10/11/06

$$S_{tot} = S + S_{g.f} + S_{ghost}$$

$S_{g.f}$ ↓ gauge fixing term

<u>Example</u> :- Take the gauge fixing fn.

$$\mathcal{H}_A \to \partial^\mu A_\mu^a(x)$$

$$S_{tot} = -\frac{1}{4}\int d^4x \, F_{\mu\nu}^a(x) F^{a\mu\nu}(x)$$
$$- \frac{1}{2\alpha}\int d^4x \, \partial^\mu A_\mu^a(x) \partial^\nu A_\nu^a(x)$$
$$+ \int d^4x \, b_a(x) \times \Box_x c_a(x)$$
$$\cancel{+ g f^{bca} \int d^4x} + g f^{bca}\int d^4x \, \partial^\mu b_a(x) A_\mu^c(x) c_b(x)$$
$$+ S_{fermion} + S_{scalar}$$

+ coupling between fermions & scalars

$$F_{\mu\nu}^c = \partial_\mu A_\nu^c - \partial_\nu A_\mu^c + g f^{abc} A_\mu^a A_\nu^b$$

<u>Step 1</u> :- Count dimension of each term.

$$S_{free} = -\frac{1}{4}\int d^4x \underbrace{(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)(\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a})}_{\text{dimension } 4}$$

Identify the term with no coupling constant ↓ then we can read out the dimn of the field

$\Rightarrow A_\nu^a$ has dimension 1.

(The term $\int d^4x \, b_a(x) \Box_x c_a(x)$ tells us) ⇒

$b_a$ has dimension $1+\lambda$
$c_a$ has dimension $1-\lambda$

We'll choose $\lambda = 0$ (for convenience)
↓
this is allowed bcos everywhere we have the combn. of $b_a$ & $c_a$ together

look at free fermion & free scalar action → det. dim. of $\phi$ & $\psi$

$\alpha$ has dimension zero
$g$ has dimension zero

$F_{\mu\nu}^{c} = \partial_\mu A_\nu^c - \partial_\nu A_\mu^c + g f^{abc} A_\mu^a A_\nu^b$

So $g$ must have dim. zero
$\rightarrow$ every term should have same dim.

---

$$F_{\mu\nu}^a F_{\mu\nu}^a = (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)(\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a})$$
$$+ 2 g f^{abc} (\partial_\mu A_\nu^c - \partial_\nu A_\mu^c) A^{\mu a} A^{\nu b}$$
$$+ g^2 f^{abc} f^{a'b'c} A_\mu^a A_\nu^b A^{\mu a'} A^{\nu b'}$$

$\boxed{\text{Dim. of every op.} \leq 4}$

(Each term has dimension 4)

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All terms have dimension 4 operators.
(Here all coupling constants have dim. $\geq 0$
$\Rightarrow$ so all op. have dim. $\leq 4$)
$\rightarrow$ First condition of power counting renormalizability is satisfied.

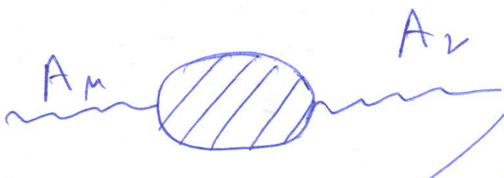
---

But terms like $\int d^4x \, A_\mu^a A^{\mu a}$, $\int d^4x \, b^a c^b A_\mu^a A^{\mu e}$ $\times$ contraction of $a, b, c, d$ (Kronecker delta or $f^{abc}$)

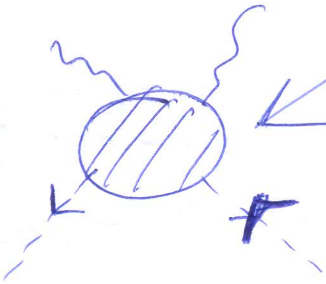
are not there.

$\therefore$

(2 Gauge field prop.) $\rightarrow$




could have uncancelled divergence



could have uncancelled divergence

this has an intrinsic div. & not subdivergences which could be cancelled by other counterterms

Also, 3-pt. coupling ($g \times$ --) of gauge fields is related to the 4-pt. coupling ($g^2 \times$ ---) of gauge fields

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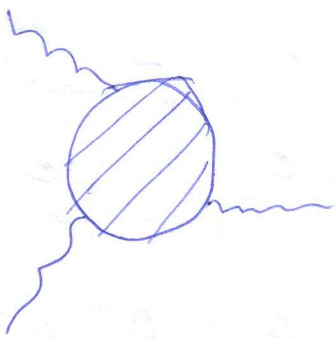
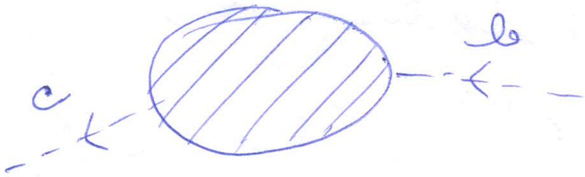


is made finite by adjusting $Z_g \tilde{Z}_A^{3/2}$

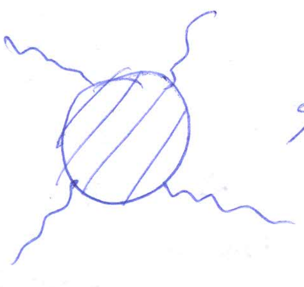

requires adjusting $\tilde{Z}_A$

(this diagram is quadratically divergent — not only the leading term, but also the term with first derivative w.r.t. k is div. in the Taylor series expn... --- adjust the mass term)



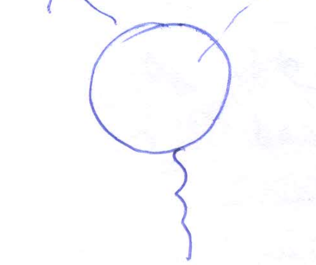

requires adjusting $\tilde{Z}_b^{1/2} \tilde{Z}_c^{1/2}$

Ghost fields should be related to renormalized ghost fields by multiplicative factors



requires adjusting $\tilde{Z}_A^2 Z_g^2 \rightarrow$ (bcos we had $g^2$)

(But this has already been det. by making the lower order diag. finite).



requires adjusting $\underbrace{\tilde{Z}_b^{1/2} \tilde{Z}_c^{1/2} \tilde{Z}_A^{1/2} Z_g}_{\text{not adjustable}} \rightarrow$ (bcos we had a $g$)

The problem becomes more complicated if we include fermions, scalars, etc.

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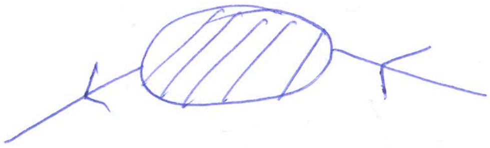
- consider fermions $\psi$, $\bar{\psi}$ with mass m.



require adjusting $\tilde{Z}_\psi$, $Z_m \hat{Z}_\psi$

require adjusting $\tilde{Z}_\psi \tilde{Z}_A^{1/2} Z_g$

($Z_A$ & $Z_g$ can no longer be independently adjusted here)

~~$\bar{\psi}m\psi$~~ $\bar{\psi} m \psi$ & $\bar{\psi}(i\not{\partial}\psi)$ will be involved

You can write $\tilde{Z}_\psi \tilde{Z}_{\bar{\psi}}$, but they always appear as $\psi\bar{\psi}$ combn. ~~so~~ — no need to consider them separately

But our original gauge inv. Lagrangian had all ~~term~~ op. with dim. $\leq 4$ — the problem is that we are not quantising that action directly — We have broken manifest gauge inv. by gauge fixing, adding ghost fields, etc. — Our Feyn. rules don't have the original gauge inv. we began with — the fact that original 3-pt. fn. was related 4-pt. fn., etc. is no longer manifest

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Ward identities that we derive from sym. various Green's fns by certain eqns — so if LHS finite, RHS must be.

We will show that the gauge fixed action is invariant under certain symmetry trs. known as **BRST symmetry**.

Symmetry trs. parameter $\xi$ is grassman number.

So it will anticommute with $\psi$, $\bar{\psi}$, $b^a$, $c^a$ etc.

(not allowed to be fn. of ~~s~~pace-time coord. — it's a single no.)

Trs. laws of gauge fields, scalars & fermions are identical to infinitesimal trs. of these fields under a gauge trs. with parameter $\xi c^a(x)$.

e.g. → $\partial_\mu \epsilon(x)$ ↓ replace by $\xi \partial_\mu c^a(x)$

$$\therefore \delta c^a(x) = -\tfrac{1}{2} f^{bda} \xi c^d(x) c^b(x)$$

gauge trs. laws for $c^a$

~~If the gauge~~

If we choose

$$S_{g.f.} = -\frac{1}{2\alpha}\int d^4x\, F^a(x) F^a(x)$$

↳ fn. of various fields

e.g. → $\partial^\mu A^a_\mu(x)$

$$\delta b^a(x) = -\tfrac{1}{\alpha} F_a(x)\, \xi$$

**Claim** :- $\delta\left(S + S_{g.f.} + S_{ghost}\right) = 0$

(invariant part)

Problem Set 2: Date Due: October 20, 2006

1. Consider an action for a scalar field $\phi$ coupled to a fermion field $\psi$:

$$S = \int d^4x \Big[ -\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2 + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + \lambda\bar{\psi}\gamma^\mu\psi\partial_\mu\phi \Big]$$

   (a) Derive the Feynman rules for this theory.

   (b) Using these Feynman rules calculate

$$\langle \tilde{\psi}_{\alpha_1}(k_1)\tilde{\psi}_{\alpha_2}(k_2)\tilde{\bar{\psi}}_{\beta_1}(p_1)\tilde{\bar{\psi}}_{\beta_2}(p_2)\rangle_c$$

   to order $\lambda^2$. Here $\langle\ \rangle_c$ denotes connected Green's function. → all external lines are connected to each other

2. In a non-abelian gauge theory based on the gauge group $G$, consider a gauge transformation of the gauge field $B^a_\mu$ by the group valued function $U_2(x)$, followed by another gauge transformation by the group valued function $U_1(x)$. Show that this is equivalent to transforming the original gauge field by the group valued function $U_1(x)U_2(x)$.

Problem Set 3: Date Due: November 25, 2006

1. Derive the Feynman rules of quantum electrodynamics if we choose the gauge fixing term in the action to be

$$-\frac{1}{2\alpha}\int d^4x\, H(x,A)H(x,A)$$

where

$$H(x,A) = \partial^\mu A_\mu + A^\mu A_\mu\,.$$

2. Consider a parametrization of the SU(2) group element as

$$U = \exp(i\alpha\sigma_3)\exp(i\beta\sigma_1)\exp(i\gamma\sigma_3)$$

where $\sigma_1$, $\sigma_2$ and $\sigma_3$ are Pauli matrices. Find an expression for the Haar measure of the group in terms of the parameters $\alpha$, $\beta$, $\gamma$.

3. Consider a field theory of a scalar field $\phi$ and a fermion field $\psi$ with action:

$$S = \int d^4x\left[-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}M^2\phi^2 + \bar\psi(i\gamma^\mu\partial_\mu - m)\psi\right]$$

   (a) Now introduce a new fermion field $\chi$ through the relation:

   $$\psi = e^{i\lambda\phi}\chi$$

   and express the action in terms of the fields $\phi$ and $\chi$. Here $\lambda$ is a constant.

   (b) Now forget about the original action and work with this new action regarding $\phi$ and $\chi$ as independent fields, and $\lambda$ as a small parameter. To order $\lambda$, calculate the S-matrix element relevant for computing the decay of a $\phi$ particle into two $\chi$ particles. Assume that $M > 2m$ so that this decay is energetically possible.

4. Consider a field theory of a fermion field $\psi$ and a scalar field $\phi$ with action:

$$S = \int d^4x\left[-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}M^2\phi^2 + i\bar\psi\gamma^\mu\partial_\mu\psi + g\,\bar\psi\psi\phi\right]$$

where $g$ is a coupling constant. Determine the minimal set of other terms you will need to add to the action to make the theory renormalizable.

$\bar\psi\gamma_5\psi\phi$ is not there for the reality condn.

$\bar\psi_L\psi_R + \bar\psi_R\psi_L$

Possible symm.??

$\psi_R \to -\psi_R$

$\psi_L \to \psi_L$

$\phi \to -\phi$

$\bar\psi\psi$, $\phi^3$, $\phi$ ruled

12/11/06

$$S_{total} = S + S_{g.f.} + S_{ghost}$$

Consider a gauge theory with gauge group $G$ & fermions $\psi_s$ in represented $R_f$ and scalars $\phi_k$ in representation $R_s$.

Infinitesimal gauge trs. $\Rightarrow$

$$\delta A^a_\mu(x) = -\partial_\mu \theta^a(x) - g f^{bca} \theta^c(x) A^b_\mu(x)$$

$$\delta \psi_s(x) = -ig\, \theta^a(x) \left(R_f(T^a)\right)_{st} \psi^t(x)$$

$$\delta \phi_k(x) = -ig\, \theta^a(x) \left(R_s(T^a)\right)_{kl} \phi^l(x)$$

(they together give the symmetry of $S$)

(It's a sym. of $S$ but not of the 2 terms $S_{g.f.}$ & $S_{ghost}$)

Choose

$$S_{g.f.} = \frac{1}{2\alpha} \int d^4x \; F^a(x) F^a(x)$$

(repeated indices are summed over)

$F^a$ → some fn. of fields

e.g. $\partial_\mu A^{\mu a}(x)$ → (for Lorentz gauge)

Claim :- $S_{total}$ has a BRST symmetry with grassman valued symmetry trs. parameter $\zeta$.

$A^a_\mu(x)$, $\psi_s(x)$ & $\phi_k(x)$ transform as if we have an infinitesimal gauge trs. with parameter $\xi\, c^a(x)$.

The prodt. $\xi c^a$ is grassman even bcos both $\xi$ & $c^a$ are grassman odd

$$\delta A^a_\mu(x) = -\partial_\mu(\xi\, c^a(x)) - g f^{abc} c^c(x) A^b_\mu(x)$$

$$\delta\psi_s(x) = -ig\,\xi\, c^a(x)\,(R_f(T^a))_{st}\,\psi^t(x)$$

$$\delta\phi_R(x) = -ig\,\xi\, c^a(x)\,(R_s(T^a))_{kl}\,\phi^l(x)$$

(Trs. of $b^a(x)$ & $c^a(x)$ $\rightarrow$ ($c$ trs. fixed by $f^{bca}$, i.e., by the gauge algebra)

$$\delta c^a(x) = -\tfrac{1}{2} g f^{bca}\,\xi\, c^c(x)\, c^b(x)$$

$$\delta b^a(x) = -\tfrac{1}{\alpha} F_a(x)\,\xi$$

($b$ trs. of $b^a$ depends on the choice of the gauge fixing fn.)

Both LHS & RHS are grassman odd - so Grassman prop. are okay

BRST trs. is a rigid trs. - global sym. of the gauge fixed action - not a gauge trs.

(Trs. of matter field is as if we are doing a gauge trs. with this strange gauge parameter) $\Rightarrow$

$\boxed{\delta S = 0}$ due to gauge invariance

$$\delta S_{gf} = -\tfrac{1}{\alpha}\int d^4x\; F_a(x)\,\delta F_a(x)$$

(depends only the matter fields - no dep. on ghost fields)

$$= -\tfrac{1}{\alpha}\int d^4x\; F_a(x)\int \left.\frac{\delta F^{\theta}_a(x)}{\delta\theta^b(y)}\right|_{\theta=0} \xi\, c^b(y)\, d^4y$$

where $F_a^{\theta}(x)$ : Transform of $F_a(x)$ under a gauge trs. $\theta$

---

$S_{ghost}$

$$= -\int d^4x\, d^4y\, b^a(x) \left.\frac{\delta F_a^{\theta}(x)}{\delta\theta^b(y)}\right|_{\theta=0} c^b(y)$$

It's a functional deriv. & so we have an integral

$F_a^{\theta}(x)$ – It's actually an infinite no. of $\theta$

Matrix in the ab-space as well as in the xy-space

$\delta S_{ghost}$

$$= -\int d^4x\, d^4y\, \delta b^a(x) \frac{\delta F_a^{\theta}(x)}{\delta\theta^b(y)} c^b(x)$$

$$- \int d^4x\, d^4y\, b^a(x)\, \delta\left(\frac{\delta F_a^{\theta}(x)}{\delta\theta^b(y)} c^b(y)\right)$$

$$= \frac{1}{\alpha}\int d^4x\, d^4y\, F_a(x)\, \xi \left.\frac{\delta F_a^{\theta}(x)}{\delta\theta^b(y)}\right|_{\theta=0} c^b(y)$$

$$- \int d^4x\, d^4y\, b^a(x)\, \delta\left(\left.\frac{\delta F_a^{\theta}}{\delta\theta^b}\right|_{\theta=0} c^b(y)\right)$$

the first term of $\delta S_{ghost}$ cancels $\delta S_{gf}$

Hence, $$\delta S_{total} = -\int d^4x\, d^4y\, b^a(x)\, \delta\left(\left.\frac{\delta F_a^{\theta}}{\delta\theta^b}\right|_{\theta=0} c^b(y)\right)$$

$$\Rightarrow \delta S_{total} = -\int d^4x\, d^4y\, b^a(x)\left[\left.\delta\left(\frac{\delta F_a^{\theta}(x)}{\delta\theta^b(y)}\right)\right|_{\theta=0} c^b(y) + \left.\frac{\delta F_a^{\theta}(x)}{\delta\theta^b(y)}\right|_{\theta=0} \delta c^b(y)\right]$$

depends on matter fields & gauge fields

$$\Rightarrow \delta S_{\text{total}} = -\int d^4x\, d^4y\, b^a(x) \Big[ \int \frac{\delta}{\delta\phi^d(z)} \left( \frac{\delta F_a^{\theta}(x)}{\delta\theta^b(y)} \Big|_{\theta=0} \right)^{\phi} \Big|_{\phi^e(0)} \xi\, c^d(z)\, d^4z \quad c^b(y)$$

$$+ \frac{\delta F_a^{\theta}(x)}{\delta\theta^b(y)} \Big|_{\theta=0} \left( -\tfrac{1}{2} g f^{dcb} \xi\, c^c(y)\, c^d(y) \right) \Big]$$

[where $\left( \frac{\delta F_a^{\theta}(x)}{\delta\theta^b(y)} \Big|_{\theta=0} \right)^{\phi}$ is the transform of

of $\frac{\delta F_a^{\theta}(x)}{\delta\theta^b(y)} \Big|_{\theta=0}$ under a gauge trs. $\phi$]

---

$\frac{\delta}{\delta\phi^d(z)} \left( \quad " \quad \right)$ involves 2 successive gauge trs.

$\frac{\delta F_a(x)}{\delta\theta^b(y)}$ involves a single deriv. — single gauge trs.

→ in general single & double deriv. aren't related

---

But commutator of gauge trs. = a single gauge trs.

bcos " " 2 generators = a single generator

---

$$\therefore \delta S_{\text{total}} = -\int d^4x\, d^4y\, d^4z\, b^a(x)\, \xi\, \underbrace{c^d(z)\, c^b(y)}_{\text{antisym. under simultaneous exchange of } b \,\&\, d, \,\&\, z \,\&\, y}$$

$$\frac{1}{2} \Big[ \frac{\delta}{\delta\phi^d(z)} \left( \frac{\delta F_a^{\theta}(x)}{\delta\theta^b(y)} \Big|_{\theta=0} \right)^{\phi} \Big|_{\phi=0}$$

$$- \frac{\delta}{\delta\theta^b(y)} \left( \frac{\delta F_a^{\phi}(x)}{\delta\phi^d(z)} \Big|_{\phi=0} \right)^{\theta} \Big|_{\theta=0} \Big]$$

$$- \int d^4x\, d^4y\, \frac{\delta F_a^{\theta}(x)}{\delta\theta^b(y)} \Big|_{\theta=0} b^a(x) \left( -\tfrac{1}{2} g f^{dcb} \xi\, c^c(y)\, c^d(y) \right)$$

(The commutator of the 2 double deriv. can be regarded as a single deriv.)

# Given any quantity $\mathcal{S}$,

$$\int \left[ \frac{\delta}{\delta \phi^d(z)} \left( \frac{\delta \mathcal{S}_a^{\theta}(x)}{\delta \theta^b(y)} \Bigg|_{\theta=0} \right)^{\phi} \right]_{\phi=0} \xi^d(z)\, \eta^b(y)\, d^4z\, d^4y$$

localised at $z$ ; localised at $y$

$$- \int \left[ \frac{\delta}{\delta \theta^b(z)} \left( \frac{\delta \mathcal{S}_a^{\phi}(x)}{\delta \phi^d(z)} \Bigg|_{\phi=0} \right)^{\theta} \right]_{\theta=0} \xi^d(z)\, \eta^b(y)\, d^4z\, d^4y$$

$$\cancel{= \int d^4z \left[ \frac{\delta \mathcal{S}^{\theta}}{\delta \theta^b} \right]_{\theta=0} \epsilon(x)}$$

$$= \int d^4w \left[ \frac{\delta \mathcal{S}^{\theta}}{\delta \theta^a(w)} \right]_{\theta=0} \tilde{\epsilon}^a(w)$$

for some ~~$\epsilon^b(x)$~~ $\epsilon^b(w)$ <u>independent of $\mathcal{S}$</u>

but depends on $y, z$, $b$, $d$

$\xi, \eta$

---

We are trying to prove these things in general irrespective of the choice of $F^a(x)$ — otherwise we could, for e.g., directly put $F^a = \partial_\mu A^a$ & show that the terms cancel

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The simplest choice of $\mathcal{S}$ is fermion or scalar fields bcos it doesn't involve deriv. — for gauge fields, trs. involves derivatives.

# Take $\mathcal{O} = \psi^s(v)$

$$\cancel{\delta\psi^s(v)\Big|_\theta = -ig\int \theta^a(x)\, d^4x \left(R_f(T^a)\right)_{st}\psi^t(x)}$$

$$\delta\psi_s(v)\Big|_\theta = -ig\, \theta^a(v)\left(R_f(T^a)\right)_{st}\psi^t(v)$$

$$\therefore \frac{\delta\psi_s^\theta(v)}{\delta\theta^a}\Bigg|_{\theta=0} = -ig\left(R_f(T^a)\right)_{st}\psi^t(v)\,\delta^{(4)}(v-y)$$

Now $$\delta\left(\frac{\delta\psi_s^\theta(v)}{\delta\theta^a(y)}\Bigg|_{\theta=0}\right)^\phi = -ig\left(R_f(T^a)\right)_{st}(-ig)\left(R_f(T^b)\right)_{tt'}\psi^{t'}(v)\,\delta\phi^d(v)\,\delta^{(4)}(v-y)$$

$$\therefore \frac{\delta}{\delta\phi^d(z)}\left(\frac{\delta\psi_s^\theta(v)}{\delta\theta^b(y)}\Bigg|_{\theta=0}\right)^\phi\Bigg|_{\phi=0}$$

$$= -g^2\left(R_f(T^b)\,R_f(T^d)\right)_{st'}\psi^{t'}(v)\,\delta^{(4)}(v-z)\,\delta^{(4)}(v-y)$$

$$\therefore \left(\frac{\delta}{\delta\phi^d(z)}\left(\frac{\delta\psi_s^\theta(v)}{\delta\theta^b(y)}\Bigg|_{\theta=0}\right)^\phi\right)_{\phi=0} - \left[\left(\frac{\delta}{\delta\theta^b(y)}\left(\frac{\delta\psi_s^\phi(v)}{\delta\phi^d(z)}\Bigg|_{\phi=0}\right)\right)\right]_{\theta=0}$$

$$= -g^2\left[R_f(T^b), R_f(T^d)\right]_{st'}\psi^{t'}(v)\,\delta^{(4)}(v-z)\,\delta^{(4)}(v-y)$$

RHS $$= -g^2\left(i f^{bda}\left(R_f(T^a)\right)_{st'}\right)\psi^{t'}(v)\,\delta^{(4)}(v-z)\,\delta^{(4)}(v-y)$$

$$= g f^{bda} \underbrace{\left(-ig R_f (T^a)_{st'} \psi^{t'}(v)\right)}_{\delta\psi_s(v)|_\theta} \delta^{(4)}(v-z) \times \delta^{(4)}(v-y)$$

∴ it is a gauge trs. with

$$\epsilon^a(v) = g f^{bda}\,\delta^{(4)}(v-z)\,\delta^{(4)}(v-y)$$

or

$$\tilde\epsilon^a(v) = \int g f^{bda}\,\delta^{(4)}(v-z)\,\delta^{(4)}(v-y)\,\xi^d(z)\,\eta^d(y)\; d^4z\, d^4y$$

(as per requirement)

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$$\frac{\delta\psi^{t'}(v)}{\delta\theta^a(w)} = -ig R_f (T^a)_{st'}\,\psi^{t'}(v)\,\delta^{(4)}(v-w)$$

with $\epsilon^a(v) = g f^{bda}\,\delta^{(4)}(v-z)\,\delta^{(4)}(v-z)\,\delta^{(4)}(v-y)$

$$\epsilon^a(w) = g f^{bda}\,\delta^{(4)}(w-z)\,\delta^{(4)}(w-y)$$

(We can choose $\xi^d(z)$ $\eta^b(y)$ equal to Dirac delta & Kronecker delta)

satisfies

$$\int\left[\frac{\delta}{\delta\phi^d(z)}\left(\frac{\delta g^a}{\delta\theta^b(y)}\Big|_{\theta=0}\right)^{\phi}\right]_{\phi=0} - \int\left[\frac{\delta}{\delta\theta^b(y)}\left(\frac{\delta g^{\phi}(z)}{\delta\phi^d(z)}\Big|_{\theta=0}\right)^{\theta}\right]_{\theta=0}$$

$$= \int d^4w \left(\frac{\delta g^\theta}{\delta\theta^a(w)}\Big|_{\theta=0}\right)\epsilon^a(w)$$

$\epsilon^a \to$ prop. of the group & it doesn't depend on what quantity you are applying on

Hence,

$$\delta S_{total} = -\int d^4x\, d^4y\, d^4z\, b^a(x)\, \xi\, c^d(z)\, c^b(y)$$

$$\frac{1}{2}\left[\int d^4w \left.\frac{\delta F_a{}^{\theta}(x)}{\delta \theta^e(w)}\right|_{\theta=0} g f^{bde}\, \delta^{(4)}(w-z)\, \delta^{(4)}(w-y)\right]$$

$$+\int d^4x\, d^4y\, b^a(x) \left.\frac{\delta F_a{}^{\theta}(x)}{\delta \theta^b(y)}\right|_{\theta=0} \left(+\tfrac{1}{2} g f^{dcb}\, \xi\, c^c(y)\, c^d(y)\right)$$

$$= -\frac{1}{2}\int d^4x\, d^4y\, b^a(x)\, \xi\, c^d(\cancel{z}y)\, c^b(y) \left.\frac{\delta F_a{}^{\theta}(x)}{\delta \theta^e(y)}\right|_{\theta=0} \times g f^{bde}$$

$$+\tfrac{1}{2}\int d^4x\, d^4y\, b^a(x) \underbrace{\left.\frac{\delta F_a{}^{\theta}(x)}{\delta \theta^b(y)}\right|_{\theta=0} g f^{dcb}\, \xi\, c^c(y)\, c^d(y)}$$

$$= 0$$

$$g \left.\frac{\delta F_a{}^{\theta}(x)}{\delta \theta^e(y)}\right|_{\theta=0} f^{dbe}\, \xi\, c^b(y)\, c^d(y)$$

$$= g \left.\frac{\delta F_a{}^{\theta}(x)}{\delta \theta^e(y)}\right|_{\theta=0} f^{bde}\, \xi\, c^d(y)\, c^b(y)$$

($b \leftrightarrow d$ ~~ban~~ & $(-1)^2$ of anti sym.)