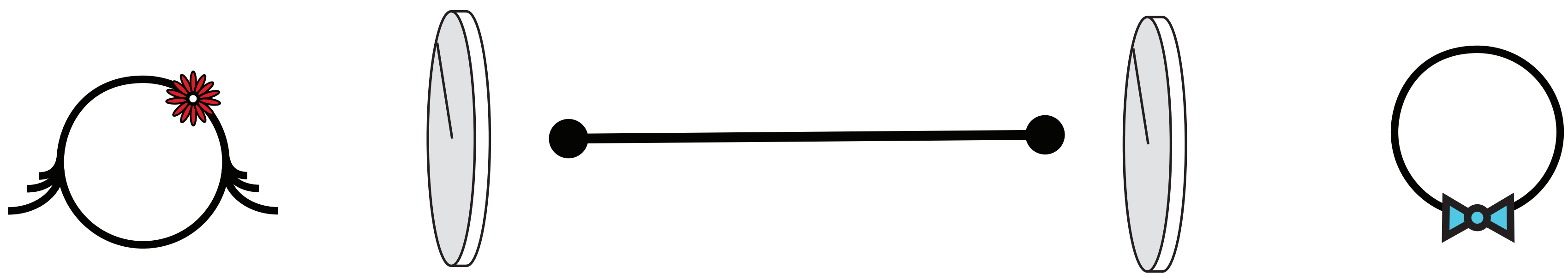
Multipartite Quantum (and Post-Quantum) Steering

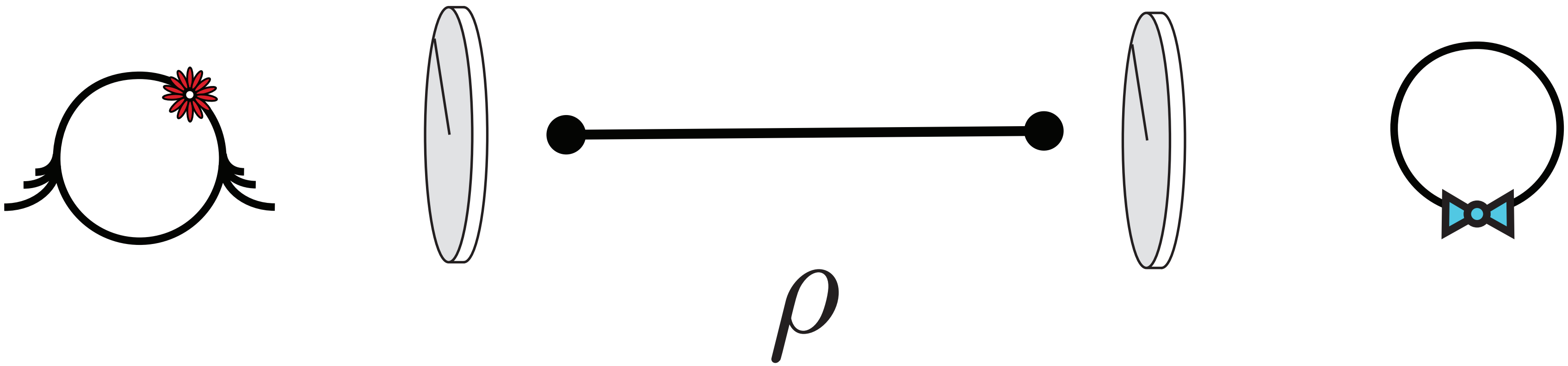
Daniel Cavalcanti (ICFO-Barcelona)

Allahabad 2015

Standard quantum information

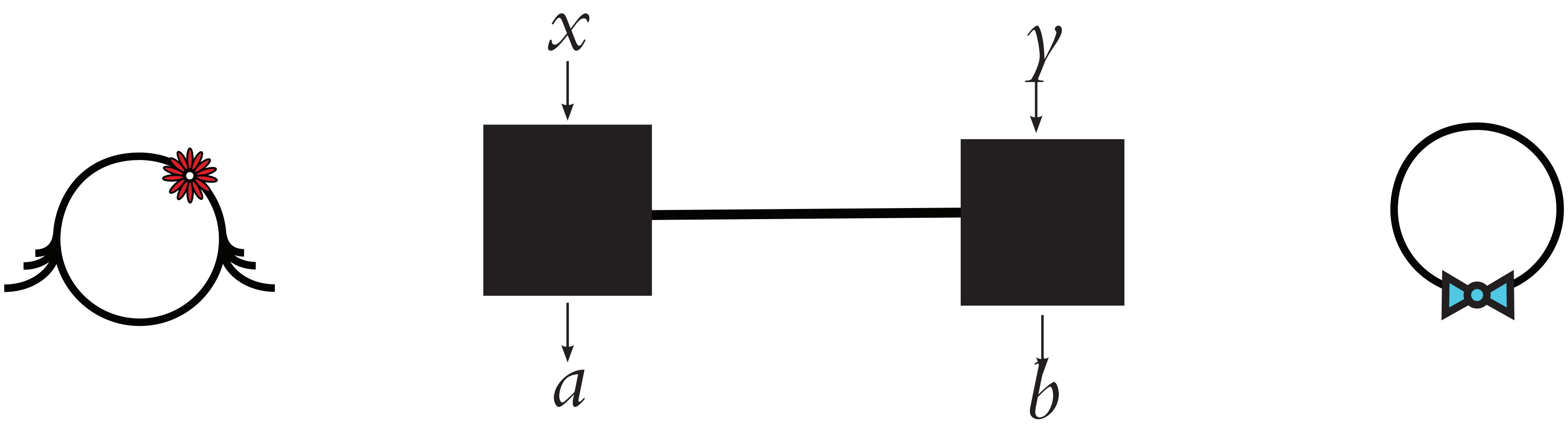


Standard quantum information



Teleportation, dense coding, entanglement,...

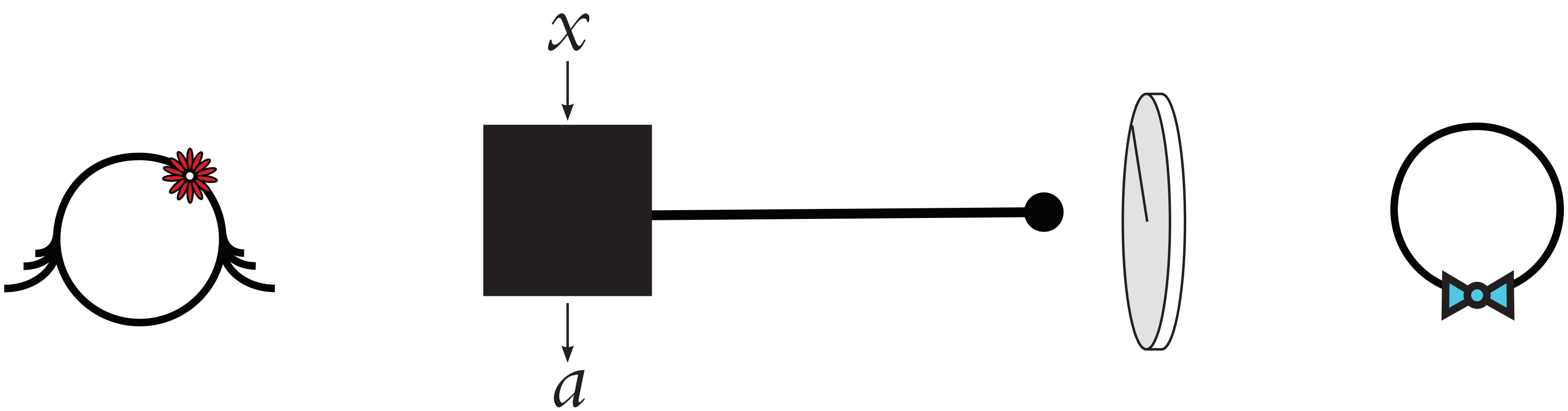
Device-independent quantum information



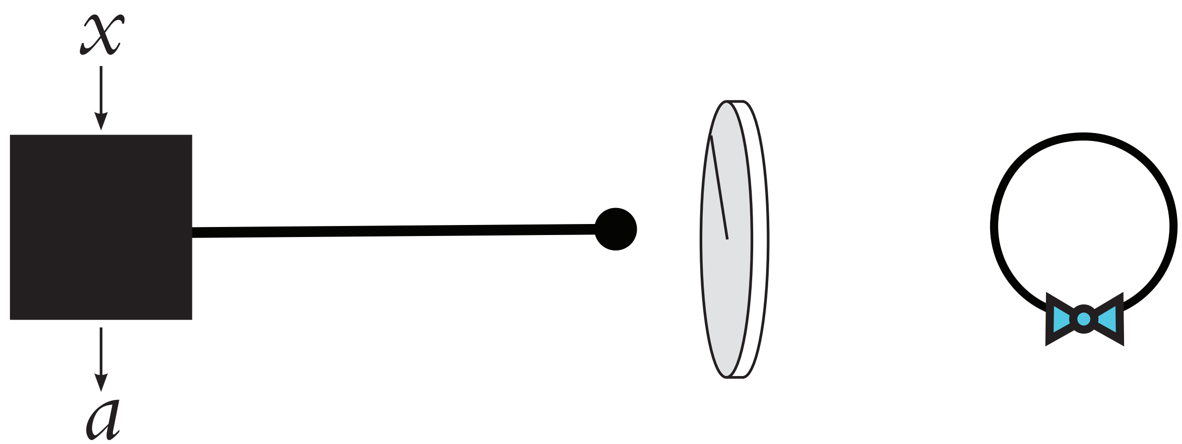
$$p(ab|xy)$$

DI QKD, entanglement, dimention witness, ...

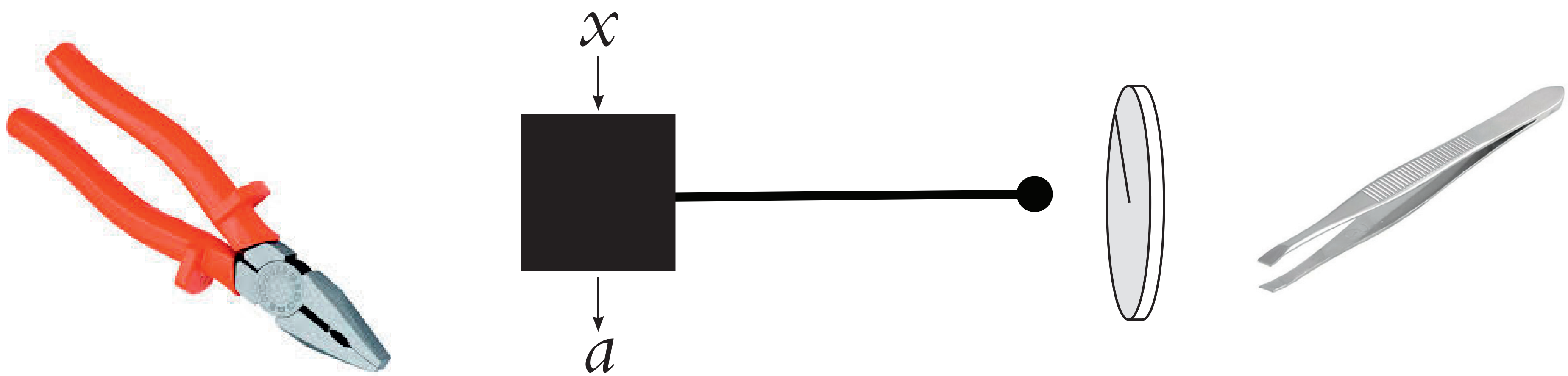
The steering scenario



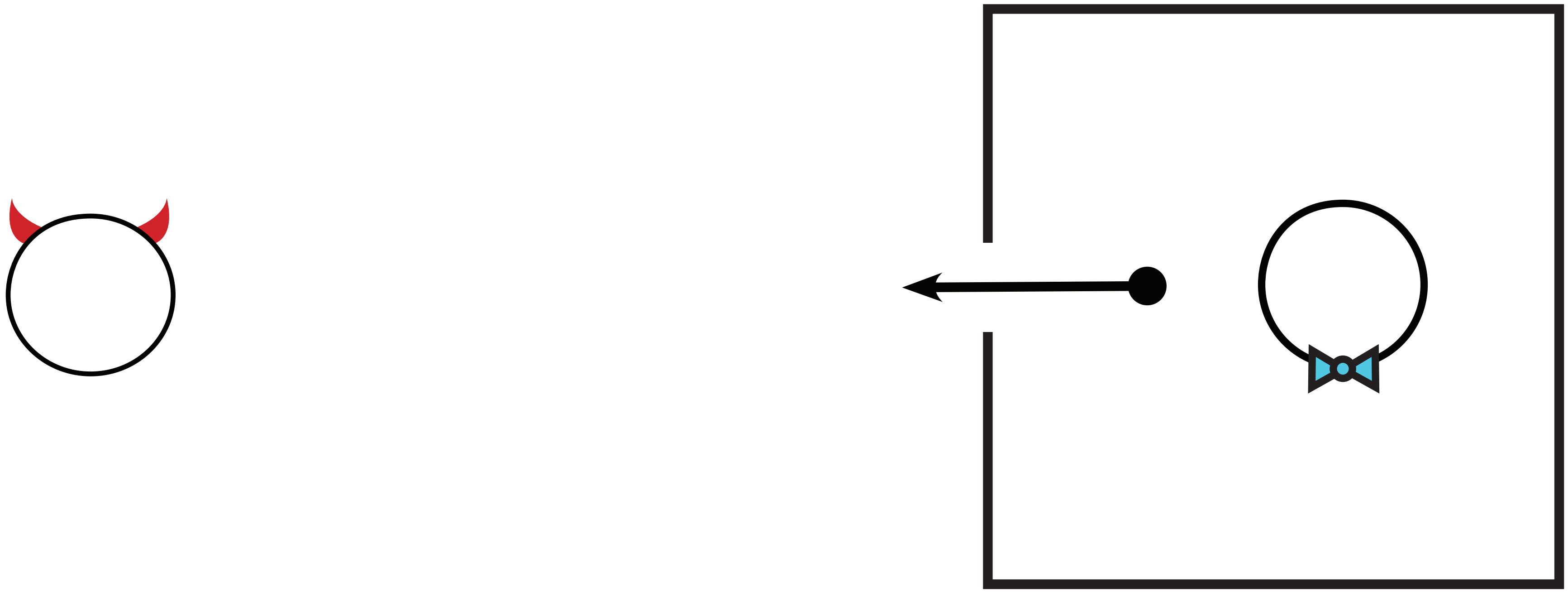
The steering scenario



The steering scenario



The steering scenario



Quantum steering

Foundational approach:

Which states can Alice remotely prepare to Bob

Schrödinger 1936

Operational approach:

Entanglement certification with an untrusted party

Wiseman, Jones, Doherty, 2007

Quantum information with untrusted parties

1. Operational approach to steering

Entanglement detection with untrusted Alice.
SDP characterization.

2. Multipartite steering

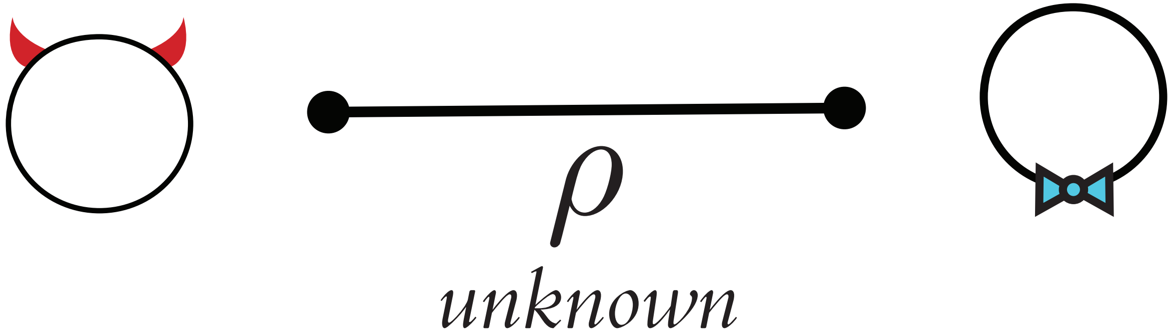
Entanglement detection with untrusted parties.

3. Post-quantum steering

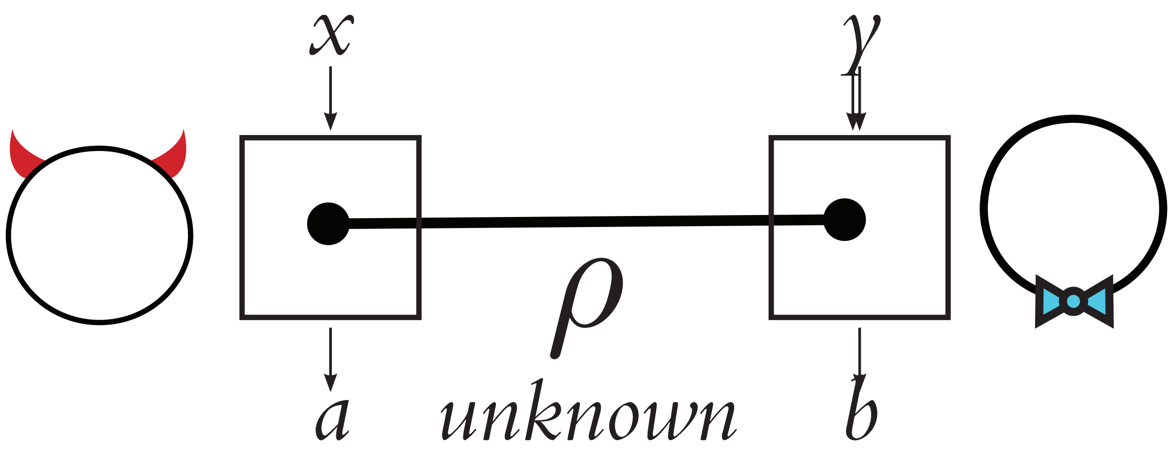
Post-quantum correlations
without post-quantum nonlocality

1.
Operational approach to steering

Operational approach to quantum steering



Operational approach to quantum steering



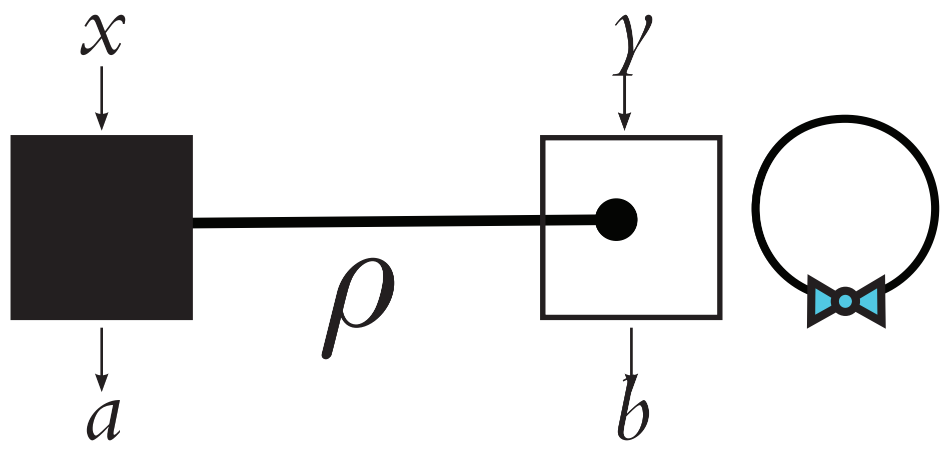
How can Bob certify entanglement?

Operational approach to quantum steering

unknown

$M_{a|x}$

in arbitrary dimension

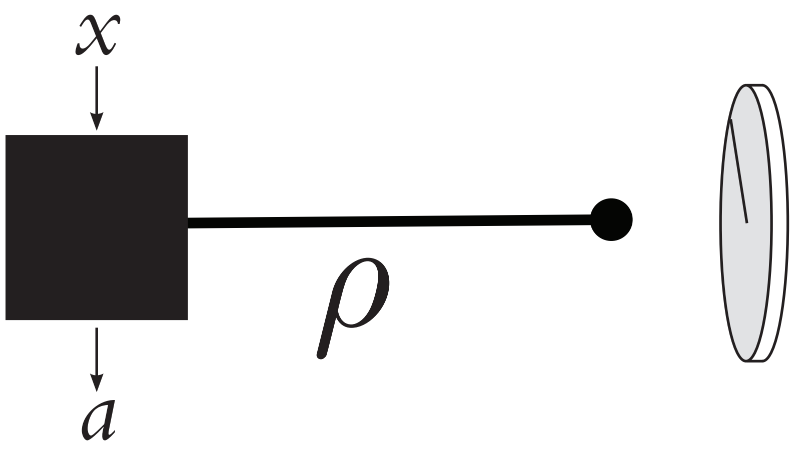


Operational approach to quantum steering

unknown

$M_{a|x}$

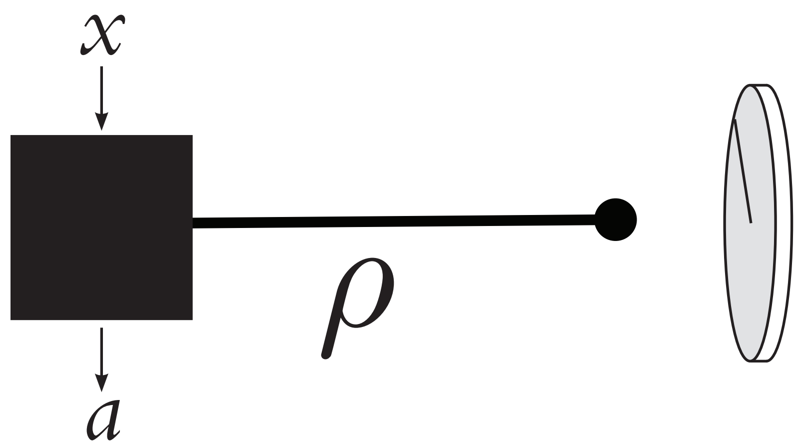
in arbitrary dimension



tomography

Operational approach to quantum steering

$M_{a|x}$



tomography

$$\begin{aligned}\sigma_{a|x} &= \mathrm{Tr}_A(M_{a|x} \otimes I \ \rho_{AB}) \\ &= p(a|x)\rho_{a|x}\end{aligned}$$

"assemblage"

Operational approach to quantum steering

Given $\sigma_{a|x}$,
is there a separable state ρ_{sep}
and measurements $M_{a|x}$
that can provide it?

Operational approach to quantum steering

Unsteerable assemblages

$$
\begin{aligned}
\sigma_{a|x}^{US} &= \mathrm{Tr}_A(M_{a|x} \otimes I \ \rho_{sep}) \\
&= \mathrm{Tr}_A(M_{a|x} \otimes I \ \sum_\lambda p_\lambda \rho_\lambda^A \otimes \rho_\lambda^B) \\
&= \sum_\lambda p_\lambda \ \mathrm{Tr}(M_{a|x} \rho_\lambda^A) \ \rho_\lambda^B \\
&= \sum_\lambda \ p(a|x,\lambda) \ \rho_\lambda^B
\end{aligned}
$$

Operational approach to quantum steering

Unsteerable assemblages

$$\begin{aligned}
\sigma_{a|x}^{US} &= \mathrm{Tr}_A(M_{a|x} \otimes I \ \rho_{sep}) \\
&= \mathrm{Tr}_A(M_{a|x} \otimes I \ \sum_\lambda p_\lambda \rho_\lambda^A \otimes \rho_\lambda^B) \\
&= \sum_\lambda p_\lambda \ \mathrm{Tr}(M_{a|x} \rho_\lambda^A) \ \rho_\lambda^B \\
&= \sum_\lambda \ p(a|x,\lambda) \ \rho_\lambda^B
\end{aligned}$$

Local hidden state model (*Wiseman, Jones, Doherty, PRL'07)*

Operational approach to quantum steering

Testing for the existence of an LHS model is a semi-definite program.

M. Pusey, PRA'13

Operational approach to quantum steering

The solution of this SDP gives the optimal steering inequality

$$\sum_{a,x} \mathrm{Tr}(F_{a|x}\sigma_{a|x}^{US}) \leq \beta_{LHS}$$

2.
Multipartite quantum steering

Operational approach to multipartite steering

Foundational approach:

Which states can Alice and Bob remotely prepare to Charlie

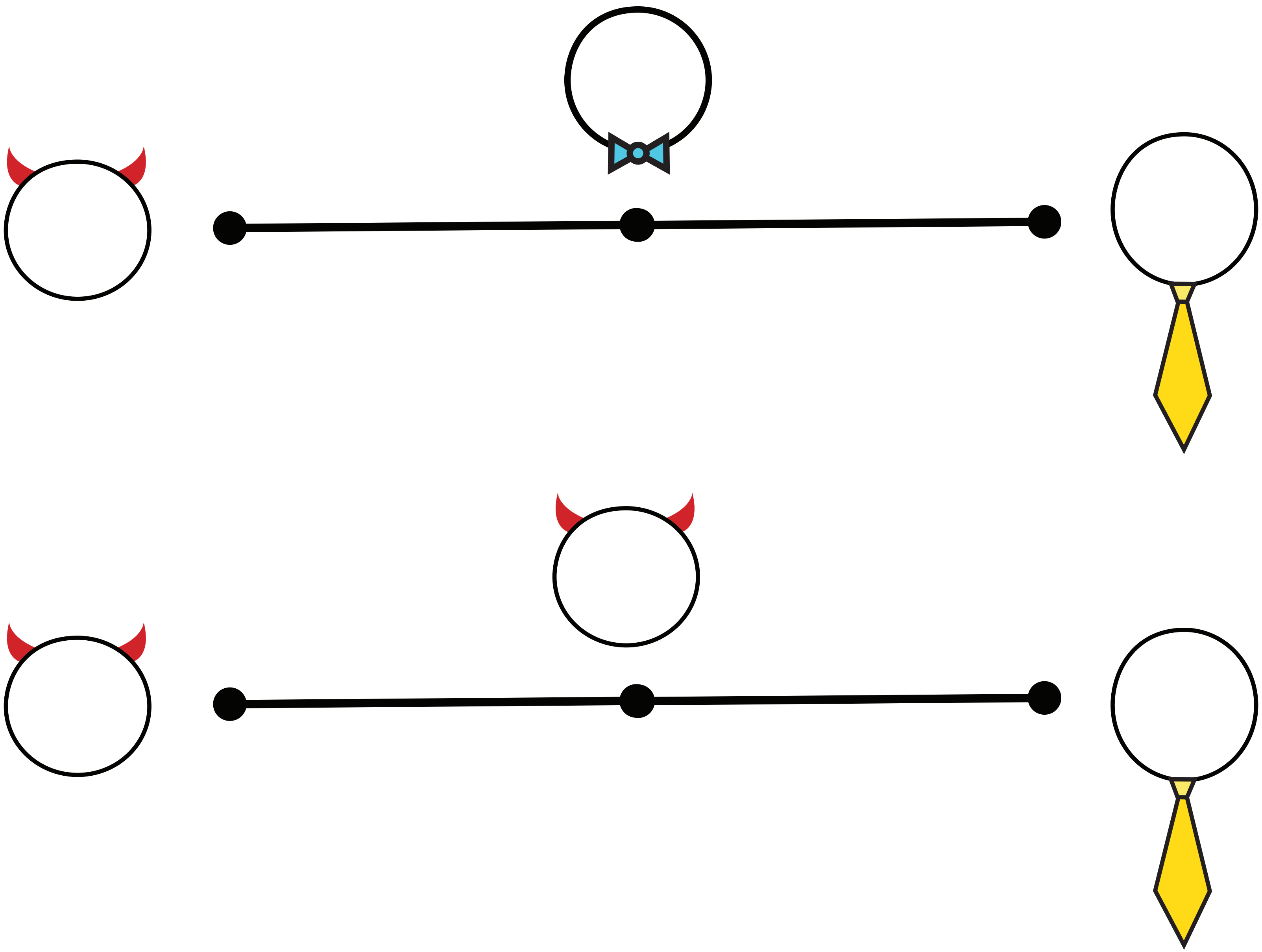
He and Reid '13

Operational approach:

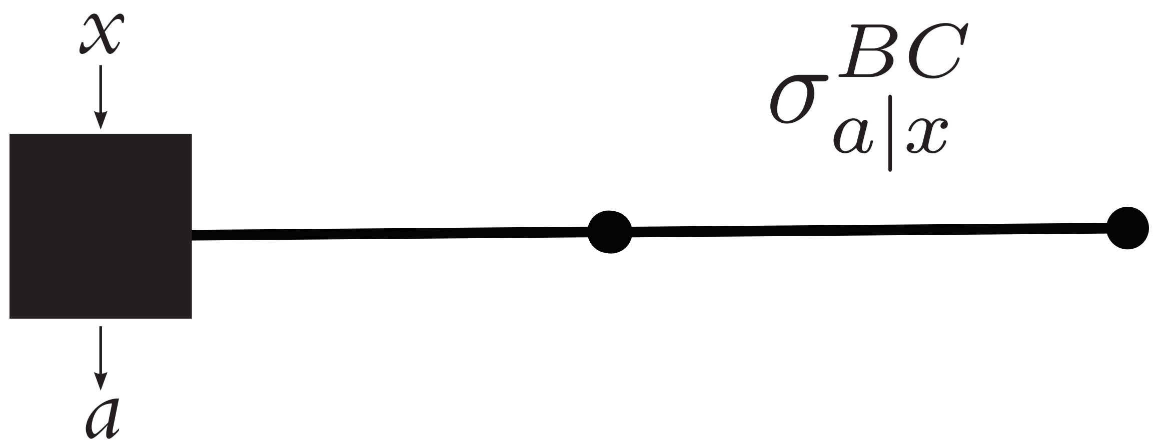
Genuine multipartite entanglement certification with untrusted parties

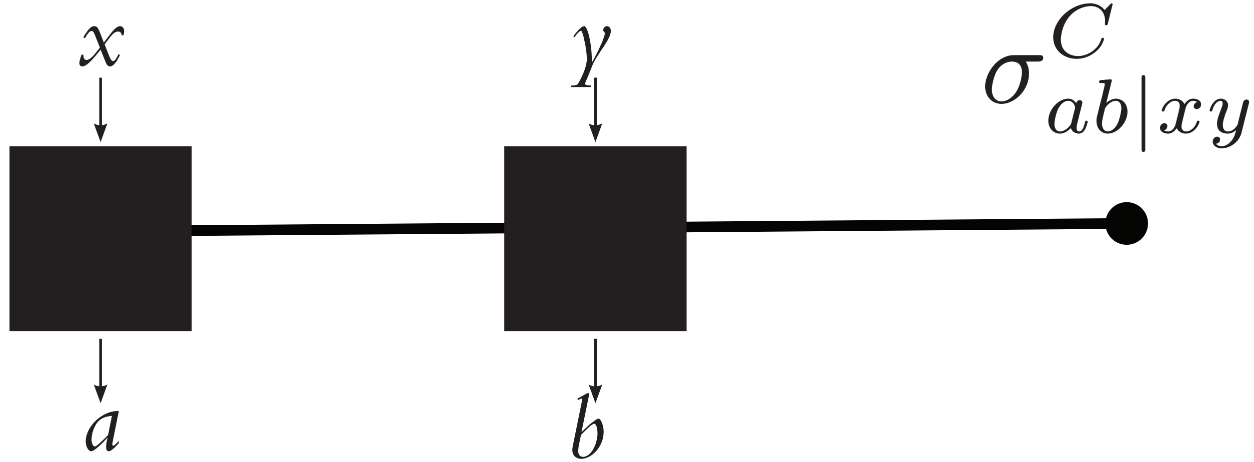
Cavalcanti, et al. '15

Operational approach to multipartite steering



Operational approach to multipartite steering





Operational approach to multipartite steering

Given $\sigma_{a|x}^{BC}$ or $\sigma_{ab|xy}^{C}$, check for the possibility of

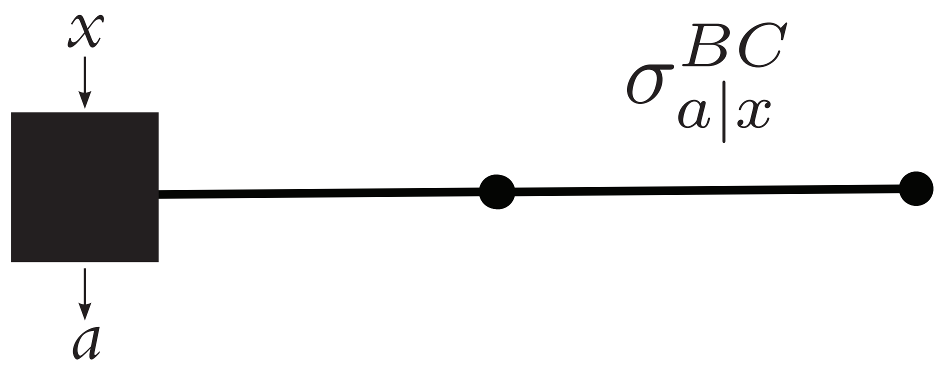
· fully separable
· biseparable
· separable in a bipartition

with semidefinite programming.

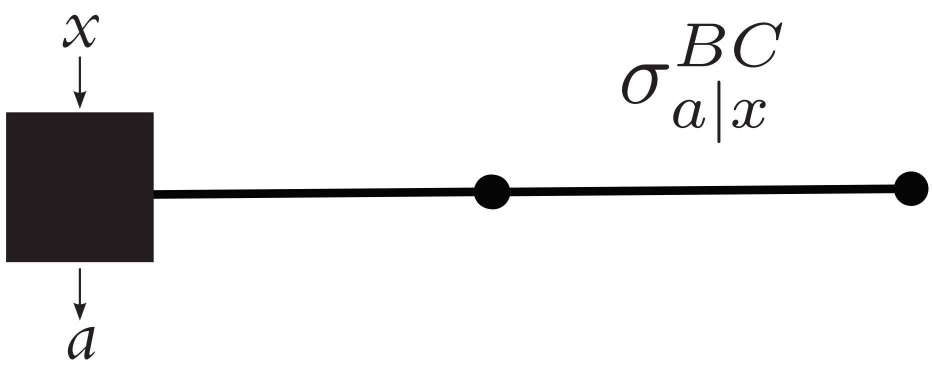
MULTIPARTITE STEERING INEQUALITIES

Cavalcanti et al., Nat Commns. '15

Genuine multipartite entanglement with one untrusted party



Genuine multipartite entanglement with one untrusted party



$$\begin{aligned}\rho^{\mathrm{ABC}} &= \sum_\lambda p_\lambda^{\mathrm{A:BC}} \rho_\lambda^{\mathrm{A}} \otimes \rho_\lambda^{\mathrm{BC}} + \sum_\mu p_\mu^{\mathrm{B:AC}} \rho_\mu^{\mathrm{B}} \otimes \rho_\mu^{\mathrm{AC}} \\ &+ \sum_\nu p_\nu^{\mathrm{AB:C}} \rho_\nu^{\mathrm{AB}} \otimes \rho_\nu^{C},\end{aligned}$$

Genuine multipartite entanglement with one untrusted party

$$
\begin{aligned}
\rho^{\mathrm{ABC}} &= \sum_\lambda p_\lambda^{\mathrm{A:BC}} \rho_\lambda^{\mathrm{A}} \otimes \rho_\lambda^{\mathrm{BC}} + \sum_\mu p_\mu^{\mathrm{B:AC}} \rho_\mu^{\mathrm{B}} \otimes \rho_\mu^{\mathrm{AC}} \\
&+ \sum_\nu p_\nu^{\mathrm{AB:C}} \rho_\nu^{\mathrm{AB}} \otimes \rho_\nu^{C},
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{a|x}^{\mathrm{BC}} &= \mathrm{tr}_A(M_{a|x} \otimes \mathbb{1}_{\mathrm{B}} \otimes \mathbb{1}_{\mathrm{C}} \rho^{\mathrm{ABC}}) \\
&= \sum_\lambda p_\lambda^{\mathrm{A:BC}} p_\lambda(a|x) \rho_\lambda^{\mathrm{BC}} \\
&+ \sum_\mu p_\mu^{\mathrm{B:AC}} \rho_\mu^{\mathrm{B}} \otimes \sigma_{a|x,\mu}^{\mathrm{C}} \\
&+ \sum_\nu p_\nu^{\mathrm{AB:C}} \sigma_{a|x\nu}^{\mathrm{B}} \otimes \rho_\nu^{\mathrm{C}}
\end{aligned}
$$

Genuine multipartite entanglement with one untrusted party

$$\begin{aligned}\rho^{\mathrm{ABC}} &= \boxed{\sum_\lambda p_\lambda^{\mathrm{A:BC}} \rho_\lambda^{\mathrm{A}} \otimes \rho_\lambda^{\mathrm{BC}}} + \sum_\mu p_\mu^{\mathrm{B:AC}} \rho_\mu^{\mathrm{B}} \otimes \rho_\mu^{\mathrm{AC}} \\ &+ \sum_\nu p_\nu^{\mathrm{AB:C}} \rho_\nu^{\mathrm{AB}} \otimes \rho_\nu^{C},\end{aligned}$$

$$\begin{aligned}\sigma_{a|x}^{\mathrm{BC}} &= \operatorname{tr}_A(M_{a|x} \otimes \mathbb{1}_{\mathrm{B}} \otimes \mathbb{1}_{\mathrm{C}} \rho^{\mathrm{ABC}}) \\ &= \boxed{\sum_\lambda p_\lambda^{\mathrm{A:BC}} p_\lambda(a|x) \rho_\lambda^{\mathrm{BC}}} \\ &+ \sum_\mu p_\mu^{\mathrm{B:AC}} \rho_\mu^{\mathrm{B}} \otimes \sigma_{a|x,\mu}^{\mathrm{C}} \\ &+ \sum_\nu p_\nu^{\mathrm{AB:C}} \sigma_{a|x\nu}^{\mathrm{B}} \otimes \rho_\nu^{\mathrm{C}}\end{aligned}$$

Unsteerable

Genuine multipartite entanglement with one untrusted party

$$\rho^{\mathrm{ABC}} = \sum_{\lambda} p_{\lambda}^{\mathrm{A:BC}} \rho_{\lambda}^{\mathrm{A}} \otimes \rho_{\lambda}^{\mathrm{BC}} + \boxed{\sum_{\mu} p_{\mu}^{\mathrm{B:AC}} \rho_{\mu}^{\mathrm{B}} \otimes \rho_{\mu}^{\mathrm{AC}}} + \sum_{\nu} p_{\nu}^{\mathrm{AB:C}} \rho_{\nu}^{\mathrm{AB}} \otimes \rho_{\nu}^{C},$$

$$\begin{aligned} \sigma_{a|x}^{\mathrm{BC}} &= \operatorname{tr}_{A}(M_{a|x} \otimes \mathbb{1}_{\mathrm{B}} \otimes \mathbb{1}_{\mathrm{C}} \rho^{\mathrm{ABC}}) \\ &= \sum_{\lambda} p_{\lambda}^{\mathrm{A:BC}} p_{\lambda}(a|x) \rho_{\lambda}^{\mathrm{BC}} \\ &+ \boxed{\sum_{\mu} p_{\mu}^{\mathrm{B:AC}} \rho_{\mu}^{\mathrm{B}} \otimes \sigma_{a|x,\mu}^{\mathrm{C}}} \\ &+ \sum_{\nu} p_{\nu}^{\mathrm{AB:C}} \sigma_{a|x\nu}^{\mathrm{B}} \otimes \rho_{\nu}^{\mathrm{C}} \end{aligned}$$

Separable

Unsteerable from Alice to Bob

Genuine multipartite entanglement with one untrusted party

$$\rho^{\mathrm{ABC}} = \sum_\lambda p_\lambda^{\mathrm{A:BC}} \rho_\lambda^{\mathrm{A}} \otimes \rho_\lambda^{\mathrm{BC}} + \sum_\mu p_\mu^{\mathrm{B:AC}} \rho_\mu^{\mathrm{B}} \otimes \rho_\mu^{\mathrm{AC}} + \boxed{\sum_\nu p_\nu^{\mathrm{AB:C}} \rho_\nu^{\mathrm{AB}} \otimes \rho_\nu^{C}},$$

$$\begin{aligned}\sigma_{a|x}^{\mathrm{BC}} &= \mathrm{tr}_A(M_{a|x} \otimes \mathbb{1}_{\mathrm{B}} \otimes \mathbb{1}_{\mathrm{C}} \rho^{\mathrm{ABC}}) \\ &= \sum_\lambda p_\lambda^{\mathrm{A:BC}} p_\lambda(a|x) \rho_\lambda^{\mathrm{BC}} \\ &+ \sum_\mu p_\mu^{\mathrm{B:AC}} \rho_\mu^{\mathrm{B}} \otimes \sigma_{a|x,\mu}^{\mathrm{C}} \\ &+ \boxed{\sum_\nu p_\nu^{\mathrm{AB:C}} \sigma_{a|x\nu}^{\mathrm{B}} \otimes \rho_\nu^{\mathrm{C}}}\end{aligned}$$

Separable

Unsteerable from Alice to Charlie

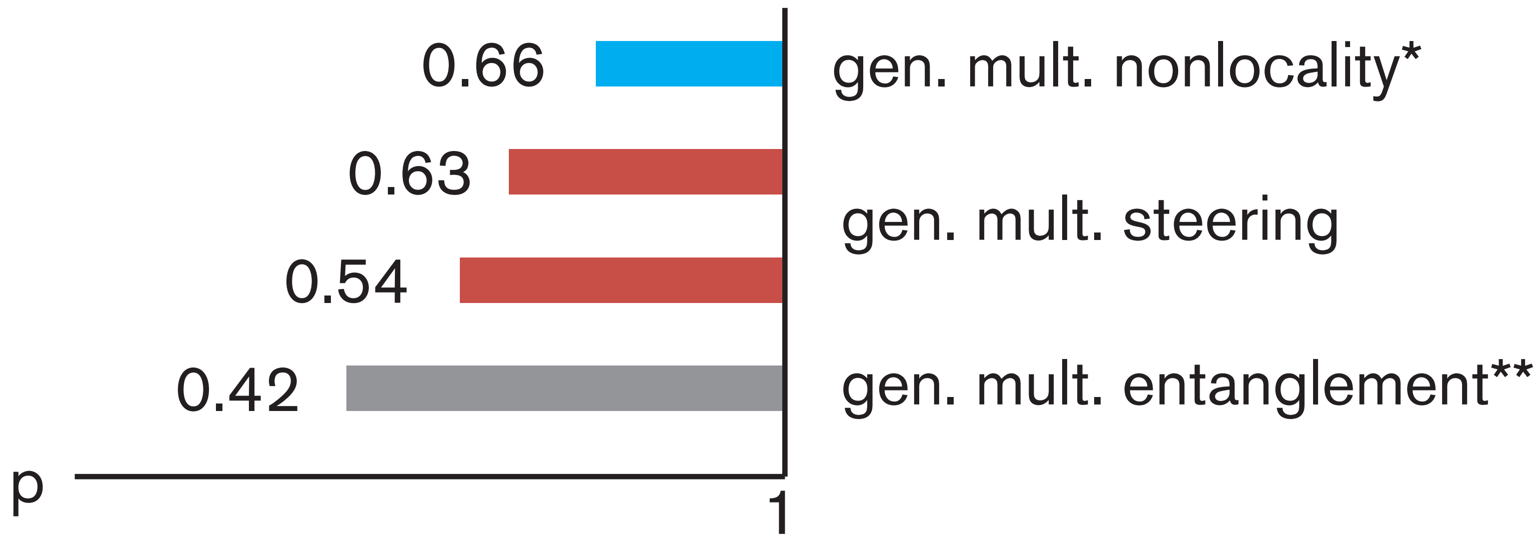
Genuine multipartite steering witness with one untrusted party

$$1 + 0.1547\langle Z_{\mathrm{B}} Z_{\mathrm{C}}\rangle - \tfrac{1}{3}\big(\langle A_3 Z_{\mathrm{B}}\rangle + \langle A_3 Z_{\mathrm{C}}\rangle + \langle A_1 X_{\mathrm{B}} X_{\mathrm{C}}\rangle - \langle A_1 Y_{\mathrm{B}} Y_{\mathrm{C}}\rangle - \langle A_2 X_{\mathrm{B}} Y_{\mathrm{C}}\rangle - \langle A_2 Y_{\mathrm{B}} X_{\mathrm{C}}\rangle\big) \geq 0$$

GHZ state achieves -0.84

Comparison with entanglement and nonlocality

$$p|GHZ\rangle\langle GHZ| + (1-p)\frac{I}{8}$$



*Bancal et al PRL'11 **Guhne and Seevink NJP'10

Tripartite steering

Form of state	Unstrusted parties	Known objects	SDP
$\sum_\lambda p_\lambda \rho_\lambda^{\mathrm{A}} \otimes \rho_\lambda^{\mathrm{B}} \otimes \rho_\lambda^{\mathrm{C}}$	A	$\sigma_{a\vert x}^{\mathrm{BC}}$	$\begin{aligned} \max \quad & p \\ \text{s.t} \quad & \textstyle\sum_\mu D_\mu(a\vert x)\sigma_\mu^{\mathrm{BC}} = \sigma_{a\vert x}^{\mathrm{BC}} - p\,\mathrm{id}_{a\vert x}^{\mathrm{BC}}, \\ & \left(\sigma_\mu^{\mathrm{BC}}\right)^{T_{\mathrm{B}}} \geq 0, \quad \sigma_\mu^{\mathrm{BC}} \geq 0. \end{aligned} \qquad (1)$
	A and B	$\sigma_{ab\vert xy}^{\mathrm{C}}$	$\begin{aligned} \max \quad & p \\ \text{s.t.} \quad & \textstyle\sum_{\mu,\lambda} D_\mu(a\vert x)D_\lambda(b\vert y)\sigma_{\mu\lambda}^{\mathrm{C}} = \sigma_{ab\vert xy}^{\mathrm{C}} - p\,\mathrm{id}_{ab\vert xy}^{\mathrm{C}}, \\ & \sigma_{\mu\lambda}^{\mathrm{C}} \geq 0. \end{aligned} \qquad (2)$
$\sum_\lambda p_\lambda \rho_\lambda^{\mathrm{A}} \otimes \rho_\lambda^{\mathrm{BC}}$	A	$\sigma_{a\vert x}^{\mathrm{BC}}$	$\begin{aligned} \max \quad & p \\ \text{s.t} \quad & \textstyle\sum_\mu D_\mu(a\vert x)\sigma_\mu^{\mathrm{BC}} = \sigma_{a\vert x}^{\mathrm{BC}} - p\,\mathrm{id}_{a\vert x}^{\mathrm{BC}}, \\ & \sigma_\mu^{\mathrm{BC}} \geq 0. \end{aligned} \qquad (3)$
	B	$\sigma_{b\vert y}^{\mathrm{AC}}$	$\begin{aligned} \max \quad & p \\ \text{s.t} \quad & \Gamma_{b\vert y}^{\mathrm{AC}} = \sigma_{b\vert y}^{\mathrm{AC}} - p\,\mathrm{id}_{b\vert y}^{\mathrm{AC}}, \\ & \mathrm{tr}_{\mathrm{C}}\,\Gamma_{b\vert y}^{\mathrm{AC}} = \textstyle\sum_\mu D_\mu(b\vert y)\sigma_\mu^{\mathrm{A}}, \\ & (\Gamma_{b\vert y}^{\mathrm{AC}})^{T_{\mathrm{A}}} \geq 0, \quad \Gamma_{b\vert y}^{\mathrm{AC}} \geq 0, \quad \sigma_\mu^{\mathrm{A}} \geq 0. \end{aligned} \qquad (4)$
	A and B	$\sigma_{ab\vert xy}^{\mathrm{C}}$	$\begin{aligned} \max \quad & p \\ \text{s.t} \quad & \textstyle\sum_\mu D_\mu(a\vert x)\sigma_{b\vert y,\mu}^{\mathrm{C}} = \sigma_{ab\vert xy}^{\mathrm{C}} - p\,\mathrm{id}_{ab\vert xy}^{\mathrm{C}}, \\ & \sigma_{b\vert y,\mu}^{\mathrm{C}} \geq 0. \end{aligned} \qquad (5)$
	B and C	$\sigma_{bc\vert yz}^{\mathrm{A}}$	$\begin{aligned} \max \quad & p \\ \text{s.t.} \quad & \textstyle\sum_\mu D^{\mathrm{NS}}(bc\vert yz,\mu)\sigma_\mu^{\mathrm{A}} = \sigma_{bc\vert yz}^{\mathrm{A}} - p\,\mathrm{id}_{bc\vert yz}^{\mathrm{A}} \\ & \textstyle\sum_\mu D^{\mathrm{NS}}(bc\vert yz,\mu)\sigma_\mu^{\mathrm{A}} \in \mathcal{Q}_{\mathrm{A}}^{(k)}, \quad \sigma_\mu^{\mathrm{A}} \geq 0 \end{aligned} \qquad (6)$
$\sum_\lambda p_\lambda^{\mathrm{A:BC}} \rho_\lambda^{\mathrm{A}} \otimes \rho_\lambda^{\mathrm{BC}} + \sum_\lambda p_\lambda^{\mathrm{B:AC}} \rho_\lambda^{\mathrm{B}} \otimes \rho_\lambda^{\mathrm{AC}} + \sum_\lambda p_\lambda^{\mathrm{AB:C}} \rho_\lambda^{\mathrm{AB}} \otimes \rho_\lambda^{C}$	A	$\sigma_{a\vert x}^{\mathrm{BC}}$	$\begin{aligned} \max \quad & p \\ \text{s.t.} \quad & \Gamma_{a\vert x}^{\mathrm{A:BC}} + \Gamma_{a\vert x}^{\mathrm{B:AC}} + \Gamma_{a\vert x}^{\mathrm{C:AB}} = \sigma_{a\vert x}^{\mathrm{BC}} - p\,\mathrm{id}_{a\vert x}^{\mathrm{BC}} \\ & \Gamma_{a\vert x}^{\mathrm{A:BC}} = \textstyle\sum_\mu D_\mu(a\vert x)\sigma_\mu^{\mathrm{BC}}, \quad \sigma_\mu^{\mathrm{BC}} \geq 0 \\ & \mathrm{tr}_{\mathrm{C}}\,\Gamma_{a\vert x}^{\mathrm{B:AC}} = \textstyle\sum_\mu D_\mu(a\vert x)\sigma_\mu^{\mathrm{B}}, \quad \sigma_\mu^{\mathrm{B}} \geq 0, \\ & \mathrm{tr}_{\mathrm{B}}\,\Gamma_{a\vert x}^{\mathrm{C:AB}} = \textstyle\sum_\mu D_\mu(a\vert x)\sigma_\mu^{\mathrm{C}}, \quad \sigma_\mu^{\mathrm{C}} \geq 0, \\ & \left(\Gamma_{a\vert x}^{\mathrm{B:AC}}\right)^{T_{\mathrm{B}}} \geq 0, \quad \left(\Gamma_{a\vert x}^{\mathrm{C:AB}}\right)^{T_{\mathrm{B}}} \geq 0. \\ & \Gamma_{a\vert x}^{\mathrm{B:AC}} \geq 0, \quad \Gamma_{a\vert x}^{\mathrm{C:AB}} \geq 0, \quad \sum_a \Gamma_{a\vert x}^{\mathrm{B:AC}} = \sum_a \Gamma_{a\vert x'}^{\mathrm{B:AC}}. \end{aligned} \qquad (7)$
	A and B	$\sigma_{ab\vert xy}^{\mathrm{C}}$	$\begin{aligned} \max \quad & p \\ \text{s.t.} \quad & \Pi_{ab\vert xy}^{\mathrm{A:BC}} + \Pi_{ab\vert xy}^{\mathrm{B:AC}} + \Pi_{ab\vert xy}^{\mathrm{C:AB}} = \sigma_{ab\vert xy}^{\mathrm{C}} - p\,\mathrm{id}_{ab\vert xy}^{\mathrm{C}} \\ & \Pi_{ab\vert xy}^{\mathrm{A:BC}} = \textstyle\sum_\mu D_\mu(a\vert x)\sigma_{b\vert y,\mu}^{\mathrm{C}}, \quad \sigma_{b\vert y,\mu}^{\mathrm{C}} \geq 0 \\ & \Pi_{ab\vert xy}^{\mathrm{B:AC}} = \textstyle\sum_\mu D_\mu(b\vert y)\sigma_{a\vert x,\mu}^{\mathrm{C}}, \quad \sigma_{a\vert x,\mu}^{\mathrm{C}} \geq 0 \\ & \Pi_{ab\vert xy}^{\mathrm{C:AB}} = \textstyle\sum_\mu D_\nu^{\mathrm{NS}}(ab\vert xy)\sigma_\nu^{\mathrm{C}}, \quad \sigma_\nu^{\mathrm{C}} \geq 0 \\ & \Pi_{ab\vert xy}^{\mathrm{C:AB}} \in \mathcal{Q}_{\mathrm{C}}^{(k)}, \quad \sum_b \sigma_{b\vert y,\mu}^{\mathrm{C}} = \sum_b \sigma_{b\vert y',\mu}^{\mathrm{C}} \end{aligned} \qquad (8)$

Cavalcanti et al., Nat Commns. '15

Stay tuned!

Characterising steering via semi-definite programming

DC, P. Skrzypczyk, A. Acín, in preparation

3.
Multipartite post-quantum steering

Post-quantum nonlocality

Quantum

$$p(ab|xy) =$$
$$\mathrm{Tr}(M_{a|x} \otimes M_{b|y}\ \rho_{AB})$$

$\Rightarrow$

No-signalling

$$\sum_a p(ab|xy) =$$
$$\sum_a p(ab|x'y) = p(b|y)$$

Post-quantum nonlocality

Quantum

$$p(ab|xy) =$$
$$\mathrm{Tr}(M_{a|x} \otimes M_{b|y} \; \rho_{AB})$$

$$\not\Leftarrow$$

No-signalling

$$\sum_a p(ab|xy) =$$
$$\sum_a p(ab|x'y) = p(b|y)$$

Post-quantum nonlocality

Quantum

$$p(ab|xy) =$$
$$\mathrm{Tr}(M_{a|x} \otimes M_{b|y} \; \rho_{AB})$$

$$\not\Leftarrow$$

No-signalling

$$\sum_a p(ab|xy) =$$
$$\sum_a p(ab|x'y) = p(b|y)$$

PR BOX

Is there post-quantum steering?

Quantum assemblages are no-signalling

Quantum

No-signalling

$$\sigma_{a|x} = \operatorname*{Tr}_A(M_{a|x} \otimes I \ \rho_{AB})$$

$\Rightarrow$

$$\sum_a \sigma_{a|x} = \sum \sigma_{a|x'} = \rho_B$$

No-signalling assemblages are quantum

Quantum

$$\sigma_{a|x} = \operatorname{Tr}_A(M_{a|x} \otimes I \ \rho_{AB})$$

$\Leftarrow$ ✓

No-signalling

$$\sum_a \sigma_{a|x} = \sum \sigma_{a|x'} = \rho_B$$

Hughston, Jozsa, Wootters, Gisin, … ‘93

Is there post-quantum steering?

bipartite: no

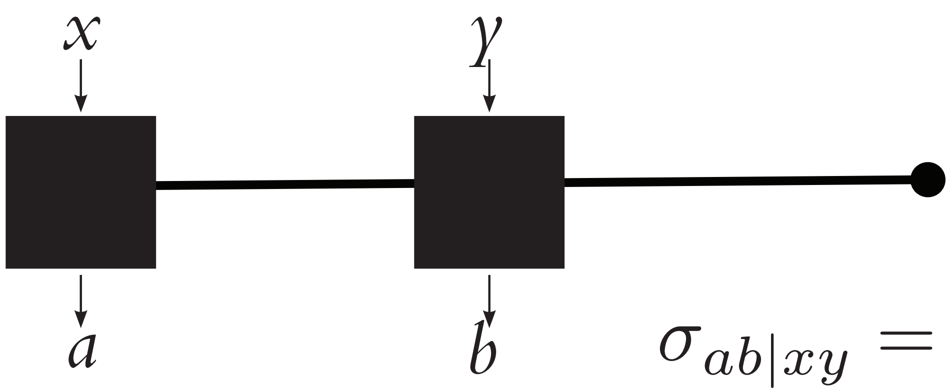
Is there post-quantum steering?

bipartite: no
tripartite: yes

Trivial post-quantum tripartite assemblage

Quantum $\not\leftarrow$ No-signalling

PR BOX



$$\sigma_{ab|xy} = PR(ab|xy)\rho_C$$

Is there post-quantum steering that doesn't come from post-quantum nonlocality?

Genuine post-quantum tripartite assemblages

We found $\sigma_{ab|xy}$

· no-signalling

· any measurement leads to local (hence quantum) probability distributions

· post-quantum

PRL115, 190403 (2015)

Outlook

Towards quantum information processing with untrusted parties

Entanglement detection, QKD
randomness certification, joint measurability
...

Steering as entanglement certification with an untrusted party.

***Multipartite* steering as *multipartite* entanglement certification with untrusted parties.**

Bipartite post-quantum steering doesn't exist.

Tripartite postquantum steering does.

Talk based on

Nat. Commun. 6, 7941 (2015)

D.C., P. Skrzypczyk, G. Aguilar, R. Nery, P. Souto Ribeiro, S. Walborn

Phys. Rev. Lett. 115, 190403 (2015)

A. Sainz, N. Brunner, D.C., P. Skrzypczyk, T. Vértesi

Talk to me for details. Thank you!