SEARCH FOR A FINAL THEORY OF MATTER[1]

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One of the questions that has puzzled human beings since the dawn of civilization is; what are we and everything around us made of? In particular, this question was asked by the Greek philosophers more than 2000 years ago. A few of them asked a more specific question: what would happen if we take a lump of some material, and keep dividing it? They concluded that if we continue this division, we would reach a stage where we cannot divide the material any more. What we get at this stage will be the smallest unit of matter of which everything else is made. They called these atoms and postulated the existence of different kinds of atoms with specific properties. Unfortunately they had no way of testing their hypothesis. Today we know a great deal more about the structure of matter. In this article I shall attempt to summarise our present knowledge of this subject.

Let us begin by asking the same question as those early philosophers: what happens if we take a lump of some material and start dividing it? Just to be more concrete, let us take a pot of water as the starting point. If we continue to divide the water into half, we shall reach our first hurdle when we encounter individual molecules of water. But this is of course not the end of this process. If we supply sufficient energy (say in the form of electrical energy), we find that the water molecule breaks up into its constituent atoms – those of hydrogen (H) and oxygen (O) – with each water molecule containing two hydrogen and one oxygen atoms. The next question would be: are the atoms indivisible? The answer is: of course not! If we take an hydrogen atom and look closely into it, we find that it has a nucleus N_H at the centre, and an electron revolving around it, as depicted in Fig.1. Similarly, oxygen atom will have a different nucleus N_O, and eight electrons revolving around the nucleus.

We can proceed and look further into the nucleus as well as the electron. Are the electrons indivisible? According to our present knowledge, the answer is yes. Indeed, all present day experiments have failed to show any substructure of the electron. Thus the electron seems to be truly indivisible. For this reason the electron is now listed as one of the elementary particles.

However, the situation with the nucleus is entirely different. When we look further into the nucleus, we discover that each nucleus is made up of two kinds of particles: protons and neutrons. The hydrogen nucleus is made of a single proton as shown in Fig.2, whereas the oxygen nucleus is made of eight protons and eight neutrons as shown in Fig.3.

[1] This is the transcript of a lecture given at the Boys' High School, Allahabad.

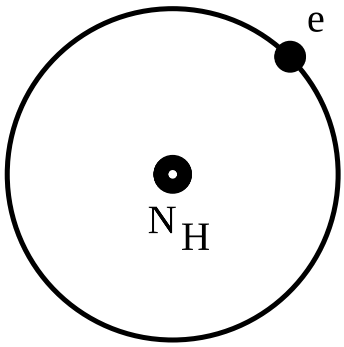

Figure 1: Inside an hydrogen atom

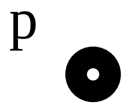

Figure 2: Inside the hydrogen nucleus

Are the protons and neutrons indivisible? It turns out that each proton (and neutron) is made of even smaller constituents, known as quarks. There are different kinds of quarks. For example a proton is made of two up (u-) quarks and one down (d-) quark. A neutron on the other hand is made of two d- and one u- quark. This has been depicted in Fig.4. Actually each of these quarks comes in three kinds. Thus there are three types of u-quark, which we shall denote by u^1, u^2 and u^3. Similarly there are three types of d-quarks, d^1, d^2 and d^3. However it is not meaningful to ask what kind of u- and d-quarks make up the proton (or the neutron) as the quarks inside a proton and a neutron continuously change their identity. Thus in one experiment the proton may appear to be made of u^1, u^2 and d^3, but on repeating the same experiment we may find that it is made of u^2, u^3 and d^1. This is counterintuitive, but is possible in the quantum theory $-$ the theory that must be used in describing the behaviour of atoms and subatomic particles.

What about substructure of the quarks? Like the electron, quarks appear to be indivisible. Indeed, all present day experiments have failed to display any substructure of the quarks.

This gives the hierarchical structure of matter as depicted in Fig.5. Although this figure neatly summarises the constituents of matter, in order to understand the various properties of matter, it is not enough to know only the constituents of matter. We also need to know how these constituents interact with each other. This will tell us, for example, what keeps the constituents together inside an atom, molecule, nucleus,

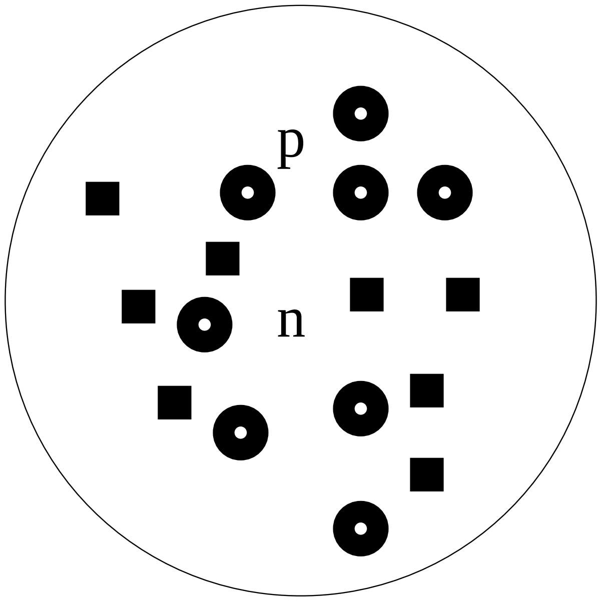

Figure 3: Inside the oxygen nucleus. The black circles represent protons and the black squares represent neutrons.

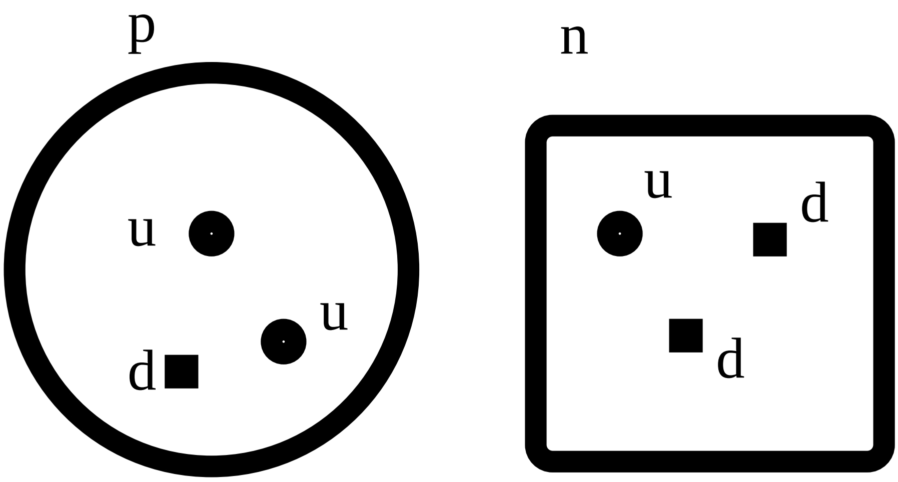

Figure 4: Inside a proton and a neutron

proton or neutron. This will also answer questions like what happens when two particles (molecules, atoms, nucleus etc.) come close to each other. The answers to these questions are relevant for understanding various reactions, *e.g.* chemical reactions, nuclear reactions

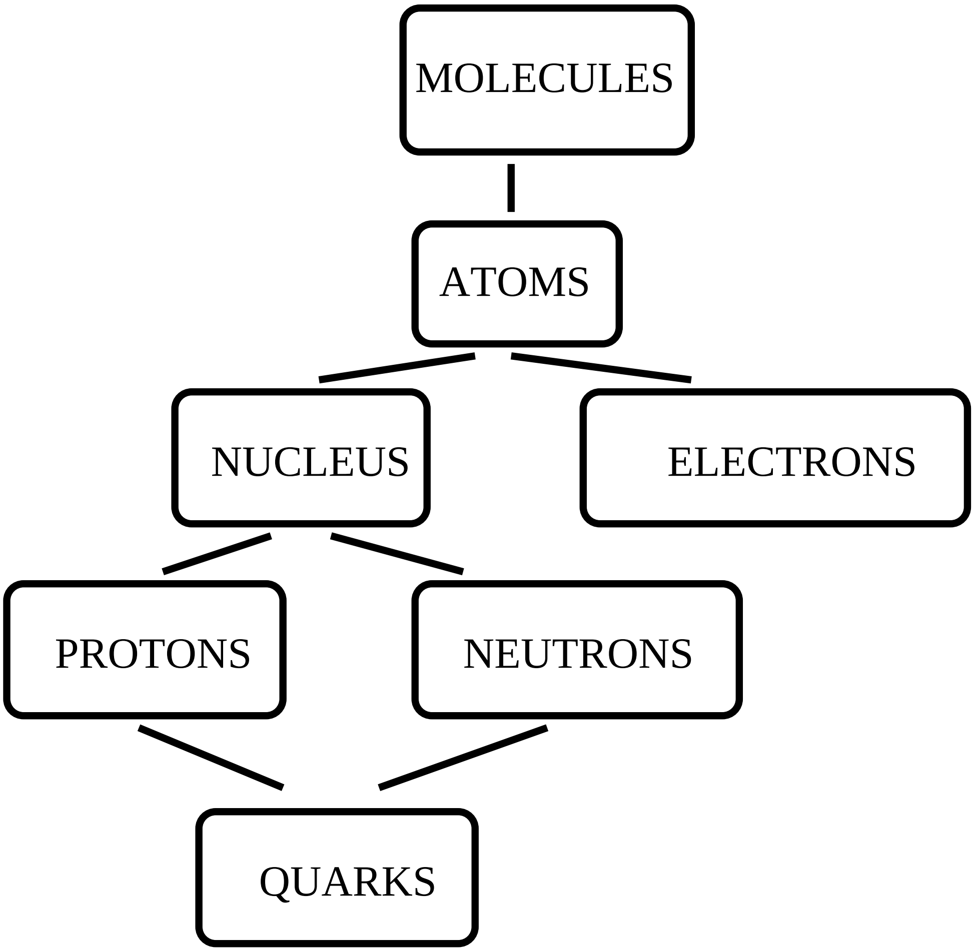

Figure 5: The hierarchical structure of matter.

etc.

Again in order to address this issue let us begin by asking a question: what kind of interactions among matter do we observe in everyday life? The answer is simple: all interactions which are visible to the naked eye can be classified into two kinds:

1. Gravitational interaction: This is the interaction that is responsible for earth's gravity that binds us to the surface of the earth. At a larger scale, this is responsible for binding the planets in the solar system to the sun, stars inside a galaxy etc.

2. Electromagnetic interaction: At an obvious level, this is responsible for the force due to a magnet, occurence of lightning, etc. However this interaction is much more versatile than one would naively guess. For example the light that is used

in illuminating this book, the force that holds all the pages of this book together, and the force that is balancing the earth's gravitational pull and is preventing you from falling to the center of the earth are all due to the electromagnetic interaction between the constituent atoms. Indeed, any interaction that we see with our naked eyes and is not due to gravity, is due to electromagnetic interaction.

It turns out that in order to describe the interaction between the elementary particles, we need to include two other kinds of interactions that are not visible in our daily life. They are

1. Strong interaction: This is the force responsible for binding the quarks inside a proton or a neutron. It is also the force responsible for binding protons and neutrons inside a nucleus.

2. Weak interaction: This is the force responsible for certain kinds of radioactivity, in particular for radioactive β-decays of nuclei.

Thus we see that altogether there are four kinds of interaction between the elementary constituents of matter. However, it turns out that in studying the physics of elementary particles, we can ignore the effect of gravitational force. To see why this is the case, let us compare the electrostatic force between two protons with the gravitational force between two protons. This is given by

$$\frac{\text{Grav. Force}}{\text{Elec. Force}} = \frac{Gm_p^2/r^2}{e_p^2/r^2} = \frac{Gm_p^2}{e_p^2}$$

where G is the Newton's constant, m_p is the mass of a proton, and e_p is the electric charge of a proton. Plugging in the known values of the constants G, m_p and e_p one finds that this ratio is extremely small, of the order of 10^{-36}. Thus the gravitational force between two protons is indeed very small compared to the electromagnetic force between the two protons. Similar analysis shows that the gravitational force is tiny compared to other forces as well. Thus we can safely ignore the gravitational interaction between elementary particles in studying their dynamics.

This accident of nature – that the gravitational force is small compared to all other interactions – has played a crucial role in the formulation of a theoretical model of elementary particles and their interactions. This is because by ignoring the gravitational interaction, we can find a mathematically consistent theory for all the elementary particles and their interactions. Indeed , this theory has been so successful in explaining all the experimental results involving elementary particles that it has been named the **standard model**.

Let us now have a look at the list of elementary particles as predicted by the standard model.

QUARKS

$$\begin{array}{cc} u^1, u^2, u^3 & d^1, d^2, d^3 \\ c^1, c^2, c^3 & s^1, s^2, s^3 \\ t^1, t^2, t^3 & b^1, b^2, b^3 \end{array}$$

LEPTONS

$$(e, \nu_e) \quad (\mu, \nu_\mu), \quad (\tau, \nu_\tau)$$

GAUGE BOSONS

gluons: $g_1, \ldots g_8$

Photon: γ

$W^+, \quad W^-, \quad Z$

HIGGS

$$\phi$$

We also need to add to this list the anti-particles of various quarks and leptons. γ, $g_1, \ldots g_8$, Z and ϕ are their own anti-particles, whereas W^- is the anti-particle of W^+. Note that this list contains the elementary particles that we encountered earlier, namely the u- and the d-quarks and the electron e. However, there are many particles in this list which did not appear in the list of elementary particles that we had found earlier. Even though these particles are not directly constituents of matter, they are produced when we throw two particles at each other with sufficiently high energy, and must be included in the list of elementary particles. Thus for example the head on collision of an electron and a proton can produce a $W^\pm$ pair, as depicted in Fig.6. During this process the kinetic energy of the initial electron and the proton gets converted to the mass of $W^\pm$ according to the famous equivalence relation between mass and energy according to the special theory of relativity.[2]

Some of these particles play specific roles as mediators of specific interactions. For example,

- The photons are mediators of electromagnetic interaction
- The gluons are mediators of strong interaction
- $W^\pm$ and Z are mediators of weak interaction.

[2]Due to the fact that new 'elementary particles' can be produced in the collision of other elementary particles, the distiction between elementary and composite particles get blurred in any theory that incorporates the principle of the special theory of relativity. A more accurate statement would be that *according to the standard model* the particles listed above are elementary, and all other particles like protons, neutrons, and various atoms and molecules are composite.

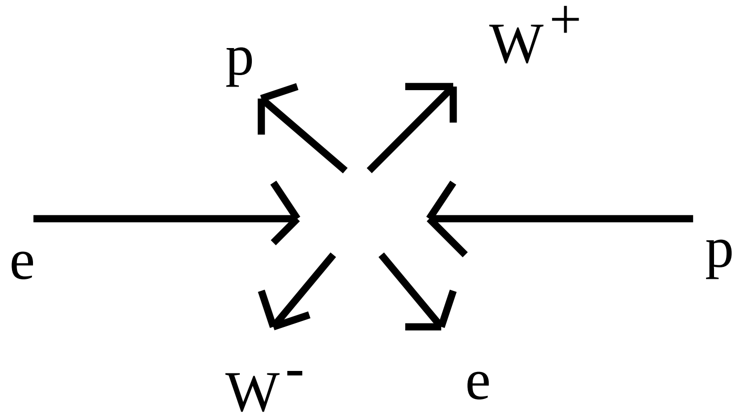

Figure 6: A possible production mode of $W^\pm$ pair

Thus for example the electromagnetic interaction between two electrons can be attributed to a specific process in which two electrons exchange a photon as depicted in Fig.7. In this figure the straight lines denote electrons and the wavy line denotes a photon.

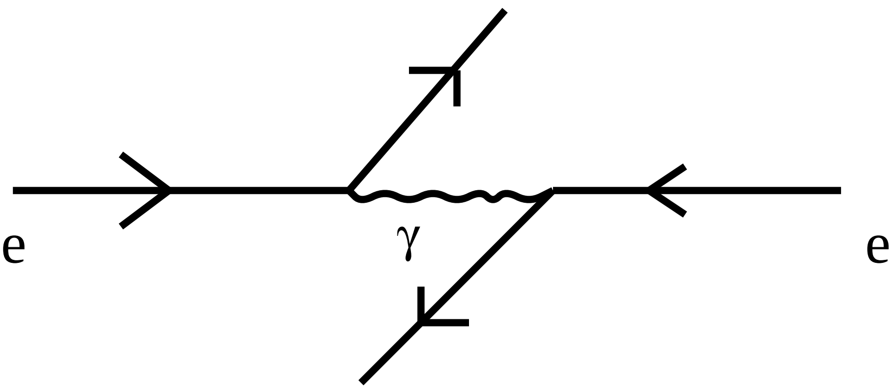

Figure 7: Electromagnetic interaction between two electrons as a result of exchange of a photon

With the help of the standard model, we can, in principle, predict the result of any experiment involving elementary particles. Many such predictions have been experimentally verified. For example, the existence of many new elementary particles, *e.g.* $W^\pm$, Z, t-quark and c-quark, was theoretically predicted before their discovery by experimentalists. At present all laboratory experiments have given rise to results consistent with the predictions of the standard model. However, one particular prediction of the standard model, the existence of a new particle known as the Higgs particle, is still to be verified experimentally. Currently many experiments are being designed to look for this particle.

Does the success of the standard model mean that we have finally found the ultimate theory that describes the world? As we shall now see, the answer to this question is: no. First of all, there is no guarantee that when we carry out the experiments at even higher energies, we will not discover new particles which are not predicted by the standard model, and / or substructures of quarks, leptons, etc. Many theorists have put forward proposals for new mathematically consistent theories where this happens. But there is a much more compelling reason why standard model cannot be the final story. This has to do with gravity. Recall that we have ignored gravitational force in our discussion of elementary particles. This means that in standard model, the particles do not have any gravitational interaction. But, however weak it may be, gravitational force is certainly present in nature as we all know from our personal experience. Thus our understanding of the world is not complete till we have a theory that describes gravity as well.

One can also consider thought experiments involving elementary particles where the gravitational force becomes strong. For this we again use the equivalence between mass and energy. According to Einstein's special theory of relativity

$$E = mc^2$$

where E is the total energy of a particle, m is the effective mass of the particle, and c is the velocity of light in free space. In a certain sense, the above equation can be regarded as the definition of the effective mass of the particle. However the general theory of relativity makes this relation more meaningful by showing that it is this effective mass that determines the strength of the gravitational interaction of a particle. From this equation we see that since c is constant, we can increase the effective mass of an elementary particle by accelerating it to a very high energy. As a result the gravitational force between the elementary particles will become stronger. We have seen earlier that the gravitational force between two protons at rest is 10^{-36} times the electromagnetic force. But now consider accelerating both protons (in opposite directions) to such an extent that each of them carry energy equal to $10^{18}m_pc^2$, m_p being the mass of a proton at rest. This increases the effective mass of each proton 10^{18}-fold. As a result the gravitational force between them increases 10^{36} fold and becomes comparable to the electromagnetic force.[3] Clearly, the standard model will give wrong predictions for the result of this experiment since it does not take into account gravitational interaction between elementary particles. Thus we see that, despite its enormous success, the standard model cannot be the final theory. If we could carry out this experiment in the laboratory and study what kind of deviations from the standard model prediction we observe in the actual experiment, we would get a clue as to how the standard model should be modified so as to include the effect of gravitational interaction. Unfortunately this experiment cannot be carried out in

[3] The electromagnetic force between protons also increases slowly as the energy increases, but this growth is only logarithmic, and hence at sufficiently high energy the gravitational force will overcome the electromagnetic force.

practice, due to the lack of accelerators powerful enough to accelerate the protons to such high energy. At present we have succeeded in accelerating them to an energy of about $10^3\, m_p c^2$, – far less than the required energy of $10^{18} m_p c^2$.

The situation we are in today, is somewhat analogous to that of the ancient philosophers. We are looking for the final theory that explains all natural phenomena, but we are unable to do the experiment that might provide a clue to this theory. But there is one crucial difference. By our experience with standard model, we now know with a reasonable degree of confidence that whatever the final theory is, there must be a mathematically consistent framework to describe it. Thus we can try to use this as a guideline in our search for the final theory.

Unfortunately (or fortunately, depending on your point of view) it is extremely difficult to construct a mathematically consistent theory that incorporates gravity, quantum mechanics and principles of relativity simultaneously. This is unfortunate on the one hand, but is also fortunate in another sense, since this indicates that if we manage to find such a consistent theory, we should take it very seriously even if it cannot be experimentally verified at present. In particular, if we find that there is only one such consistent theory, then it must be the final theory we are looking for. It turns out that there is at least one consistent theory that can incorporate gravity, quantum mechanics and relativity simultaneously. This makes it a serious candidate for the final theory. This theory is known as string theory.

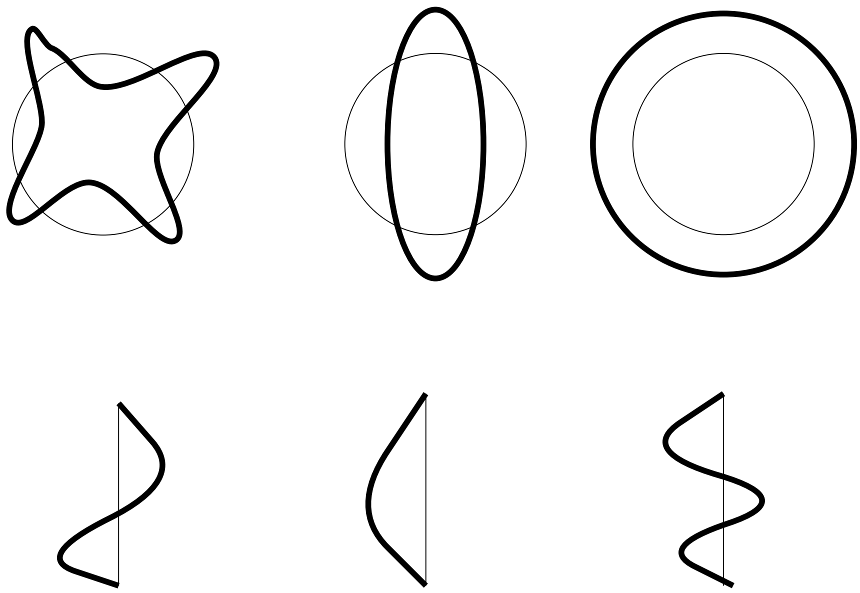

Figure 8: Vibrating strings

The basic idea in string theory is quite simple. It says that different elementary 'particles' that we observe in nature, instead of being point-like objects, are different vibrational modes of a string as depicted in Fig.8. This seems to contradict what we have said earlier, that the elementary constituents of matter *e.g.* electrons, quarks etc. do not show any structure and behave like point objects. There is however, no contradiction, as the typical size of such a string is of order $10^{-33}cm$. In order to see this structure we need extremely powerful microscopes, or equivalently, extremely powerful accelerators which can accelerate particles to an energy of order $10^{19}m_pc^2$. As we have already stated earlier, present day accelerators cannot produce such high energy particles. As a result this string-like structure will be invisible to the present day experimentalist, and so to them these vibrating strings will appear point-like.

In string theory, when one calculates the force between these vibrating strings, one finds that they have gravitational interaction. Thus string theory automatically contains gravity – a fact that has been the main cause of so much excitement among theoretical physicists about string theory. However, if we want to show that string theory is the final theory of nature, then we must also show that at low energy, when gravitational interactions are small, string theory could be approximated by standard model. So far nobody has been able to show this, although there has been a lot of progress in this direction.

This is where we stand today. Let me end by summarising the main points. According to our present world-view, all matter is made of a set of elementary particles. We have an extremely successful theory, known as the standard model, for describing the interactions among these elementary particles. However, this theory does not include gravity, and hence cannot be the final theory of nature. Finding a theory that explains all phenomena involving elementary particles, including their gravitational interaction, remains an open and challenging problem. Progress in this direction based on string theory has been encouraging.

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