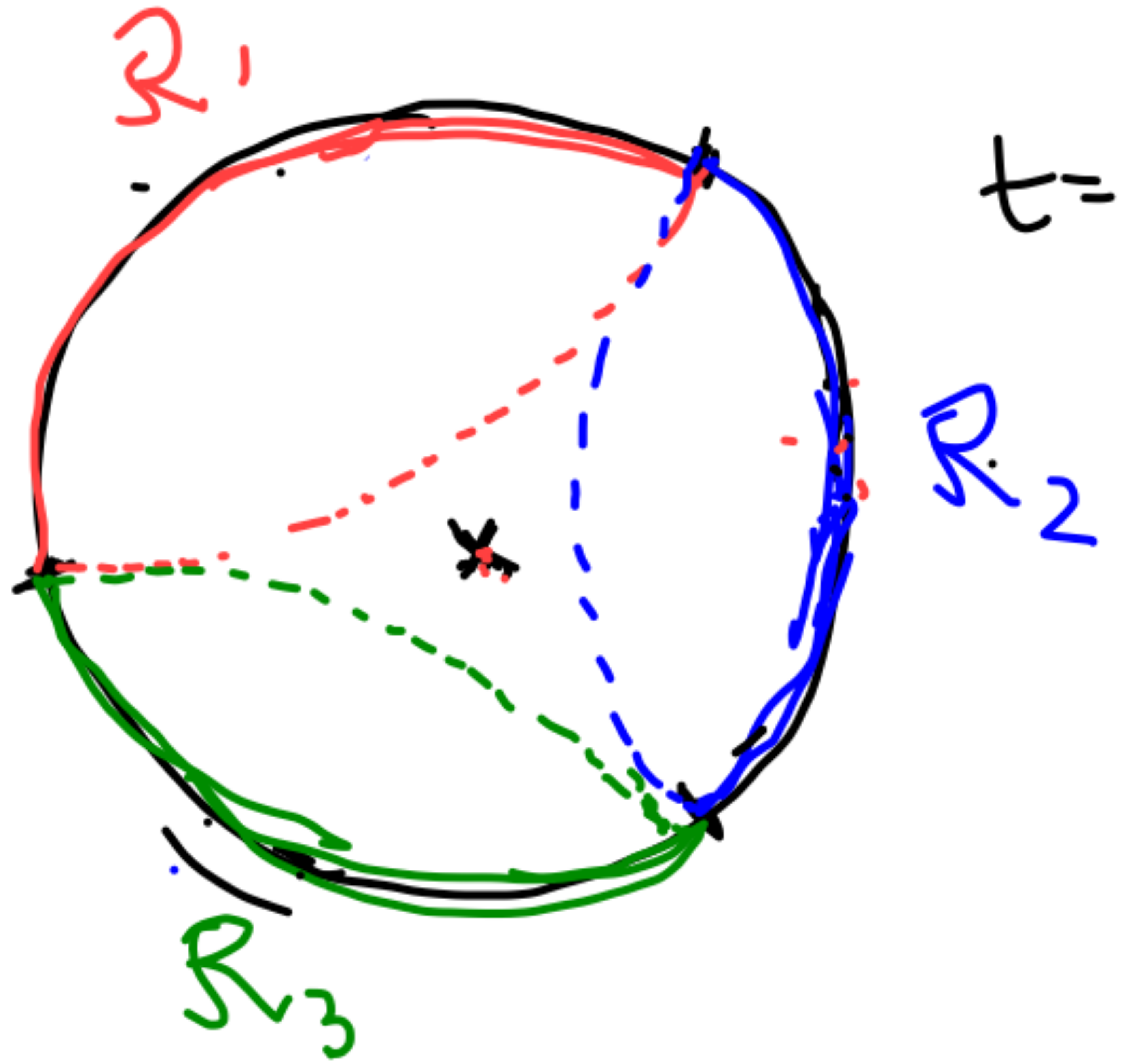
In AdS_{d+1}, R' can be any ball shaped region.

Causal wedge of R' will be a small region "close" to the boundary.

Bulk operators in this wedge can be constructed in terms of boundary operators in R.

$\tan^{-1}\frac{r}{R}$ coordinate

$0 \leq \tan^{-1}\frac{r}{R} \leq \frac{\pi}{2}$



$t=0$.

R_1, R_2 and R_3 are individually dispensible.

$\phi(0)$ cannot be reconstructed from operators in R_1 or in R_2 or in R_3

can be reconstructed from ops. in $R_1 \cup R_2$ or $R_2 \cup R_3$ or $R_1 \cup R_3$

Quantum error correction code.

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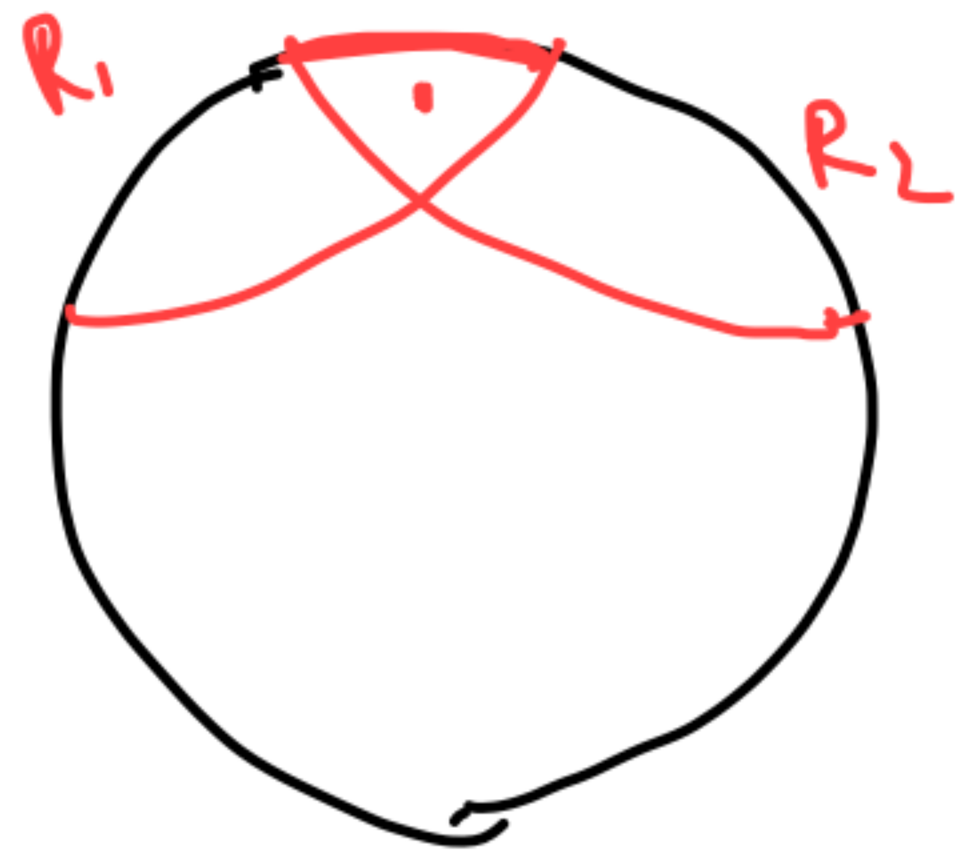

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