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# Probabilistic quantum teleportation

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## Abstract

We consider a generalized quantum teleportation protocol for an unknown qubit using non-maximally entangled state as a shared resource. Without recourse to local filtering or entanglement concentration, using standard Bell-state measurement and classical communication one cannot teleport the state with unit fidelity and unit probability. We show that using non-maximally entangled measurements one can teleport an unknown state with unit fidelity albeit with reduced probability. We also give a generalized protocol for entanglement swapping using non-maximally entangled states.

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## 1. Introduction

Many of the quantum information processing protocols typically involve sending quantum states from sender to receiver using quantum and classical channels. Transmission of an intact unknown state from one place to another is very important in the field of quantum information. One amazing discovery in this context is teleportation of an unknown quantum state with the help of a maximally entangled channel, local operation and classical communications [1]. In standard teleportation protocol, Alice performs a Bell-state measurement on the unknown state and one-half of the maximally entangled pair and depending on the measurement outcome Bob applies a local unitary op-

eration to recover the unknown state. This has also been experimentally verified in recent years [2–4]. The study of quantum teleportation protocol is not only limited to qubits and qudits (systems in  $d$ -dimensional Hilbert spaces) but also to quantum systems in infinite dimensional Hilbert spaces [5,6]. Quantum teleportation can also be understood as a quantum computation [7] and it has been even suggested that quantum teleportation will play an important role as a primitive subroutine in quantum computation [8].

In real situations sender and receiver may not have shared maximally entangled state but some form of non-maximally entangled pure state (due to some imperfection at the source). Usually if one follows the standard protocol, one will not be able to complete the teleportation process with unit fidelity and unit probability. Rather, the fidelity will depend on the parameters of the the unknown state and the teleportation will not be reliable. Of course, if one

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has several non-maximally entangled pairs one can first perform entanglement concentration [9] and then recover fewer perfect maximally entangled pairs, and then use one of them to teleport an unknown state using the standard protocol. If Alice and Bob have only one pair, they can perform local filtering [10] first, and convert a non-maximally entangled pair to maximally entangled pair with certain probability. Then they can follow standard protocol.

In this Letter, we consider the question of teleporting an unknown state with *unit fidelity* but less than unit probability when two parties share a non-maximally entangled state. We should mention that there has been a proposal to teleport an unknown state using any pure entangled state but using generalized measurements such as POVMs [11]. This has been termed as conclusive teleportation. Also, there has been a qubit assisted conclusive teleportation process [12]. It has also been mentioned that teleportation with unit fidelity but less than unit probability is possible for a qubit encoded in a coherent state [13]. However, those protocols are different than ours as we will see below. We provide a simple protocol that uses single shot standard orthogonal projections in *non-maximally entangled basis* and able to teleport an arbitrary state with unit fidelity albeit less than unit probability, hence probabilistic teleportation. We discuss various special cases from probabilistic to deterministic teleportation of unknown states. Further, we generalize entanglement swapping protocol for non-maximally entangled states.

## 2. Teleportation with non-maximally entangled state

In this section we present our simple scheme to teleport an unknown state using non-maximally entangled state. Let us consider two observers A and B (conventionally called Alice and Bob) who share a pure non-maximally entangled state as a resource:

$$|\Phi_{\text{res}}\rangle_{12} = \frac{1}{\sqrt{1+|n|^2}}(|00\rangle_{12} + n|11\rangle_{12}), \quad (1)$$

where  $n$  is a known complex number. It is understood that qubits ‘1’ and ‘2’ are with Alice and Bob, respectively. Notice that because of the existence of Schmidt decomposition [14,15] any two qubit

entangled state  $|\Phi\rangle \in \mathcal{H}^2 \otimes \mathcal{H}^2$  such as

$$|\Phi\rangle = a|00\rangle + b|11\rangle + c|01\rangle + d|10\rangle, \quad (2)$$

can be written as a superposition of two basis vectors. In general given an arbitrary two-qubit state (2), the computational basis  $|0\rangle$  and  $|1\rangle$  need not be the Schmidt basis, but we assume that Alice and Bob know the Schmidt basis and coefficients. Then (1) is the most general non-maximally entangled state up to local unitary transformations relating Schmidt basis and computational basis states. Now Alice receives a qubit in an unknown state  $|\psi\rangle_a = (\alpha|0\rangle_a + \beta|1\rangle_a)$  with  $|\alpha|^2 + |\beta|^2 = 1$ . Alice wishes to teleport this state to Bob using the non-maximally entangled resource, local operations and classical communications (LOCCs).

In order to send the state, Alice will make a joint measurement on the the two qubits: qubit ‘1’ that is in the entangled state  $|\Phi_{\text{res}}\rangle_{12}$  with particle ‘2’ and the other that is in the state  $|\psi\rangle_a$ . If Alice performs a measurement in the Bell basis, then the state  $|\psi\rangle_a$  cannot be teleported faithfully, i.e., with unit fidelity and unit probability. However if the measurement is in a non-maximally entangled basis then it is possible for Alice to send the state with unit fidelity, though not with unit probability. Therefore we call our protocol probabilistic quantum teleportation. We will also discuss how it is important to use non-maximally entangled measurements having same amount of entanglement as that of the shared resource. This is like taking out a nail by another nail! To see this we carry out the following analysis.

First we give the most general set of basis vectors for two qubit Hilbert space possessed by Alice. Since Alice can do whatever physical operations within her laboratory we allow the general entangled basis states. If we denote a set of basis vectors as  $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ , known as computational basis, then we can define another set of mutually orthogonal basis vectors as:

$$|\varphi_\ell^+\rangle = \frac{1}{\sqrt{1+|\ell|^2}}(|00\rangle + \ell|11\rangle), \quad (3)$$

$$|\varphi_\ell^-\rangle = \frac{1}{\sqrt{1+|\ell|^2}}(\ell^*|00\rangle - |11\rangle), \quad (4)$$

$$|\psi_p^+\rangle = \frac{1}{\sqrt{1+|p|^2}}(|01\rangle + p|10\rangle), \quad (5)$$

$$|\psi_p^-\rangle = \frac{1}{\sqrt{1+|p|^2}}(p^*|01\rangle - |10\rangle). \quad (6)$$

Here  $\ell$  and  $p$  are complex numbers in general. We notice that when  $\ell = p = 0$ , this basis reduces to the computational basis which is not entangled. For  $\ell = p = 1$ , it reduces to the Bell basis which is maximally entangled. Therefore this set interpolates between unentangled and maximally entangled set of basis vectors. Also note that the set  $|\varphi_\ell^\pm\rangle$  and  $|\psi_p^\pm\rangle$  have different amount of entanglement. As measured by von Neumann entropy [16], the entanglement of  $E(|\varphi_\ell^\pm\rangle) = (-L^2 \log_2 L^2 - L^2 |\ell|^2 \log_2 L^2 |\ell|^2)$  and of  $E(|\psi_p^\pm\rangle) = (-P^2 \log_2 P^2 - P^2 |p|^2 \log_2 P^2 |p|^2)$ , respectively, are different for these sets with  $L = \frac{1}{\sqrt{1+|\ell|^2}}$  and  $P = \frac{1}{\sqrt{1+|p|^2}}$  as real numbers. However, when  $\ell = p$ , then all basis vectors have identical von Neumann entropy.

We can invert the above transformations and we see:

$$|00\rangle = \frac{1}{\sqrt{1+|\ell|^2}}(|\varphi_\ell^+\rangle + \ell|\varphi_\ell^-\rangle), \quad (7)$$

$$|11\rangle = \frac{1}{\sqrt{1+|\ell|^2}}(\ell^*|\varphi_\ell^+\rangle - |\varphi_\ell^-\rangle), \quad (8)$$

$$|01\rangle = \frac{1}{\sqrt{1+|p|^2}}(|\psi_p^+\rangle + p|\psi_p^-\rangle), \quad (9)$$

$$|10\rangle = \frac{1}{\sqrt{1+|p|^2}}(p^*|\psi_p^+\rangle - |\psi_p^-\rangle). \quad (10)$$

Using the non-maximally entangled basis states given in (3)–(6) we can rewrite the combined state of the input and resource state as:

$$\begin{aligned} |\psi\rangle_a |\Phi_{\text{res}}\rangle_{12} &= N(\alpha|0\rangle_a + \beta|1\rangle_a)(|00\rangle_{12} + n|11\rangle_{12}) \\ &= N(\alpha|00\rangle_{a1}|0\rangle_2 + \alpha n|01\rangle_{a1}|1\rangle_2 \\ &\quad + \beta|10\rangle_{a1}|0\rangle_2 + \beta n|11\rangle_{a1}|1\rangle_2) \\ &= N[|\varphi_\ell^+\rangle_{a1}(L\alpha|0\rangle_2 + Ln\beta\ell^*|1\rangle_2) \\ &\quad + |\varphi_\ell^-\rangle_{a1}(L\ell\alpha|0\rangle_2 - nL\beta|1\rangle_2) \\ &\quad + |\psi_p^+\rangle_{a1}(P\beta p^*|0\rangle_2 + P\alpha n|1\rangle_2) \\ &\quad + |\psi_p^-\rangle_{a1}(-P\beta|0\rangle_2 + P\alpha n p|1\rangle_2)]. \quad (11) \end{aligned}$$

Here  $N = \frac{1}{\sqrt{1+|n|^2}}$ ,  $L = \frac{1}{\sqrt{1+|\ell|^2}}$  and  $P = \frac{1}{\sqrt{1+|p|^2}}$  are real numbers. Above expression is the most general way of rewriting an unknown state and two qubit

entangled state. We now wish to have teleportation with unit fidelity and nonzero probability. Let us consider several scenarios involving various choices of the parameters  $\ell$  and  $p$ , given the value of  $n$ . Choice is at the disposal of Alice.

(i) *Standard teleportation protocol.* Let us choose  $\ell = \frac{1}{\ell^*} = p^* = \frac{1}{p} = n$ . In this case  $\ell, n$  and  $p$  can be pure phases, i.e., complex numbers of unit modulus. Then faithful teleportation is possible with unit fidelity and unit probability. This is classic teleportation [1]. Bob can regenerate the state  $|\psi\rangle_a$  by applying the local unitary transformation  $\sigma_0 = I, \sigma_z, \sigma_x$ , or  $i\sigma_y$  on his qubit. These transformations will correspond to the Alice's result of measurement  $|\varphi_\ell^+\rangle, |\varphi_\ell^-\rangle, |\psi_p^+\rangle$ , or  $|\psi_p^-\rangle$ , respectively. Alice can communicate the results of her measurement to Bob using 2 cbits of information over a classical channel. Then Bob with his knowledge of the shared resource state can find out the unitary operation needed to convert the state of his qubit to  $|\psi\rangle_a$ . The unitary operations  $\sigma_x, i\sigma_y$ , and  $\sigma_z$  correspond to the rotation by  $180^\circ$  around the  $x, y$ , and  $z$  axis, respectively. We would also like to emphasize that in a slightly modified version of the classic teleportation protocol, Bob does not have to know the shared resource state; only Alice has to know the shared resource state. In this modified version of the protocol, Alice will encode in two cbits the unitary transformation (instead of the state she has got after the measurement), that Bob has to apply on his qubit to complete the teleportation.

(ii) *Probabilistic teleportation protocol.* If we make the choice  $\ell = n = p^*$ , or  $\ell = n = \frac{1}{p}$ , or  $\ell^* = \frac{1}{n} = p$ , or  $\ell^* = \frac{1}{n} = \frac{1}{p^*}$ , then for any of these choices, reliable (i.e., with unit fidelity) teleportation is possible for only two out of four possible results of the measurement. An interesting observation here is that the above choice of parameters refers to the situation where the basis used for joint measurements and the resource state have the same amount of quantum entanglement, namely,  $E(|\Phi_{\text{res}}\rangle) = (-N^2 \log_2 N^2 - N^2 |n|^2 \log_2 N^2 |n|^2)$ . For example, in the case of first choice, when the outcome is  $|\varphi_{\ell=n}^-\rangle$ , then the state at Bob's hand will be  $(\alpha|0\rangle - \beta|1\rangle)$  and when the outcome is  $|\psi_{p=n}^+\rangle$ , then the state at Bob's hand is  $(\beta|0\rangle + \alpha|1\rangle)$ . Therefore, when Alice sends two classical bits to Bob he will apply  $\sigma_z$  in the former and  $\sigma_x$  in the later case to recover

the unknown state with unit fidelity. The total probability of this successful teleportation will be given by

$$P_{\text{succ}} = \frac{2|n|^2}{(1 + |n|^2)^2}. \quad (12)$$

Thus, we can say that using  $E(|\Phi_{\text{res}}\rangle) = (-N^2 \log_2 N^2 - N^2|n|^2 \log_2 N^2|n|^2)$  amount of entanglement and two classical bits Alice can teleport an unknown state with unit fidelity and probability given in (12). This is one of the main result of our Letter. We see that this probability goes from zero for unentangled  $|\Phi_{\text{res}}\rangle$  to one-half for the maximally entangled resource state. (Other one-half will come from the other two possible outcomes of the measurement when the shared resource and joint measurement are maximally entangled states.) In this sense we can regard our protocol as a generalized quantum teleportation protocol (GQTP) that goes from probabilistic one to deterministic one. Furthermore, we can amplify the probability statistically by repetitions. We can say that the reciprocal of the average success probability must be the number of repetitions  $R$  that are required in order to successfully (all the time) teleport an unknown state with unit fidelity. We see that one shall need on the average at least  $R = (1 + |n|^2)^2/|n|^2$  repetitions to get a faithful teleportation with unit probability. Therefore, if Alice and Bob share  $RE(|\Phi_{\text{res}}\rangle)$  pairs of non-maximally entangled state they can successfully teleport an arbitrary state using local operation and  $2R$  bits of classical communication. We also notice that as the degree of entanglement increases, the number of required repetitions decreases and becomes one for maximally entangled states as expected. It becomes infinite for the unentangled resource state. Thus when no prior shared entanglement exist howsoever many times one tries, it will be impossible to teleport an unknown state with unit fidelity.

Our approach is similar to the filtering approach, however, there is one important difference. In the filtering approach one cannot proceed with the protocol if the filtering does not succeed. In principle, the unknown qubit can be put into memory until the next entangled pair is sent, and so on until a successful filtering event happens. Then one can proceed with the standard teleportation protocol. Such an option is not available in our protocol as it is not till the actual measurement of the qubit has taken place that success or failure is known, and by that time the unknown qubit has been

destroyed. Therefore, we also need  $R$  copies of the unknown state in order to get faithful (unit fidelity and unit probability) teleportation. Since the state is provided by a third party (say Victor), who knows the state, there is no ‘cost’ involved as he can make one copy or several of them.

There are also four different choices of parameters when only one out of four results of the measurement would lead to reliable teleportation. This choice of parameter values is given by  $\ell = \frac{1}{n^*}$ , or  $\ell = n$ , or  $p = n^*$ , or  $p = \frac{1}{n}$ . In this case the total probability of faithful teleportation will be  $|n|^2/(1 + |n|^2)^2$ . This is half of the scenario given above. We note that unlike in the case of standard teleportation (where only Alice has to know), in probabilistic teleportation both Alice and Bob have to know the shared resource state. Only then, Bob will know what basis Alice has used for making the measurement after he receives classical communication.

(iii) *No teleportation.* If the values of  $p$  and  $\ell$  are not related with that of  $n$ , then teleportation is not possible with unit fidelity. This brings out an interesting point: in order that an arbitrary quantum entangled channel is useful for teleportation we must use an entangled measurement containing the same amount of entanglement as the shared resource state. Even though shared entanglement is regarded as a resource and local entanglement is not (as Alice can create or destroy entanglement), still the above point is worth observing.

Before ending this section it may be noted that present experiments have reported teleportation of qubit with certain success probability less than unity, in spite of the sharing of maximally entanglement [2]. This limitation is a practical limitation on Bell-state detection [17]. Though our protocol behaves in a similar way to the experiments in the sense that teleportation is only successful for a limited subset of the possible measurement results, fundamentally they are different.

### 3. Entanglement swapping with non-maximally entangled states

Another important prediction of quantum theory is that conditional upon suitable joint measurement, two

particles can be found in an entangled state that have never interacted in the past. If there are two maximally entangled pairs then making a Bell measurement on two halves makes other two halves maximally entangled. This is known as entanglement swapping [18, 19]. In this section we will discuss how to generate entanglement between two independent particles using non-maximally entangled states as the starting resource and non-maximal measurements. Let us consider two pairs of qubits ‘ $ab$ ’ and ‘ $12$ ’. Let them be in non-maximally entangled states  $|\varphi\rangle_{ab}$  and  $|\psi\rangle_{12}$ . Here

$$\begin{aligned} |\varphi\rangle_{ab} &= M(|00\rangle + m|11\rangle)_{ab}, \\ |\psi\rangle_{12} &= N(|01\rangle + n|10\rangle)_{12}. \end{aligned} \quad (13)$$

If an observer, Alice, makes a measurement on the qubit pair ‘ $a1$ ’, then we wish to analyze the state of the particle ‘ $b$ ’ at Bob’s location and the particle ‘ $2$ ’ at Charlie’s location. For this, we consider the state of the combined system:

$$\begin{aligned} &|\varphi\rangle_{ab}|\psi\rangle_{12} \\ &= MN(|00\rangle + m|11\rangle)_{ab}(|01\rangle + n|10\rangle)_{12} \\ &= MN \left[ LP'(|\varphi_\ell^+\rangle_{a1} + \ell|\varphi_\ell^-\rangle_{a1}) \right. \\ &\quad \times (|\psi_{p'}^+\rangle_{b2} + p'|\psi_{p'}^-\rangle_{b2}) \\ &\quad + mnLP'(\ell^*|\varphi_\ell^+\rangle_{a1} - |\varphi_\ell^-\rangle_{a1}) \\ &\quad \times (p'^*|\psi_{p'}^+\rangle_{b2} - |\psi_{p'}^-\rangle_{b2}) \\ &\quad + nPL'(|\psi_p^+\rangle_{a1} + p|\psi_p^-\rangle_{a1}) \\ &\quad \times (|\varphi_{\ell'}^+\rangle_{b2} + \ell'|\varphi_{\ell'}^-\rangle_{b2}) \\ &\quad \left. + mPL'(p^*|\psi_p^+\rangle_{a1} - |\psi_p^-\rangle_{a1}) \right. \\ &\quad \left. \times (\ell'^*|\varphi_{\ell'}^+\rangle_{b2} - |\varphi_{\ell'}^-\rangle_{b2}) \right] \\ &= MN \left[ LP'|\varphi_\ell^+\rangle_{a1} \{ |\psi_{p'}^+\rangle_{b2} (1 + mnp'^*\ell^*) \right. \\ &\quad \left. + |\psi_{p'}^-\rangle_{b2} (p' - mn\ell^*) \} \right. \\ &\quad + LP'|\varphi_\ell^-\rangle_{a1} \{ |\psi_{p'}^+\rangle_{b2} (\ell - mnp'^*) \\ &\quad \left. + |\psi_{p'}^-\rangle_{b2} (p'\ell + mn) \} \right. \\ &\quad + PL'|\psi_p^+\rangle_{a1} \{ |\varphi_{\ell'}^+\rangle_{b2} (n + mp^*\ell'^*) \\ &\quad \left. + |\varphi_{\ell'}^-\rangle_{b2} (n\ell' - mp^*) \} \right. \\ &\quad \left. + PL'|\psi_p^-\rangle_{a1} \{ |\varphi_{\ell'}^+\rangle_{b2} (np - m\ell'^*) \right. \\ &\quad \left. + |\varphi_{\ell'}^-\rangle_{b2} (np\ell' + m) \} \right]. \end{aligned} \quad (14)$$

Here  $N = \frac{1}{\sqrt{1+|n|^2}}$ ,  $M = \frac{1}{\sqrt{1+|m|^2}}$ ,  $L = \frac{1}{\sqrt{1+|\ell|^2}}$ ,  $P = \frac{1}{\sqrt{1+|p|^2}}$ ,  $L' = \frac{1}{\sqrt{1+|\ell'|^2}}$ , and  $P' = \frac{1}{\sqrt{1+|p'|^2}}$  are real numbers. We are given the parameters  $m$  and  $n$ , and we can choose  $\ell$ ,  $\ell'$ ,  $p$  and  $p'$ . In rewriting the four particle state above, we use the basis  $\{|\varphi_\ell^\pm\rangle, |\psi_{p'}^\pm\rangle\}$  for pair ‘ $a1$ ’, while for the pair ‘ $b2$ ’ the basis  $\{|\varphi_{\ell'}^\pm\rangle, |\psi_p^\pm\rangle\}$  is used.

For faithful entanglement swapping parameters  $\ell$ ,  $p$ ,  $\ell'$ , and  $p'$  must satisfy a set of conditions. As an illustration, we choose the following set of conditions: (1)  $p' = mn\ell^*$  and  $\ell = mnp'^*$  for  $p'$  and  $\ell$ ; (2)  $n\ell' = mp^*$  and  $np = m\ell'^*$  for  $\ell'$  and  $p$ . A different set of conditions should lead to similar conclusions. We now consider following situations.

(i) *Standard entanglement swapping.* First, we state the conditions under which standard entanglement swapping is possible. If we choose  $\ell = \frac{1}{\ell'^*} = p^* = \frac{1}{p} = \ell' = \frac{1}{\ell'^*} = p' = \frac{1}{p'^*} = m = n$ , then in this case faithful swapping is possible with unit probability. And all parameters are pure phases and all the considered entangled states are maximally entangled. However, we note that only  $m$  and  $n$  need be pure phases, i.e., two initial pairs must be maximally entangled. Measurement basis need not be Bell basis. It can be non-maximally entangled basis with the requirement:  $|\ell| = |p'|$  and  $|p| = |\ell'|$ . The resulting state at Bob and Charlie’s location will be non-maximally entangled. Note that if the observer for the pair ‘ $a1$ ’ and the pair ‘ $b2$ ’ use the same basis, then this means  $\ell = p$ , and all basis vectors have same degree of entanglement.

(ii) *Probabilistic entanglement swapping.* Two conditions (1) and (2) given above cannot be satisfied simultaneously if the two initial states are not maximally entangled, i.e., when  $m$  or  $n$  are not pure phases. In such a case two out of four measurements will lead to reliable entanglement swapping. There are many possible choices for the values of the parameters that would lead to the reliable swapping but with probability less than unity. One such choice is  $\ell = \frac{1}{n^*}$ ,  $p' = m$ ,  $p = \frac{1}{m^*}$ , and  $\ell' = \frac{1}{n}$ . In this case successful swapping probability will be given by

$$P_{\text{succ}} = M^4 N^4 [ |n|^2(1 + |m|^2)^2 + |m|^2(1 + |n|^2)^2 ]. \quad (15)$$

This reduces to one-half, when the two initial states are maximally entangled (and other half will come from the two other outcomes).

There is an interesting possibility if the entanglement of the two initial states is not maximal, but identical. This happens when  $|m| = \frac{1}{|n|}$  or  $|m| = |n|$ . In this case three out of four possible measurement results would lead to reliable entanglement swapping and the swapping probability will be

$$P'_{\text{succ}} = 3|n|^2 N^8 (1 + |n|^2)^2. \quad (16)$$

For lesser constraints on the parameter values, as in the case of a qubit state teleportation, only one out of four possible measurement results will lead to faithful entanglement swapping. An example of these parameter values is  $\ell = \frac{1}{n^*}$ ,  $p' = m$ . And there are no constraints on other parameter values.

(iii) *No swapping*. In this most general case when the parameters values are not related to original resource, entanglement swapping is not possible.

Thus the scheme presented here tries to capture probabilistic and deterministic entanglement swapping protocols for qubits. Since entanglement swapping can be understood as a teleportation of an entangled state, we have generalized to such scenarios as well.

#### 4. Conclusions

In conclusion, we have shown that it is possible to teleport an unknown state with unit fidelity but less than unit probability using non-maximally entangled states. The difference between the present protocol and the existing ones is that neither we use local filtering nor entanglement concentration and then follow the standard teleportation protocol. It is a *single shot teleportation protocol* for non-maximally entangled resource without first converting to a maximally entangled pair. The key to this generalization is if one uses non-maximally entangled state as a resource use non-maximally entangled-state measurement containing same amount of entanglement as that of the shared

resource instead of the Bell measurement. This also points, perhaps, to a link between global and local entanglement. In some sense ours is a generalized quantum teleportation protocol that encompasses in a simple way probabilistic as well as deterministic (standard) teleportation protocols. In addition we have presented a scheme how to perform entanglement swapping using these resources. In future it will be interesting to extend these probabilistic teleportation schemes for higher dimensional Hilbert space and continuous variable systems. We hope that with the existing technology it may be possible to implement the probabilistic quantum teleportation protocol with ease.

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