Brundaban Sahu

Research Summary:

1. The derivative of a modular form is not a modular form. But there are many interesting connections between differential operators and the theory of modular forms. In 1956, R. A. Rankin gave a general description of the differential operators which send modular forms to modular forms. In 1977, H. Cohen constructed modular forms from derivatives of two modular forms. In 1990, Zagier studied the algebraic properties of these bilinear operators and called them Rankin-Cohen brackets. Following Rankin's method, Zagier computed the n-th Rankin-Cohen bracket of a modular form g of weight k_1 with the Eisenstein series of weight k_2 and then computed the inner product of this Rankin-Cohen brackets with a cusp form f of weight $k = k_1 + k_2 + 2n$ and showed that this inner product gives the special value of the Rankin-Selberg convolution of f and g upto a constant.

   The theory of the classical Jacobi forms on $\mathcal{H} \times \mathbb{C}$ has been extensively studied by Eichler and Zagier. The study of Rankin-Cohen bracket for Jacobi forms using the heat operator started with the works of YoungJu Choie. Following the work of Zagier, Y.J Choie and Kohnen generalized the above result of Zagier to Jacobi forms.They computed the Petersson scalar product $\langle F, [G, E_{k,m}]_\nu \rangle$ of a Jacobi cusp form F against the Rankin-Cohen bracket $[G, E_{k,m}]_\nu$ of a Jacobi form G and an Eisenstein series $E_{k,m}$ explicitly under a certain assumption on the weight of G and k. Though the concept of Rankin-Selberg convolution has not been done yet in the case of Jacobi forms, the above mentioned work of Choie and Kohnen gives the special value of a kind of Rankin-Selberg type convolution of the Jacobi forms F and G.

   We study the similar results for Jacobi forms of higher degree, that is Jacobi forms on $\mathcal{H} \times \mathbb{C}^{(g,1)}$ (Preprint [1]). It is also possible to extend this method/result to Hermitian Jacobi forms, that is Jacobi forms arising from number fields (of degree two). The work is in progress for the case Hermitian Jacobi form.

2. The congruences between modular forms is another interesting topics in modular forms. First it was studied by J. Sturm in 1987. The question is

to find the least positive integer n such that their $n-$th Fourier coefficients must differ if the two modular forms are not identical. Recently there are improvements by Ram Murty and Kohnen. Using some relationship of Jacobi forms and modular forms, we study the congruences between Jacobi forms. The work is in progress.

3. We study the distribution of quadratic non-residues which are not primitive roots (QNRNP) (Preprint [2]). We prove the following theorems.

   Theorem 1. Let $\varepsilon \in (0, 1/2)$ be fixed and let N be any positive integer. Then for all primes $p \geq \exp((2\varepsilon^{-1})^{8N})$ satisfying

   $$\frac{\phi(p-1)}{p-1} \leq \frac{1}{2} - \varepsilon,$$

   we can find N consecutive QNRNP's modulo p.

   The above theorem generalizes a result of A. Brauer and a result of our previous paper "Distribution of Quadratic non-residues which are not primitive roots, Math. Bohem. 130 (2005), no. 4, 387–396 (with S. Gun, R. Thangadurai and B. Ramakrishnan).

   Given a prime number p, we let

   $$k := \frac{p-1}{2} - \phi(p-1)$$

   denote the number of QNRNP's modulo p and we write $g_1 < g_2 < \ldots < g_k$ for the increasing sequence of QNRNP's.

   Corollary. For any given $\varepsilon \in (0, 1/2)$ and natural number N, for all primes $p \geq \exp((2\varepsilon^{-1})^{8N})$ and satisfying $\phi(p-1)/(p-1) \leq 1/2 - \varepsilon$, the sequence $g_1 + N, g_2 + N, \ldots, g_k + N$ contains at least one QNRNP.

   Theorem 2. There exists an absolute constant $c_0 > 0$ such that for almost all primes p, there exist a string of $N_p = \left\lfloor c_0 \frac{\log p}{\log\log p} \right\rfloor$ of quadratic non-residues which are not primitive roots.

   Theorem 3. For every positive integer N there are infinitely many primes p for which $1,\ 2,\ \ldots,\ N$ are quadratic residues modulo p, and there exist both a string of N consecutive $QNRNP$'s as well as a string of N consecutive primitive roots. The smallest such prime can be chosen to be $< \exp(\exp(c_1 N^2))$, where $c_1 > 0$ is an absolute constant.

Publications:

1. B.Sahu and B. Ramakrishnan, *On the Fourier Expansions of Jacobi Forms of Half-Integral Weight*, Int. J. Math. Math. Sci. Vol 2006.

Preprints:

1. B. Sahu, B. Ramakrishnan, *Differential operators on Jacobi form of several variable*
2. B. Sahu, S. Gun, Florian Luca, P. Rath and R. Thangadurai, *Distribution of Residues Modulo p.*

Conferences/Workshops Attended:

1. *International Conference on Number Theory and Cryptography* (Feb 23-27, 2007), HRI, Allahabad
2. *International Conference on Number Theory and Applications* (Dec 27-29, 2006), RKM Vivekananda College, Chennai
3. *International Conference on Number Theory* (Dec 1-5, 2006), HRI, Allahabad
4. *India-UK Number Theory Conference* (Sept 18-23, 2006), IMSc, Chennai

Invited Lectures/Seminars:

1. *On the Rankin-Cohen Brackets for Jacobi Forms of higher degree*, International Conference on Number Theory and Applications (Dec 27-29, 2006) in RKM Vivekananda College, Chennai.
2. *On the Fourier expansion of Jacobi Forms of half-integral weights*, International Conference on Number Theory (Dec 1-5, 2006) in HRI, Allahabad.
3. *Basic Algebra*, Interactive Mathematics Training Camp, Regional Institute of Education, June 2006, Bhubaneswar (sponsored by NBHM), organized by Institute of Mathematics and Applications, Bhubaneswar.
4. *Lectures on Elementary Number Theory*, Interactive Mathematics Training Camp , Regional Institute of Education, June 2006, Bhubaneswar (sponsored by NBHM), organized by Institute of Mathematics and Applications, Bhubaneswar.

Academic recognition/Awards:

1. I became a Reviewer of Mathematical Reviews, American Mathematical Society from March 2007.