B. Ramakrishnan

Research Summary:

1. **Rankin-Cohen Brackets and Jacobi forms of several variables** (with B. Sahu):

   The Rankin-Cohen bracket is a differential operator, which sends modular forms to modular forms. Following Rankin's method, D. Zagier computed the n-th Rankin-Cohen bracket of a modular form g of weight k_1 with the Eisenstein series of weight k_2 and then computed the inner product of this Rankin-Cohen brackets with a cusp form f of weight $k = k_1 + k_2 + 2n$ and showed that this inner product gives the special value of the Rankin convolution of f and g upto a constant. Following the work of Zagier, Y. Choie and W. Kohnen generalized the above result to Jacobi forms.They computed the Petersson scalar product $\langle F, [G, E_{k,m}]_\nu \rangle$ of a Jacobi cusp form F against the Rankin-Cohen bracket $[G, E_{k,m}]_\nu$ of a Jacobi form G and an Eisenstein series $E_{k,m}$ explicitly under a certain assumption on the weight of G and k. Though the concept of Rankin-Selberg convolution has not been done yet in the case of Jacobi forms, the above mentioned work of Choie and Kohnen gives the special value of a kind of Rankin-Selberg type convolution of the Jacobi forms F and G. In this work, we study similar results for Jacobi forms of higher degree, that is Jacobi forms on $\mathcal{H} \times \mathbb{C}^{(g,1)}$.

2. **A characterization of the space of new forms of half-integral weight and a conjecture of Zagier** (with S. Gun and M. Manickam).

   In our earlier work we showed the existence of a subspace of $M_{k+1/2}(4)$ which is mapped to $M_{2k}(1)$ under the first Shimura map. By characterizing the space of new forms in $S_{k+1/2}(4N)$ with respect to the prime $p = 2$, we show that the space $M^{(\pm,2)}_{k+1/2}(4N)$, which is the subspace of $M_{k+1/2}(4N)$ consisting of forms which are eigenfunctions under the $W(4)$ operator with eigenvalue ± 1, is mapped to $M_{2k}(N)$ under a class of Shimura maps when N is odd and square-free. This

generalizes our earlier work. The present work gives new formulas for $r_{2k+1}(|t|n^2)$, where $t \equiv 1 \pmod 4$ is a square-free integer with $(-1)^k t > 0$.

3. **Twisted Averages of L-functions** (with M. Manickam and V. Kumar Murty).

   W. Luo and D. Ramakrishnan used twisted averages of L-functions to characterize modular forms. More precisely, let f be a new form of integral weight k, level N with trivial character. They proved that for any real $\sigma \in \left(\frac{k-1}{2}, \frac{k+1}{2}\right)$,

$$\lim_{j\to\infty} \frac{1}{p^{j-1}} \sum_{\substack{\chi \bmod p^j \\ \mathrm{ord}(\chi) = p^a}} \overline{\chi(m)}\ L(f,\chi,\sigma) = \frac{1}{p}\left(1-\frac{1}{p}\right)\frac{a_f(m)}{m^\sigma}.$$

   They also proved a result for averages of quadratic twists. In this case, on the right hand side, one gets not $a_f(m)$ itself, but rather a rational function in $a_f(m)$ from which the Fourier coefficient can be recovered. It should be noted that they proved that the coefficients of the L-function (and hence the L-function itself) is completely determined by certain special values of twists of the L-function. In the present work, we derive similar implications for the L-functions associated to new forms in the case of both integer and half-integer weight, by proving appropriate results with explicit error terms. This work is in progress.

Publications:

1. (with B. Sahu) *On the Fourier Expansions of Jacobi Forms of Half-Integral Weight*, Int. J. Math. Math. Sci. Vol 2006.

Preprints:

1. (with B. Sahu) *Differential operators on Jacobi forms of several variable*

2. (with S. Gun, M. Manickam) *A Characterization of the space of new forms of half-integral weight and a conjecture of Zagier.*

3. (with M. Manickam and V. Kumar Murty) *Twisted averages of L-functions.*

Conferences/Workshops Attended:

1. Attended the International Congress of Mathematicians (ICM) 2006 held at Spain during August 2006.

2. Attended and gave an invited talk in the International Conference on Number Theory and Applications, RKM Vivekananda College, Chennai during December 27–29, 2006.

3. Attended and gave a talk in the International Conference on Number Theory and Cryptography, held at HRI, Allahabad during Feb 2007.

4. Attended and gave an invited talk in the 21st Automorphic Forms Workshop held at the University of California at Santa Barbara, Santa Barbara during March 2007.

Visits to other Institutes:

1. IHES, Paris, France. April 2006.

2. University of Toronto at Mississauga, Mississauga, Canana. June-July, 2006.

Invited Lectures/Seminars:

1. *On a conjecture of Zagier*, Aachen-Köln-Lille-Siegen seminar on Automorphic Forms, RWTH, Aachen, April 2006.

2. *An estimate for a certain average of the special values of character twists of modular L-functions*, Number Theory Seminar, Queen's University, Kingston, July 2006.

3. *On a conjecture of Zagier*, Symposium in Mathematics (in the memory of Professor I.S. Luthar), Panjab University, s Chandigarh, March 2007.

Other Activities:

1. Guiding one student for his Ph.D and another student is doing a second year project under me.
2. Member of Faculty Recruitment Committee (Mathematics).
3. One of the organizers of the International Conference on Number Theory held at HRI during December 2006.
4. Dean of Administration (since October 2005).