R. Thangadurai

Research Summary:

(1) **Relative Davenport's Constant.** (Reference. [1] R. Thangadurai, *The variant of Davenport's constant*, Proc. Indian Acad. Sci. (Math. Sci.), **117**, 1-11, (2007)).

Let G be a finite abelian group of exponent n. Let A be a non-empty subset of $\{1, 2, \cdots, n\}$. By $d_A(G)$, we denote the least positive integer t such that for any sequence $g_1, g_2, \cdots, g_t$ (not necessarily distinct) in G, we can find a subsequence $g_{i_1}, g_{i_2}, \cdots, g_{i_\ell}$ satisfying

$$\sum_{j=1}^{\ell} a_j g_{i_j} = 0 \text{ in } G.$$

When $A = \{1\}$, then $d_A(G) = D(G)$, the well-known Davenport's constant. Hence $d_A(G)$ generalizes the classical Davenport's constant. In [1], we have proved the following result.

Theorem 1. *Let $G \sim \mathbb{Z}_{p^{e_1}} \oplus \mathbb{Z}_{p^{e_2}} \oplus \cdots \oplus \mathbb{Z}_{p^{e_l}}$ where $1 \leq e_1 \leq e_2 \leq \cdots \leq e_l$ are integers. Then, for any non-empty subset A of $\{1, 2, \cdots, p^{e_l}\}$ such that the elements of A are incongruent modulo p and non-zero modulo p, we have,*

$$d_A(G) \leq \left\lceil \frac{1}{|A|} \left(1 + \sum_{i=1}^{l} (p^{e_i} - 1) \right) \right\rceil$$

where $\lceil x \rceil$ denotes the smallest positive integer greater than or equal to x.

Corollary 1.1. *Let $G \sim \mathbb{Z}_p^d$ where $d \geq 1$ integer. Then for any non-empty subset A of $\{1, 2, \cdots, p-1\}$, we have,*

$$d_A(G) \leq \left\lceil \frac{1}{|A|}(d(p-1)+1) \right\rceil.$$

For any integer $x \geq 1$, we denote $]x[$ by $\{1, 2, \cdots, x\}$.

Corollary 1.2. *Let $G \sim \mathbb{Z}_p^d$ where $d \geq 1$ integer. If*
(a) $A =]p-1[$, then $d_A(G) = d+1$;
(b) $A_1 = \{a \in]p-1[: a \equiv x^2 \pmod p$ for some $x \in]p[\}$, then $d_{A_1}(G) = 2d+1$;
(c) $A_2 = \{a \in]p-1[: a \not\equiv x^2 \pmod p$ for all $x \in]p[\}$, then $d_{A_2}(G) = 2d+1$;
(d) $A_3 = \{a \in]p-1[: a$ generates $\mathbb{F}_p^*\}$, then $d_{A_3}(G) = 2d+1$;
(e) $A_4 = \{a \in]p-1[: a \in A_2 \backslash A_3\}$, then $d_{A_4}(G) = 2d+1$ holds for all primes $p \neq 2^{2^m}+1$ for some $m \in \mathbb{N}$.
(f) $A_5 \subset]p-1[$ such that $|A_5| = (p-1)/2$ and if $x \in A_5$, then $p - x \notin A_5$, then $d_{A_5}(\mathbb{Z}_p^d) = 2d+1$.

(g) *$A =]r[\subset]p[$ and $r \geq d$, then*

$$d_A(G) = \left\lceil \frac{d(p-1)+1}{r} \right\rceil.$$

Other than this study, along with Gao, Hou and W. Schmid, we proved some results in the classical zero-sum lattice point constant for the group $G \sim \mathbb{Z}_n^3$ when $n = 3, 5, 2^a 3^b$. Also, we proved some inverse problem related to this lattice point problem too.

(2) **Distribution of quadratic non-residues which are not primitive roots modulo** p. (Reference. [2] S. Gun, F. Luca, P. Rath, B. Sahu and R. Thangadurai, *Distribution of residues modulo p*, Preprint, 2006.)
In [2], we studied the distribution of quadratic non-residues which are not primitive roots modulo p. More precisely, we studied that whether for large enough primes p has arbitrarily long string of consecutive quadratic non-residues which are not primitive roots modulo p? We abbreviate quadratic non-residues which are not primitive roots modulo p by QNRNP. Indeed, we proved, in [2], the following results.

Theorem 2. *Let $\varepsilon \in (0, 1/2)$ be fixed and let N be any positive integer. Then for all primes $p \geq \exp((2\varepsilon^{-1})^{8N})$ satisfying*

$$\frac{\phi(p-1)}{p-1} \leq \frac{1}{2} - \varepsilon,$$

we can find N consecutive QNRNP's modulo p.

Corollary 2.1. *For any given $\varepsilon \in (0, 1/2)$ and natural number N, for all primes $p \geq \exp((2\varepsilon^{-1})^{8N})$ and satisfying $\phi(p-1)/(p-1) \leq 1/2 - \varepsilon$, the sequence $g_1 + N, g_2 + N, \ldots, g_k + N$ contains at least one QNRNP.*

Theorem 3. *There exists an absolute constant $c_0 > 0$ such that for almost all primes p, there exist a string of $N_p = \left\lfloor c_0 \frac{\log p}{\log\log p} \right\rfloor$ of quadratic non-residues which are not primitive roots.*

Theorem 4. *For every positive integer N there are infinitely many primes p for which $1, 2, \ldots, N$ are quadratic residues modulo p, and there exist both a string of N consecutive QNRNP's as well as a string of N consecutive primitive roots. The smallest such prime can be chosen to be $< \exp(\exp(c_1 N^2))$, where $c_1 > 0$ is an absolute constant.*

Apart from these results, we continue our study of this distribution problems of these residues. For instance, we are tackling the following two problems. (i) For a given integer a which is not a square, can one find infinitely many primes p such that a is a QNRNP modulo p?

(ii) Whether the spacing distribution of these residues modulo p behaves like Poissonian with mean spacing tending to infinity?

(3) **Collected works of Dr. S. S. Pillai.** Most of the papers of Dr. S. S. Pillai has been typed during this period. Also, we have collected many old letters of Dr. S. S. Pillai from Dr. S. Chowla, T. Vijayaraghavan etc. This work is expected to complete by the end of 2007.

Publications:

1. W. D. Gao and R. Thangadurai, *On zero-sum Sequences of prescribed length*, Aequationes Math., **72**, 201-212, (2006)

2. R. Thangadurai, *A variant of Davenport's Constant*, Proc. Indian. Acad. Sci. (Math. Sci.), **117**, 1-11, (2007)

3. W. D. Gao, Q. H. Hou, W. A. Schmid and R. Thangadurai, *On short zero-sum subsequences - II*, Integers, **7** A-21, 22pp, (2007)

Preprints:

1. S. Gun, F. Luca, P. Rath, B. Sahu and R. Thangadurai, *Distribution of residues modulo p*, Submitted for Publication.

2. R. Thangadurai, *Distribution of quadratic non-residues which are not primitive roots modulo p - II*, in preparation.

3. P. Moree and R. Thangadurai, *Distribution of quadratic non-residues which are not primitive roots modulo p - III*, in preparation.

Conference/Workshops Attended:

1. *Number Theory and Harmonic Analysis*, University of Lille, France, June, 2006

2. *Additive Number Theory*, University of Saint-Etienne, France, June, 2006

Visits to other Institutes:

1. Center for Combinatorics, Nankai University, Tianjin, P. R. China, April, 2006.

2. University of Lille, Lille, France, June, 2006.

3. University of Saint-Etienne, Saint-Etienne, France, June, 2006.

Invited Lectures/Seminars:

1. *Davenport's Constant*, Combinatorics Seminar, Center for Combinatorics, Nankai University, Tianjin, P.R. China, April, 2006.

2. *Davenport's Constant*, Number Theory Seminar, University of Lille, Lille, France, June, 2006.

Other Activities:

1. Seminar Course for the first year graduate students, August-December, 2006.

2. Galois Theory for the first year graduate students, January-May, 2006.

3. Office Furniture Committee, Convener, 2006-07.

4. Library Committee, Member, 2006-07.