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$$G^{(n)R}_{s_1\ldots s_n}(k_1,\ldots,k_n,\vec{g}_R,\mu) = \left\langle \prod_{i=1}^{n} \tilde{\Phi}_{s_i R}(k_i) \right\rangle$$

$$\left\{ \mu\frac{\partial}{\partial\mu} + \sum_\alpha \beta_\alpha(\vec{g}_R,\mu)\frac{\partial}{\partial g_{\alpha R}} + \sum_i \gamma_{s_i}(\vec{g}_R,\mu)\right\} G^{(n)R}_{s_1\ldots s_n} = 0$$

Assume : $\gamma_{st} = \gamma_s \delta_{st}$

$$G^{(n)R}_{s_1\ldots s_n}(\lambda k_1,\ldots \lambda k_n, \lambda^{\Delta_\alpha} g_{\alpha R}, \lambda\mu) = \lambda^{\sum d_{s_i}} \times G^{(n)}_{s_1\ldots s_n}(k_1,\ldots k_n;\{g_{\alpha R}\},\mu)$$

where d_s : dimension of $\tilde{\Phi}_s(k)$

and Δ_α : dimension of $g_{\alpha R}$

$$g_{\alpha R} \to \lambda^{-\Delta_\alpha} g_\alpha \quad , \quad \mu \to \lambda^{-1}\mu$$

$$G^{(n)R}_{s_1\ldots s_n}(\lambda k_1,\ldots,\lambda k_n,\{g_{\alpha R}\},\mu) = \lambda^{\sum_{i=1}^{n} d_{s_i}} G^{(n)R}_{s_1\ldots s_n}(k_1,\ldots,k_n;\{\lambda^{-\Delta_\alpha} g_{\alpha R}\},\lambda^{-1}\mu)$$

we get,

$$\lambda \frac{\partial G^{(n)R}_{s_1\ldots s_n}(\lambda k_1,\ldots,\lambda k_n,\{g_{\alpha R}\},\mu)}{\partial\lambda} = \left(\sum_{i=1}^{n} d_{s_i} - \sum_\alpha \Delta_\alpha g_{\alpha R}\frac{\partial}{\partial g_{\alpha R}} - \mu\frac{\partial}{\partial\mu}\right) \times \underbrace{\lambda^{\sum d_{s_i}} \times G^{(n)R}_{s_1\ldots s_n}(k_1,\ldots,k_n;\{\lambda^{-\Delta_\alpha} g_{\alpha R}\},\lambda^{-1}\mu)}_{G^{(n)R}_{s_1\ldots s_n}(\lambda k_1,\ldots,\lambda k_n;\{g_{\alpha R}\},\mu)}$$

$$\lambda \frac{\partial}{\partial \lambda} G^{(n)R}_{s_1 \dots s_n}(\lambda k_1, \dots, \lambda k_n, \{g_{\alpha R}\}, \mu)$$

$$= \left(\sum_{i=1}^{n} d_{s_i} - \sum_{\alpha} \Delta_\alpha g_{\alpha R} \frac{\partial}{\partial g_{\alpha R}} - \mu \frac{\partial}{\partial \mu} \right) G^{(n)R}_{s_1 \dots s_n}(\lambda k_1, \dots, \lambda k_n; \{g_{\alpha R}\}, \mu) \quad \text{— (A)}$$

$$= \Big[\sum_{i=1}^{n} \{ d_{s_i} + \gamma_{s_i}(\vec{g}_R, \mu) \} + \sum_{\alpha} \{ \beta_\alpha(\vec{g}_R, \mu) - \Delta_\alpha g_{\alpha R} \} \times \frac{\partial}{\partial g_{\alpha R}} \Big]$$

$$\times G^{(n)R}_{s_1 \dots s_2}(\lambda k_1, \dots, \lambda k_n; \{g_{\alpha R}\}, \mu) \quad \text{— (B)}$$

eqn. where there is no μ-derivative

- tells us how the Green's fn. when we scale out the momenta

→ But why is this possible? → bcos by dimensional analysis, we can transfer the change in momenta to other quantities

\# Suppose there was no need to renormalise the theory. In this case $\beta_\alpha = 0, \gamma_s = 0$

$$\lambda \frac{\partial}{\partial \lambda} G^{(n)R}_{s_1 \dots s_n}(\lambda k_1, \dots, \lambda k_n, \{g_{\alpha R}\})$$

$$= \left(\sum_{i=1}^{n} d_{s_i} - \sum_{\alpha} \Delta_\alpha g_{\alpha R} \frac{\partial}{\partial g_{\alpha R}} \right) \times G^{(n)R}_{s_1 \dots s_n}(\lambda k_1, \dots, \lambda k_n; \{g_{\alpha R}\})$$

The renorm. parameters/fields can be taken to be equal to non-renorm. parameters/fields

(This eqn should reflect the dimensional analysis)

[Suppose we change the mass scale by $(1+\epsilon)$]

constructing the diff. eqn. from the int. version

Then $G^{(n)R}_{s_1\dots s_n}\left((1+\epsilon)\lambda k_1, \dots, (1+\epsilon)\lambda k_n, (1+\epsilon)^{\Delta_\alpha} g_{\alpha R}\right)$

$$= (1+\epsilon)^{\sum d_{s_i}} \underbrace{G^{(n)R}_{s_1\dots s_n}\left(\lambda k_1, \dots, \lambda k_n, g_{\alpha R}\right)}$$

$\lambda \to (1+\epsilon)\lambda = \lambda + \epsilon\lambda$

[Expand out both sides upto $O(\epsilon)$] $\to$

$$\Rightarrow \left(\epsilon\lambda\frac{\partial}{\partial\lambda} + \sum_\alpha \Delta_\alpha \, \epsilon \, g_{\alpha R}\frac{\partial}{\partial g_{\alpha R}}\right) G = \epsilon \sum d_{s_i} \, G$$

$$\Rightarrow \boxed{\lambda\frac{\partial}{\partial\lambda} G = \left(\sum d_{s_i} - \sum_\alpha \Delta_\alpha \, g_\alpha \frac{\partial}{\partial g_\alpha}\right) G}$$

$g_R \to (1+\Delta_\alpha\epsilon) g_{\alpha R} = g_{\alpha R} + \Delta_\alpha \epsilon g_{\alpha R}$

(This relation is the same as the special case of (B), when we don't need to renormalize)

From (C) of previous copy, we get RHS:- $(1+\epsilon)^{\sum d_{s_i}} \lambda^{\sum d_{s_i}} G^{(n)R}_{s_1,\dots,s_n}(k_1,\dots,k_n, g_{\alpha R}\lambda^{-\Delta_\alpha}(1-\epsilon)^{\Delta_\alpha}, \mu\lambda^{-1}(1+\epsilon)^{-1})$

$= (1+\epsilon)^{\sum d_i} \lambda^{\sum d_{s_i}} \left[G^{(n)R}_{s_1,\dots,s_n}(k_1,\dots,k_n, g_{\alpha R}\lambda^{-\Delta_\alpha}, \mu\lambda^{-1}) + O(\epsilon)\right]$

$= (1+\epsilon)^{\sum d_i} G^{(n)R}_{s_1,\dots,s_n}(\lambda k_1,\dots,\lambda k_n, \{g_{\alpha R}\}, \mu) + O(\epsilon^2)$

(Though (A) is qualitatively a diff. eqn. than the one we obtd. here, by renorm. grp. eqns we converted the μ-deriv. term into other terms)

(B) $\Rightarrow$ generalization of the case when we don't need to renormalise the theory)

$d_{s_i} + \gamma_{s_i}$ doesn't mean that the field has changed dimensions, it is as if we need to assign a dim. $d_{s_i} + \gamma_{s_i}$ to the fields if we want to arrive at this eqn. by dimensional analysis — this additional term is coming from elimination of μ-deriv.

$$d_{s_i} + \gamma_{s_i}$$

as if ϕ_{sR} has dimension

$d_s + \gamma_s(\vec{g}_R, \mu)$

anomalous dimension

In the more general case we get an anomalous dim. matrix which we will have to diagonalise going to an appropriate basis.

So we have

$$\lambda \frac{\partial}{\partial\lambda} G^{(n)R}_{s_1 \ldots s_n}(\lambda k_1, \ldots, \lambda k_n, \vec{g}_R, \mu)$$

$$= \left[\left(\sum_{i=1}^{n} d_{s_i} + \gamma_{s_i}(\vec{g}_R, \mu)\right) + \sum_\alpha \underbrace{\left(\beta_\alpha(\vec{g}_R, \mu) - \Delta_\alpha g_{\alpha R}\right)}_{\tilde{\beta}_\alpha(\vec{g}_R, \mu)} \frac{\partial}{\partial g_{\alpha R}}\right]$$

$$\times G^{(n)R}_{s_1, \ldots, s_n}(\lambda k_1, \ldots, \lambda k_n, \vec{g}_R, \mu)$$

[We would want to int. this diff. eqn w.r.t. λ & express $G^{(n)R}(\lambda k_i)$ to

$G^{(n)R}(k_i) \rightarrow \lambda = 1$]

Solution :- Define : $\vec{\bar{g}}_{\alpha R}(\lambda, \vec{g}_R)$ to be the solution to

$$\lambda \frac{\partial}{\partial\lambda} \bar{g}_{\alpha R}(\lambda, \vec{g}_R) = \tilde{\beta}_\alpha\left(\vec{\bar{g}}_{\alpha R}(\lambda, \vec{g}_R)\right)$$

where $\tilde{\beta}_\alpha = \beta_\alpha - \Delta_\alpha g_{\alpha R}$

$$\bar{g}_{\alpha R}(\lambda, \vec{g}_R) = g_{\alpha R} \quad \text{at } \lambda = 1$$

(Dep. of $\bar{g}_{\alpha R}$ on λ comes bcos it satisfies a diff. eqn. in λ) $\Rightarrow$

$$\lambda \frac{\partial}{\partial \lambda} \bar{g}_{\alpha R} = \tilde{\beta}_\alpha (\vec{\bar{g}}_R) \longrightarrow$$ so $\bar{g}_{\alpha R}$ depends on λ

($\bar{g}_{\alpha R}$ depends on $\vec{g}_R$ through boundary condns; otherwise the above diff. eqn knows nothing about $\vec{g}_R$) $\Longrightarrow$

$$\bar{g}_{\alpha R} \text{ at } \lambda = 1 = g_{\alpha R} \rightarrow \text{Boundary condn}$$

Example $\Rightarrow$ suppose $\tilde{\beta}(g_R, \mu) = -A g_R^3$

$$\therefore \lambda \frac{\partial}{\partial \lambda} \bar{g}_R = -A \bar{g}_R^3$$

$\Rightarrow$ (It is the same as) $\Rightarrow$ $$\lambda \frac{\partial}{\partial \lambda}\left(\frac{1}{2\bar{g}_R^2}\right) = A$$

$$\Rightarrow \frac{1}{2\bar{g}_R^2} = A \ln \lambda + c$$

c → integration constant

(Apply B. C.) $\Rightarrow$ $\bar{g}_R = g_R$ at $\lambda = 1$

$$\Rightarrow c = \frac{1}{2 g_R^2}$$

$$\therefore \frac{1}{2\bar{g}_R^2} = A \ln \lambda + \frac{1}{2g_R^2} = \frac{1 + 2A g_R^2 \ln \lambda}{2 g_R^2}$$

$$\boxed{\bar{g}_R^2 = \frac{g_R^2}{1 + 2A g_R^2 \ln \lambda}}$$

$\bar{g}_R(\lambda, g_R)$

In other cases, suppress μ-dep. bcos there is no μ-deriv. — that we take μ to be fixed

[We need one initial condn per variable bcos we have first order diff. eqns — this is supplied by $\vec{g}_R$]

$\boxed{\text{CLAIM}}$ ⇒

<u>Solution</u> for $G^{(n)R}_{s_1\ldots s_n}(\lambda k_1, \ldots, \lambda k_n, \vec{g}_R, \mu)$

Drop this out i.e, take this equal to some fixed const.

$$= G^{(n)R}_{s_1\ldots s_n}\left(k_1, \ldots, k_n; \bar{g}_{\alpha R}(\lambda, \vec{g}_R)\right)$$

$$\exp\left[\sum_{i=1}^{n} d_{s_i} \ln\lambda\right]$$

$$\times \exp\left[\sum_{i=1}^{n}\int_1^{\lambda}\frac{d\lambda'}{\lambda'}\gamma_{s_i}\left(\{g_\alpha(\lambda', \vec{g}_R)\}\right)\right] \quad \text{— ©}$$

(We'll do the proof later but we can do some)

<u>Consistency check</u> ⇒

[Take the case when we don't need to renormalise the theory]

Suppose $\gamma_{s_i} = 0$, $\beta_\alpha = 0$

Then, $\tilde{\beta}_\alpha = -\Delta_\alpha g_{\alpha R}$; $\lambda\frac{\partial}{\partial\lambda}\bar{g}_{\alpha R} = -\Delta_\alpha \bar{g}_{\alpha R}$

[Bcos $\tilde{\beta}_\alpha = \beta_\alpha - \Delta_\alpha g_{\alpha R}$] ⇒ $\boxed{\bar{g}_{\alpha R} = \lambda^{-\Delta_\alpha} g_{\alpha R}}$

Now from ©, we have

$$G^{(n)R}_{s_1\ldots s_n}(\lambda k_1, \ldots, \lambda k_n, \{g_R\})$$

$$= G^{(n)R}_{s_1\ldots s_n}\left(k_1, \ldots, k_n, \{\lambda^{-\Delta_\alpha} g_{\alpha R}\}\right)\lambda^{\sum d_{s_i}}$$

⟶ (this is precisely the result from dimensional analysis ⟶ so consistent)

[Once we know the β's, we know γ's & $\bar{g}_\alpha$'s — so we know $G^{(n)R}$ at the scaled momenta with coupling const. $\vec{g}_R$ in terms of $G^{(n)R}$ at $\lambda = 1$ with coupling const. $\bar{g}_{\alpha R}$]

As $\lambda \to \infty$, on the RHS we have amplitudes at finite momentum but at coupling constant $\bar{g}_{\alpha R}(\lambda, \vec{g}_R)$ for large λ.

for Non-abelian gauge theories :—

$$\bar{g}_R(\lambda, g_R) \to 0 \quad \text{for } \lambda \to \infty \quad \text{if } A > 0$$

$$\bar{g}_R^2 = \frac{g_R^2}{1 + 2A g_R^2 \ln\lambda}$$

Perturbation theory becomes more accurate ← ASYMPTOTIC FREEDOM

otherwise we can't apply pert. theory

small mom. means large distance scale → pert. th. can't be applied

e.g. → what is the force if we want 2 pull apart 2 fermions (or constituents)?

→ These questions can't be addressed bcos coupling consts. becomes large — these ques. are imp. for the formation of bound states

Running of $\bar{\alpha}_R$ is not important for calculation of gauge invariant quantities.

But running of $\bar{m}_R$ is important.

$$\lambda \frac{d\bar{m}_R}{d\lambda} = \tilde{\beta}_m(\bar{m}_R, \bar{g}_R)$$

$$\Rightarrow \lambda \frac{\partial \overline{m}_R}{\partial \lambda} = \left(\beta_m(\overline{m}_R, \overline{g}_R) - \overline{m}_R \right)$$

$$\Downarrow$$

$$\sim \overline{g}_R^{\,2}\, \overline{m}_R$$

(can be ignored compared to $\overline{m}_R$ for small $\overline{g}_R$)

$$\Rightarrow \lambda \frac{\partial \overline{m}_R}{\partial \lambda} \simeq - \overline{m}_R$$

Now, $\overline{m}_R \simeq \frac{m_R}{\lambda}$ since at $\lambda = 1$ $\overline{m}_R = m_R$

$\therefore$ as $\lambda \to \infty$, $\overline{m}_R \to 0$

effective mass of fermion going to zero

$\downarrow$

this can cause problem becos div. may come from $p = 0$ region

$\longrightarrow$ infrared divergence

RHS = zero times infinity is not of much use (bcos $\overline{g}_R = 0$)

- $\to$ This can cause potential IR divergence on the Right-hand side.
- $\to$ Then the RG eqns are not very useful to get large λ behaviour.

We need to look for quantities which remain finite as $\overline{m}_R \to 0$

$\to$ these corr. to free field theory

(Infrared safe quantities)

$\downarrow$

e.g. $\to$ Deep inelastic scattering amplitude

Sum over all possible/large no. of possible final states

But looking a single proton, etc. will have IR div.

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$$\lambda \frac{\partial}{\partial \lambda} G^{(n)R}_{s_1 \ldots s_n}(\lambda k_1, \ldots, \lambda k_n; \{g_{\alpha R}\}, \mu)$$

$$= \left\{ \sum_{i=1}^{n} \left(d_{s_i} + \gamma_{s_i}(\vec{g}, \mu) \right) + \sum_{\alpha} \tilde{\beta}_\alpha(g_R, \mu) \frac{\partial}{\partial g_{\alpha R}} \right\}$$

$$G^{(n)R}_{s_1 \ldots s_n}(\lambda k_1, \ldots, \lambda k_n; \{g_{\alpha R}\}, \mu)$$

where $\tilde{\beta}_\alpha = \beta_\alpha - \Delta_\alpha g_{\alpha R}$ (no sum over α)

Solution :-

$$\boxed{\begin{aligned} &G^{(n)R}_{s_1 \ldots s_n}(\lambda k_1, \ldots, \lambda k_n; \{g_{\alpha R}\}, \mu) \\ &= \exp\left(\sum_{i=1}^{n} d_{s_i} \ln \lambda \right) \exp\left(\sum_{i=1}^{n} \int_1^\lambda \frac{d\lambda'}{\lambda'} \gamma_{s_i}(\{\bar{g}_{\alpha R}(\lambda', \vec{g}_R, \mu)\}, \mu) \right) \\ &\quad G^{(n)R}_{s_1 \ldots s_n}(k_1, \ldots, k_n; \{\bar{g}_{\alpha R}(\lambda, \vec{g}_R, \mu)\}, \mu) \end{aligned}}$$

where $\bar{g}_{\alpha R}(\lambda, \vec{g}_R, \mu)$ is defined through:

$$\lambda \frac{\partial}{\partial \lambda} \bar{g}_{\alpha R}(\lambda, \vec{g}_R, \mu) = \tilde{\beta}_\alpha(\bar{g}_{\alpha R}(\lambda, \vec{g}_R, \mu), \mu),$$

& $\bar{g}_{\alpha R}(1, \vec{g}_R, \mu) = g_{\alpha R}$ (B.C.)

Simplify notation :-

(1) Drop all μ factors. (bcos this eqn. doesn't involve any deriv. of μ; but all fns appearing there can, in principle, depend on μ)

(2) $G^{(n)}_{s_1 \ldots s_n} \longrightarrow G$

(3) $\partial_\alpha G(\lambda k_1, \ldots, \lambda k_n; \vec{g}_R) \equiv \dfrac{\partial}{\partial g_{\alpha R}} G(\lambda k_1, \ldots, \lambda k_n; \vec{g}_R)$

↳ (derivative w.r.t. coupling const. argument)

We will use this identity

Identity :- $\displaystyle\sum_\gamma \frac{\partial \bar{g}_{\alpha R}}{\partial g_{\gamma R}} \tilde{\beta}_\gamma(\vec{g}_R) = \tilde{\beta}_\alpha(\vec{g}_R)$

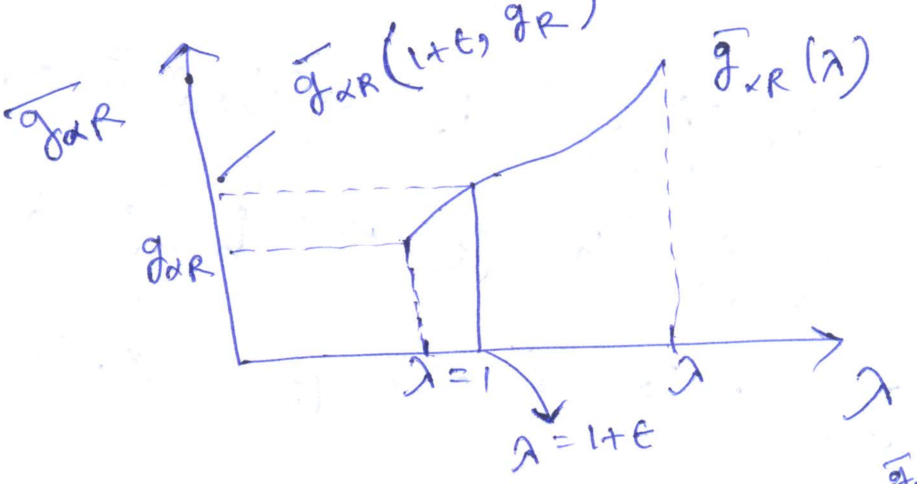
Proof :-

Begin with the equation

$$\lambda \frac{\partial}{\partial \lambda} \bar{g}_{\alpha R}(\lambda, \vec{g}_R) = \tilde{\beta}_\alpha(\vec{\bar{g}}_\alpha(\lambda, \vec{g}_R))$$

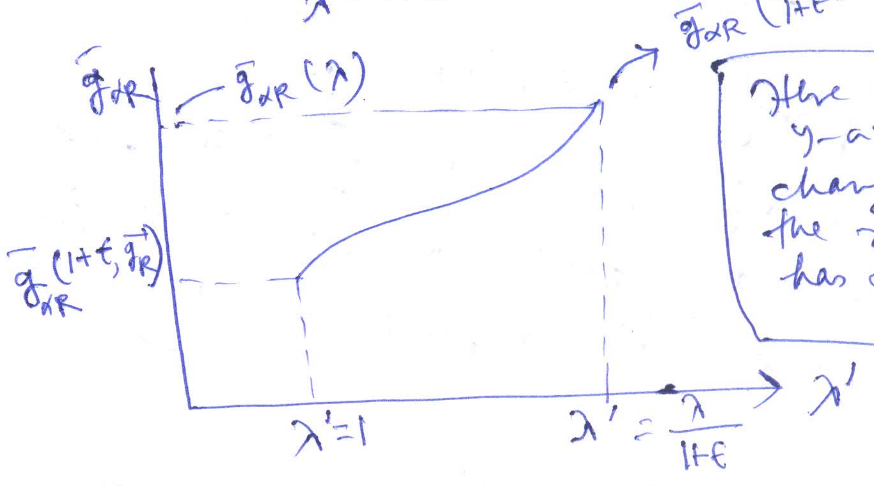
(this eqn is scale invariant in λ) → diff. eqn. is unchanged under a rescaling of λ

If $\bar{g}_{\alpha R}(\lambda, \vec{g}_R)$ is a soln., so is $\bar{g}_{\alpha R}(c\lambda, \vec{g}_R)$ for any constant c.



Also, can be thought of as evolving from $\lambda = 1+\epsilon$ & $\bar{g}_{\alpha R}(1+\epsilon, \vec{g}_R)$

Define :- $\lambda' = \dfrac{\lambda}{1+\epsilon}$



Here the y-axis hasn't changed but the x-axis has changed

(Forget about the first graph & consider the 2^{nd} graph only – as if we are solving the diff. eqn. in the λ'- plane) $\longrightarrow$

We get the identity (evolving from 2 diff. initial pts ~~~~ to the same pt.)

$$\bar{g}_{\alpha R}(\lambda, \vec{g}) = \bar{g}_{\alpha R}\left(\frac{\lambda}{1+\epsilon}, \bar{g}_{\alpha R}(1+\epsilon, g_R)\right)$$

$$\simeq \bar{g}_{\alpha R}\Big(\lambda - \epsilon\lambda, \bar{g}_{\alpha R}(1, \vec{g}_R) + \epsilon \frac{\partial}{\partial \lambda'} \bar{g}_{\alpha R}(\lambda', \vec{g}_R)\Big|_{\lambda'=1}\Big)$$

(Taylor series exp. the argument)

$$\Rightarrow \bar{g}_{\alpha R}(\lambda, \vec{g}) = \bar{g}_{\alpha R}\left(\lambda - \epsilon\lambda, g_{\alpha R} + \epsilon\tilde{\beta}_\gamma(\vec{g}_R)\right)$$

Bcos $\tilde{\beta}_\gamma(\vec{g}_R(\lambda'))\Big|_{\lambda'=1} = \tilde{\beta}_\gamma(\vec{g}_R)$

Using $\bar{g}_{\alpha R}(1, \vec{g}_R, \mu) = g_{\alpha R}$ & $\lambda \frac{\partial}{\partial \lambda} \bar{g}_{\alpha R} = \tilde{\beta}_\alpha(\ldots)$

$$\cancel{\Rightarrow \bar{g}_{\gamma R}(\lambda, \vec{g}_R) = \bar{g}_{\gamma R}}$$

$$\Rightarrow \bar{g}_{\alpha R}(\lambda, \vec{g}_R) = \bar{g}_{\alpha R}(\lambda, \vec{g}_R) - \epsilon\lambda \frac{\partial}{\partial\lambda} \bar{g}_{\alpha R}(\lambda, \vec{g}_R) + \epsilon\tilde{\beta}_\gamma(\vec{g}_R)\frac{\partial \bar{g}_{\alpha R}(\lambda, \vec{g}_R)}{\partial g_{\gamma R}}$$

$$= \bar{g}_{\alpha R}(\lambda, \vec{g}_R) - \epsilon\tilde{\beta}_\alpha\left(\vec{\bar{g}}_R(\lambda, \vec{g}_R)\right) + \epsilon\tilde{\beta}_\gamma(\vec{g}_R)\frac{\partial \bar{g}_{\alpha R}(\lambda, \vec{g}_R)}{\partial g_{\gamma R}}$$

(compare L.H.S. & RHS.) $\longrightarrow$

Coeffs. of $O(\epsilon)$ terms give the reqd. identity. (proved)

LHS is encoding the change in initial condns while the RHS involves $\lambda \frac{\partial}{\partial \lambda} g_{\alpha R}$

Why these 2 be related?

Analogy → consider an eqn which is time-translation invariant

→ We can compensate for the change in initial condns, evolve over a diff. range of λ — since it is scale invariant in λ — on one end you are changing the λ & on the other, you are " the initial condns → these 2 are correlated

(e.g. → Evolve for 58 sec. instead of 1 min.
→ Evolve from 2 sec. initial condns)

$$\sum_{\gamma} \frac{\partial g_{\alpha R}(\lambda, \vec{g_R})}{\partial g_{\gamma R}} \beta_{\gamma}(\vec{g_R}) = \beta_{\alpha}(\vec{g_R}(\lambda, \vec{g_R}))$$

For the solution given, we calculate the l.h.s. of the diff. eqn which is

$$\lambda \frac{\partial}{\partial \lambda} G(\lambda k, \ldots, \lambda k_n, \vec{g_R}))$$

$$= \left\{ \left(\sum_i d_{s_i} \right) + \sum_i \gamma_{s_i}(\vec{g_R}(\lambda, \vec{g_R})) G \ldots \right\} G[\ldots]$$

so, LHS

$$\lambda \frac{\partial}{\partial \lambda} G(\lambda k_1, \ldots, \lambda k_n, \vec{g}_R))$$

$$= \exp\left(\sum_{i=1}^{n} d_{s_i} \ln \lambda\right) \exp\left(\sum_{i=1}^{n} \int_1^{\lambda} \frac{d\lambda'}{\lambda'} \gamma_{s_i}\left(\{\bar{g}_{\alpha R}(\lambda', \vec{g}_R, \mu)\right)\right)$$

$$\times \left[\left(\sum_{i=1}^{n} d_{s_i} + \sum_{i=1}^{n} \gamma_{s_i}(\vec{\bar{g}}_R(\lambda, \vec{g}_R)) \right) \times G(k_1, \ldots, k_n; \vec{\bar{g}}_R(\lambda, \vec{g}_R)) \right.$$

$$\left. + \partial_\gamma G(k_1, \ldots, k_n; \vec{\bar{g}}_R(\lambda, \vec{g}_R)) \times \lambda \frac{\partial \bar{g}_R(\lambda, \vec{g}_R)}{\partial \lambda} \right]$$

replace by $\tilde{\beta}_\gamma(\vec{\bar{g}}_R(\lambda, \vec{g}_R))$

R.H.S. of the differential eqn:

$$= \exp\left(\sum_i d_{s_i} \ln \lambda\right) \exp\left(\sum_{i=1}^{n} \int_1^{\lambda} \frac{d\lambda'}{\lambda'} \gamma_{s_i}(\{\bar{g}_{\alpha R}(\lambda', \vec{g}_R)\})\right)$$

$$\times \left[\sum_{i=1}^{n} \left(d_{s_i} + \gamma_{s_i}(\vec{g}_R, \mu) \right) G(k_1, \ldots, k_n; \vec{\bar{g}}_R(\lambda, \vec{g}_R)) \right.$$

$$+ G(k_1, \ldots, k_n, \bar{g}_R(\lambda, \vec{g}_R)) \sum_{i=1}^{n} \tilde{\beta}_\alpha(\vec{g}_R, \mu) \int_1^{\lambda} \frac{d\lambda'}{\lambda'} \partial_\gamma \gamma_{s_i}(\vec{\bar{g}}_{\alpha R}(\lambda', g_{\alpha R})) \frac{\partial \bar{g}_{\gamma R}(\lambda', \vec{g}_R)}{\partial g_{\alpha R}}$$

$$\left. + \partial_\gamma G(k_1, \ldots, k_n; \vec{\bar{g}}_R(\lambda, \vec{g}_R)) \frac{\partial \bar{g}_{\gamma R}(\lambda, \vec{g}_R)}{\partial g_{\alpha R}} \tilde{\beta}_\alpha(\vec{g}_R) \right]$$

(deriv. from the 2nd exponential)

(Using the identity we proved, we have) →

RHS

$$= \exp\left(\sum_i d_{s_i} \ln\lambda\right) \exp\left(\sum_i \int_1^{\lambda} \frac{d\lambda'}{\lambda'} \gamma_{s_i}\left(\vec{g}_R(\lambda', \vec{g}_R)\right)\right)$$

$$\Big[G\left(k_1, \ldots, k_n; \vec{g}_R(\lambda, \vec{g}_R)\right)$$

$$\times \sum_{i=1}^{n} \left(d_{s_i} + \gamma_{s_i}(\vec{g}_R) + \int_1^{\lambda} \frac{d\lambda'}{\lambda'} \partial_\gamma \gamma_{s_i}\left(\vec{g}_R(\lambda', \vec{g}_R)\right) \times \tilde{\beta}_\gamma\left(\vec{g}_R(\lambda')\right) \right)$$

$$+ \partial_\gamma G\left(k_1, \ldots, k_n; \vec{g}_R(\lambda, \vec{g}_R)\right) \times \tilde{\beta}_\gamma\left(\vec{g}_R(\lambda, \vec{g}_R)\right)\Big]$$

Everything on RHS & LHS matches except for

$\sum_i \gamma_{s_i}\left(\vec{g}_R(\lambda, \vec{g}_R)\right)$ on LHS &

$\sum_i \gamma_{s_i}(\vec{g}) + \sum_i \int_1^{\lambda} \frac{d\lambda'}{\lambda'} \partial_\gamma \gamma_{s_i}\left(\vec{g}_R(\lambda', \vec{g}_R)\right) \tilde{\beta}_\gamma\left(\vec{g}_R(\lambda')\right)$

on the RHS

Now, $\tilde{\beta}_\gamma\left(\vec{g}_R(\lambda')\right) = \lambda' \frac{\partial}{\partial\lambda'} g_{\gamma R}(\lambda', \vec{g}_R)$

$$\therefore \int_1^{\lambda} \frac{d\lambda'}{\lambda'} \partial_\gamma \gamma_{s_i}\left(\vec{g}(\lambda', \vec{g}_R)\right) \tilde{\beta}_\gamma\left(\vec{g}_R(\lambda')\right)$$

$$= \int_1^{\lambda} d\lambda' \frac{\partial}{\partial\lambda'} \gamma_{s_i}\left(\vec{g}_R(\lambda', \vec{g}_R)\right)$$

$$= \gamma_{s_i}\left(\vec{g}_R(\lambda, \vec{g}_R)\right) - \gamma_{s_i}\left(\vec{g}_R(1, \vec{g}_R)\right)$$

(Using the identity we proved, we have) →

RHS

$$= \exp\left(\sum_i d_{s_i} \ln\lambda\right) \exp\left(\sum_i \int_1^{\lambda} \frac{d\lambda'}{\lambda'} \gamma_{s_i}\left(\vec{g}_R(\lambda', \vec{g}_R)\right)\right)$$

$$\Big[G(k_1, \ldots, k_n; \vec{g}_R(\lambda, \vec{g}_R))$$

$$\times \sum_{i=1}^{n} \left(d_{s_i} + \gamma_{s_i}(\vec{g}_R) + \int_1^{\lambda} \frac{d\lambda'}{\lambda'} \partial_\gamma \gamma_{s_i}(\vec{g}_R(\lambda', \vec{g}_R)) \times \tilde{\beta}_\gamma(\vec{g}_R(\lambda')) \right)$$

$$+ \partial_\gamma G(k_1, \ldots, k_n; \vec{g}_R(\lambda, \vec{g}_R)) \times \tilde{\beta}_\gamma(\vec{g}_R(\lambda, \vec{g}_R)) \Big]$$

Everything on RHS & LHS matches except for

$$\sum_i \gamma_{s_i}(\vec{g}_R(\lambda, \vec{g}_R)) \text{ on LHS \&}$$

$$\sum_i \gamma_{s_i}(\vec{g}) + \sum_i \int_1^{\lambda} \frac{d\lambda'}{\lambda'} \partial_\gamma \gamma_{s_i}(\vec{g}_R(\lambda', \vec{g}_R)) \tilde{\beta}_\gamma(\vec{g}_R(\lambda'))$$

on the RHS

Now, $\tilde{\beta}_\gamma(\vec{g}_R(\lambda')) = \lambda' \frac{\partial}{\partial\lambda'} g_{\gamma R}(\lambda', \vec{g}_R)$

$$\therefore \int_1^{\lambda} \frac{d\lambda'}{\lambda'} \partial_\gamma \gamma_{s_i}(\vec{g}(\lambda', \vec{g}_R)) \tilde{\beta}_\gamma(\vec{g}_R(\lambda'))$$

$$= \int_1^{\lambda} d\lambda' \frac{\partial}{\partial\lambda'} \gamma_{s_i}(\vec{g}_R(\lambda', \vec{g}_R))$$

$$= \gamma_{s_i}(\vec{g}_R(\lambda, \vec{g}_R)) - \gamma_{s_i}(\vec{g}_R(1, \vec{g}_R))$$

$$= \gamma_{s_i}\left(\vec{g}_R(\lambda, \vec{g}_R)\right) - \underbrace{\gamma_{s_i}(\vec{g}_R)}$$

this cancels $\sum_i \gamma_{s_i}(\vec{g}_R)$

So $\boxed{LHS = RHS}$ (proved)

24/11/06

Spontaneous symmetry breaking

$$S \equiv \int d^4x \left[-\tfrac{1}{2} \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

Suppose $V(\phi) = \frac{1}{2} m^2 \phi^2 + \lambda/4 \, \phi^4$

We have taken $1/4$ insteade of $\frac{1}{4!}$ for convenience

Here, $S[\phi] = S[-\phi]$

$$\Rightarrow \langle \phi(x_1) \cdots \phi(x_n) \rangle$$

$$= \frac{\int [D\phi] \, e^{iS[\phi]} \phi(x_1) \cdots \phi(x_n)}{\int [D\phi] \, e^{iS[\phi]}}$$

{change of variable} $\to$ $\phi(x) = -X(x)$: change of variable

$[D\phi] = [DX] \times \text{sign}$

$S[\phi] = S[X]$ due to symmetry

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle$$

$$= \frac{\int [DX] \, e^{iS[X]} (-1)^n X(x_1) \cdots X(x_n)}{\int [DX] \, e^{iS[X]}}$$

$\downarrow$ rename X as ϕ

$$= (-1)^n \frac{\int [D\phi] \, e^{iS[\phi]} \phi(x_1) \cdots \phi(x_n)}{\int [D\phi] \, e^{iS[\phi]}}$$

$$= (-1)^n \langle \phi(x_1) \cdots \phi(x_n) \rangle$$

$$\Rightarrow \langle \phi(x_1) \cdots \phi(x_n) \rangle = (-1)^n \langle \phi(x_1) \cdots \phi(x_n) \rangle$$

$\Rightarrow \langle \phi(x_1) \ldots \phi(x_n) \rangle = 0$ if n is odd

---

Take a new action

$$S = \int d^4x \left[-\tfrac{1}{2} \eta^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V[\phi] \right]$$

where $V[\phi] = -\tfrac{1}{2} \mu^2 \phi^2 + \lambda/_4 \, \phi^4$

(We have replaced) $m^2 \xrightarrow{\text{by}} -\mu^2$

Naive pert. of this th. leads to inconsistencies

Perturbation theory in λ:

$\lambda = 0 \Rightarrow$ free field theory

Single particle state $|\vec{p}\rangle$

has $\hat{H} |\vec{p}\rangle = \sqrt{\vec{p}^{\,2} - \mu^2} \, |\vec{p}\rangle$

$\searrow$ replaces

$\sqrt{\vec{p}^{\,2} + m^2} \, |\vec{p}\rangle$

$\sqrt{\vec{p}^{\,2} - \mu^2}$ tells us that as if we have a particle of $-$ve $(\text{mass})^2$.

Velocity $v = \dfrac{dE}{dp} = \dfrac{|\vec{p}|}{\sqrt{\vec{p}^{\,2} - \mu^2}} > 1$

$\Rightarrow$ particle propagates faster than the speed of light $\longrightarrow$ TACHYONS

$\longrightarrow$ not a consistent theory

this kind of particles are called tachyons

Physical origin of the problem

Perturbation theory is valid if ϕ is small

so that $\left| \dfrac{\lambda \phi^4}{4} \right| << |\mu^2 \phi^2|$

(To see if the above is a consistent assumption)

Consider a small fluctuation in ϕ

(B.C. $\rightarrow$ $\phi(\vec{x}, t) = \epsilon$ at $t = 0$,

$\dot{\phi}(\vec{x}, t) = 0$ at $t = 0$

(Such fluctuations of ϕ will be there bcos of quantum fluctuations ~~but~~ typically such fluctuations are small — we will see the evolution acc. to classical eqns.)

Classical eqn of motion of ϕ:

$$(\partial_t^2 - \vec{\nabla}^2 - \mu^2)\phi = 0 \text{ for small } \lambda$$

similar to K.G. eqn except for $-\mu^2$ in place of $+m^2$

For space-independent ϕ, $(\partial_t^2 - \mu^2)\phi = 0$,

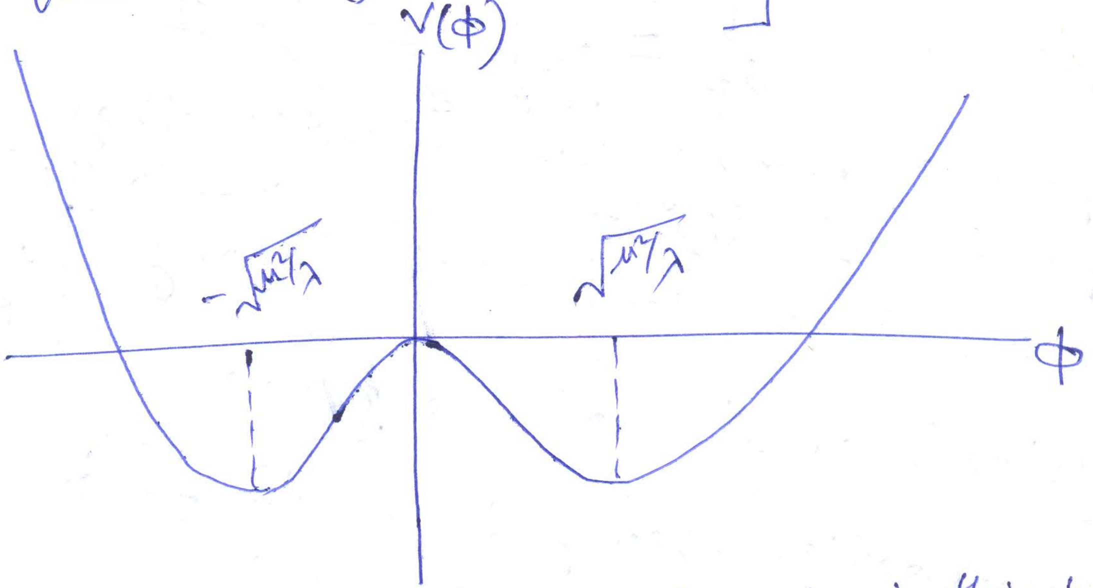
$$\phi = Ae^{\mu t} + Be^{-\mu t}$$

$\parallel$ b.c.

~~Let's assume~~ $\epsilon \cosh \mu t \to \infty$ as $t \to \infty$

Fair $\phi = Ae^{\mu t} + Be^{-\mu t} \underset{\text{b.c.}}{=} \epsilon \cosh \mu t \to \infty$ as $t \to \infty$

[$\therefore$ taking ϕ to be small is not a consistent assumption — so until you take into account the effect of $\lambda \phi^4/4$, ϕ grows uncontrollably — $\therefore$ ϕ^4 term becomes as imp. as ϕ^2 term bcos of quantum fluctuations]

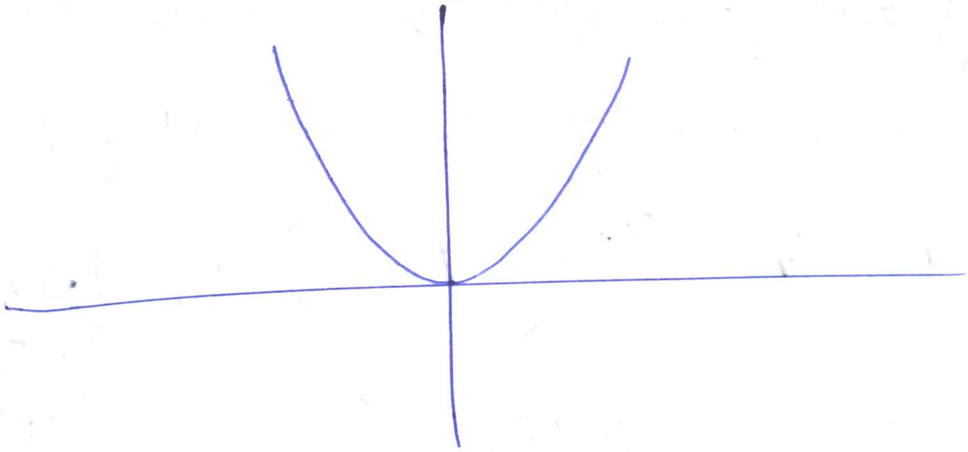


(Motion of ϕ will be the same as a particle in this potential)

(pert. th. isn't good here — consider a slight displacement from the origin ......)

Contrast this case with

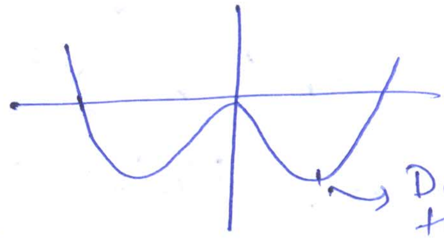
$\frac{1}{2} m \phi^2 + \lambda/4 \, \phi^4$ $\quad m, \lambda > 0$



(pert. th. is good here, bcos displacement from origin ----)

Remedy :-

Carry out perturbation theory around a minimum of $V(\phi)$.



→ Do pert. th. here

– small fluctuation

------ → consistent

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \lambda/4 \, \phi^4$$

$$V'(\phi) = -\mu^2\phi^2 + \lambda\phi^3$$

$$\therefore \quad V'(\phi) = 0 \Rightarrow \phi^2 = \mu^2/\lambda, \ \phi = \pm\sqrt{\mu^2/\lambda}$$

[We can do pert. at $\phi = \sqrt{\mu^2/\lambda}$ or at $\phi = -\sqrt{\mu^2/\lambda}$ $\Rightarrow$ 2 choices

→ let's choose $\phi = \sqrt{\mu^2/\lambda}$]

Define a new field $\chi = \phi - \sqrt{\mu^2/\lambda}$

$$\therefore \ \phi = \chi + \sqrt{\mu^2/\lambda}$$

$$\therefore \ V(\phi) = -\lambda/4\left(\mu^2/\lambda\right)^2 + \mu^2\chi^2 + \lambda\sqrt{\mu^2/\lambda}\,\chi^3 + \lambda/4\,\chi^4$$

$$\equiv \tilde{V}(\chi)$$

$$S = \int d^4x \left[-\tfrac{1}{2}\, \partial_\mu \chi\, \partial^\mu \chi - \tilde{V}(\chi) \right]$$

(The th. in terms of χ is a perfectly good th.)

χ has $(\text{mass})^2 = 2\mu^2 > 0$ $\Rightarrow$ A well defined perturbation theory

regular field (not a tachyonic field)

χ^3 & χ^4 are interactions

However this does not have a manifest symmetry of the correlation functions.

The $\phi \to -\phi$ symmetry is spontaneously broken.

(Even though the original action had this sym., the physics that we get out of it won't have this sym. — bcs corrln. fns. of χ are the physical quantities & they don't have this sym.)

---

e.g. 3 or 4 pt. fn. of χ won't see this sym.

Reason for symmetry breaking:—

① Path integral formalism:—

For a conventional theory of +ve $(\text{mass})^2$ we use a boundary condition $\phi \to 0$ as $x \to \infty$

(otherwise we get various kinds of boundary terms which are hard to deal with — so make the path int. well defined by b.c.

typically For an infinite vol. sys., what b.c. we have put on the fields don't affect the physis)

In a spontaneously broken symmetric theory, the natural b.c. is

$$\chi \to 0 \quad \text{as} \quad x \to \infty$$

$$\phi \to \sqrt{\mu^2/\lambda} \quad \text{as} \quad x \to \infty$$

$\phi \to -\sqrt{\mu^2/\lambda}$ is also possible

symmetry (relates these 2 b.c.)

[Physically both b.c. are equivalent
- In path int., the b.c. should also be inv.
- But there is a one-to-one map betn. the corrln. fns. for the 2 b.c.
- But the corrln. fn. by themselves don't exhibit this sym. as ~~bo~~ b.c. are diff. & path int. will give diff. results for diff. b.c.]

Canonical formalism

$$\langle \phi(x_1) \dots \phi(x_n) \rangle$$

$$= \langle \Omega | T(\hat{\phi}(x_1) \dots \hat{\phi}(x_n)) | \Omega \rangle$$

(the symmetry) $\phi(x) \to -\phi(x)$ is implemented by a unitary operator $\hat{U}$:-

$$\hat{U} \hat{\phi}(x) \hat{U}^{-1} = -\hat{\phi}(x)$$

It's a discrete sym. - for a continuous sym. you would have a continuous parameter

$$\langle \Omega | \hat{U} (T(\hat{\phi}(x_1) \dots \hat{\phi}(x_n)) \hat{U}^{-1} | \Omega \rangle$$

$$= \langle \Omega | T (\underbrace{U \hat{\phi}(x_1) \hat{U}^{-1}}_{-\hat{\phi}(x_1)} \underbrace{U \hat{\phi}(x_2) \hat{U}^{-1}}_{-\hat{\phi}(x_2)} \dots) | \Omega \rangle$$

$\Rightarrow \langle \Omega | \hat{U} [T(\hat{\phi}(x_1) \ldots \hat{\phi}(x_n))] \hat{U}^{-1} | \Omega \rangle$

$= (-1)^n \langle \Omega | T(\hat{\phi}(x_1) \ldots \hat{\phi}(x_n)) | \Omega \rangle$

Now, $\hat{U} \hat{\mathcal{H}} \hat{U}^{-1} = \mathcal{H}$ → since $\hat{U}$ is a symmetry.

$\therefore \hat{U} \hat{\mathcal{H}} \hat{U}^{-1} | \Omega \rangle = \hat{\mathcal{H}} | \Omega \rangle = \underbrace{E_\Omega}_{\text{vacuum energy}} | \Omega \rangle$

$\Rightarrow \hat{\mathcal{H}} \hat{U}^{-1} | \Omega \rangle = E_\Omega \hat{U}^{-1} | \Omega \rangle$

In other words, $\hat{U}^{-1} | \Omega \rangle$ also has energy E_Ω (same as that of $| \Omega \rangle$)

Now there <u>two</u> possibilities :—

(1) $\hat{U}^{-1} | \Omega \rangle = | \Omega \rangle \Longrightarrow \langle \Omega | \hat{U} = \langle \Omega |$ (In this case we are done)

(Then) $\Rightarrow \langle \Omega | T(\hat{\phi}(x_1) \ldots \hat{\phi}(x_n)) | \Omega \rangle$

$= (-1)^n \langle \Omega | T(\hat{\phi}(x_1) \ldots \hat{\phi}(x_n)) | \Omega \rangle$

(This is the case when sym. is unbroken — the action of sym. trs. leaves the vacuum unchanged & we get the same sym. in the corrln. fn. bcos of this)

(2) $\hat{U}^{-1} | \Omega \rangle = | \Omega' \rangle$ → different state

($| \Omega' \rangle$ can also be called a vacuum bcos it has the same energy E_Ω as $| \Omega \rangle$ — degeneracy in vacuum state — typically

Sym. trs. will take you from one to the other — mix .... — doesn't leave the vacuum invariant)

Why is it not the gr. state is a superposition of a wave fns centred around the 2 minima as in H.O. ?

$\rightarrow$ In QFT, we have an ∞ no. H.O.'s

In Q. Mech

$$\langle \psi_1 | \hat{H} | \psi_2 \rangle = a \rightarrow \text{small no.}$$

perturbative ground states

1st order pert.

$$\hat{H} = \begin{pmatrix} \langle \psi_1 | \hat{H} | \psi_1 \rangle & \langle \psi_1 | \hat{H} | \psi_2 \rangle \\ \langle \psi_2 | \hat{H} | \psi_1 \rangle & \langle \psi_2 | \hat{H} | \psi_2 \rangle \end{pmatrix}$$

In QFT, these are raised to infinite power $\rightarrow$ overlap is zero $\rightarrow$ true ground state is not a superposition of the two — either $|\Omega\rangle$ or $|\Omega'\rangle$

$$\hat{H} = \hat{H}_0 + \hat{H}_1$$

$$\langle \Omega_0 | \hat{H}_1 | \Omega_0' \rangle = 0$$

pert. gr. states

becos it will involve prd. of ∞ no. of terms each of which is less than 1

In path int., b.c. fixing means as if freezing has ~~occurred~~ occurred at the boundary

analogy $\rightarrow$ Stat. mech.

$m^2 > 0 \rightarrow$ like in paramagnetic state & system doesn't know about boundary

$\mu^2 < 0 \rightarrow$ " " ferromag. phase & b.c. makes a diff. — you have frozen spins at the boundary & this will align the rest of the spins

Sym. is broken by dynamics though the Lag. has the sym. — spontaneous sym. breaking

$\therefore$ we have moved to a new vacuum — imposed certain b.c. — now if we reconstruct corrln. fn. of ϕ ~~in terms~~ fn. after calculating " " " χ, these corrln. of ϕ also don't obey the sym. — bcos the b.c. are diff.

Exp. we see only χ - particles

$a, a^\dagger$ have meaning only if you can apply pert. th. — otherwise meaningless

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Note $\rightarrow$ $\langle \Omega | \hat{\phi}(x) | \Omega \rangle \neq 0$

while $\langle \Omega | \hat{\chi} | \Omega \rangle = 0$

Spontaneous breaking of continuous symmetries

$$S = \int d^4x \left[-\eta^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi - V(\phi) \right]$$

where $V(\phi) = -\mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2$

$\phi \equiv \frac{1}{\sqrt{2}} (\phi_1 + i \phi_2)$ is complex

$$S = \int d^4x \left[-\tfrac{1}{2} \eta^{\mu\nu} (\partial_\mu \phi_1 \partial_\nu \phi_1 + \partial_\mu \phi_2 \partial_\nu \phi_2) - V \right]$$

where $V = -\frac{1}{2} \mu^2 (\phi_1^2 + \phi_2^2) + \frac{\lambda}{4} (\phi_1^2 + \phi_2^2)^2$

Symmetry of S

$$\phi \rightarrow e^{i\alpha} \phi, \quad \phi^* \rightarrow e^{-i\alpha} \phi^*$$

or in terms of real fields
$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

(∵ potential has an axial sym. — corr. to rotation in xy-plane)

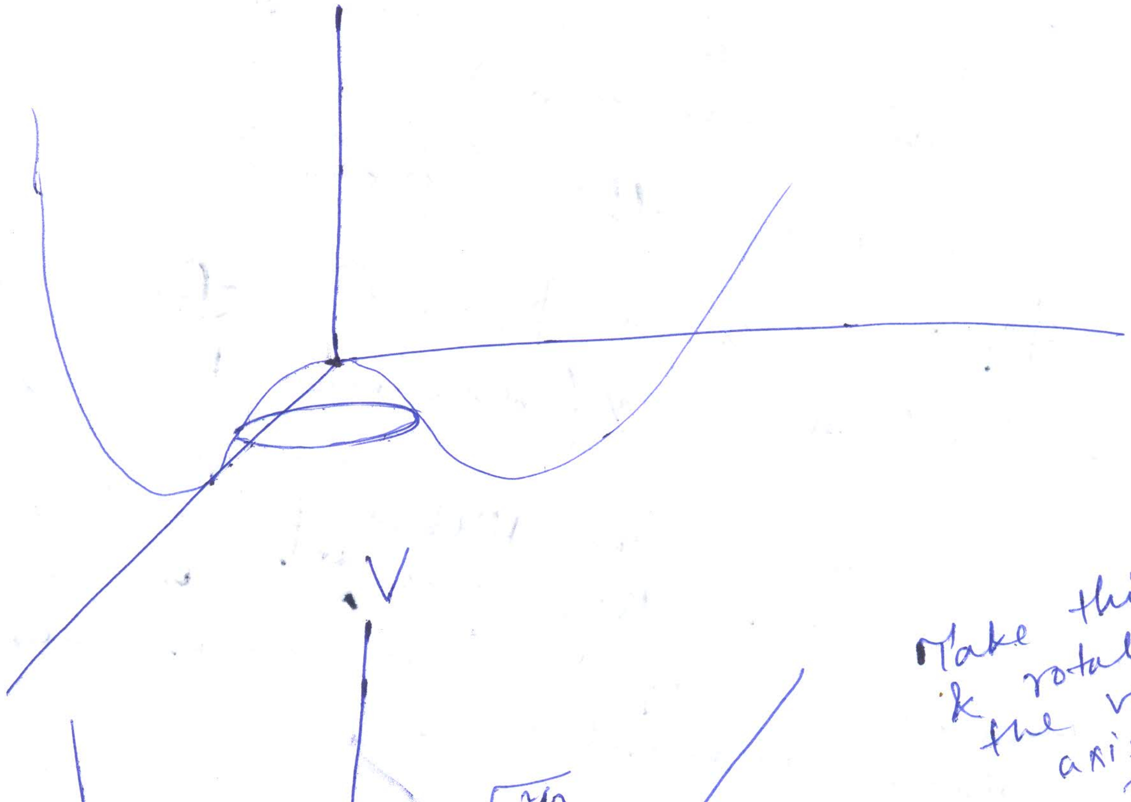
$$V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$$

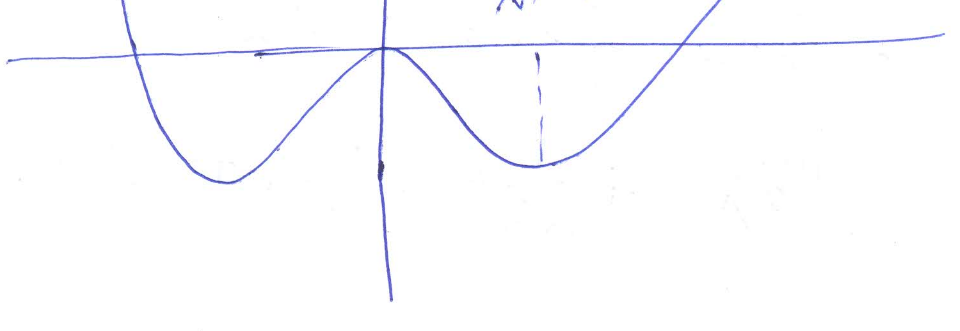
$$\frac{\partial V}{\partial|\phi|} = -2\mu^2|\phi| + 4\lambda|\phi|^3 \Rightarrow |\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$$

$$\therefore \quad \phi = \sqrt{\mu^2/2\lambda}\; e^{i\theta}, \text{ where } \theta = \text{arbitrary}$$

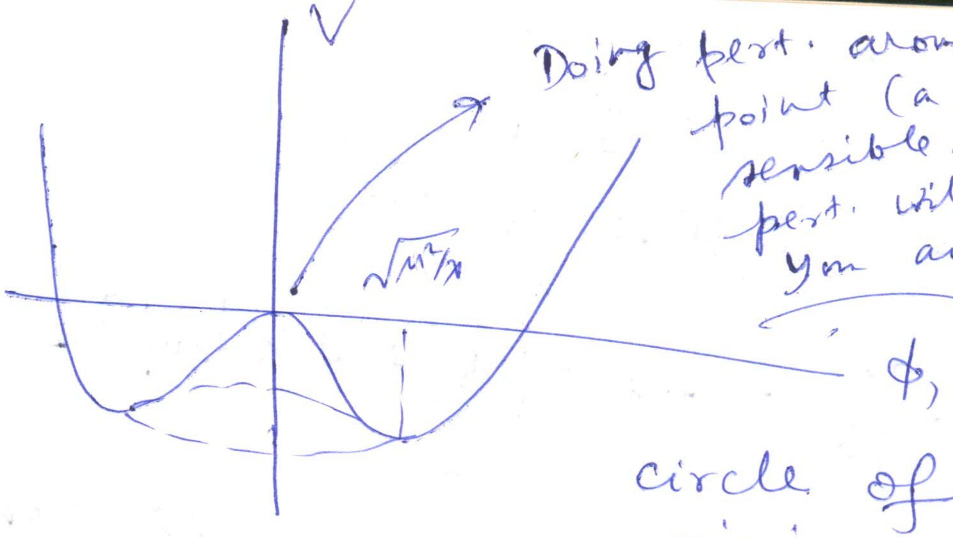
(Minimise w.r.t. the real field or the complex field → will get the same ans.)

$$\phi_1 = \sqrt{\mu^2/\lambda}\,\cos\theta$$
$$\phi_2 = \sqrt{\mu^2/\lambda}\,\sin\theta$$





Make this figure & rotate about the vertical axis by 360° → generate in the full ϕ_1–ϕ_2 phase



Doing pert. around this point (a max.) isn't sensible — a small pert. will drive you away from this pt.

circle of minimum

$$\left.\begin{aligned} \phi_1 &= \sqrt{\mu^2/\lambda}\ \cos\theta \\ \phi_2 &= \sqrt{\mu^2/\lambda}\ \sin\theta \end{aligned}\right\}$$ labels a circle of minimum

(Physically it doesn't matter which pt. on the circle we choose for pert. — we can map this corrln. fn. to others about diff. pts. of minima)

Let us choose the point

$$\phi_1 = \sqrt{\mu^2/\lambda}\ ,\ \phi_2 = 0$$ for the perturbation expansion $\phi = \sqrt{\mu^2/2\lambda}$

(There is no overlap bet. the Hilbert spaces bcos of infinite vol. — otherwise the correct vacuum would be a superposition of the diff. vacuum states — Hilbert spaces are distinct)

Define a new field χ through

$$\phi = \sqrt{\mu^2/2\lambda} + \frac{1}{\sqrt{2}}\left(\chi_1 + i\chi_2\right)$$

$$S = \int d^4x \left[-\tfrac{1}{2}\eta^{\mu\nu}\partial_\mu X_1 \partial_\nu X_1 - \tfrac{1}{2}\eta^{\mu\nu}\partial_\mu X_2 \partial_\nu X_2 - \tilde{V}(X_1, X_2) \right]$$

where $\tilde{V}[X_1, X_2] = V[\phi] = -\lambda/4\,(\mu^2/\lambda)^2 + \mu^2 X_1^2$ + cubic & higher order terms

$\boxed{\text{Ex. Prove this}}$

X_1 has $(\text{mass})^2 = 2\mu^2$

X_2 has $(\text{mass})^2 = 0$ → this feature is generic in a th. of spontaneously broken sym.

GOLDSTONE BOSONS

Pot. is flat along the angular dirn. — 2nd deriv. of pot. along this dirn. is zero — In orthogonal dirn., pot. changes — Shifting X_1 means moving in orthogonal dirn. & we get a (mass)2 — X_2 corr. to a massless particle & this in turn corr. to moving in the angular dirn.

Sym. trs. parameter was a <u>bosonic</u> ~~coordinate~~ <u>parameter</u> → these fluctuations are bosonic & the corr. massless particles are <u>bosons</u>.

Spontaneously broken fermionic sym. will give fermions.

$\cancel{\phi = \frac{1}{\sqrt{2}}\, \rho\, e^{i\theta}}$ $\quad \phi = \left(\sqrt{\frac{\mu^2}{2\lambda}} + \rho\right) e^{i\theta}$

→ It becomes clear that $V(\phi)$ is indep. of θ

→ take ρ & θ as our fields.

To the leading order, χ_2 is the same as θ upto a normalisation (bcos χ_2 is the imag. part of ϕ)

Temperature can restore spontaneously broken sym.

→ for e.g. in ferromagnet.