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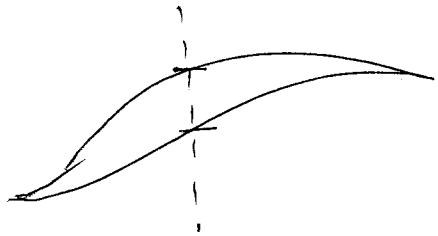

$q_r \longrightarrow q_r + \delta q_r$

$v_r \longrightarrow v_r + \delta v_r$

We will take restricted variation

where $\delta v_r = \frac{\delta q_r}{dt}$ $\longrightarrow$ Read variation.

If there is a symmetry $\longrightarrow$ then there is a conserved charge.

If there is a conserved charge. then there will be symmetry that doesn't depend on eqn. of motion

Symmetry :- Given one trajectory we can define another trajectory that has the same action.

(1) Fields without using eqn. of motion

(2) Ignore all boundary term

$$S[\tilde{\phi}] - S[\phi_r] = \int \frac{df}{dt} \cdot dt$$

$\longrightarrow$ we will still define action symmetry.

$S[\hat{\phi}] - S[\phi_r]$ - generate some eqn. of motion.

Conservation law :- $\frac{d\phi}{dt} = 0$.

provided eqn. of motion is satisfied.

$\partial_\mu J^\mu = 0$. (in case of QFT).

Noether theorem says that that symmetry $\longrightarrow$ conservation laws.

It also goes other way, if we have canonical ~~eqn~~ formulation & we can identify ϕ, then that implies we have same ~~eqn. of~~ action without ~~motion~~ using eqn. of motion.

$$\frac{\partial \phi}{\partial q_r}\frac{\partial H}{\partial p^r} - \frac{\partial \phi}{\partial p_r}\frac{\partial H}{\partial q^r} + \frac{\partial \phi}{\partial t} = 0 \quad \text{--- (i)}$$

if ϕ is conserved with help of eqn. of motion then --(i) is valid without eqn. of motion.

$$\cancel{\frac{\partial \mathcal{L}}{\partial q}\frac{\partial}{\partial q} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right)}$$

Rotation or Lorentz transformation

$$\tilde{\phi}(x) = \phi(Mx)$$

$$Mx = M^\mu{}_\nu x^\nu$$

M - Lorentz transformation.

$$M \eta M^T = \eta = M^T \eta M$$

$\longrightarrow$ Lorentz transformation matrix.

$$\mathcal{L} = \frac{1}{2}\left(-\eta^{\mu\nu}\partial_\mu \partial_\nu \phi\right) - \frac{1}{2}m^2\phi^2$$

$$S = -\int d^4x \left[\frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} m^2 \phi^2 \right]$$

$$S[\tilde{\phi}] = -\int d^4x \left[\frac{1}{2} \eta^{\mu\nu} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} + \frac{1}{2} m^2 \tilde{\phi}^2 \right]$$

change variables:-

$$y^\mu = M^\mu{}_\nu x^\nu .$$

$$\tilde{\phi}(x) = \phi\left(M^\mu{}_\nu x^\nu\right) = \phi(y)$$

$$\cancel{\frac{\partial_\mu \tilde{\phi}}{\partial}} \quad \frac{\partial \tilde{\phi}}{\partial x^\mu} = \frac{\partial \tilde{\phi}}{\partial y^\rho} \cdot \frac{\partial y^\rho}{\partial x^\mu} .$$

$$= \frac{\partial \tilde{\phi}}{\partial y^\rho} \cdot M^\rho{}_\mu .$$

$$\partial_\nu \tilde{\phi} = \frac{\partial \tilde{\phi}}{\partial y^\sigma} M^\sigma{}_\nu .$$

$$\eta^{\mu\nu} M^\rho{}_\mu M^\sigma{}_\nu \cancel{\Rightarrow (M^\nu{}_\rho)^T \eta^{\mu\nu} M^\rho{}_\mu}$$

$$\cancel{\Rightarrow (M^\nu{}_\rho)^T \eta^{\nu\mu} M^\rho{}_\mu} .$$

$$= M^\rho{}_\mu \eta^{\mu\nu} M^\sigma{}_\nu . \quad \left\{ \begin{array}{l} \underline{M^a{}_b \eta^{bc}} = (M^{Ta})^c{}_c \\ M^a{}_b \eta^{bc} M^c{}_d = \eta^{ad} \end{array} \right.$$

$$= \eta^{\rho\sigma} .$$

$$\int d^4x \longrightarrow \int d^4y \quad \text{as } |\det M| = 1 .$$

$$\tilde{\Phi}(x) = \Phi(Mx)$$

Actually $\Omega \to$ is infinitesimal parameter.

infinitesimal Φ version

$$M \eta M^T = \eta .$$

$$M = I + \epsilon \Omega$$

$\epsilon \to$ infinitesimical parameter

$\Omega \to$ some matrix.

$$M \eta M^T = \eta$$

$$(I + \epsilon \Omega) \eta (I + \epsilon \Omega)^T = \eta$$

$$\Longrightarrow \cancel{I \eta I} \quad \cancel{\epsilon \Omega \eta \epsilon \Omega^T}$$

$$\epsilon \Omega \eta + \eta \epsilon \Omega^T = 0$$

$$\Longrightarrow \epsilon \left[\Omega \eta + \eta \Omega^T \right] = 0 .$$

$$\Longrightarrow \Omega \eta + (\Omega \eta)^T = 0 . \quad O(\epsilon) \text{ term.}$$

$$M = I + \epsilon \Omega$$

$$M^\mu{}_\nu = \delta^\mu{}_\nu + \epsilon \Omega^\mu{}_\nu$$

$$\Omega^\mu{}_\nu \eta^{\nu \rho} = \Omega^{\mu \rho} .$$

$$\cancel{\eta^{\mu \rho} (\Omega \partial^\mu{}_\nu)^T = \eta^{\rho \mu} \Omega^\nu{}_\mu}$$

$$\cancel{= \Omega^{\nu \rho}} .$$

$$\cancel{\Omega^\nu{}_\mu \eta} (\Omega \eta)^{T \mu \rho} = \cancel{\Omega^\mu{}_\nu} (\Omega^T)^\mu{}_\sigma \eta^{\sigma \rho} .$$

$$= M^\sigma{}_\mu \eta^{\sigma \rho} .$$

$$=$$

$$(\Omega\eta)^T = (\Omega^{\mu}{}_{\nu}\eta^{\nu\rho})^T = (\Omega^{\mu\rho})^T = \Omega^{\rho\mu}$$

So Ω is antisymmetric

antisymmetric multiplied by i

$\equiv$ Hermitian matrix.

$$\begin{bmatrix} 0 & -a & -b \\ a & 0 & c \\ b & -c & 0 \end{bmatrix} = \begin{bmatrix} 0 & -ia & -ib \\ ia & 0 & ic \\ ib & -ic & 0 \end{bmatrix}$$

Lorentz transformation matrices are real matrix, so ϵ - real.

$$M^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} + \epsilon\,\Omega^{\mu}{}_{\nu}$$

$\Omega^{\mu}{}_{\nu}$ - antisymmetric matrix.

~~Thus~~

$$\tilde{\phi}(x) = \phi(Mx)$$

$$\tilde{\phi}(x) = \phi((I + \epsilon\,\Omega)x)$$

$$= \phi(x + \epsilon\,\Omega x).$$

$$= \phi(x) + \epsilon\,\Omega^{\mu}{}_{\nu}\, x^{\nu}\, \partial_{\mu}(\phi(x))$$

ϵ independent no.

So ϵ conserved charges.

1. make $\Omega^{\mu}{}_{\nu}(x)$.

Once we make $\Omega^{\mu}{}_{\nu}(x)$, this is no longer a sym.

S doesn't remain same any more.

(2) $S[\tilde{\phi}] - S[\phi]$

$$\int d^4x \, (\partial_\rho \Omega^\mu{}_\nu) N^\nu{}_\mu{}^\rho .$$

$$= \int d^4 \, (\partial_\rho \Omega^\mu{}_\nu) N^\nu{}_\mu{}^\rho \longrightarrow \text{Current index.}$$

this indices are fixed

depend upon which Ω we take.

Conservation law

$$\partial_\rho N^\nu{}_\mu{}^\rho = 0 .$$

$$\partial_\rho \in J^\mu .$$

$$\partial_\rho N^\nu{}_\mu{}^\rho = 0 .$$

we will always consider it as if coordinates have changed. Old field is replaced by new field. ν_μ only keeps track that there are multiple parameter.

for deriving energy mom. tensor

$$\tilde{\phi}(x) = \phi(x) + \epsilon^\mu(x) \partial_\mu \phi(x)$$

$$S[\tilde{\phi}] - S[\phi] = -\int \partial_\rho \epsilon^\mu(x) T_\mu{}^\rho$$

Think Ω an infinitesimal parameter

$$S[\tilde{\Phi}] - S[\Phi] = -\int \partial_\rho \epsilon^\mu(x) T_\mu{}^\rho .$$

$$S[\tilde{\Phi}] - S[\Phi] = -\int \partial_\rho \epsilon^\mu(x) T_\mu{}^\rho .$$

Take $\epsilon^\mu(x) = \Omega^\mu{}_\nu(x) x^\nu$

$$S[\tilde{\Phi}] - S[\Phi] = -\int \left[\partial_\rho \left(\Omega^\mu{}_\nu x^\nu\right) T_\mu{}^\rho\right] d^4x$$

$$= -\int d^4x \left(\partial_\rho \Omega^\mu{}_\nu(x)\right) x^\nu \; T_\mu{}^\rho .$$

$$- \int d^4x \; \Omega^\mu{}_\nu \; \delta^\nu{}_\rho \; T_\mu{}^\rho .$$

$$= -\int d^4x \left(\partial_\rho \Omega^\mu{}_\nu(x)\right) x^\nu \; T_\mu{}^\rho$$

$$- \int d^4x \; \Omega^\mu{}_\rho(x) T_\mu{}^\rho .$$

$\Rightarrow$ we can use this formalism irrespective of any x.

$$= -\int d^4x \; \partial_\rho \Omega^\mu{}_\nu \; x^\nu T_\mu{}^\rho .$$

$$- \int d^4x \; \underbrace{\Omega_{\mu\rho}}_{\text{antisym.}} T^{\mu\rho} .$$

$\rightarrow$ vanishes if $T^{\mu\rho}$ is symmetric

$$N^{(\nu)}{}_{(\mu)}{}^{\rho} = -x^{\nu} T_{\mu}{}^{\rho}$$

$$= -\int d^4x \, \partial_\rho \Omega_{\mu\nu} x^{\nu} T^{\mu\rho}$$

$$= -\int d^4x \, \partial_\rho \Omega^{\mu}{}_{\nu} x^{\nu} \cdot T_{\mu}{}^{\rho} .$$

$$= -\int d^4x \, \eta^{\alpha\mu} \partial_\rho \Omega_{\alpha\nu} x^{\nu} T_{\mu}{}^{\rho}$$

$$= -\int d^4x \, \partial_\rho \Omega_{\mu\nu} x^{\nu} T^{\mu\rho}$$

$$= -\frac{1}{2} \int d^4x \left(\partial_\rho \Omega_{\mu\nu}\right)\left(x^{\nu} T^{\mu\rho} - x^{\mu} T^{\nu\rho}\right)$$

$$- \Omega_{\mu\nu} x^{\mu} T^{\nu\rho}$$

$$= -\Omega_{\nu\mu} x^{\nu} T^{\mu\rho}$$

$$= +\Omega_{\mu\nu} x^{\nu} T^{\mu\rho} .$$

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(2) $S[\tilde{\phi}] - S[\phi]$

$$= \int d^4x \left(\partial_\rho \Omega_{\mu\nu}\right) N^{\nu\mu\rho} .$$

$$N^{(\nu\mu)\rho} = -\frac{1}{2}\left(x^{\nu} T^{\mu\rho} - x^{\mu} T^{\nu\rho}\right)$$

$$\partial_\rho N^{(\nu\mu)\rho}$$

$$= -\frac{1}{2}\partial_\rho \left(x^\nu T^{\mu\rho} - x^\mu T^{\nu\rho} \right)$$

$$= -\frac{1}{2}\left[\delta^\nu_\rho T^{\mu\rho} - x^\nu \partial_\rho T^{\mu\rho} - \delta^\mu_\rho T^{\nu\rho} - x^\mu \partial_\rho T^{\nu\rho} \right]$$

$$\Rightarrow \quad \partial_\rho N^{(\nu\mu)\rho} = 0 .$$

~~Suppose~~ we did not antisymmetrize

$$S[\hat{\varphi}] - S[\varphi] = -\int d^4x \, \partial_\rho(\Omega_{\mu\nu}) x^\nu T^{\mu\rho}$$

$$= \int d^4x \, \Omega_{\mu\nu}(x) \underbrace{\partial_\rho \left(x^\nu T^{\mu\rho} \right)}_{\text{if symmetric}} .$$

So $S[\hat{\varphi}] - S[\varphi] = 0$ (always)

but once we antisymmetrise.

~~then~~ ~~that~~ $S[\hat{\varphi}] - S[\varphi] = 0$ we can ~~infer~~ conclude

~~that the~~ antisymmetric part is 0.

The symmetric part is not concerned

The conserved charge for the

~~boost~~ J^{01}

$$\frac{d\varphi}{dt} = [\varphi, H] + \frac{\partial \varphi}{\partial t} .$$

when $\nu / \mu = 0$ the $\frac{\partial \varphi}{\partial t} \neq 0$.

~~so~~ $[\varphi, H] \neq 0$

measurable, but boost generator are not eigenstate of Hamiltonian, as we

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Boundary terms are not important

2nd variation of the action can be seen.

Req. eqn. of motion unchanged
→ 2nd variation should be zero.

For testing the eqn. of invariance of eqn. of motion, we would see that

$$\delta F_i = \sum_i c_{ij} F_j = 0,$$

i.e variation of the eqn. of motion is the linear comb. of eqn. of motion.

if we check double variation of x around the extremum at the boundary $\delta\phi = 0$.

Boundary terms are not going to affect the eqn. of motion has certain symmetry.

it is imp. when we do Feynman Path integral

If mom. is conserved, then by canonical trans. $p \to q$.
then q will be transformed.

$$\Phi(\{q_\mu\}, \{p_\mu\}, \tau)$$

$$\delta q_\mu = \frac{\partial \Phi}{\partial p_\mu} \quad \& \quad \delta p_\mu = -\frac{\partial \Phi}{\partial q_\mu}.$$

$$\hat{\phi}_\mu(x) = \phi_\mu(Mx)$$

$$M \eta M^T = \eta = M^T \eta M$$

Infinitesimal version

$$M^\mu{}_\nu = \delta^\mu{}_\nu + \omega^\mu{}_\nu \longrightarrow \text{symmetry transformation parameter.}$$

$$\omega_{\mu\nu} = -\omega_{\nu\mu} \quad (\text{antisymmetric}),$$

$$\omega_{\mu\nu} = \eta_{\mu\sigma}\, \omega^\sigma{}_\nu,$$

$$\hat{\phi}_\mu(x) - \phi_\mu(x) = \phi_\mu(x + \omega x) - \phi_\mu(x)$$

$$= \omega^\mu{}_\nu x^\nu \partial_\mu \phi_\mu + O(\omega^2).$$

Conserved charge:—
see more $\omega^\mu{}_\nu$ x dependent.
make $\omega_{\mu\nu}$ x dependent.

(1) make $\omega_{\mu\nu}$

$$\delta S = \int (\partial_\rho \omega_{\mu\nu}) M^{\nu\mu\rho}.$$

for some $M^{\nu\mu\rho}$

$M^{\nu\mu\rho}$ should satisfy

$\partial_\rho M^{\nu\mu\rho} = 0$ when eqn. of motion hold.

If we know energy mom. tensor then we don't need any-thing else

$$\Phi'_\mu(x) - \Phi_\mu(x)$$

$$= \underbrace{\omega^\mu{}_\nu(x)\, x^\nu}_{\epsilon^\mu(x)} \partial_\mu \Phi_\mu .$$

~~inf~~ infinitesimal $\Rightarrow$ when we expand we will keep it in 1st order

$$\delta S = -\int \partial_\rho \epsilon^\mu(x)\, T_\mu{}^\rho \, d^4x .$$

$$= -\int d^4x\, \partial_\rho \left(\omega^\mu{}_\nu(x)\, x^\nu\right) T_\mu{}^\rho \, d^4x .$$

$$= -\int d^4x \left(\partial_\rho \omega_{\mu\nu}\right) x^\nu T^{\mu\rho}$$

$$-\int d^4x\, \partial \omega_{\mu\rho} T^{\mu\rho} .$$

if T is symmetric.

then $\omega_{\mu\rho} T^{\mu\rho} = 0$.

$$= -\frac{1}{2}\int d^4x \left(\partial_\rho \omega_{\mu\nu}\right) \left\{ x^\nu T^{\mu\rho} - x^\mu T^{\nu\rho} \right\} .$$

Only in this case it is conserved.

if T is not symmetric then we will rewrite it some how as the total diff.

$x^{\nu} T^{\mu\rho} - x^{\mu} T^{\nu\rho}$ should be conserved.

$$\partial_\rho \left(x^{\nu} T^{\mu\rho} - x^{\mu} T^{\nu\rho} \right)$$

$$= \cancel{x^{\nu} \partial_\rho T^{\mu\rho}} + \cancel{T^{\mu\nu}} - \cancel{x^{\mu} \partial_\rho T^{\nu\rho}} - \cancel{T^{\nu\mu}}$$

$$= 0.$$

$\partial_\rho T^{\mu\rho} = 0$ & $T^{\mu\nu} = T^{\nu\mu}$

if $\delta S = -\frac{1}{2} \int d^4x \, (\partial_\rho \omega_{\mu\nu}) N^{\nu\mu\rho}$

~~then~~

$\cancel{x^{\nu} T^{\mu\rho} - x^{\mu} T^{\nu\rho}}$

Then define

$$J^{\mu\nu} = \int d^3x \, N^{\nu\mu 0}.$$

$$\Longrightarrow \frac{d}{dT} J^{\mu\nu} = 0 \longrightarrow$$ provided boundary terms are 0.

J^{ij} $\quad i, j = 1, 2, 3$

$\longrightarrow$ we can recognize this as angular mom.

$J^{13} \longrightarrow J^3$

$J^{23} \longrightarrow J^1$

$J^{31} \longrightarrow J^2$

$$J^{12} = \int d^3x \left(x^1 T^{20} - x^2 T^{10} \right)$$

$\longrightarrow$ mom. density.

J^{0i} : Boost generators.

they are conserved but

$\{ J^{0x}, H \} \neq 0$

as $\dfrac{\partial J^{0i}}{\partial t} \neq 0$

e.g. where $Q(\{q_k\}, \{p_k\}, t)$.

$$\frac{dQ}{dt} = 0 \longrightarrow \frac{\partial Q}{\partial t} + \{Q, H\} = 0.$$

Because Boost changes energy.

So it won't commute with Hamiltonian.

$\dfrac{dQ}{dt} \longrightarrow$ time evolution of Q.

e.g:- Harmonic oscillator

$q^2 + p^2 + t^2 \longrightarrow$ at any time $Q(t)$ can be calculated.

~~Lorentz~~ state of time.

General case :- render Lorentz transform ~~they~~ they don't transform like scaler.

Take a field theory with fields $\phi_1, \phi_2, \ldots$

General Lorentz transformation

$$\hat{\phi}_r(x) = S_{rs}(M)\,\phi_s(Mx).$$

given a M

it gives S_{rs} that tells ϕ_s how to transform.

$\longrightarrow$ if we define some arbitrary fields that are fc. of original field ($\hat{a}$ - ~~field~~ will not transform nicely like vector field).

Ex:- Vector.

$$\hat{A}_\mu(\hat{x}) = \frac{\partial x^\rho}{\partial \hat{x}^\mu} A_\rho(x).$$

$$\cancel{\hat{A}_\mu(\hat{x}) \quad \frac{\partial \hat{x}^\mu}{x^\rho}}$$

$$x^\mu = M^\mu{}_\nu \hat{x}^\nu$$

$$\hat{A}_\mu(\hat{x}) = M^\rho{}_\mu A_\rho(x).$$

$$= M^\rho{}_\mu A_\rho(M\hat{x})$$

$$\longrightarrow \hat{A}_\mu(x) = M^\rho{}_\mu A_\rho(Mx).$$

$$S_{\mu s}(M) = (M P_\mu)^{\prime}$$

Prop. of S:

Suppose we made another Lorentz transformation by a matrix N.

$$\tilde{\Phi}_\mu(x) = S_{\mu s}(N)\,\tilde{\Phi}_s(\underline{Nx}).$$

$$\tilde{\Phi}_s(\underline{Nx}) = S_{st}(M)\,\Phi_t(\underline{MNx}).$$

$$\tilde{\Phi}_s(\underline{y}) = S_{st}(M)\,\Phi_t(\underline{My})$$

Argument is multiplied by M.

So

$$\tilde{\Phi}_\mu(x) = S_{\mu s}(N)\,\tilde{\Phi}_s(Nx)$$

$$= S_{\mu s}(N)\,S_{st}(M)\,\Phi_t(MNx)$$

$$\tilde{\Phi}_\mu(x) = S_{\mu t}(MN)\,\Phi_t(MNx).$$

$$S(MN) = S(N)\,S(M).$$

S^T is a representation.

$$S^T(MN) = S^T(M)\,S^T(N).$$

As the second index is contract with Φ.

$M, N \longrightarrow$ two unitary matrix.

we associate $R(M)$, $R(N)$.

we call it representation

$$R(M)R(N) = R(MN). \quad \Big\} R(I) \longrightarrow I$$

$R(M) = I \longrightarrow$ ~~trin~~ trivial representation

$$\Phi \longrightarrow e^{i\alpha}\Phi.$$

~~we define it by~~

we can relate it by symm.

$$S(MN) = S(N)\,S(M) \quad \Big\} \Rightarrow$$

$$\cancel{= S^T(M)\,S^T} \quad \Big\} S^T(M) = S^T(I)\,S^T(M)$$

$$S^T(MN) = S^T(M)\,S^T(N)$$

$\longrightarrow$ representation.

Infinitesimal transformation

$$\hat{\Phi}_r(x) = S_{rs}(M)\,\Phi_s(Mx)$$

$M = I + \omega$ identity maps to itself.

$M^\mu{}_\nu = \delta^\mu{}_\nu + \omega^\mu{}_\nu$,

for infinitesimal transf. $S \to \omega$.

$$S_{rs}(I+\omega) = \delta_{rs} + \omega_{\mu\nu}\Sigma^{\mu\nu}{}_{rs}$$

$$S_{rs}(I+\omega) = \delta_{rs} + \omega_{\mu\nu}\Sigma^{\mu\nu}{}_{rs}$$

$$\hat{\Phi}_r(x) = \left(\delta_{rs} + \omega_{\mu\nu}\Sigma^{\mu\nu}{}_{rs}\right)\left(\Phi_s(x) + \omega^{\mu}{}_{\nu}x^{\nu}\partial_{\mu}\Phi_s(x)\right)$$

$$= \Phi_r(x) + \omega_{\mu\nu}\Sigma^{\mu\nu}{}_{rs}\Phi_s(x) + \omega^{\mu}{}_{\nu}x^{\nu}\partial_{\mu}\Phi_r(x) + O(\omega^2)$$

independently they are not symm.

S - knows whether the field's vector / scalar / fermion.

$\longrightarrow$ Σ. (in the infinitesimal)

here are not distinguishing between upper & lower as they are up Lorentz indices.

so

$\delta\Phi_r(x)$ = If action is Lorentz invariant, then δS should be zero under $\Phi \to \delta\Phi$.

Take $\omega_{\mu\nu}(x)$ & calculate δS

$$\hat{\Phi}_r(x) - \Phi_r(x)$$

$$= \omega_{\mu\nu}(x)\Sigma^{\mu\nu}{}_{rs}\Phi_s(x) + \omega^{\mu}{}_{\nu}(x)x^{\nu}\partial_{\mu}\Phi_r(x)$$

$$= \omega_{\mu\nu}\Sigma^{\mu\nu}{}_{rs}\Phi_s(x) + \omega^{\mu}{}_{\nu}(x)x^{\nu}\partial_{\mu}\Phi_r(x).$$

$$\delta S = -\frac{1}{2}\int d^4x\, \partial_{\rho}\omega_{\mu\nu} N^{\nu\mu\rho}$$

$$N^{\nu\mu\rho} = x^{\nu} \cdot T^{\mu\rho} - x^{\mu} T^{\nu\rho} + R^{\nu\mu\rho}$$

$\hookrightarrow$ extra term.

if $T^{\mu\nu}$ is not sym.

then we will always have $T^{\mu\nu} w_{\mu\nu}$

$\hookrightarrow R^{\nu\mu\rho}$.

then that will have to combine with

$$w_{\mu\nu}(x) \Sigma^{\mu\nu}{}_{\kappa s} + T^{\mu\nu} w_{\mu\nu}$$

$$J^{\nu\mu} = \int N^{\nu\mu 0} d^3x.$$

$\rightarrow$ Lorentz group acts on the internal index

$\rightarrow$ internal spin $\longrightarrow$ under Boost spin

Lorentz ~~transformat~~ will mix with the temporal part.

if we take 0 mom. particle but because of internal spin we will have angular mom.

for e.m. field.

Ex:- Take the free K.G field & express $J^{\mu\nu}$ in terms of creation/annihilation op. (given in terms of various integrals).

Check:- After subtracting infinite constants we have

$$J^{\mu\nu}|0\rangle = 0 \longrightarrow \text{Vacuum is Lorentz invariant}$$

P^i : $a^\dagger a^\dagger$, $a\,a$

infinite const doesn't matter as we only measure diff.

Infinite const. we are allowed to subtract ~~p~~ but ~~for operator~~

$$J^{0i} = \int \left[x^0 T^{i0} - x^i T^{00} \right] d^3x .$$

$$= x^0 \int T^{i0} d^3x - \underbrace{\int x^i T^{00} d^3x}_{\to 0}$$

$$\cancel{J^{ij} = \int x^i T^{ij}}$$

$$J^{ij} = \int \left(x^j \underline{T^{i0}} - x^i \underline{T^{j0}} \right)$$

Lorentz ~~transf.~~ symmetry of theory & also under vacuum.

~~Suppose that~~ Φ is a conserved charge

if $\{\Phi, \Phi_\mu(x)\}_{P.B} = \partial_\mu [\{\Phi\}, x]$.

$\delta \Phi_\mu(x) = \epsilon \partial_\mu$.

is symmetry of S.

we can generate a symmetry transf. using Poisson ~~Bracket~~ Bracket.

In quantum ~~re~~ theory

$$[\Phi, \Phi_x] = i f_x([\Phi_s], x).$$

$$\Rightarrow [\Phi, \Phi_x] = i f_x([\Phi_s], x).$$

if subtract const. from Φ it doesn't affect the scan.

Suppose,
if Φ annihilates vacuum.

$$\Rightarrow \Phi|0\rangle = 0.$$

$$\langle 0|[\Phi, \Phi_x(x_1) \cdots \Phi_{x_k}(x_k)]|0\rangle$$

$$= \langle 0|\Phi \Phi_x(x_1) \cdots \Phi_{x_k}(x_k)|0\rangle$$

$$- \langle 0|\Phi_x(x_1) \cdots \Phi_{x_k}(x_k)\Phi|0\rangle$$

$$= 0.$$

$$\langle 0|[\Phi, \Phi_{x_1}(x)]\Phi_{x_2} \cdots \Phi_{x_k}(x_k)|0\rangle$$

$$+ \langle 0|\Phi_{x_1}(x_1)[\Phi, \Phi_{x_2}(x_2)]\Phi_{x_3} \cdots |0\rangle$$

$$+ \cdots = 0$$

$$\Rightarrow i\langle 0|f_{x_1}(x_1)\Phi_{x_2}(x_2) \cdots \Phi_{x_k}(x_k)|0\rangle$$

$$+ i\langle 0|\Phi_{x_1}(x_1) f_{x_2}(x_2)\Phi_{x_3}(x_3) \cdots |0\rangle$$

$$+ \cdots = 0$$

$$\delta \phi_r(x) = \epsilon f_r(x)$$

So

if we multiply it by ϵ.

then

$$i\langle 0| f_{r_1}(x_1)\, \phi_{r_2}(x_2) \cdots \phi_{r_k}(x_k)|0\rangle$$
$$+ \langle 0| \phi_{r_1}(x_1)\, f_{r_2}(x_2) \cdots |0\rangle$$

$$= \frac{i}{\epsilon}\, \delta \langle 0| \phi_{r_1}(x_1) \cdots \phi_{r_k}(x_k)|0\rangle$$

if ϕ annihilates the vacuum

then

$$\langle 0| \phi_{r_1}(x_1) \cdots \phi_{r_k}(x_k)|0\rangle$$

is invariant under the infinitesimal symmetry transformation

if $x_i \to x_i + a$.

then translation invariant ~~too~~ suggest that

$$\langle 0| \phi_{r_1}(x_1) \cdots \phi_{r_k}(x_k)|0\rangle$$

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If the charge is not conserved then it is very hard to ensure that ~~on acting~~ Φ it is maximum invariant

$$\{\Phi, \Phi_r(x)\} = f_r$$
$$[\Phi, \Phi_r] = i f_r$$

} holds only for time argument is same & for all time.

$$\langle 0 | [\Phi, \Phi_r(x_1) \cdots \Phi_{r_n}(x_n)] | 0 \rangle$$

$$[\Phi(t), \Phi_r] = i f_r .$$

$\longrightarrow$ fixed time.

$$[\Phi(t_0), \Phi_r(x_1)] = i f_r .$$

$\longrightarrow$ even this time an diff, then it is valid only when Q is conserved.

$\Phi \longrightarrow$ real object in classical Mechanics but it goes to Hermitian operator

Φ if it is not Hermitian then we can symmetrize.

$\longrightarrow$ complex.

$$\psi = e^{i\alpha} \psi$$

$$\psi = \psi_1 + i \psi_2 .$$

then ~~$\psi_1 + i\psi_2$~~ $\psi_1 \longrightarrow \cos\alpha\, \psi_1 + \sin\alpha\, \psi_2$

$\psi_2 \longrightarrow -\sin\alpha\, \psi_1 + \cos\alpha\, \psi_2$

$S \longrightarrow$ ordinary matrix.

~~In the classical~~

In the classical theory there is a conserved charge Q such that it generates

$$\{ Q, \phi_x(x) \}_{PB} = f_x .$$

$$\langle 0 | [Q, \phi_{x_1}(x_1) \cdots \phi_{x_k}(x_k)] | 0 \rangle .$$

$$= 0$$

if $\quad Q|0\rangle = 0 ,$

Since $Q = Q^{\dagger}$.

$$\Longrightarrow \langle 0 | Q = 0$$

$$\epsilon \Big[\langle 0 | f_{x_1}(x_1) \phi_{x_2}(x_2) \cdots \phi_{x_k}(x_k) | 0 \rangle$$

$$+ \langle 0 | \phi_{x_1}(x_1) f_{x_2}(x_2) \phi_{x_3}(x_3) \cdots | 0 \rangle$$

$$+ \cdots \Big] = 0 .$$

$$\Longrightarrow S \langle 0 | \phi_{x_1}(x_1) \cdots \phi_{x_k}(x_k) | 0 \rangle$$

$$= 0$$

$$\Longrightarrow \langle 0 | \phi_{x_1}(x_1 + \epsilon f_{x_1}(x_1)) \phi_{x_2}((x_2) + \epsilon f_{x_2}(x_2)) \cdots | 0 \rangle$$

$$= \langle 0 | \phi_{x_1}(x_1) \cdots \phi_{x_k}(x_k) | 0 \rangle$$

$$\langle 0 | \hat{\Phi}_{r_1}(x_1) \cdots \hat{\Phi}_{r_k}(x_k) | 0 \rangle$$

$$= \langle 0 | \Phi_{r_1}(x_1) \cdots \Phi_{r_k}(x_k) | 0 \rangle$$

true for

$\longrightarrow$ infinitesimal transformation.

~~same as we can exponentiate~~

true for finite transformation, as we can think of it as many infinitesimal trans.

Φ - operators.

$$\langle 0 | \Phi_{r_1}(x_1) \cdots \Phi_{r_k}(x_k) | 0 \rangle$$

- pure fnc.

$$\langle 0 | \Phi_{r_1}(x_1) \cdots \Phi_{r_k}(x_k) | 0 \rangle$$

$$= H_{r_1 \cdots r_k}(x_1 \cdots x_k)$$

an ordinary fnc.

symmetry imposes a restriction on this function.

Translation symmetry :-

$$\hat{\Phi}_r(x) = \Phi_r(x+a)$$

$$\langle 0 | \Phi_{r_1}(x_1 + a)\, \Phi_{r_2}(x_2 + a) \cdots | 0 \rangle$$

$$= \langle 0 | \Phi_{r_1}(x_1)\, \Phi_{r_2}(x_2) \cdots | 0 \rangle$$

$$H_{r_1 r_2 \cdots r_k}(x_1, x_2, \cdots x_k, x)$$

$$= H_{r_1 r_2 \cdots r_k}(x_1 + a, x_2 + a, \cdots x_k + a)$$

H should depend on the arguements in term of differences.

2) If we know H in any coordinates then we can find it by all other coordinates shifted by equal dist.

Rotational Symmetry :-

$$S_{x_1 s_1} S_{x_2 s_2} \cdots S_{x_k s_k}$$

$$H(s_1 \ldots s_k (Mx_1, \ldots Mx_k))$$

$$= H_{x_1 \ldots x_k}(x_1 \ldots x_k)$$

Significance of H :- all operators are build of ϕ. to everything we are going to calculate can be calculated from H.

We can also calculate it in ~~terms of integrals of~~ any arbitrary state as they are action of a^+ which can be rewritten in terms of ϕ.

Go back to Klein Gordon theory

$$\Delta_+(x,y) = \langle 0 | \phi(x)\phi(y) | 0 \rangle .$$

$\phi \longrightarrow -\phi$ in K.G. theory is a symmetry. ~~odd~~ power fn. should be zero

$\cancel{\phi^3, =}$ $\phi \to -\phi$

$L \longrightarrow L$

$$\Delta_+(x,y) = \langle 0 | \phi(x)\phi(y) | 0 \rangle$$

(1) should depend on $(x-y)$

$$\Delta_+(x,y) = f(x-y)$$

2) rotn. sym

$$f(x-y) = f(M(x-y))$$

ϕ will create particle in position space.

index basically say that there are diff. fields.

$L = \phi \longrightarrow -\phi$.

$\cancel{f(\langle 0 | \phi \phi | 0 \rangle}$

each comb. of index

fell have diff. fn.

$$\phi(x) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \phi(\vec{k}, t)$$

$\hookrightarrow \vec{x}, x_0$ ..

$$\phi(\vec{k}, t) = \frac{\omega_{\vec{k}}^{-1/2}}{\sqrt{2}} \left\{ a(\vec{k}, t) + a^+(-\vec{k}, t) \right\}$$

$$\phi(y) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{y}} \phi(\vec{k}, t)$$

$\hookrightarrow y, y_0$ ..

$$a|0\rangle = 0$$

$$\langle a^+ | 0 = 0$$

$$= \int \frac{d^3k}{(2\pi)^{3/2}} \int \frac{d^3k'}{(2\pi)^{3/2}} \frac{e^{i\vec{k}\cdot\vec{x} + i\vec{k}'\cdot y}}{2\sqrt{\omega_{\vec{k}}\omega_{\vec{k}'}}}$$

$$\langle 0 | a(\vec{k}, x^0) a^+(-\vec{k}', y^0) | 0 \rangle$$

$$e^{i\vec{k}\cdot\vec{x}} + e^{i\vec{k}'\cdot y}$$

$$= \int \frac{d^3k}{(2\pi)^{3/2}} \int \frac{d^3k'}{(2\pi)^{3/2}} \frac{e^{i\vec{k}\cdot\vec{x} + i\vec{k}'\cdot\vec{y}}}{2\sqrt{\omega_{\vec{k}}\omega_{\vec{k}'}}}$$

$$\langle 0 | a(\vec{k}, x^0) a^+(-\vec{k}', y^0) | 0 \rangle$$

$$e^{i\vec{k}\cdot\vec{x} + i\vec{k}'\cdot\vec{y}}$$

$$H = \int d^3k\, \omega_{\vec{k}}\, a^+(\vec{k}, t)\, a(\vec{k}, t)$$

$$[H, H] = 0$$

So we can use equal time com. reln.

$$[H, a(\vec{k}, t)] = -\omega_{\vec{k}}\, a(\vec{k}, t)$$

$$[H, a^+(\vec{k}, t)] = \omega_{\vec{k}}\, a^+(\vec{k}, t)$$

$$i\frac{\partial a(\vec{k}, t)}{\partial t} = [a, H].$$

$$= \omega_k\, a(\vec{k}, t).$$

$$a(\vec{k}, t) = a(\vec{k}, 0)\, e^{-i\omega_k t}.$$

$$a^+(\vec{k}, t) = a^+(\vec{k}, 0)\, e^{i\omega_{\vec{k}} t}.$$

$$= \frac{1}{(2\pi)^3} \int d^3k \int d^3k' \frac{1}{2\sqrt{\omega_k \omega_{k'}}}$$

$$e^{i(\vec{k}\cdot\vec{x} + \vec{k}'\cdot\vec{y} - \omega_k x^0 + \omega_{k'} y^0)}$$

$$\langle 0 | a(\vec{k}, 0)\, a^+(-\vec{k}', 0) | 0 \rangle$$

$$\delta^{(3)}(\vec{k} + \vec{k}')$$

$$= \frac{1}{(2\pi)^3} \int d^3k \, \cancel{\int d^3k'} \, \frac{e^{i\vec{k}\cdot(x-y)}\, e^{-i\omega_k (x^0 - y^0)}}{2\omega_k}$$

$$= \frac{1}{(2\pi)^3} \int \frac{e^{-i\omega_k (x^0 - y^0)}}{2\omega_k} \cdot d^3k\, e^{i\vec{k}\cdot(\vec{x} - \vec{y})}$$

$\longrightarrow$ rotn. + translation invariance

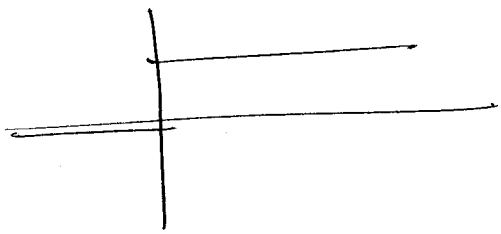
$= \Phi$

$$\frac{1}{(2\pi)^3}\int \frac{d^3k}{2\omega_k}\, e^{i\vec{k}\cdot(\vec{x}-\vec{y}) - i\omega_{\vec{k}}(x^0-y^0)}$$

$$\cancel{\delta^2(k^2 -}$$

$$\int dk^0\, \delta\left((k_0)^2 - k^2 - m^2\right)$$

$$\frac{\delta\left(k_0 - (k-m)\right) + \delta\left(k_0 - (k+m)\right)}{k-m}$$



$$\int_{-\infty}^{+\infty} \delta\left((k^0)^2 - \vec{k}^2 - m^2\right)\theta(k^0)$$

$$= \frac{1}{2\omega_k}.$$

$$k^0 = \pm\omega_k.$$

$$\frac{1}{(2\pi)^3}\int d^3k \int dk^0\, \delta\left((k^0)^2 - \vec{k}^2 - m^2\right)\cdot\theta(k^0)\, e^{i\vec{k}\cdot(\vec{x}-\vec{y}) - i\omega_{\vec{k}}(x^0-y^0)}$$

if we rewrite it as k_0 then it is ~~not~~ Lorentz invariant.

$$k^2 = m^2$$

$$-k^2 = -m^2$$

$$+k_0^2 - k^2 = m^2 \Rightarrow k_0^2$$

$\delta((k^0)^2 - \vec{k}^2 - m^2)$ has solu. only

with $k^0 \longrightarrow \omega_k$

$$\frac{1}{(2\pi)^3}\int d^3k \, e^{i\vec{k}\cdot(\vec{x}-\vec{y}) - ik^0(x^0-y^0)}$$

if we put have $\Theta(k)$ fac.

the

$$\int dk^0 \, \delta((k^0)^2 - \vec{k}^2 - m^2)\, e^{-ik^0(x^0-y^0)}$$

$$\neq \int dk^0 \, \delta((k^0)^2 - \vec{k}^2 - m^2)\, e^{-i\omega_k(x^0-y^0)}$$

if we multiply ~~by~~ in by $\Theta(k)$

then the equality holds.

$$k^2 = -k_0^2 + k^2$$

$$= \frac{1}{(2\pi)^3}\int d^4k \, \delta(-k^2 - m^2)\, \Theta(k^0)\, e^{ik\cdot(x-y)}$$

$$k\cdot(x-y) = -k^0(x^0-y^0) + \vec{k}\cdot(\vec{x}-\vec{y})$$

$$k^2 = -k_0^2 + \vec{k}^2,$$

$$\longrightarrow \frac{1}{(2\pi)^3}\int d^4k \, \delta(-k^2-m^2)\, \Theta(k^0)\, e^{-i\vec{k}\cdot(\vec{x}-\vec{y})}$$

Because of this delta fnc. it is Lorentz invariant

$$x \longrightarrow Mx \qquad \cancel{M^T \eta M = \eta}.$$
$$y \longrightarrow My$$

$$\Delta_+(Mx, My) = \Delta_+(x, y)$$

$$= \frac{1}{(2\pi)^3} \int d^4k \; \delta(-k^2 - m^2)\, \theta(k^0)\, e^{i\hat{k}\cdot(x-y)}$$

or

$$= \frac{1}{(2\pi)^3} \int d^4k \; \delta(-k^2 - m^2)\, \theta(k^0)\, e^{ik_\mu M^\mu{}_\nu (x^\nu - y^\nu)}$$

$$k_\mu M^\mu{}_\nu = \hat{k}_\nu.$$

$\cancel{+k_0^2 - k^2 = m^2}$

$k_0^2 - k^2 = m^2$

$$d^4k = d^4\hat{k}.$$

$$\delta(-k^2 - m^2) = \delta(-\hat{k}^2 - m^2)$$

$\cancel{\theta(}$

$\cancel{\theta(k^0)}$ if under transformation

k^0 & $\hat{k}^0$ has same sign

$$\theta(k^0)\, \delta(-k^2 - m^2) = \theta(\hat{k}^0)\, \delta(-\hat{k}^2 - m^2)$$

if k is space like.

~~the~~ $\theta(k_0)\, \delta(-k^2 - m^2) = 0$.

when k is space like / light like then it is 0.

So they are same.

If ~~from~~ we have time like

$$(k^0)^2 > k^2$$

then under any cont. Lorentz transformation

k^0 can not go to zero.

So under a cont transformation k^0 can not go to 0

So it never changes sign.

$$\Delta_+(M_x, M_y) = \frac{1}{(2\pi)^3} \int d^4\tilde{k}\, \delta(-\tilde{k}^2 - m^2)\, \Theta(\tilde{k}^0)\, e^{i\tilde{k}\cdot(x-y)}.$$

$$\Delta_+(x,y) = \langle 0 | \phi(x)\phi(y) | 0 \rangle.$$

$$\Delta(x,y) = \langle 0 | [\phi(x), \phi(y)] | 0 \rangle.$$

$$= \Delta_+(x,y) - \Delta_+(y,x)$$

$$\frac{1}{(2\pi)^3} \int d^4k\, \delta(-k^2 - m^2)\, \Theta(k^0) \left\{ e^{ik\cdot(x-y)} - e^{-ik\cdot(y-x)} \right\}.$$

$k^{0\,2} - \omega_{\vec{k}}^2$

$$= \frac{1}{(2\pi)^3} \int \frac{d^3k}{2\omega_{\vec{k}}} \left[e^{i\vec{k}\cdot(\vec{x}-\vec{y}) - i\omega_{\vec{k}}(x^0-y^0)} - e^{-i\vec{k}\cdot(\vec{x}-\vec{y}) + i\omega_{\vec{k}}(x^0-y^0)} \right]$$

~~$\delta(x$~~

$\Delta(x, y)$ - should vanish if $x - y$ is space like.

$\longrightarrow$ it is Lorentz invariant

$$\int_{-\infty}^{+\infty} dk_x \longrightarrow \int_{-\infty}^{+\infty} d\hat{k}_x$$

$$\hat{k}_x \longrightarrow -k_x.$$

$\Delta(x, y) = 0$ for space like.

if $x^0 - y^0$ is too big then it becomes time like.

$\Phi(x)$ - coordinates.

$\Delta_F(x,y)$

$= \Theta(x^0 - y^0) \langle 0| \phi(x)\phi(y)|0\rangle$

$+ \Theta(y^0 - x^0) \langle 0|\phi(y)\phi(x)|0\rangle$

$= \langle 0| T(\phi(x)\phi(y))|0\rangle$

$x^0 > y^0$

$x^0 < y^0$

$x^0 > y^0$ put $\phi(x)$ first

if $y^0 > x^0$ put $\phi(y)$ first.

if $x^0 = y^0$, then

$\langle 0|\phi(x)\phi(y)|0\rangle = \langle 0|\phi(y)\phi(x)|0\rangle$

Feynman's propagator

$\Delta_F(x,y)$

$= \Theta(x^0 - y^0)\Delta_+(x,y) + \Theta(y^0 - x^0)\Delta_+(y,x)$

for time like event

$\Theta(x^0 - y^0)$ & $\Theta(y^0 - x^0)$ are

Lorentz invariant

for space like

Define :-

$$\Delta_F(x,y) = i \int \frac{d^4k}{(2\pi)^4} \frac{1}{-k^2 - m^2 + i\epsilon} e^{ik\cdot(x-y)}$$

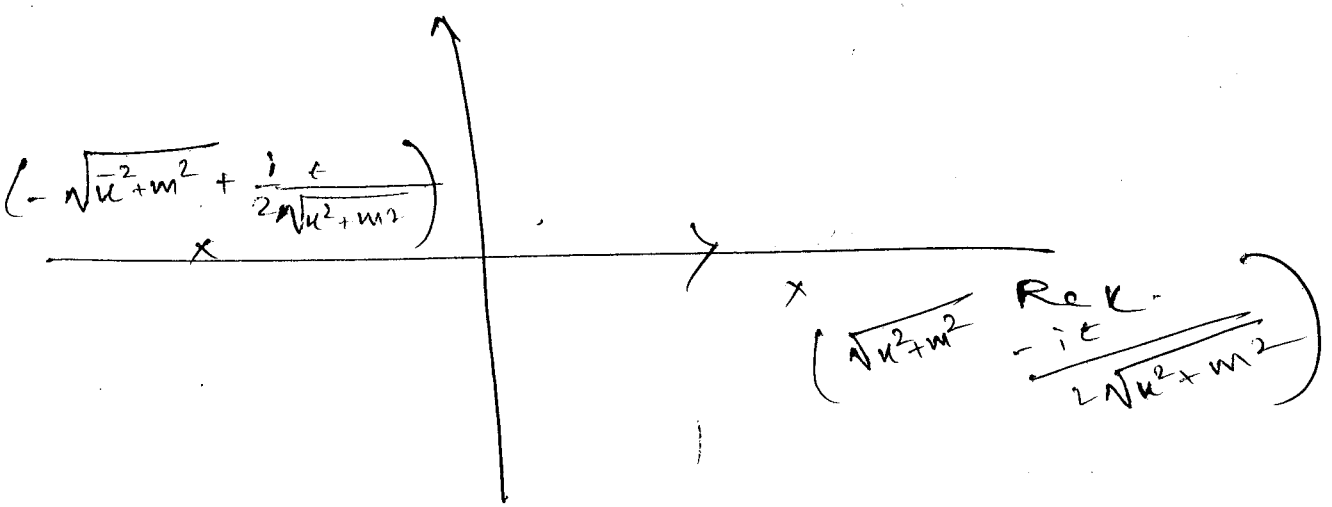
ϵ - small +ve no. which should be taken to zero at the end.

$$i \int \frac{d^3k}{(2\pi)^3} \int \frac{dk_0}{2\pi} \frac{1}{(k_0)^2 - (\vec{k}^2 + m^2 - i\epsilon)}$$

$$e^{-ik_0(x^0 - y^0) + i\vec{k}\cdot(\vec{x} - \vec{y})}$$

$$= i \int \frac{d^3k \; e^{i\vec{k}\cdot(\vec{x}-\vec{y})}}{(2\pi)^3} \frac{1}{(2\pi)} \int \frac{dk_0 \; e^{-ik_0(x^0 - y^0)}}{\left(k^0 - \sqrt{\vec{k}^2 + m^2 - i\epsilon}\right)\left(k_0 + \sqrt{\vec{k}^2 + m^2 - i\epsilon}\right)}$$

Contour integral



$$k^0 = \pm\sqrt{\vec{k}^2 + m^2 - i\epsilon}$$

$$= \pm\sqrt{\vec{k}^2 + m^2}\left(1 - \frac{i\epsilon}{2(k^2 + m^2)}\right)$$

$$= \pm\sqrt{\vec{k}^2 + m^2} \mp \frac{i\epsilon}{2\sqrt{k^2 + m^2}}$$

if $x^0 - y^0 > 0$

close the contour from below

$$\int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} \frac{1}{2\pi} \times -2\pi i \; e^{-i\omega_k (x^0 - y^0) + i\vec{k}\cdot(\vec{x}-\vec{y})}$$

$$= \Delta_+ (x - y).$$

~~$\Delta - m$~~

Ex :- For $x^0 - y^0 < 0$,

$$\Delta_F (x, y) = \Delta_+ (y, x).$$

Check when ϵ is -ve.

Lecture 14 24/02/2011

$$\Delta_+ (x,y) = \langle 0 | \phi(x) \phi(y) | 0 \rangle$$

$$\Delta_F (x-y) = \theta(x^0 - y^0) \Delta(x,y) + \theta(y^0 - x^0) \Delta(y,x).$$

K.G. eqn :-

$$\eta^{\mu\nu} \partial_\mu \partial_\nu \phi - m^2 \phi = 0$$

$$(\Box - m^2) \phi = 0.$$

classical eqn. for ϕ.

$(\Box_x - m^2)\phi(x) = 0$ → operator

$\langle 0 | (\Box_x - m^2) \phi(x) | 0 \rangle = 0$

In classical ~~[illegible]~~ mechanics, Heisenberg ~~[illegible]~~ of QM satisfies the eqn. operator

$$\langle 0 | \{ (\Box_x - m^2) \phi(x) \} \phi(y) | 0 \rangle$$

$$\Box = \eta^{\mu\nu} \partial_\mu \partial_\nu$$

$$(\Box_x - m^2) \langle 0 | \phi(x) \phi(x) | 0 \rangle = 0$$

state (time not dependent)

$$(\Box_x - m^2)\, \Delta_+(x,y) = 0 \longrightarrow$$ it doesn't specify boundary cond. So that each

$$\Delta_+(x,y) = \int \frac{d^4k}{(2\pi)^3}\, \delta(-k^2-m^2)\, \theta(k_0)\, e^{ik\cdot x}$$

$$\cancel{(\Box_x - m^2)}$$

States doesn't depend on x.

All time dependence is in operator.

$$(\Box_x - m^2)\, \Delta_+(x,y)$$

even in spatial derivative can be taken out

even in spatial derivative that can be taken out

$$(\Box_x - m^2)\, \Delta_+(x,y) = 0$$

$$(\Box_x - m^2) \cdot \left[\int (-k^2-m^2)\, \frac{d^4k}{(2\pi)^3}\, \delta(-k^2-m^2)\, \theta(k_0)\, e^{ikx}\right]$$

$$= 0$$

because of δ fnc.

$$\Box_x - m^2 = -\frac{\partial^2}{\partial x_0^2} + \vec{\nabla}_x^2 - m^2$$

$$\frac{\partial}{\partial x_0}\Delta_+(x,y) = \cancel{\delta(x^0-y^0)\, \Delta_+(x,y)} + \theta(x^0-y^0)\frac{\partial}{\partial x_0}\Delta_+(x,y)$$

$$- \cancel{\delta(x^0-y^0)\, \Delta_+(y,x)} + \theta(x^0-y^0)\frac{\partial}{\partial x_0}\Delta_+(y,x)$$

$\delta(x^0-y^0)$ is non vanishing when $x^0 = y^0$.

$\Delta_+(x,y) = \Delta_+(y,x)$

$$\frac{\partial}{\partial x_0}\Delta_F(x,y) = \theta(x^0-y^0)\frac{\partial}{\partial x_0}\Delta_+(x,y) + \theta(x^0-y^0)\frac{\partial}{\partial x_0}\Delta_+(y,x)$$

$$\frac{\partial^2}{\partial x_0^2}\Delta_F(x,y) = \delta(x^0-y^2)\frac{\partial^2}{\partial x_0^2}\Delta_+(x,y) + \theta(x^0-y^0)\frac{\partial^2}{\partial x_0^2}\Delta_+(x,y)$$

$$- \delta(x^0-y^0)\frac{\partial}{\partial x_0}\Delta_+(y,x) + \theta(x^0-y^0)\frac{\partial^2}{\partial x_0^2}\Delta_+(y,x)$$

$$\left(-\frac{\partial}{\partial x_0^2}\Delta_F(x,y) + \vec{\nabla}_x^2 - m^2\right)\Delta(x,y)$$

$$= -\delta(x^0-y^0)\frac{\partial}{\partial x_0}\left(\Delta_+(x,y)\right).$$

$$+\theta(x^0-y^0)\cancel{\left(-\frac{\partial^2}{\partial(x^0)^2} + \vec{\nabla}_x^2 - m^2\right)}\Delta_+(x,y)$$

$$+ \delta(x^0-y^0)\frac{\partial}{\partial x_0}\Delta_+(y,x)$$

$$+\theta(y^0-x^0)\cancel{\left(-\frac{\partial}{\partial(x^0)^2} + \nabla_x^2 - m^2\right)}\Delta_+(y,x)$$

$$(\Box_x - m^2)\Delta_F(x,y)$$

$$= -\delta(x^0-y^0)\langle 0|\frac{\partial\phi}{\partial x^0}(x)\phi(y)|0\rangle$$

$$+ \delta(x^0-y^0)\langle 0|\phi(y)\frac{\partial\phi}{\partial x^0}|0\rangle$$

$$\frac{\partial \Phi_x}{\partial x^0} = \pi(x) \qquad \pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

$$= \delta(x^0 - y^0)\, i\, \delta^{(3)}(\vec{x} - \vec{y}) \qquad -k_0^2 + k^2 = k^2$$

$$= i\, \delta^{(4)}(x - y)$$

because of $(\Box_x - m^2)$ there is $-k^2 - m^2$ in the numerator which cancels the $-k^2 - m^2$ in the denominator. So ϵ doesn't have any role.

Next we'll calculate

$$\langle 0 | T(\Phi(x_1) \ldots \Phi(x_k)) | 0 \rangle$$

increasing time

Free Klein Gordon theory → free field theory.

If we add interaction, then we will treat the interaction as perturbation theory

In QM

Result in the perturbation theory comes from the matrix element of the unperturbed state

In perturbation we calculate matrix element operator

Thus we need to calculate the matrix element various operator

Let us assume some time order,

suppose

$$x^0_{i_1} > x^0_{i_2} \cdots > x^0_{i_k}$$

$(i_1, i_2, \cdots i_k)$ some permutation

of $1, 2, \cdots k$.

In this case

$$\langle 0 | T(\phi(x_1) \cdots \phi(x_k)) | 0 \rangle$$

$$= \langle 0 | \phi(x_{i_1}) \phi(x_{i_2}) \cdots \phi(x_{i_k}) | 0 \rangle$$

rewrite each ϕ as creation annihilation operator,

$$\int (a + a^+) \quad (a + a^+)$$

$$\Delta_+(x_{i_1}, x_{i_2})$$

it should be same as

$$\langle 0 | \phi_{i_1}(x_{i_1}) \phi_{i_2}(x_{i_2}) | 0 \rangle$$

for x_{i_3} x_{i_4}

$$\Delta_+(x_{i_1}, x_{i_2}) \Delta_+(x_{i_3}, x_{i_4}) \cdots \Delta_+(x_{i_k}, x_{i_k})$$

$$+ \Delta_+(x_{i_1}, x_{i_3}) \Delta_+(x_{i_2}, x_{i_4}) \Delta_+(x_{i_5}, x_{i_6}) \cdots$$

all possible pairings.

$\binom{k}{2}\frac{1}{2!}$ no. of terms is

Ex:- Count ~~then~~ based on the one ~~reader~~ large r time will come ~~first~~

But if we use a diff time order, the ordering will change & it won't be ~~invariant~~ same form

But if we use Feynman propagator it automatically keeps the order.

$= \Delta_F(x_1, x_2)\,\Delta_F(x_3, x_4) \cdots \Delta_F(x_{k-1}, x_k)$

+ all possible order pairings

Ex:- check this for $\langle 0 | T\,\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) | 0 \rangle$

$(k-1)(k-3)(k-5)\cdots 1$

~~$k-1\,C_2$ $k-2$~~

${}^{k}C_2 \quad {}^{k-2}C_2 \quad {}^{k-4}C_2 \cdots$

Diagrammatic way :-

Represent each ~~pt~~ ϕ as a pt. x.

$x^1 \quad x^2 \quad x^3 \quad x^k$

Every pt. should have 1 line coming out of it.

Any line joining i to j

→ a factor $\Delta_F(x_i, x_j)$

(2) In any ~~like~~ graph — take the product over contribution from all lines.

There is no constraint over x_2 & x_i

So $x_2 = x_1$

$$|\psi(x)|^2 = \langle \psi(x) | \psi(y) \rangle$$

$$\langle 0 | x \rangle \quad \langle 0 | \phi(x)$$

Coherent states

$$\exp(\lambda a_n^\dagger)|0\rangle$$

For fermions

$a_n^\dagger$ | applied only once.

$(+1)\frac{u^* b u}{2!}$ → N!

$\frac{N!}{2}$ (+), $\frac{N!}{2}$ (−)

Saturday – 11:00

Monday – 5:00 – 9:00 – Lecture 13, Lecture 14, Lecture 15, Lecture 16

Tuesday – 7:00 – 11:00 – Lecture 17, Lecture 18, Lecture 19

Wednesday & Lecture 20

Thursday – Revision

Friday

μ_1

$\mu_2 u$

$\frac{N!}{2}$ – + , –

$\frac{N!}{2}$ – –

21:00 – Lecture 8

01:00 – Lecture 9

9:00 – 4:00 – Lecture 10

8:30 – 12:30 – Lecture 11

6:30 – 10:30 – Lecture 12