D. Surya Ramana

Research Summary:

In a preprint titled *Arithmetical applications of an identity for the Vandermonde determinant* we address a general question particular cases of which are the problem of determining upper bounds for the number of integer points on small arcs of conics considered in a number of works of J. Cilleruelo *et. al.* (Proc. A.M.S., Vol. 115, 1992, Duke Math. Journal, Vol. 76, 1994, Journal of Number Theory, Vol. 63, 1997, Acta Arith., Vol. 85, 1998) and problem of showing that the number of divisors of an integer N lying in certain arithmetical progressions is bounded independently of N, considered in H. Lenstra (Math. of Comp., Vol. 42, 1984). These problems are of interest not only for their intrinsic appeal but also for their relation to a number of other questions on the interface between harmonic analysis and number theory, as described recently by J. Cilleruelo and A. Granville (Math. Arxiv, NT/0608109, August, 2006).

In the preprint cited, we present an identity for the Vandermonde determinant that provides rather simple proofs for a number of results given in the works of J. Cilleruelo *et. al.* as well as the result of Lenstra mentioned above. We also observe that this identity may be used to deduce a special case of the overlapping theorem of J. Cilleruelo and G. Tenenbaum (preprint, 2005), by which means, or by closely related methods, the aforementioned results are obtained in the literature.

A particular case of the results on integer points on arcs of conics dealt with in the preprint described above shows that there is a constant $c(d)$ such any arc of the conic $X^2 + dY^2 = R$ of length $c(d)|R|^{1/6}$ contains no more than two integer points, where d and R are integers. In a work in preparation titled *Arcs of Conics Containing Three Integer Points* we provide a family of examples of arcs of lengths $c'(d)R^{1/6}$ containing exactly three integer points for some $c'(d) > c(d)$. In particular, when d is not congruent 2 modulo 4 we show that such arcs exist for any $c > c(d)$ and that the triples of integer points lying on such arcs may all be found in single orbit under the action of the group $\Gamma(4d)$ on triples of integer points in the plane. These results generalise the result of J. Cilleruello (Acta Arith., Vol. 51, 1991) where the case of the circle is studied.

A subset of an abelian group is said to be sum-free if the sum of any pair of elements of this subset lies in the complement of this subset. In the study of sum-free subsets of finite abelain groups, it turns out to be convenient to classify the groups according to the residue classes of the primes dividing their orders modulo 3. A finite abelian group is called type III when all the divisors of the order of G are congruent to 1 modulo 3. In earlier work with Gyan Prakash, we obtained, with the aid of a classification theorem for largest sum-free subsets of type III due to R. Balasubramanian and Gyan Prakash, a formula for the number of orbits of $\mathcal{L}$, the set of all largest sum-free subsets of G, when G is of type III.

In a joint work in preparation with R. Balasubramanian and Gyan Prakash titled *Sum-free Subsets of Finite Abelian groups of Type III* we describe the work mentioned above and give a further application of the classification theorem. More precisely, for a finite abelian group H let $S(k, l, H)$ denote the set $\{\mathcal{B} \subset H : |\mathcal{B}| = k,\ |\mathcal{B} + \mathcal{B}| = l\}$, where $\mathcal{B} + \mathcal{B}$ is the set of sums $\{x + y | (x, y) \in \mathcal{B} \times \mathcal{B}\}$. The aforementioned classification theorem enables us to obtain a relation between the cardinality of the set $SF(G)$, where $SF(G)$ is the set of all sum-free subsets of G, and that of $S(k, l, H)$, where H is a supplement of a copy of $\mathbf{Z}/m\mathbf{Z}$ in G with m being the exponent of G. These results are obtained by refining the methods developed by Ben Green and Imre Ruzsa (Israel J. Math., Vol. 147, 2005) for the study of sum-free subsets in finite abelian groups.

When G is the additive group of a finite dimensional vector space over a finite field, we improve the estimate for $S(k, l, G)$ provided by Ben Green (Combinatorica, Vol. 25, 2005). This, together with the results mentioned above, then allows us to obtain a substantial improvement for the upper bound of the cardinality of $SF(G)$, when $G = (\mathbf{Z}/p\mathbf{Z})^r$ with p being a prime congruent to 1 modulo 3. The work in preparation mentioned above ends with an account of this improvement.

An important requirement in the context of inequalities of the large sieve type is to obtain estimates for the sum $\sum_{x \in \mathcal{X}} | \sum_{i \in I} u_i e(xy_i))|^2$, where $\mathcal{X}$ is a well-spaced set of real numbers, I is a finite set, $\{u_i\}_{i \in I}$ are complex numbers and $\{y_i\}_{i \in I}$ is a *sparse subsequence* of the integers.

Basic examples of sparse sequences of integers are provided the sequence of values of polynomials of degree ≥ 2 with integer coefficients. In a recent note, Liangyi Zhao (Monatschefte fur Mathematik, to appear in 2007) obtained, by an application of the double large sieve inequality of Bombieri and Iwaniec, a non-trivial inequality of the large sieve type when $\{y_i\}_{i \in I}$ is the sequence of values of polynomials of degree 2 with integer coefficients. Zhao also shows that his estimate is essentially the best possible, except for the dependence of this estimate on the coefficients of the polynomial and the initial point of the interval I.

In a preprint authored together with Gyan Prakash titled *The Large Sieve Inequality for Quadratic Polynomial Amplitudes*, we combine Zhao's method with an interpolation argument due to Heath-Brown, to show that a modest improvement of Zhao's inequality is still possible in respect of its dependence on the coefficients of the polynomial and the initial point of I. In a work in preparation titled *Baier's variant of the Large Sieve Inequality for Quadratic Amplitudes* we show that a variant of Zhao's inequality due to S. Baier may also be refined.

Preprints:

1. D.S. Ramana,*Arithmetical applications of an identity for the Vandermonde determinant,* http://arxiv.org/abs/0705.1739.
2. D.S. Ramana, *Arcs of Conics Containing Three Integer Points,*in preparation .
3. R. Balasubramanian, Gyan Prakash and D.S. Ramana, *Sum-free subsets of finite abelian groups of type III,* in preparation .
4. Gyan Prakash and D.S. Ramana, *The Large Sieve Inequality for Quadratic Polynomial Amplitudes,* http://arxiv.org/abs/0705.1739 .
5. Gyan Prakash and D.S. Ramana, *Baier's Variant of the Large Sieve Inequality for Quadratic Polynomial Amplitudes,* in preparation .

Conference/Workshops Attended:

1. *The First Indo-U.K. conference in Number Theory*, India, September 2006.
2. *Conference in Number Theory, India, October 2006.*
3. *International Conference on Number Theory, India, December 2006.*
4. *International Conference in Number Theory and Cryptography, India, February 2007.*

Visits to other Institutes:

1. Indian Institute of Technology, Mumbai, India, January, 2007.

Invited Lectures/Seminars:

1. *Orbits of Largest Sum-free Subsets in Abelian Groups of Type III*, First Indo-U.K. conference, Institute of Mathematical Sciences , Chennai, September, 2006.
2. *Large Sieve Inequality for Quadratic Polynomials*, Conference on Number Theory, Panjab University, Chandigarh, October, 2006.

Other Activities:

1. Assisted the Director, H.R.I. in the selection process for the institute's engineering consultant.
2. Conducted the mathematics part of the Annual Science Talent Test of H.R.I..
3. Served on the Local Works Committee of H.R.I. for the year 2006 to 2007.