Gyan Prakash

Research Summary:

A subset of an abelian group is said to be sum-free if the sum of any pair of elements of this subset lies in the complement of this subset. In the study of sum-free subsets of finite abelian groups, it turns out to be convenient to classify the groups according to the residue classes of the primes dividing their orders modulo 3. A finite abelian group is called type III when all the divisors of the order of G are congruent to 1 modulo 3. In earlier work with R. Balasubramanian we had obtained a classification theorem for largest sum-free subsets of type III. Using this in a work with D.S. Ramana we had obtained a formula for the number of orbits of $\mathcal{L}$, the set of all largest sum-free subsets of G, when G is of type III.

In a joint work in preparation with R. Balasubramanian and D.S. Ramana titled *Sum-free Subsets of Finite Abelian groups of Type III* we describe the work mentioned above and give a further application of the classification theorem. More precisely, for a finite abelian group H let $S(k, l, H)$ denote the set $\{\mathcal{B} \subset H : |\mathcal{B}| = k,\ |\mathcal{B} + \mathcal{B}| = l\}$, where $\mathcal{B} + \mathcal{B}$ is the set of sums $\{x + y | (x, y) \in \mathcal{B} \times \mathcal{B}\}$. The aforementioned classification theorem enables us to obtain a relation between the cardinality of the set $SF(G)$, where $SF(G)$ is the set of all sum-free subsets of G, and that of $S(k, l, H)$, where H is a supplement of a copy of $\mathbf{Z}/m\mathbf{Z}$ in G with m being the exponent of G. These results are obtained by refining the methods developed by Ben Green and Imre Ruzsa (Israel J. Math., Vol. 147, 2005) for the study of sum-free subsets in finite abelian groups.

When G is the additive group of a finite dimensional vector space over a finite field, we improve the estimate for $S(k, l, G)$ provided by Ben Green (Combinatorica, Vol. 25, 2005). This, together with the results mentioned above, then allows us to obtain a substantial improvement for the upper bound of the cardinality of $SF(G)$, when $G = (\mathbf{Z}/p\mathbf{Z})^r$ with p being a prime congruent to 1 modulo 3. The work in preparation mentioned above ends with an account of this improvement.

A finite subset of an abelian group is said to be incomplete if there is an element in G which can not be written as a sum of distinct elements from G and is zero-free if the identity element of G is such an element. In a preprint authored together with Jean-Marc Deshouillers titled *Large zero-free and incomplete subsets of* $\mathbf{Z}/p\mathbf{Z}$, we study the structure of zero-free and incomplete subsets of $\mathbf{Z}/p\mathbf{Z}$ the cardinality of which is close to largest possible, when p is sufficiently large a prime. In an earlier work, recorded in the same preprint, we had determined the cardinality of the largest zero-free subset of $\mathbf{Z}/p\mathbf{Z}$, when p is sufficiently large a prime.

An important requirement in the context of inequalities of the large sieve type is to obtain estimates for the sum $\sum_{x \in \mathcal{X}} | \sum_{i \in I} u_i e(xy_i))|^2$, where $\mathcal{X}$ is a well-spaced set of real numbers, I is a finite set, $\{u_i\}_{i \in I}$ are complex numbers and $\{y_i\}_{i \in I}$ is a *sparse subsequence* of the integers.

Basic examples of sparse sequences of integers are provided the sequence of values of polynomials of degree ≥ 2 with integer coefficients. In a recent note, Liangyi Zhao (Monatschefte fur Mathematik, to appear in 2007) obtained, by an application of the double large sieve inequality of Bombieri and Iwaniec, a non-trivial inequality of the large sieve type when $\{y_i\}_{i \in I}$ is the sequence of values of polynomials of degree 2 with integer coefficients. Zhao also shows that his estimate is essentially the best possible, except for the dependence of this estimate on the coefficients of the polynomial and the initial point of the interval I.

In a preprint authored together with D.S. Ramana titled *The Large Sieve Inequality for Quadratic Polynomial Amplitudes*, we combine Zhao's method with an interpolation argument due to Heath-Brown, to show that a modest improvement of Zhao's inequality is still possible in respect of its dependence on the coefficients of the polynomial and the initial point of I. In a work in preparation titled *Baier's variant of the Large Sieve Inequality for Quadratic Amplitudes* we show that a variant of Zhao's inequality due to S. Baier may also be refined.

Publications:

1. R. Balasubramanian and Gyan Prakash, *Asymptotic formula for sum-free sets in abelian groups*, Acta Arithmetica, **Vol. 127, No. 2**, 115-124, 2007.

Preprints:

1. R. Balasubramanian, Gyan Prakash and D.S. Ramana, *Sum-free subsets of finite abelian groups of type III,* in preparation.
2. Jean-Marc Deshouillers and Gyan Prakash, *Large zero-free and incomplete subsets of* $\mathbf{Z}/p\mathbf{Z}$, in preparation.
3. Gyan Prakash and D.S. Ramana, *The Large Sieve Inequality for Quadratic Polynomial Amplitudes,* http://arxiv.org/abs/0705.1739.
4. Gyan Prakash and D.S. Ramana, *Baier's Variant of the Large Sieve Inequality for Quadratic Polynomial Amplitudes,* in preparation.

Conference/Workshops Attended:

1. *The First Indo-U.K. conference in Number Theory*, India, September 2006.
2. *International Conference on Number Theory*, India, December 2006.

3. *International Conference in Number Theory and Cryptography*, India, February 2007.

Visits to other Institutes:

1. Institute of Mathematical Sciences, Chennai, India, September, 2006.

2. Institute of Mathematical Sciences, Chennai, India, March, 2007.

Invited Lectures/Seminars:

1. *Large zero-free subsets of* $\mathbf{Z}/p\mathbf{Z}$, First Indo-U.K. conference, Institute of Mathematical Sciences, Chennai, September, 2006.

Other Activities:

1. I have been a referee for a paper submitted to the journal *Proceedings- Mathematical Sciences* published by Indian Academy of Sciences.