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HRI – 2017

A short course in effective theories

Introduction

The basic ideas behind effective field theory

Introduction

The SM is incomplete

- No ν masses
- No DM
- No gravity

Presumably due to new physics

… but who knows where it lurks.

Two possibilities: look for new physics:

- directly $\rightarrow$ energy limited
- in deviations form the SM $\rightarrow$ luminosity limited

The goal is to find the $\mathcal{L}_{NP}$ – easier if the NP is observed directly

SM deviations usually restrict but do not fix the NP.

In particular, two interesting possibilities:

- **NP = SM extension**: The SM fields $\in \mathcal{L}_{NP}$ (example: SUSY)
- **NP = UV realization**: the SM fields are generated in the IR (example: Technicolor)

Basic EFT for the SM idea

Begin with S_{light}[light-fields]

Assume the NP is not directly observable

$\Rightarrow$ virtual NP effects will generate deviations from S_{light} predictions

The EFT approach is a way of studying this possibility systematically

THE GENERAL EFT RECIPE

- Choose the light symmetries
- Choose the light fields (& their transformation properties)
- Write down *all* local operators $\mathcal{O}$ obeying the symmetries using these fields & their derivatives

$$\mathcal{L}_{\text{eff}} = \sum c_{\mathcal{O}}\, \mathcal{O}$$

The sum is infinite; yet the problem is *not* renormalizablity, but predictability

$\mathcal{L}_{\text{eff}}$ is renormalizable. Any divergence:

- polynomial in the external momenta
- obeys the symmetries

$\Rightarrow$ corresponds to an $\mathcal{O}$

$\Rightarrow$ renormalizes the corresponding $c_{\mathcal{O}}$

The real problem: at first sight, $\mathcal{L}_{\text{eff}}$ has no predictive power

∞ coefficients $\Rightarrow$ ∞ measurements

However, there is a hierarchy:

$$\{\mathcal{O}\} = \{\mathcal{O}\}_{\text{leading}} \cup \{\mathcal{O}\}_{\text{subleading}} \cup \{\mathcal{O}\}_{\text{subsubleading}} \cdots$$

Eventually the effects of the $\mathcal{O}$ are below the experimental sensitivity.

The hierarchy depends on classes of NP:

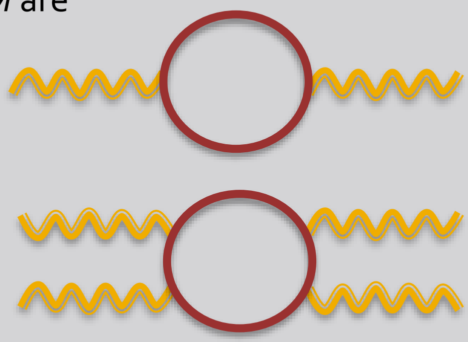
- UV completions: a derivative expansion
- Weakly-coupled SM extensions: dimension

$$\vdots$$

Example

Imagine QED with a heavy fermion Ψ of mass M

All processes at energies below M are



etc.

- Each term is separately gauge invariant
- There are no unitarity cuts since energies < M

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\not{\partial} - M + e\not{A})\Psi$$

$$e^{iS_\Psi} = \int [d\Psi\, d\bar{\Psi}] \exp\left[i \int d^4x\, \bar{\Psi}\left(i\not{\partial} - M + e\not{A}\right)\Psi\right]$$

$$\begin{aligned} S_\Psi &= \ln\det[i\not{\partial} - M + e\not{A}] + \text{const} \\ &= -i\,\text{trln}\left[\mathbb{1} + \frac{1}{i\not{\partial} - M} e\not{A}\right] \\ &= i\sum_{n=1}^{\infty} \frac{(-e)^n}{nM^n} \text{tr}\left(\frac{1}{i\not{\partial}/M - 1}\not{A}\right)^n \end{aligned}$$

$$n = 2: \qquad \frac{i}{2}e^2 \int d^4x\, d^4y\, A^\mu(x) G_{\mu\nu}(x-y) A^\nu(y)$$

$$G_{\mu\nu} = G_{\nu\mu}\,, \qquad \partial^\mu G_{\mu\nu} = 0$$

Example (cont.)

Since the full theory is known $G_{\mu\nu}$ can be obtained explicitly

There is a divergent piece $\propto$ $C_{UV} = 1/(d-4)$ + finite

The divergent piece is unobservable: absorbed in WF renormalization

Observable effects are:

- $\propto 1/M^{2n} \Rightarrow$ ***Hierarchy***
- $\propto e^{2n}/(16\,\pi^2)$

$\Rightarrow$ all observable effects vanish as $M \to \infty$

The expansion is useful only if energy < M

Loop suppression factor: relevant since the theory is weakly coupled

$$G_{\mu\nu}(x) = \int \frac{d^4k}{(2\pi)^4}\, e^{ik.x}\, (k^2\eta_{\mu\nu} - k_\mu k_\nu)\mathcal{G}(k^2)$$

Required by gauge invariance

$$\begin{aligned}\mathcal{G}(k^2) &= \frac{1}{2\pi^2}\left\{\frac{1}{6}C_{UV} - \int_0^1 du\, u(1-u)\ln\left[1 - u(u-1)\frac{k^2}{M^2}\right]\right\} \\ &= \frac{C_{UV}}{12\pi^2} + \frac{1}{60\pi^2}\frac{k^2}{M^2} + \frac{1}{560\pi^2}\left(\frac{k^2}{M^2}\right)^2 - \frac{1}{3780\pi^2}\left(\frac{k^2}{M^2}\right)^3 + \cdots\end{aligned}$$

$$S_{\text{eff}} = \int d^4x \left[-\frac{1 + 2\alpha C_{UV}/3}{4} F_{\mu\nu}F^{\mu\nu} + \frac{\alpha}{30M^2}F_{\mu\nu}\Box F^{\mu\nu} - \frac{\alpha}{280M^4}F_{\mu\nu}\Box^2 F^{\mu\nu} + \cdots\right] + O(e^4)$$

Example (concluded)

If we don't know the NP:

- Symmetries: U(1) & SO(3,1)
- Fields: A_μ

U(1): $A_\mu \to F_{\mu\nu}$
[Wilson loops: non-local]

F^2 terms: change the refraction index

F^4 terms $\supset$ Euler-Heisenberg Lagrangian (light-by-light scattering).

NP chiral $\Rightarrow \mathcal{L}_{\text{eff}} \supset \epsilon_{\mu\nu\rho\sigma}$

NP known: $c_{\mathcal{O}}$ are predicted

NP unknown: $c_{\mathcal{O}}$ parameterize all possible new physics effects

EFT fails: energies $\geq \Lambda$

$$\mathcal{L}_{\text{eff}}^{(2)} = \sum \frac{c_n^{(2)}}{\Lambda^{2n}} F_{\mu\nu} \Box^n F^{\mu\nu} , \quad \left[F_{\mu\nu} \Box^n \tilde{F}^{\mu\nu} = 2\partial_\mu \left(2A_\nu \Box^n \tilde{F}^{\mu\nu} \right) \to \text{drop} \right]$$

$$\mathcal{L}_{\text{eff}}^{(4)} = \frac{c_1^{(4)}}{\Lambda^2} (F_{\mu\nu}F^{\mu\nu})^2 + \frac{c_2^{(4)}}{\Lambda^2} (F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu}) + \frac{c_3^{(4)}}{\Lambda^2} (F_{\mu\nu}F^{\mu\nu}) \left(F_{\mu\nu}\tilde{F}^{\mu\nu} \right) + \frac{c_2^{(4)}}{\Lambda^2} \left(F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}\tilde{F}^{\sigma\mu} \right)$$

$\mathcal{L}_{\text{eff}}$ for the SM

Construct all $\mathcal{O}$ assuming:

- low-energy Lagrangian = $\mathcal{L}_{\text{SM}}$
- The $\mathcal{O}$ are gauge invariant
- The $\mathcal{O}$ hierarchy is set by the canonical dimension
- Exclude $\mathcal{O}'$ if $\mathcal{O}' \propto \mathcal{O}$ on shell (justified later)

("on shell" means when the equations of motion are imposed)

CONVENTIONS

Gauge fields

group	symbol	generator
$SU(3)_c$	G_μ^A	T^A
$SU(2)_L$	W_μ^I	τ^I
$U(1)_Y$	B_μ	

Indices

group	symbol
$SU(3)_c$	$A,\ B, \cdots$
$SU(2)_L$	$I,\ J, \cdots$
family	$p,\ q,\ r, \cdots$

Matter fields

fields	symbol	$SU(3)_c$ irrep	$SU(2)_L$ irrep	$U(1)_Y$ irrep
LH lepton doublet	l	1	2	-1/2
RH charged lepton	e	1	1	-1
LH quark doublet	q	3	2	1/6
RH up-type quark	u	3	1	2/3
RH down-type quark	d	3	1	-1/3
scalar doublet	ϕ	1	2	1/2

Dimension 5 :

$$\mathcal{O}^{(5)} = \left(\bar{l}_p \tilde{\phi}\right) \left(\phi^\dagger l_q^c\right)$$

Family index

1 operator
L-violating

Dimension 6:

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
$\mathcal{O}_G$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_\varphi$	$(\varphi^\dagger\varphi)^3$	$\mathcal{O}_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi)$
$\mathcal{O}_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$\mathcal{O}_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r \widetilde{\varphi})$
$\mathcal{O}_W$	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star\left(\varphi^\dagger D_\mu\varphi\right)$	$\mathcal{O}_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p d_r \varphi)$
$\mathcal{O}_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$\mathcal{O}_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W_{\mu\nu}^I$	$\mathcal{O}_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{l}_p\gamma^\mu l_r)$
$\mathcal{O}_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\,\widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$\mathcal{O}_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\,\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$\mathcal{O}_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uG}$	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{\varphi}\, G_{\mu\nu}^A$	$\mathcal{O}_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{e}_p\gamma^\mu e_r)$
$\mathcal{O}_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\,\widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{\varphi}\, W_{\mu\nu}^I$	$\mathcal{O}_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{q}_p\gamma^\mu q_r)$
$\mathcal{O}_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{uB}$	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{\varphi}\, B_{\mu\nu}$	$\mathcal{O}_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\,\varphi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$\mathcal{O}_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\,\widetilde{B}_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{dG}$	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)\varphi\, G_{\mu\nu}^A$	$\mathcal{O}_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{u}_p\gamma^\mu u_r)$
$\mathcal{O}_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphi\, W_{\mu\nu}^I$	$\mathcal{O}_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{d}_p\gamma^\mu d_r)$
$\mathcal{O}_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\,\widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}_p\sigma^{\mu\nu}d_r)\varphi\, B_{\mu\nu}$	$\mathcal{O}_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu\varphi)(\bar{u}_p\gamma^\mu d_r)$

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{ll}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	$\mathcal{O}_{ee}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	$\mathcal{O}_{le}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{uu}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{lu}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{dd}$	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{ld}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$\mathcal{O}_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{eu}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{qe}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{ed}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$	
$\mathcal{O}_{ledq}$	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

59 operators
(assuming B conservation)

Dimension 7:

$$(\overline{\ell^c}\epsilon D^\mu\phi)(\ell\epsilon D_\mu\phi), \quad (\overline{e^c}\gamma^\mu N)(\phi\epsilon D_\mu\phi), \quad (\overline{\ell^c}\epsilon D_\mu\ell)(\phi\epsilon D^\mu\phi), \quad \overline{N^c}(D_\mu\phi\epsilon D^\mu\ell),$$

$$(\overline{N^c}\ell)\epsilon(\bar{e}\ell), \quad (\overline{N^c}N)|\phi|^2, \quad [\overline{N^c}\sigma^{\mu\nu}(\phi\epsilon\mathbf{W}_{\mu\nu}\ell)], \quad (\overline{N^c}\sigma^{\mu\nu}N)B_{\mu\nu},$$

$$(\bar{d}q)\epsilon(\overline{N^c}\ell), \quad [(\overline{q^c}\phi)\epsilon\ell)(\bar{d}\ell), \quad (\overline{N^c}q)\epsilon(\bar{d}\ell), \quad (\overline{\ell^c}\epsilon q)(\bar{d}N),$$

$$(\bar{d}N)(u^TCe), \quad (\overline{N^c}\ell)(\bar{q}u), \quad (\bar{u}d^c)(\bar{d}N), \quad [\overline{q^c}(\phi^\dagger q)]\epsilon(\bar{\ell}d),$$

$$(\overline{q^c}\epsilon q)(\bar{N}d), \quad (\bar{d}d^c)(\bar{d}E), \quad (\bar{e}\phi^\dagger q)(\overline{d^c}d), \quad (\bar{u}N)(\bar{d}d^c).$$

where

$$N = \tilde{\phi}^T l\,, \qquad E = \phi^\dagger l\,, \qquad \mathbf{W}_{\mu\nu} = W^I_{\mu\nu}\tau^I$$

20 operators
All violate B-L

Formal Developments

Renormalization
Gauge invariance
Decoupling thm.
PTG operators
Equivalence thm.

Equivalence theorem

Low-energy theory with action $S_o = \int d^4x\, \mathcal{L}_o$

Effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{light}} + \sum c_{\mathcal{O}} \mathcal{O}$$

Two effective operators $\mathcal{O}$, $\mathcal{O}'$ such that

$$a\mathcal{O} - \mathcal{O}' = \mathcal{A}(\phi) \frac{\delta S_{\text{light}}}{\delta \phi}$$

Some constant

A local operator

Generic light field

Then the S-matrix depends only on

$$c_{\mathcal{O}} + a\, c_{\mathcal{O}'}$$

Not on $c_{\mathcal{O}}$, $c_{\mathcal{O}'}$ separately.

Without loss of generality one an drop either $\mathcal{O}$ or $\mathcal{O}'$ from $\mathcal{L}_{\text{eff}}$

What this means: the EFT *cannot distinguish* the NP that generates $\mathcal{O}$ from the one that generates $\mathcal{O}'$

Example: 1d QM

Simple classical Lagrangian

$$L = \frac{1}{2} m\dot{x}^2 - V$$

Add a term vanishing on-shell

$$\begin{aligned} L &\to L - \epsilon A(x)(m\ddot{x} + V') + O(\epsilon^2) \\ &\to L + \epsilon(mA'\dot{x}^2 - AV') + \text{tot. der.} + O(\epsilon^2) \end{aligned}$$

Find the canonical momentum and Hamiltonian

$$p = \left(\frac{\partial L}{\partial \dot{x}}\right) = m(1 - 2\epsilon A')\dot{x}$$

$$\begin{aligned} H &= p\dot{x} - L = \frac{1}{2m}p^2 + V + \epsilon\left(-\frac{1}{m}A'p^2 + AV'\right) + O(\epsilon^2) \\ &= H_0 + \epsilon H' + O(\epsilon^2) \end{aligned}$$

1d QM (concluded)

Quantize as usual (with an appropriate ordering prescription)

$$A'p^2 \to \frac{1}{4}\{\{p, A'\}, p\} = \frac{1}{4}(p^2 A' + 2pA'p + A'p^2)$$

The quantum Hamiltonian is then

$$H = \frac{1}{2m}p^2 + V + \epsilon\left(-\frac{1}{4m}\{\{p, A'\}, p\} + AV'\right) + O(\epsilon^2)$$

Which is equivalent to the original one

$$H = UH_0U^\dagger + O(\epsilon^2)\,, \qquad U = \exp\left(-\frac{i}{2}\epsilon\{p, A\}\right)$$

Also:

$$UxU^\dagger = x + \epsilon A + O(\epsilon^2)$$

$$UpU^\dagger = p - \frac{1}{2}\epsilon\{p, A'\} + O(\epsilon^2)$$

QFT: Sketch of proof

Suppose $\mathcal{O}$, $\mathcal{O}'$ are leading effective operators (the other cases are similar)

$$\mathcal{L}_{\rm eff} = \mathcal{L}_0 + \epsilon\,(c'\mathcal{O}' + c\mathcal{O} + \cdots) + O(\epsilon^2)$$

Make a change of variables

$$\phi \to \phi + \epsilon c' \mathcal{A}$$

To leading order

$$\mathcal{L}_{\rm eff} \to \mathcal{L}_0 + \epsilon \left(c'\,\mathcal{A}\frac{\delta S_0}{\delta\phi} + c'\mathcal{O}' + c\mathcal{O} + \cdots \right) + O(\epsilon^2)$$

(annotation on the term $c'\,\mathcal{A}\frac{\delta S_0}{\delta\phi}$: = c′ $\mathcal{O}$)

$$\to \mathcal{L}_0 + \epsilon\,[(c + ac')\mathcal{O} + \cdots] + O(\epsilon^2)$$

There is also a Jacobian

- $\mathcal{A}$ local $\Rightarrow$ $\mathbf{J} \propto \delta^{(4)}(0)$ & its derivatives
- $\Rightarrow \mathbf{J} \to 0$ in dim. reg.

$$[d\phi] \to {\rm Det}\left[1 + \epsilon c' \frac{\delta \mathcal{A}}{\delta\phi}\right] [d\phi]$$

$$\to \left\{1 + \epsilon c' {\rm Tr}\left[\frac{\delta\mathcal{A}}{\delta\phi}\right]\right\} [d\phi] = (1 + \epsilon c'\,\mathbf{J})[d\phi]$$

[in general: $\mathbf{J}$ = renormalization effect]

Gauge invariance

In all extensions of the SM

$$\underbrace{\mathbf{G}_{\rm SM}}_{\rm SM\ gauge\ group} \subset \underbrace{\mathbf{G}_{\rm tot}}_{\rm Full\ gauge\ group}$$

$$\Rightarrow \quad \mathcal{O} \text{ invariant under } \mathbf{G}_{\rm SM}$$

$$\mathcal{O}_{\rm gauge-variant} \stackrel{\rm rad.corrections}{\longrightarrow} \text{ALL gauge variant couplings}$$

$\Rightarrow$ a non-unitary theory

There is, however, a way of interpreting this.

Stuckelberg trick

Model with N vectorbosons W^n_μ (n=1,2, … , N) and other fields χ

$$\mathcal{L} = \mathcal{L}(W, \chi)$$

Choose *any* Lie group **G** of dim. L $\geq$ N, generated by $\{T^n\}$ and add L-N non-interacting vectors W^n_μ (n=N+1, … , L)

$$T^n = -T^{n\dagger}\,, \qquad \mathrm{tr} T^n T^m = -\delta_{nm}$$

Define a derivative operator

$$D_\mu = \partial_\mu + i \sum_{n=1}^{L} T^n W^n_\mu$$

Introduce an auxiliary unitary field *U* in the fund. rep. of **G**

$$\delta U = \sum_{n=1}^{L} \epsilon_n T^n U$$

Define gauge-invariantized gauge fields $\mathcal{W}^n_\mu$

$$\mathcal{W}^n_\mu = -\mathrm{tr}\left(T^n U^\dagger D_\mu U\right)$$

Gauge invariant Lagrangian

$$\mathcal{L}_{G.I.} = \mathcal{L}(\mathcal{W}, \chi) \qquad [\mathcal{L}(\mathcal{W}, \chi)|_{U=\mathbb{1}} = \mathcal{L}(W, \chi)]$$

- *Any* $\mathcal{L}$ equals some $\mathcal{L}_{\text{G.I.}}$ in the unitary gauge… but the χ (matter fields) <u>are gauge singlets</u>

- Also $\mathcal{L}_{\text{G.I.}}$ is non-renormalizable
  $\Rightarrow$ valid at scales below $\sim 4\pi\, m_W$

- The <u>same</u> group should be used throughout:

$\mathcal{L}_{\text{dim} < 5}$ **G**-invariant $\Rightarrow$ *all* $\mathcal{L}_{\text{G.I.}}$ is **G**-invariant

So gauge invariance ***has*** content:

- It predicts relations between matter couplings (most χ are *not* singlets)

- If we assume a part of the Lagrangian is invariant under a **G**, *all* the Lagrangian has the same property

$$\Rightarrow S_{\text{eff}} \text{ is invariant under } \mathbf{G}_{\text{SM}}$$

Example

$\mathcal{L}_{light} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4$

Z_2 symmetry + Lor. invariance

$\mathcal{O} \sim \partial^{2n}\phi^{2k}$; dim $= n+k$

smallest dim : $n+k = 3$

$\mathcal{O} = \phi^6,\ \phi^3\Box\phi,\ (\Box\phi)^2$

eom: $\Box\phi + m^2\phi + \frac{1}{6}\lambda\phi^3 = 0$

$\Rightarrow \phi^3\Box\phi = -\phi^3(m^2\phi + \frac{1}{6}\lambda\phi^3)$ + eom

$\to -\frac{\lambda}{6}\phi^6$ + eom + ren (of λ)

$(\Box\phi)^2 \to \frac{\lambda^2}{36}\phi^6$ + eom + ren (of λ & m^2)

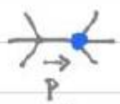
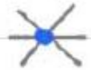
$\Rightarrow \mathcal{L}_{eff} = \mathcal{L}_{light} + \frac{c}{\Lambda^2}\phi^6 + \cdots$

If $\mathcal{O} = \phi^3(\Box\phi + m^2\phi + \frac{1}{6}\lambda\phi^3)$

 $= -\frac{6i}{\Lambda^2}\sum(k_i^2 - m^2)$

 $= +i\frac{5!c}{\Lambda^2}\lambda$

1 $\mathcal{O}$ insertion, 6 ext. legs, $\mathcal{O}(\lambda)$

 + 

$-i\lambda\frac{i}{p^2-m^2}\cdot 20\cdot\left(-\frac{6i}{\Lambda^2}\right)(p^2-m^2)$ → no single part. pole

$-i\lambda\frac{i}{p^2-m^2}\left(-\frac{6i}{\Lambda^2}\right)\sum(k_i^2-m^2)$ → cancel.

$+i\frac{120c}{\Lambda^2}\lambda$

· The factor of 20: let $\chi = (\Box + m^2)\phi$

$\Rightarrow$ need to contract $\phi^3\chi$ & $\frac{1}{4!}\phi^4$

There are 4 ways of getting $\phi^3\chi\ \phi\phi^3$ and the contraction gives 1; then

$\to \frac{4}{4!}\phi^3\cdot 1\cdot\phi^3 = \frac{1}{6}\phi^6 = \frac{120}{6!}\phi^6$

which cancels the $c\lambda\phi^6$ term contribution

Renormalization

For a generic operator

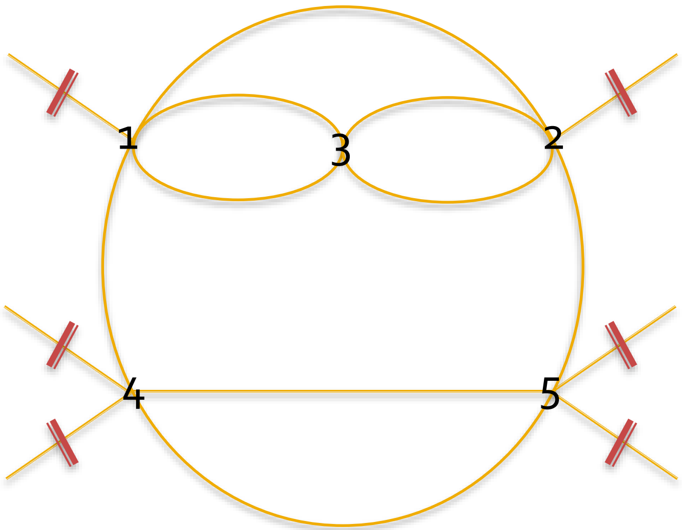
$$\mathcal{O} \sim D^d B^b F^c$$

B=boson field, F=fermion field.
Its coefficient will be the form

$$c_{\mathcal{O}} \sim \lambda(b,f)\Lambda^{-\Delta_{\mathcal{O}}}$$

$$\Delta_{\mathcal{O}} = \dim(\mathcal{O}) - 4 = b + \frac{3}{2}f + d - 4$$

A divergent L-loop graph generated by $\mathcal{O}_v$ renormalizing $\mathcal{O}$:



Naïve degree of divergence

$\Delta_{\mathcal{O}} = \dim(\mathcal{O}) - 4$

$$\mathrm{N}_{\mathrm{div}} = 4L - 2I_b - I_f + \sum d_v - d = \sum \Delta_{\mathcal{O}_v} - \Delta_{\mathcal{O}}$$

Power of Λ:

$$\left.\begin{array}{ll} \text{each } \mathcal{O}_v : & -\Delta_{\mathcal{O}_v} \\ \text{divergence} : & \mathtt{N}_{\text{div}} \end{array}\right\} \to \mathtt{N}_{\text{div}} - \sum \Delta_{\mathcal{O}_v} = -\Delta_{\mathcal{O}}$$

Use Λ as a cutoff

Radiative corrections to $\lambda(b,f)$

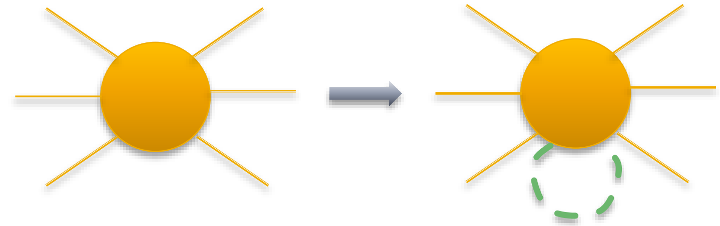
$$\delta\lambda(b,f) \sim (16\pi^2)^{-L} \prod_v \lambda(b_v, f_v)$$

Naturality: for *any* graph

$$\lambda(b,f) \sim \delta\lambda(b,f)$$

ESTIMATING λ

Replace $\mathcal{O}_v \rightarrow B^2\,\mathcal{O}_v$



$$\lambda(b_v+2, f_v) \times \frac{1}{16\pi^2} = \lambda(b_v, f_v) \quad \Rightarrow \quad \lambda(b,f) = (4\pi)^{b-1}\lambda(1,f)$$

Similarly, for fermions

$$\lambda(b,f) = (4\pi)^{f-2}\lambda(b,2)$$

Combining everything:

#fields - 2

$$\lambda(b,f) = (4\pi)^{N_{\mathcal{O}}}\,, \qquad N_{\mathcal{O}} = b+f-2$$

TWO TYPES OF DIVERGENCES

Logarithmic divergences generate the RG

- If $\mathrm{N}_{\mathtt{div}}$=0

$$\delta c_{\mathcal{O}} \sim \frac{(4\pi)^{N_{\mathcal{O}}}}{\Lambda^{\Delta_{\mathcal{O}}}} \times (\text{power of } \ln\Lambda)$$

- If $\mathrm{N}_{\mathtt{div}}$>0 there is a log <u>sub</u>divergence

$$\delta c_{\mathcal{O}} \sim \frac{(4\pi)^{N_{\mathcal{O}}}}{\Lambda^{\Delta_{\mathcal{O}}}} \times \left(\frac{m}{\Lambda}\right)^{N_{\mathtt{div}}} \times (\text{power of } \ln\Lambda)$$

Light mass

Leading RG effects from $\mathtt{N_{div}}$=0

$$\sum_v \Delta_{\mathcal{O}_v} = \Delta_{\mathcal{O}}\,; \qquad (\mathtt{N_{div}} = 0).$$

Super-renormalizable (SR) vertices:

- $\Delta_{\mathcal{O}} \geq 0$ except SR vertices: Δ_{SR}=-1
- If the SR vertex $\sim\Lambda\,\phi^3$ then $m_\phi \sim \Lambda$
- Natural theories: SR vertices ∝ light scale
- Natural theories: SR vertices → subleading RG effects

Ignoring SR vertices → $\Delta_{\mathcal{O}} \geq 0$

THE OPERATOR INDEX AND THE RG

The index of an operator is defined by

$$s_{\mathcal{O}}(u) = \Delta_{\mathcal{O}} + \frac{u-4}{2} N_{\mathcal{O}} = \frac{u-2}{2} b + \frac{u-1}{2} f + d - u$$

Real parameter:
$0 \leq u \leq 4$

Then

$$\mathrm{N}_{\mathrm{div}} = \sum s_{\mathcal{O}_v} - s_{\mathcal{O}} + (4-u)L$$

RG:

- $\mathrm{N}_{\mathtt{div}}$=0
- $\Delta_{\mathcal{O}} \geq 0$

$$\Rightarrow \quad s_{\mathcal{O}} = \sum s_{\mathcal{O}_v} + (4-u)L \geq \sum s_{\mathcal{O}_v} \geq s_{\mathcal{O}_v}$$

The RG running of $c_{\mathcal{O}}$ is generated by operators or lower or equal indexes.

If

$$\mathcal{L}_{\text{eff}} = \sum_{\text{index}=s} \mathcal{L}_s$$

RG evolution of $\mathcal{L}_s$ generated by $\mathcal{L}_{s'}$ with s′ ≤ s

Special cases

$$s_{\mathcal{O}} = (\text{dim of } \mathcal{O} - 4) + \frac{u-4}{2}(\#\text{ fields in } \mathcal{O}) - u$$
$$= d + \left(\frac{u}{2} - 1\right) b + \frac{u-1}{2} f - u$$

For u=1, d≥1, and b=0: s = d-1

- Λ_ψ: natural scale
- Hierarchy: der. expansion
- Higher *s* → subdominant

$$\mathcal{O} \sim \frac{1}{\Lambda_\psi^\Delta (4\pi)^{2s/3}} \psi^f D^d , \quad \Lambda_\psi = \frac{\Lambda}{(4\pi)^{2/3}}$$

s independent of *f*

For u=2: s = d + f/2 - 2

- Λ_ϕ: natural scale
- Hierarchy: der. & ferm. # expansion
- Higher *s* → subdominant

$$\mathcal{O} \sim \frac{1}{\Lambda_\phi^\Delta (4\pi)^{s}} \phi^b \psi^f D^d , \quad \Lambda_\phi = \frac{\Lambda}{4\pi}$$

s independent of *b*

For u=4: s = d + b + (3/2)f - 4

- Λ: natural scale
- No suppression factor

$$\mathcal{O} \sim \frac{1}{\Lambda^\Delta} \phi^b \psi^f D^d$$

This approach also gives a natural estimate for the $c_{\mathcal{O}}$ (aside from power of a scale)

Examples

- *Nonlinear SUSY*:

$$\mathcal{L} = -\frac{1}{2\kappa^2}\det A\,, \qquad A^a_\mu = \delta^a_\mu + i\kappa^2\,\psi\sigma^a\,\overset{\leftrightarrow}{\partial}_\mu\,\bar{\psi}$$

$$\mathcal{O} \sim \psi^f D^d\,, \quad c_{\mathcal{O}} \lesssim \frac{1}{\Lambda_\psi^\Delta\,(3\pi)^{2(d-1)/3}} \quad \Rightarrow \quad \kappa \lesssim \frac{1}{(4\pi)^{1/3}\,\Lambda_\psi^2}$$

- *Chiral theories (low-energy hadron dynamics):*

Simplest case: no fermions

$$U = \exp\left(\frac{i}{f_\pi}\boldsymbol{\sigma}.\boldsymbol{\pi}\right)$$

$$\mathcal{L} = -f_\pi^2 \mathrm{tr}\partial_\mu U^\dagger \partial^\mu U + \bar{c}_4^{(1)}\left[\mathrm{tr}\partial_\mu U^\dagger \partial^\mu U\right]^2 + \cdots + \frac{\bar{c}_{2n}}{f_\pi^{2n-4}} \times \left[\partial^{2n}\mathrm{terms}\right] + \cdots$$

$$\mathcal{O} \sim \phi^b \psi^f D^d\,, \quad c_{\mathcal{O}} \lesssim \frac{1}{\Lambda_\phi^\Delta\,(4\pi)^{d-2}} \quad \Rightarrow \quad f_\pi = \Lambda_\phi\,,\ \bar{c}_d \lesssim (4\pi)^{2-d}$$

PTG operators

- Strongly coupled NP: NDA estimates of $c_{\mathcal{O}}$

- For weakly coupled NP: $c_{\mathcal{O}} < 1/\Lambda^{\Delta}$

  … but we can do better.

  - If $\mathcal{O}$ is generated at tree level then

    $$c_{\mathcal{O}} = \prod (\text{couplings})/\Lambda^{\Delta}$$

  - If $\mathcal{O}$ is generated by at **L** loops then

    $$c_{\mathcal{O}} \sim \prod(\text{couplings})/[\mathbf{(16\pi^2)^L}\,\Lambda^{\Delta}]$$

Assume the SM extension is a gauge theory.

We can then find out the $\mathcal{O}$ that are *always* loop generated.

The remaining $\mathcal{O}$ may or may not be tree generated: I call them "**P**otentially **T**ree **G**enerated" (**PTG**) operators.

To find the PTG operators we need the allowed vertices.

NB: I assume there are no heavy-light quadratic mixings (can always be ensured)

Vector interactions

Multi-vector vertices come from the kinetic Lagrangian

$$\mathcal{L}_V = -\frac{1}{4} V^a_{\mu\nu} V^{a\,\mu\nu}\,, \quad V^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu - g f_{abc} V^b_\mu V^c_\nu$$

Cubic vertices $\propto$ f
Quartic vertices $\propto$ f f

V = { A (light), X(heavy)}

Light generators close

$$[T_l, T_l] = T_l \quad \Rightarrow \quad f_{AAX} = 0$$

This leads to the list of allowed vertices

$$\begin{aligned} \text{cubic}: &\quad AAA,\ AXX,\ XXX \\ \text{quartic}: &\quad AAAA,\ AAXX,\ AXXX,\ XXXX \end{aligned}$$

In particular this implies that pure-gauge operators are loop generated

$$\text{loop generated}: \ \epsilon_{IJK} W^{I\,\nu}_{\mu} W^{J\,\rho}_{\nu} W^{K\,\mu}_{\rho} \quad \&\ \text{etc.}$$

Vector-fermion interactions

Vertices with vectors and fermions come form the fermion kinetic term in $\mathcal{L}$

χ = {ψ (light), Ψ (heavy)}

The *unbroken* generators T_l do not mix light and heavy degrees of freedom $\Rightarrow$ **no $\psi\Psi$ A vertex**

Allowed vertices

$$\bar{\chi} i \not{D} \chi \,, \quad D_\mu = \partial_\mu + i g T^a V^a_\mu$$

$$\begin{array}{ll} \text{with } A: & \psi\psi A,\ \Psi\Psi A \\ \text{with } X: & \psi\psi X,\ \Psi\Psi X,\ \psi\Psi X \end{array}$$

Scalar-vector interactions (begin)

These come form the scalar kinetic term in L

$$|D\vartheta|^2, \quad D_\mu = \partial_\mu + igt^a V_\mu^a$$

ϑ = {ϕ (light), Φ (heavy) }

Terms VV$\vartheta \propto \langle \Phi \rangle$

$$\left(\langle \Phi \rangle \, t^a t^b \vartheta \right) V_\mu^a V^{b\mu} \,, \quad t_{\text{light}} \, \langle \Phi \rangle = 0$$

The (unbroken) t_l do not mix ϕ and Φ

The vectors $t_h \langle \Phi \rangle$ point along the Goldstone directions then

- $t_h \langle \Phi \rangle \perp \phi$ (physical) directions
- $t_h \langle \Phi \rangle \perp \Phi$ (physical) directions

Gauge transformations do not mix ϕ (light & physical) with the Goldstone directions

$$\langle \Phi \rangle \, t_{\text{heavy}} t^a \phi = 0$$

Scalar-vector interactions (conclude)

This leaves 14 allowed vertices (out of 25)

$$\begin{aligned}
\vartheta\vartheta V:&\quad \phi\phi A,\ \Phi\Phi A\\
&\quad \phi\phi X,\ \Phi\Phi X,\ \phi\Phi X\\
\vartheta\vartheta VV:&\quad \phi\phi AA,\ \Phi\Phi AA\\
&\quad \phi\phi AX,\ \Phi\Phi AX,\ \phi\Phi AX\\
&\quad \phi\phi XX,\ \Phi\Phi XX,\ \phi\Phi AX\\
\vartheta VV:&\quad \phi XX,\ \Phi XX
\end{aligned}$$

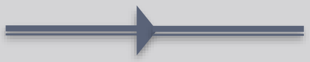
The forbidden vertices are

$$\begin{aligned}
\text{cubic}:&\quad \begin{array}{ccc} \phi\phi\phi & \phi\Phi A & \psi\Psi A\\ \phi AA & \phi AX & \phi XX\\ \Phi AA & \Phi AX & AAX \end{array}\\
\text{quartic}:&\quad \phi\Phi AA \quad AAAX
\end{aligned}$$

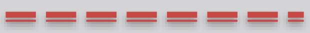
Application: tree graphs suppressed by $1/\Lambda^2$ or $1/\Lambda$

Notation:

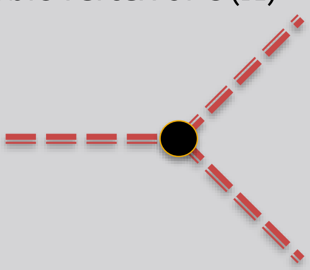
Fermion:

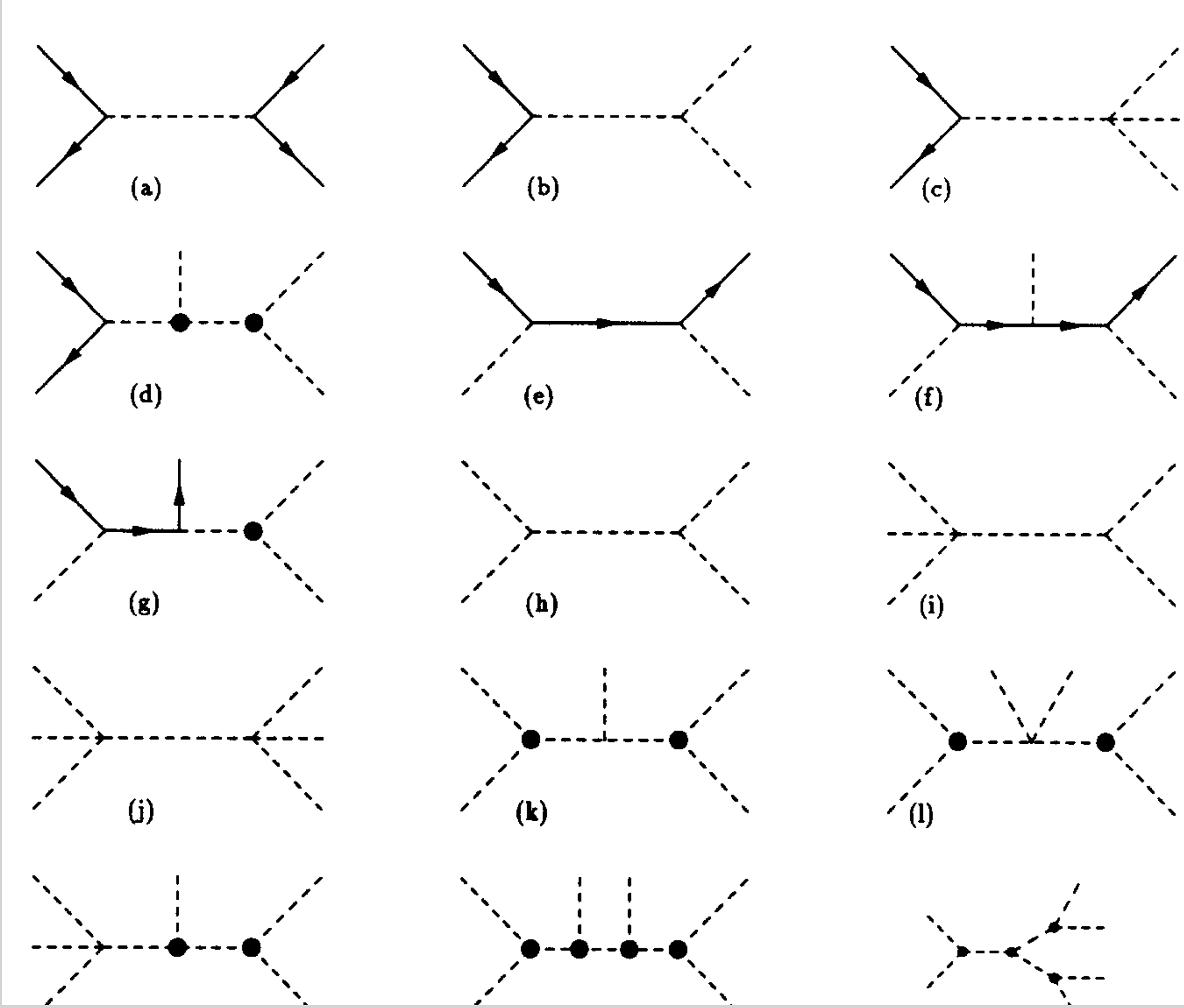


Bosons:



Cubic vertex of O(Λ)





PTG dimension 6 operators:

X^3		ϕ^6 and $\phi^4 D^2$		$\psi^2\phi^3$	
$\mathcal{O}_G$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_\phi$	$(\phi^\dagger\phi)^3$	$\mathcal{O}_{e\phi}$	$(\phi^\dagger\phi)(\bar{l}_p e_r \phi)$
$\mathcal{O}_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{\phi\Box}$	$(\phi^\dagger\phi)\Box(\phi^\dagger\phi)$	$\mathcal{O}_{u\phi}$	$(\phi^\dagger\phi)(\bar{q}_p u_r \widetilde{\phi})$
$\mathcal{O}_W$	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{\phi D}$	$\left(\phi^\dagger D^\mu\phi\right)^\star\left(\phi^\dagger D_\mu\phi\right)$	$\mathcal{O}_{d\phi}$	$(\phi^\dagger\phi)(\bar{q}_p d_r \phi)$
$\mathcal{O}_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				
$X^2\phi^2$		$\psi^2 X\phi$		$\psi^2\phi^2 D$	
$\mathcal{O}_{\phi G}$	$\phi^\dagger\phi\, G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\phi W_{\mu\nu}^I$	$\mathcal{O}_{\phi l}^{(1)}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\bar{l}_p\gamma^\mu l_r)$
$\mathcal{O}_{\phi\widetilde{G}}$	$\phi^\dagger\phi\,\widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)\phi B_{\mu\nu}$	$\mathcal{O}_{\phi l}^{(3)}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu^I\phi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$\mathcal{O}_{\phi W}$	$\phi^\dagger\phi\, W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uG}$	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{\phi}\, G_{\mu\nu}^A$	$\mathcal{O}_{\phi e}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\bar{e}_p\gamma^\mu e_r)$
$\mathcal{O}_{\phi\widetilde{W}}$	$\phi^\dagger\phi\,\widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{\phi}\, W_{\mu\nu}^I$	$\mathcal{O}_{\phi q}^{(1)}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\bar{q}_p\gamma^\mu q_r)$
$\mathcal{O}_{\phi B}$	$\phi^\dagger\phi\, B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{uB}$	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{\phi}\, B_{\mu\nu}$	$\mathcal{O}_{\phi q}^{(3)}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu^I\phi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$\mathcal{O}_{\phi\widetilde{B}}$	$\phi^\dagger\phi\,\widetilde{B}_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{dG}$	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)\phi\, G_{\mu\nu}^A$	$\mathcal{O}_{\phi u}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\bar{u}_p\gamma^\mu u_r)$
$\mathcal{O}_{\phi WB}$	$\phi^\dagger\tau^I\phi\, W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\phi\, W_{\mu\nu}^I$	$\mathcal{O}_{\phi d}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\bar{d}_p\gamma^\mu d_r)$
$\mathcal{O}_{\phi\widetilde{W}B}$	$\phi^\dagger\tau^I\phi\,\widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}_p\sigma^{\mu\nu}d_r)\phi\, B_{\mu\nu}$	$\mathcal{O}_{\phi ud}$	$i(\widetilde{\phi}^\dagger D_\mu\phi)(\bar{u}_p\gamma^\mu d_r)$

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{ll}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	$\mathcal{O}_{ee}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	$\mathcal{O}_{le}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{uu}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{lu}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{dd}$	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{ld}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$\mathcal{O}_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{eu}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{qe}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{ed}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$	
$\mathcal{O}_{ledq}$	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

39 PTG operators (assuming B conservation)

PTG operators are of 5 types

- Only Higgs $\sim \phi^6\,, \quad \phi^2 \Box \phi^2$
- T parameter $\sim |\phi^\dagger D \phi|^2$
- Yukawa like $\sim |\phi|^2 (\bar{\psi} \psi' \phi)$
- W,Z couplings $\sim (\phi^\dagger D_\mu \phi)(\bar{\psi} \gamma^\mu \psi')$
- 4 fermion $\sim (\psi_1 \Gamma_a \psi_2)(\psi_3 \Gamma^a \psi_4)$

Some phenomenology

Phenomenologically: the amplitude for an observable receives 3 types of contributions

Generic observable

👓 = (👓)$_{\text{SM tree}}$ + (👓)$_{\text{SM loop}}$ + (👓)$_{\text{eff}}$

where

- (👓)$_{\text{SM loop}}$ $\sim (\alpha/4\pi)$ (👓)$_{\text{SM tree}}$
- (👓)$_{\text{eff}}$ $\sim (E^2\, c_{\mathcal{O}}/\Lambda^2)$ (👓)$_{\text{SM tree}}$

Easiest to observe the NP for PTG operators

Some limits on Λ are very strict:

for $\mathcal{O} \sim$ eedd: $\Lambda > 10.5$ TeV

$\Rightarrow$ is NP outside the reach of LHC?

Not necessarily. Simplest way: a new symmetry

- All heavy particles transform non-trivially
- All SM particles transform trivially

$\Rightarrow$ <u>all</u> dim=6 $\mathcal{O}$ are loop generated (no PTG ops)

and the above limit becomes $\Lambda > 840$ GeV

Examples:

- SUSY: use R-parity
- Universal higher dimensional models: use translations along the compactified directions

Decoupling theorem (w/o proof)

Theory with light (ϕ) and heavy (Φ) fields of mass O(Λ)

- $S = S_l[\phi] + S_h[\Phi,\phi]$
- $S_l \rightarrow$ renormalizable
- $\exp(\, i\, S[\phi]) = \int [d\Phi] \exp(\, i\, S_h)$

Then

- $S = S_{divergent} + S_{eff}$
- $S_{divergent}$ renormalizes S_l
- For large Λ
  - $S_{eff} = \int d^4x \sum c_{\mathcal{O}} \mathcal{O}$
  - $c_{\mathcal{O}}$ finite
  - $c_{\mathcal{O}} \to 0$ as $\Lambda \to \infty$

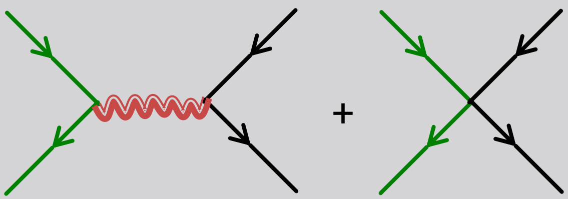
Limitations

The formalism fails if

- $\mathcal{L}_{\text{eff}}$ is used in processes with E > Λ
- If some $\mathcal{O}$ breaks a local symmetry
- If some $c_{\mathcal{O}}$ are impossibly large

If E > Λ

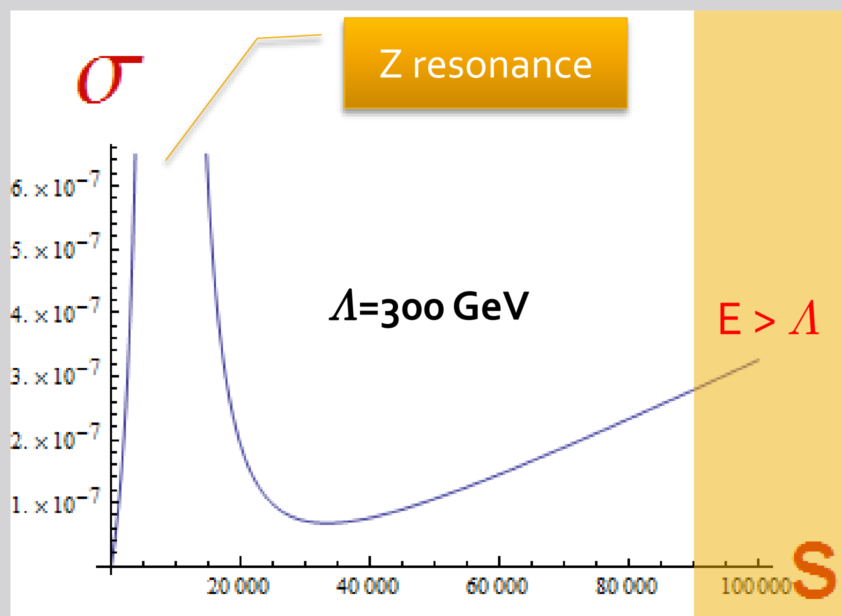
Consider ee $\to \mu\nu$



Then $\sigma \to \infty$ as $E_{CM} \to \infty$

$$\sigma(e^+e^- \to \nu_\mu \bar{\nu}_\mu) = \frac{A\,s}{(s-m_Z^2)^2} + \frac{B\,s}{s-m_Z^2} + C\,s$$

$$A = \frac{1}{4\pi}\left(\frac{g}{4c_W}\right)^2 (1-4s_W^2)^2 \quad B = -\frac{1}{4\pi}\frac{g}{2c_W}\frac{c_{\mathcal{O}}}{\Lambda^2}(1-2s_W^2) \quad C = \frac{c_{\mathcal{O}}{}^2}{8\pi\Lambda^4}$$

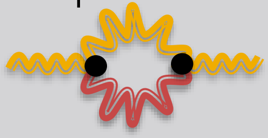


Very large coefficients

A simple example: choose

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} - i\frac{\lambda e}{m_W^2} W^+_{\mu\nu} W^{-\nu\rho} \left[F_\rho{}^\mu + Z_\rho{}^\mu \right]$$

And calculate the 1-loop W vacuum polarization Π_W



$$\Pi_W = \frac{\lambda^2 g^2}{48\pi^2} \left(\frac{\Lambda}{m_W}\right)^2 \left[\frac{q^2}{5m_W^2} + 2 - \frac{6}{\lambda} - \frac{1}{2}\frac{\Lambda^2}{m_W^2}\right] q^2$$

Then get the propagator poles s_1 and s_2

$$q^2 \to s_1,\ s_2 \quad \Rightarrow \quad \langle T W^\mu W^\nu \rangle(q) = -\frac{\eta^{\mu\nu}}{q^2 + \Pi_W - m_W^2} \to \infty$$

If λ is independent of Λ: no light W

$$s_1 \propto -\frac{m_W^4}{\Lambda^2} \qquad s_2 \propto \Lambda^2$$

If $\lambda \propto 1/\Lambda^2$ the poles make physical sense

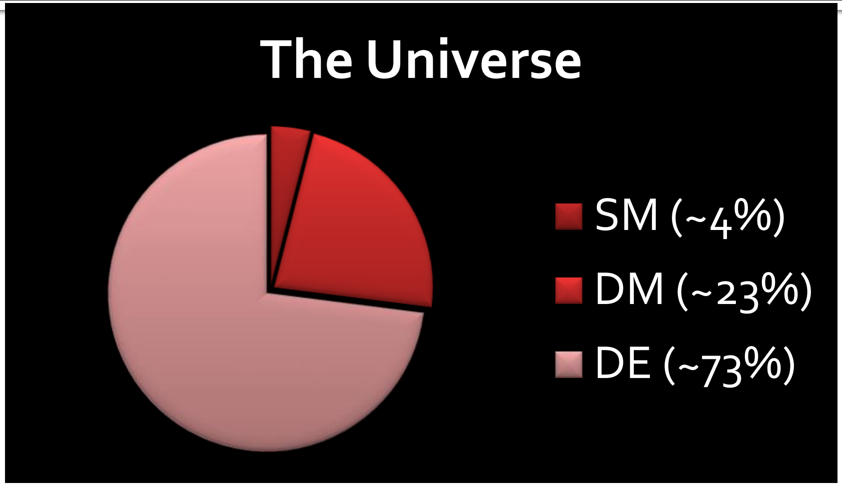
$$s_1 \sim m_W^2 \qquad s_2 \sim \Lambda^2$$

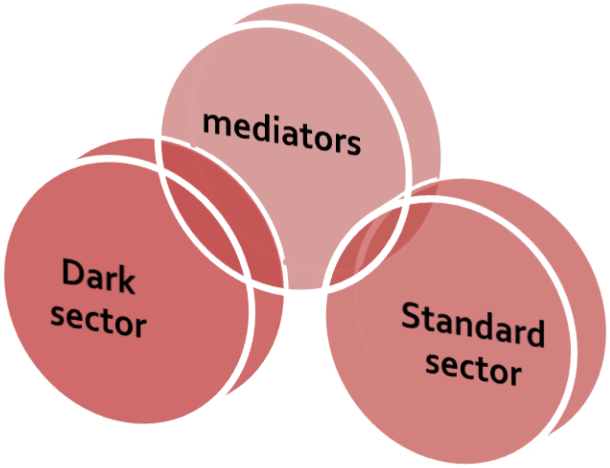
Applications

Collider phenomenology LNV

DM Higgs couplings

DM





Assumptions:

- standard & dark sectors interact via the exchange of heavy mediators
- DM stabilized against decay by some symmetry G_{DM}
- SM particles: $\mathbf{G}_{DM}$ singlets
- Dark particles: $\mathbf{G}_{SM}$ singlets
- Weak coupling

EFFECTIVE THEORY OF DM-SM INTERACTIONS

Within the paradigm:

$$\mathcal{L} \sim \wp\mathcal{O}_{\rm SM} + \wp\mathcal{O}_{\rm DM} + \mathcal{L}_\wp$$

Mediator fields; singlets under DM & SM symmetries

$$\mathcal{L}_{\rm eff} \sim \frac{1}{M^k}\mathcal{O}_{SM}\mathcal{O}_{DM} + \frac{1}{M^l}\mathcal{O}_{SM}\mathcal{O}_{SM} + \frac{1}{M^n}\mathcal{O}_{DM}\mathcal{O}_{DM}$$

Mediator mass

LEADING INTERACTIONS

Leading interactions:
Lowest dimension (smallest M suppression)
Weak coupling ⇒ Tree generated (no loop suppression factor)

dim	$\mathcal{O} \times \mathcal{O}$	mediator
4	$\lvert\phi\rvert^2 \Phi^2$	$-$
	$\lvert\phi\rvert^2 \bar\Psi \Psi$	S (scalar)
5	$\lvert\phi\rvert^2 \Phi^3$	S
	$(\bar\ell \tilde\phi)(\Phi^\dagger \Psi)$	N (fermion)

Higgs portal

Φ : dark scalar
Ψ : dark fermion
ϕ : SM scalar doublet
ℓ : SM lepton doublet

(2)Loop generated :
$B_{\mu\nu} X^{\mu\nu} \Phi \quad B_{\mu\nu} \bar\Psi \sigma^{\mu\nu} \Psi$

***N*-generated:**

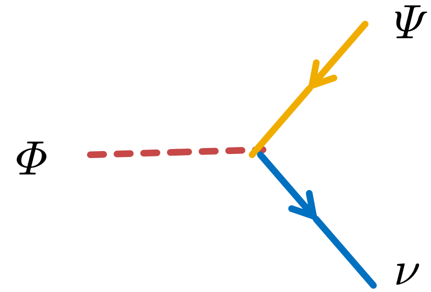
- ≥ 2 component dark sector
- Couple DM (Φ, Ψ) to neutrinos
- (Φ,Ψ)-Z,h coupling @ 1 loop

ν PORTAL SCENARIO

Dark sector: at least Φ & Ψ

$m_\Phi > m_\Psi \Rightarrow$ all Φ's have decayed: fermionic DM.

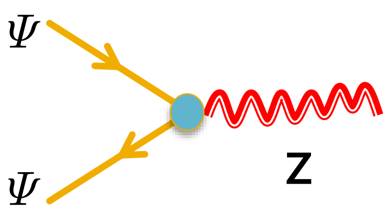
$$(\bar{\ell}\tilde{\phi})(\Phi^\dagger\Psi) \to \frac{v}{\sqrt{2}}\bar{\nu}_L\Phi^\dagger\Psi$$

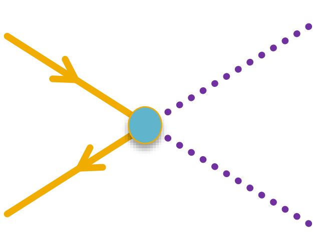


Important loop-generated couplings

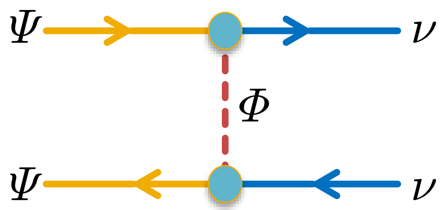
$$i(\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\bar{\Psi}_{L,R}\gamma^\mu\Psi_{L,R})$$

$$|\phi|^2(\bar{\Psi}\Psi)$$

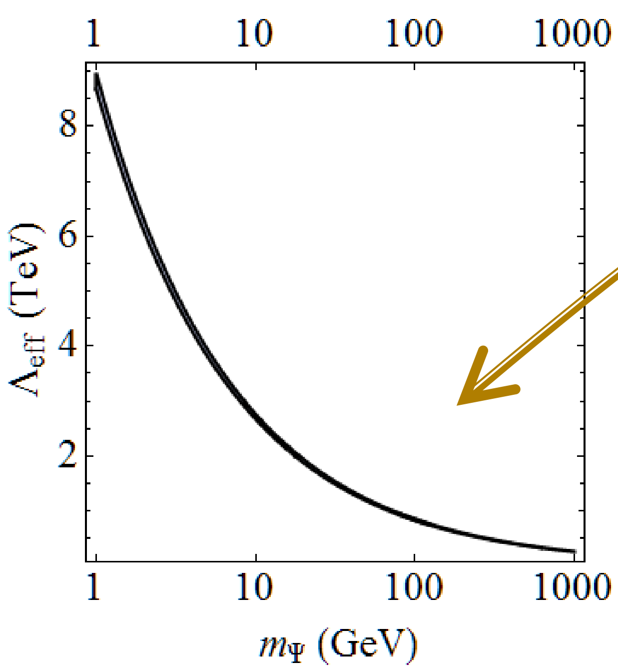




RELIC ABUNDANCE



$$\langle \sigma v \rangle_{\Psi\Psi \to \nu\nu} \simeq \frac{(v/\Lambda_{\rm eff})^4}{128\pi m_\Psi^2}\,, \qquad \Lambda_{\rm eff} = \frac{\Lambda}{f}\sqrt{1+\frac{m_\Phi^2}{m_\Psi^2}}$$



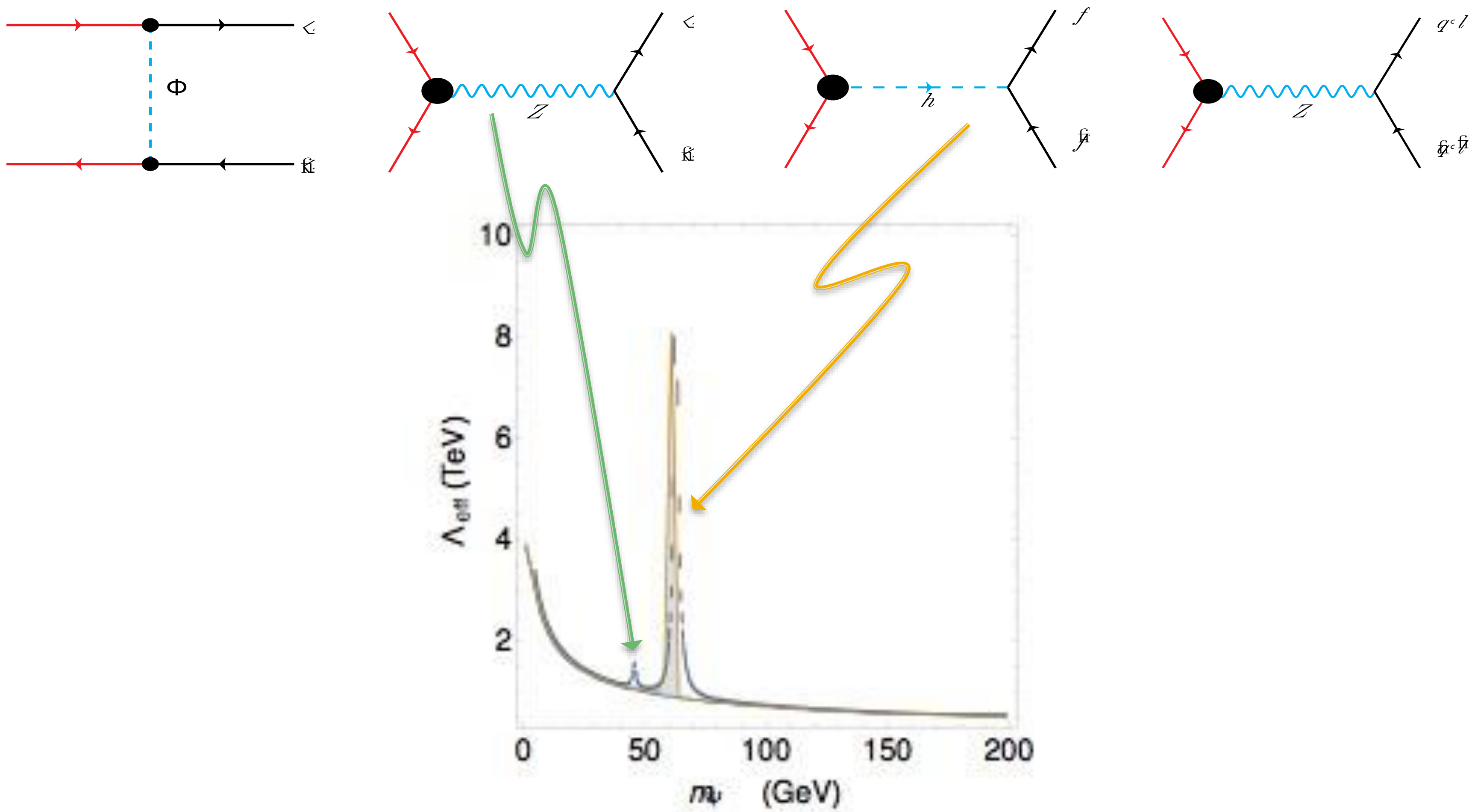
The Planck constraints fix

$$\Lambda_{\rm eff} = \Lambda_{\rm eff}(m_\Psi)$$

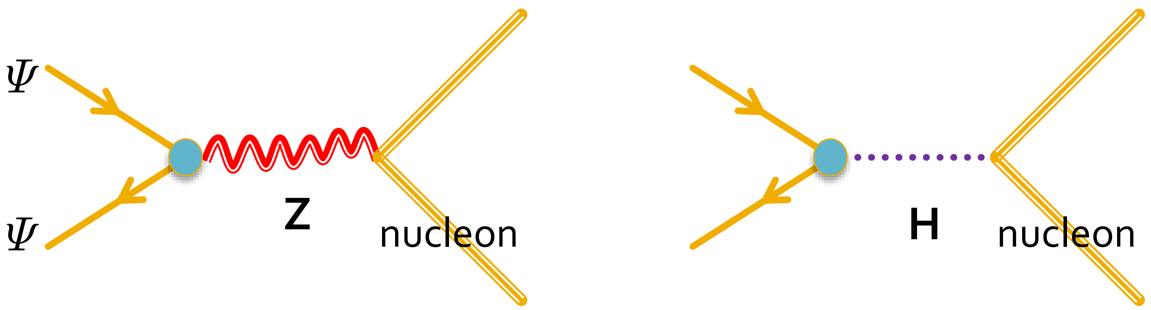
NB:

Large $\Lambda_{\rm eff} \Rightarrow$ small m_Ψ
Small $\sigma \Rightarrow$ small m_Ψ

More refined treatment: include Z and H resonance effects.



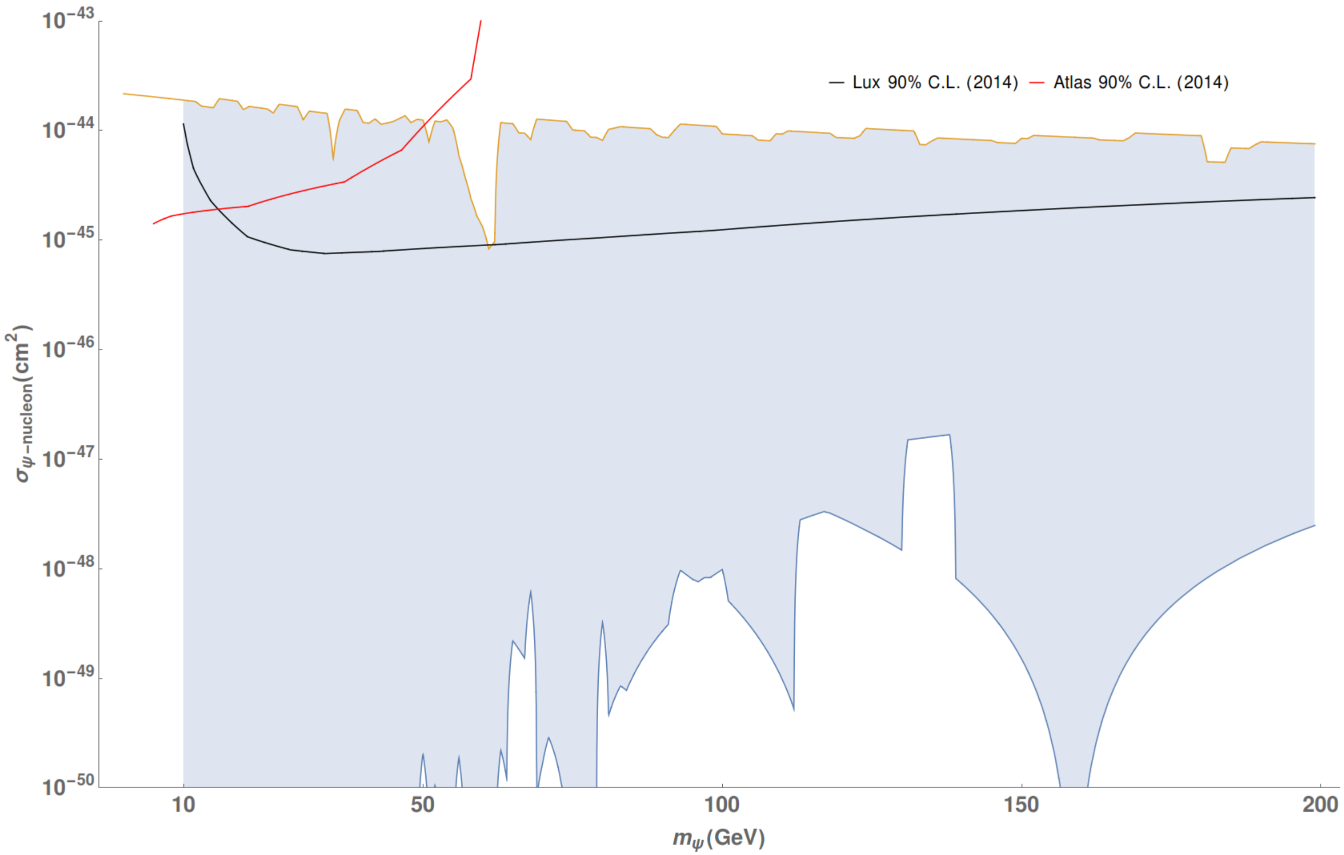
DIRECT DETECTION



$$\mathcal{L} = \frac{\epsilon_h}{v^2}(\bar{\Psi}\Psi)(\bar{\mathcal{N}}\mathcal{N}) + \frac{1}{v^2}\bar{\Psi}\gamma_\mu(\epsilon_L P_L + \epsilon_R P_R)\Psi J^\mu_{\mathcal{N}}$$

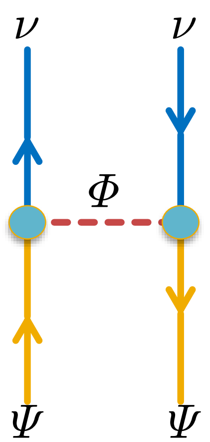
Nucleonic weak current

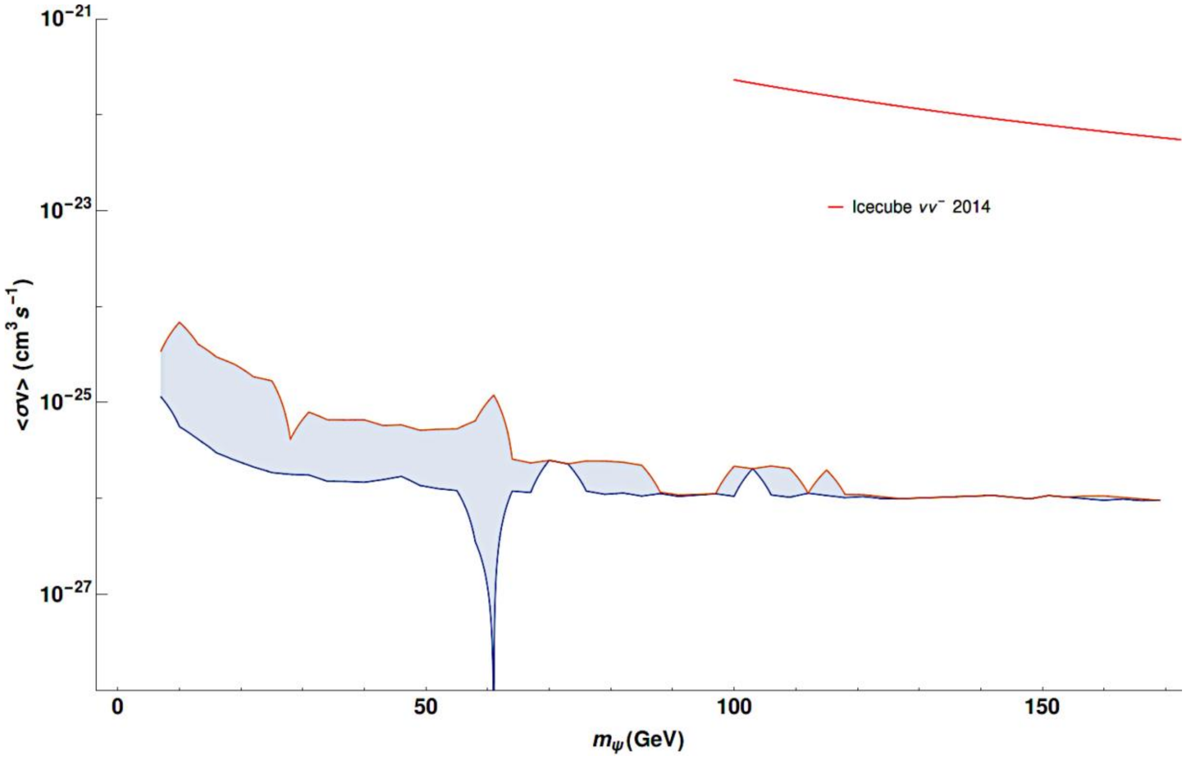
Results: easy to accommodate LUX (and other) limits.



INDIRECT DETECTION

Expect monochromatic neutrinos of energy m_Ψ ;





UV COMPLETION

Add neutral fermions ***N*** to the SM:

$$\mathcal{L} = \bar{N}(i\not{\partial} - m_o)N + (y\,\bar{\ell}\tilde{\phi}N + \mathrm{H.c}) + (z\bar{N}\Phi^\dagger\Psi + \mathrm{H.c})$$

Mass eigentsates: $\boldsymbol{n_L}$ (mass=0), and χ (mass=***M***)

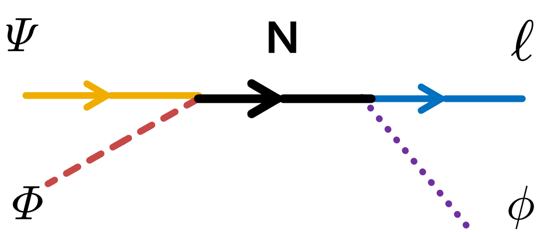
$$N = -s_\theta\, n_L + (c_\theta P_L + P_R)\chi\,, \qquad \nu = c_\theta\, n_L + s_\theta\, \chi_L$$

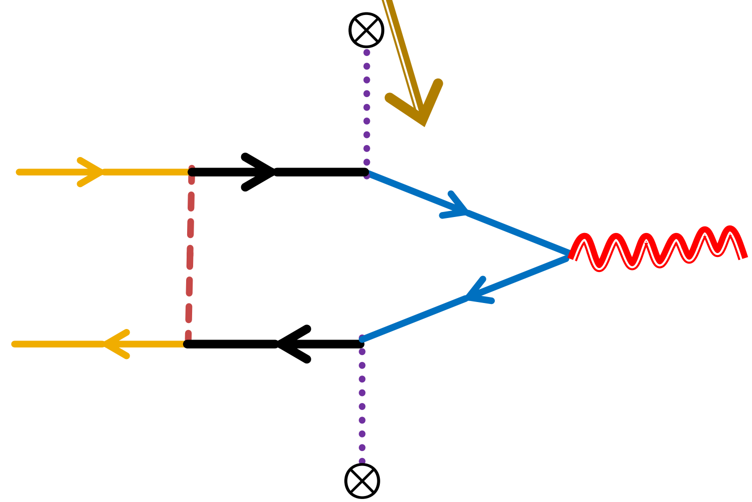
$$\tan\theta = yv/m_o\,; \quad M = \sqrt{m_o^2 + (yv)^2}$$

$$\epsilon_L = \left|\frac{yvz}{4\pi m_o}\right|^2, \quad \epsilon_R = \left|\frac{yvz}{4\pi m_o}\right|^2 \ln\left|\frac{m_\Phi}{m_o}\right|$$

Large m_o:

$$\mathcal{L}_{\rm eff} = \left[\frac{yz}{m_o}(\bar\ell\tilde\phi)(\Phi^\dagger\Psi) + {\rm H.c.}\right] - \frac{g}{2c_{\rm w}}\bar\Psi\ \not{Z}(\epsilon_L P_L + \epsilon_R P_R)\Psi$$





In a model the c_O may be correlated $\Rightarrow$ more stringent bounds

For this model a strong constraint comes from

$$\Gamma\,(\mathrm{Z} \rightarrow \text{invisible})$$

This rules out $m_\Psi > 35$ GeV unless $m_\Phi \sim m_\Psi$

Higgs - simplified

Phenomenological description:

$$\mathcal{L}_{eff} = \frac{H}{v}\left[(2c_W M_W^2 W_\mu^- W_\mu^+ + c_Z M_Z^2 Z_\mu^2) + c_t m_t t\bar{t} + c_b m_b b\bar{b} + c_\tau m_\tau \tau\bar{\tau}\right] + \frac{H}{3\pi v}\left[c_\gamma \frac{2\alpha}{3} F_{\mu\nu}^2 + c_g \frac{\alpha_S}{4} G_{\mu\nu}^2\right].$$

Experiments measure the c_i

$\Rightarrow$ need to relate these couplings to the $c_{\mathcal{O}}$

The relevant $\mathcal{O}$ can be divided into 3 groups

- Pure Higgs
- $\mathcal{O}$ affecting the H-W and H-Z couplings
- $\mathcal{O}$ affecting the couplings of H, Z and W to the fermions

Pure Higgs operators

There are two of them

$$\mathcal{O}_{\partial\varphi} = \tfrac{1}{2}(\partial_\mu|\varphi|^2)^2 \quad \mathcal{O}_\varphi = |\varphi|^6 \qquad \varphi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

The first changes the normalization of H

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c_{\partial\varphi}}{\Lambda^2}\mathcal{O}_{\partial\varphi} + \cdots \approx \frac{1}{2}(1+\epsilon c_{\partial\varphi})(\partial h)^2 + \cdots$$

Canonically normalized field

$$H = \sqrt{1 + c_{\partial\varphi}\epsilon}\; h \approx \left(1 + \tfrac{1}{2}c_{\partial\varphi}\epsilon\right) h$$

$$\epsilon = \frac{v^2}{\Lambda^2}$$

Must replace h $\rightarrow$ H ***everywhere***

The second operator changes v: absorbed in finite renormalizations

This operator can be probed only by measuring the Higgs self-coupling.

O modifying H-W and H-Z couplings

There is one PTG operator.

Contributes to the T oblique parameter.

The constraints on δT imply this cannot affect the c_i within existing experimental precision

All the rest are loop generated
⇒ neglect to a first approximation

⇒ HZZ & HWW couplings are SM to lowest order.

$$\mathcal{O}_{\varphi D} = |\varphi^\dagger D\varphi|^2$$

$$\delta T = \left| \frac{\epsilon c_{\varphi D}}{\alpha} \right| \leq 0.1$$

$\mathcal{O}_{\varphi G}$	$\varphi^\dagger \varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	$\mathcal{O}_{\varphi B}$	$\varphi^\dagger \varphi\, B_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{\varphi \widetilde{G}}$	$\varphi^\dagger \varphi\, \widetilde{G}^A_{\mu\nu} G^{A\mu\nu}$	$\mathcal{O}_{\varphi \widetilde{B}}$	$\varphi^\dagger \varphi\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{\varphi W}$	$\varphi^\dagger \varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	$\mathcal{O}_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi\, W^I_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{\varphi \widetilde{W}}$	$\varphi^\dagger \varphi\, \widetilde{W}^I_{\mu\nu} W^{I\mu\nu}$	$\mathcal{O}_{\varphi \widetilde{W}B}$	$\varphi^\dagger \tau^I \varphi\, \widetilde{W}^I_{\mu\nu} B^{\mu\nu}$

H, W, Z coupling to fermions (begin)

First: **vector or tensor couplings.**

These are PTG or loop generated.

Limits on FCNC coupled to the Z suggest Λ is very large unless *p=r*

For c~1:

- $\mathcal{O}_{\varphi\psi}$ involving leptons: $\Lambda >$ 2.5 TeV
- $\mathcal{O}_{\varphi\psi}$ involving quarks except the top: $\Lambda >$ O(1 TeV)
- $\mathcal{O}_{\varphi ud}$: $\Lambda >$ O(1 TeV)

O(1%) corrections to the SM: ignore

PTG		LG	
$\mathcal{O}_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$	$\mathcal{O}_{eW}$	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W_{\mu\nu}^I$
$\mathcal{O}_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$	$\mathcal{O}_{eB}$	$(\bar{l}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$
$\mathcal{O}_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$	$\mathcal{O}_{uG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{\varphi} G_{\mu\nu}^A$
$\mathcal{O}_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_r)$	$\mathcal{O}_{uW}$	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \widetilde{\varphi} W_{\mu\nu}^I$
$\mathcal{O}_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{q}_p \tau^I \gamma^\mu q_r)$	$\mathcal{O}_{uB}$	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{\varphi} B_{\mu\nu}$
$\mathcal{O}_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{u}_p \gamma^\mu u_r)$	$\mathcal{O}_{dG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r)\varphi G_{\mu\nu}^A$
$\mathcal{O}_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{d}_p \gamma^\mu d_r)$	$\mathcal{O}_{dW}$	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I \varphi W_{\mu\nu}^I$
$\mathcal{O}_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}_p \gamma^\mu d_r)$	$\mathcal{O}_{dB}$	$(\bar{q}_p \sigma^{\mu\nu} d_r)\varphi B_{\mu\nu}$

Family index

H coupling to fermions (concluded)

There are also **scalar couplings**

In unitary gauge
$\varphi\,|\varphi|^2 = (\epsilon/2)\,(v + 3\,H + \cdots\,)/\sqrt{2}$

ϵ v contributions: absorbed in finite renormalization. GIM mechanism survives.

ϵ H contributions: observable deviations form the SM

$$(\mathcal{O}_{e\varphi})_{pr} = |\varphi|^2 \bar{\ell}_p e_r \varphi,$$
$$(\mathcal{O}_{u\varphi})_{pr} = |\varphi|^2 \bar{q}_p u_r \tilde{\varphi},$$
$$(\mathcal{O}_{d\varphi})_{pr} = |\varphi|^2 \bar{q}_p d_r \varphi,$$

LG operators

In most cases these are ignored, but since H $\to \gamma\gamma$, Zγ, GG are LG in the SM, $\mathcal{O}_{LG}$ whose contributions interfere with the SM should be included.

Operators containing the dual tensors do not interfere with the SM: they are subdominant

$$\mathcal{O}_{\varphi X} = \frac{1}{2}|\varphi|^2 X_{\mu\nu}X^{\mu\nu}\,, \quad X = \{G^A,\, W^I,\, B\}$$

$$\mathcal{O}_{WB} = \left(\varphi^\dagger \tau^I \varphi\right) W^I_{\mu\nu} B^{\mu\nu}$$

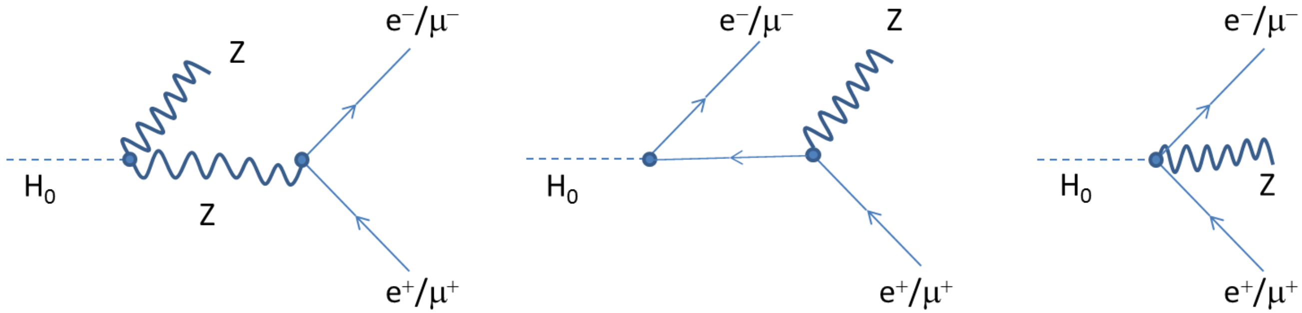
PHENOMENOLOGICAL IMPLICATIONS

$\mathrm{H} \to \psi\psi$

$$\Gamma(H \to \bar{\psi}\psi) = \kappa_\psi^2 \Gamma_{SM}(H \to \bar{\psi}\psi)$$

$$\kappa_\psi^2 = \left(1 - c_{\partial\phi}\epsilon + \frac{\sqrt{2}\, v}{m_\psi} c_{\psi\varphi}\epsilon\right).$$

$\mathrm{H} \to V V^*$ (V=Z, W)



$$\Gamma(H \to VV^*) = \kappa_V^2 \Gamma_{SM}(H \to VV^*)$$

$$\kappa_V^2 = (1 - c_{\partial\phi}\epsilon)$$

H $\to \gamma\gamma$, γ Z, GG

$$\Gamma(H \to \gamma\gamma) = \kappa^2_{\gamma\gamma}\Gamma_{SM}(H \to \gamma\gamma) \qquad \kappa^2_{\gamma\gamma} = 1 - \epsilon\,(c_{\partial\phi} - 0.30\tilde{c}_{\gamma\gamma} - 0.28c_{t\varphi})$$

$$\Gamma(H \to Z\gamma) = \kappa^2_{Z\gamma}\Gamma_{SM}(H \to Z\gamma) \qquad \kappa^2_{Z\gamma} = 1 - \epsilon\,(c_{\partial\phi} - 1.82\tilde{c}_{Z\gamma} - 1.46c_{t\varphi})$$

$$\Gamma(H \to GG) = \kappa^2_{GG}\Gamma_{SM}(H \to GG) \qquad \kappa^2_{GG} = 1 - \epsilon\,(c_{\partial\phi} - 2.91\tilde{c}_{GG} - 4c_{t\varphi})$$

where

$$\tilde{c}_{\gamma\gamma} = \frac{16\pi^2}{g^2}c_{\varphi W} + \frac{16\pi^2}{g'^2}\tilde{c}_{\varphi B}$$

$$\tilde{c}_{Z\gamma} = \frac{16\pi^2}{eg}\left[\frac{1}{2}(c_{\phi W} - c_{\phi B})s_{2\mathrm{w}} - c_{WB}c_{2\mathrm{w}}\right]$$

$$\tilde{c}_{GG} = \frac{16\pi^2}{g_s^2}c_{\varphi G}$$

A SPECIAL CASE

If there are no tree-level generated operators:

$$\Rightarrow \quad c_{\mathcal{O}} \sim 1/(16\pi^2) \quad \tilde{c}_{\gamma\gamma,\gamma Z,GG} \sim 1$$

and

$$\frac{\sigma^{\rm prod}}{\sigma^{\rm prod}_{SM}} - 1 = 2.91\,\epsilon\,\tilde{c}_{GG}$$

$$\frac{B(H \to VV^*)}{B_{SM}(H \to VV^*)} - 1 = -0.25\,\epsilon\,\tilde{c}_{GG}$$

$$\frac{B(H \to \gamma\gamma)}{B_{SM}(H \to \gamma\gamma)} - 1 = \epsilon\,(0.3\tilde{c}_{\gamma\gamma} - 0.249\tilde{c}_{GG})$$

$\Rightarrow$ $\Lambda > 1.4$ TeV

LNV & EFT

There is a single dimension 5 operator that violates lepton number (LN) – assuming the SM particle content:

$$\mathcal{O}^{(5)}_{rs} = N_r^T C N_s \quad N_r = \phi^T \epsilon \ell_r, \ \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Note that it involves only left-handed leptons!

Different chiralities have different quantum numbers, different interactions and different scales. The scale for $\mathcal{O}^{(5)}$ is large , what of the scales when fermions of other chiralities are involved?

Operator with ℓ and e:

$$\mathcal{O} \sim \ell e \phi^a \tilde{\phi}^b D^c \text{ with } a - b = 3 \quad (\text{dim} = 3 + a + b + c = 2a + c).$$

Opposite chiralities $\Rightarrow$ need an odd number of γ matrices $\Rightarrow$ c=odd.

Try the smallest value: c=1. If the D acts on ℓ and e:

$$\not{D}\ell \to 0 \qquad \not{D}e \to 0.$$

because of the equations of motion and the equivalence theorem.

The smallest number of scalars needed for gauge invariance is a=3, b=0. Then the smallest-dimensional operator has dimension 7:

$$\mathcal{O}^{(7)}_{rs} = \left(e_r^T C \gamma^\mu N_s\right) \left(\phi^T \epsilon D_\mu \phi\right).$$

Operator with two e:

$$\mathcal{O} \sim ee\phi^a\tilde{\phi}^b D^c \text{ with } a-b=4 \quad (\text{dim} = 3+a+b+c = 2a+c).$$

Same chiralities $\Rightarrow$ need an even number of γ matrices $\Rightarrow$ c=even. Try the smallest number of ϕ: a=4

Cannot have c=0: SU(2) invariance then requires the ϕ contract into

$$\phi^T\epsilon\phi = 0.$$

Then try c=2; each must act on a ϕ and must not get a factor of $\phi^T \epsilon \phi$. The only possibility is then

$$\mathcal{O}_{rs}^{(9)} = (e_r^T C e_s)\left(\phi^T D_\mu \phi\right)^2 .$$

that has dimension 7:

0ν - $\beta\beta$ decay: introduction

Some nuclei cannot undergo β decay, but can undergo 2β decay because

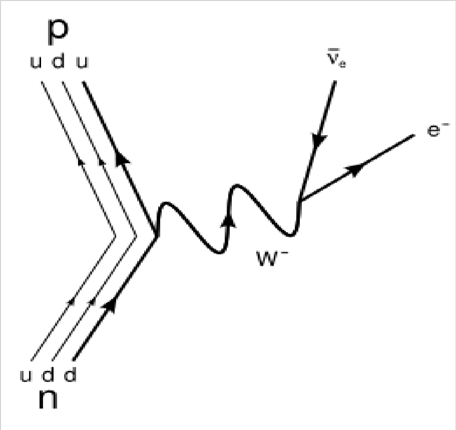
- $E_{bind}(Z) > E_{bind}(Z+1)$
- $E_{bind}(Z) < E_{bind}(Z+2)$

There are 35 nuclei exhibiting 2β decay:

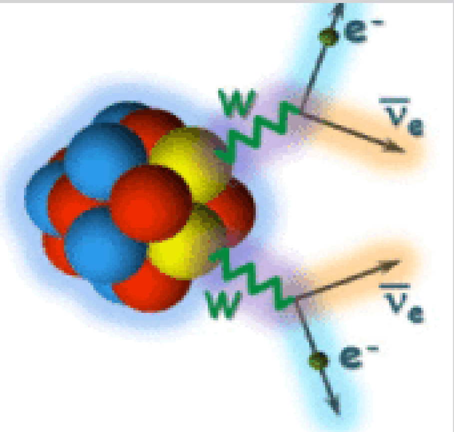
^{48}Ca, ^{76}Ge, ^{82}Se , ^{96}Zr , ^{100}Mo , ^{116}Cd , ^{128}Te , ^{130}Te , ^{136}Xe , ^{150}Nd, ^{238}U

It may be possible to have no ν on the final state (LNV process)

Best limits: Hidelberg-Moscow experiment



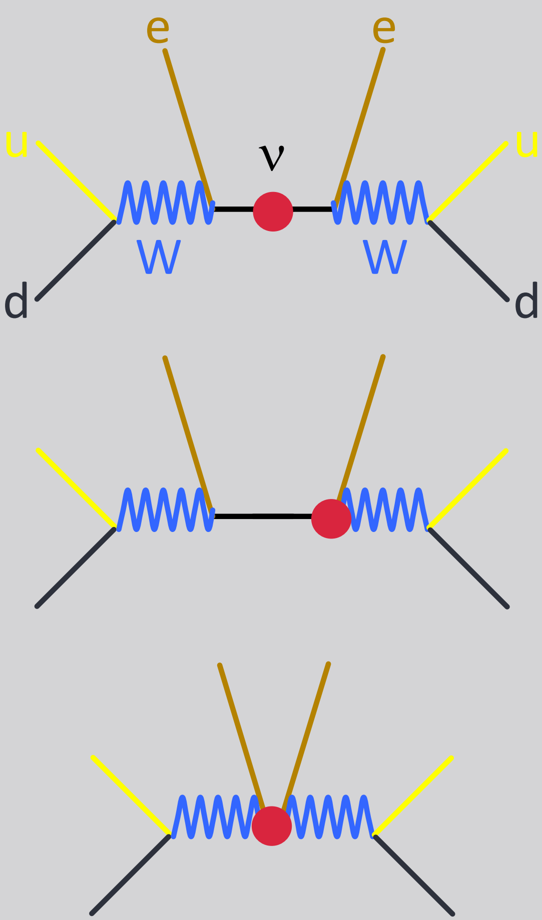
$$A_Z \to A_{Z+1} + e^- + \bar{\nu}_e$$



$$A_Z \to A_{Z+2} + 2e^- + 2\bar{\nu}_e$$

$$T_{1/2}(\not{\nu} - \beta\beta) > 1.8 \times 10^{25}\text{years}$$

0ν - $\beta\beta$ decay: operators, vertices & amplitudes



$$\text{Amplitude} \simeq \mathcal{A}/(Q^2 v^3)$$
$$\epsilon = v/\Lambda$$
$$\eta = Q/v \simeq 2 \times 10^{-4}$$

$$(\bar{\ell}\tilde{\phi})C(\bar{\ell}\tilde{\phi}) \to \mathcal{A} = \epsilon$$

$$(\phi^\dagger D_\mu \tilde{\phi})\left[\bar{e}\gamma^\mu(\tilde{\phi}^T \ell^c)\right] \to \mathcal{A} = \eta\epsilon^3$$

$$(\phi^\dagger D^\mu \tilde{\phi})^2(\bar{e}e^c) \to \mathcal{A} = \eta^2\epsilon^3$$

The implications of the lifetime limit depend strongly on the type of NP.

$$\text{Amplitude} \simeq \mathcal{A}/(Q^2 v^3)$$
$$\epsilon = v/\Lambda$$
$$\eta = Q/v \simeq 2 \times 10^{-4}$$

$$\text{Limit: } \mathcal{A} < 1.4 \times 10^{-12} \quad \Rightarrow$$

dim of $\mathcal{O}$	$\mathcal{A}$	$\Lambda_{\rm min}(\text{TeV})$
5	ϵ	1.8×10^{11}
7	$\eta\epsilon^3$	130
9	$\eta^2\epsilon^3$	3

If the NP generates the ee operator @ tree level it may be probed at the LHC

Flavor physics: b parity

b – quark production in $e^+ e^-$ machines

$$e^+ e^- \to n\, b + X$$

In the SM model the 3rd family (t,b) mixes with the other families, however

$$\mathcal{L}_{\rm SM-mix} = -\frac{g}{\sqrt{2}} \left(\overline{u_L}, \overline{c_L}, \overline{t_L}\right) \not{W}^+ \mathbb{V}_{\rm CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$$

$$|V_{ub}| = (4.15 \pm 0.49) \times 10^{-3} \quad |V_{cb}| = (40.9 \pm 1.1) \times 10^{-3}$$
$$|V_{td}| = (8.4 \pm 0.06) \times 10^{-3} \quad |V_{ts}| = (42.9 \pm 2.6) \times 10^{-3}$$

$\Rightarrow$ neglecting $V_{ub,\, cb,\, td,\, ts}$ there is a discrete symmetry:

$$\mathbf{(-1)^{(\#\ of\ b\ quarks)}\ is\ conserved}$$

In particular $e^+ e^- \to (2n+1)\, b + X$ is forbidden in the SM!

For non-zero V's this "b-parity" is almost conserved.

NP effects that violate b-parity are easier to observe because the SM ones are strongly suppressed.

Looked at the reaction

$e^+ e^- \to$ n b + m c + l j   (j=light-quark jet)

Let

- ϵ_b = efficiency in tagging (identifying) a b jet
- t_j = probability of mistaking a j-jet for a b-jet
- t_c = probability of mistaking a c-jet for a b-jet
- $\sigma_{n\,m\,l} = \sigma\,(e^+ e^- \to$ n b + m c + l j)

Cross section for detecting k b-jets (some misidentified!):

$$\bar\sigma_k = \sum_{u+v+w=k} \binom{n}{u}\binom{m}{v}\binom{l}{w} \left[\epsilon_b^u (1-\epsilon_b)^{n-u}\right] \left[t_c^v (1-t_c)^{m-v}\right] \left[t_j^w (1-t_j)^{l-w}\right] \sigma_{nml}$$

Let

N_{kJ} = # of events with k b-jets and J total jets (k=odd)

Then a 3-sigma deviation from the SM requires

$$|N_{kJ} - N_{kJ}^{SM}| > 3\,\Delta$$

Where Δ = error = $[\Delta_{stat}^2 + \Delta_{syst}^2 + \Delta_{theo}^2]^{1/2}$

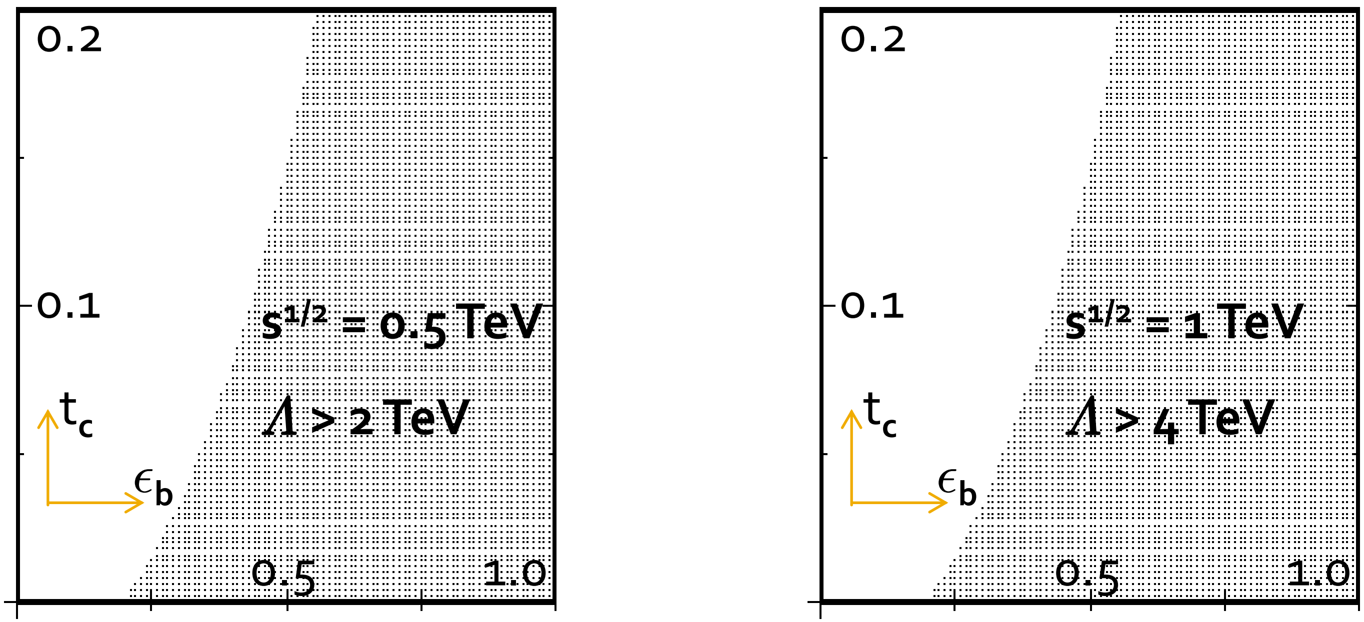
- $\Delta_{stat} = (N_{kJ})^{1/2}$
- $\Delta_{syst} = N_{kJ}\,\delta_s$
- $\Delta_{theo} = N_{kJ}\,\delta_t$

New physics:

$$\mathcal{L} = \frac{1}{\Lambda^2}(\bar{\ell}\gamma^\mu \ell)(\bar{q}_i \gamma_\mu q_j)\,; \quad i,j = 1,2,3$$

$\delta_s = 0.05,\ \delta_t = 0.05,\ t_c = 0.1$ and $t_j = 0.02$				
$\sqrt{s}$ (GeV)	L (fb^{-1})	$\epsilon_b = 0.25$	$\epsilon_b = 0.4$	$\epsilon_b = 0.6$
200	2.5	0.68	0.74	0.81
500	100	1.81	1.96	2.15
1000	200	3.61	3.91	4.36

3σ limits on Λ (in TeV) derived from $N_{k=1,\ J=2}$



3σ allowed regions derived from $N_{k=1,\ J=2}$ when $\delta_s = \delta_t = 0.05$, $t_j = 0.02$

Because of the SM suppression, even for moderate efficiencies and errors one can probe up to $\Lambda \sim 3.5\sqrt{s}$