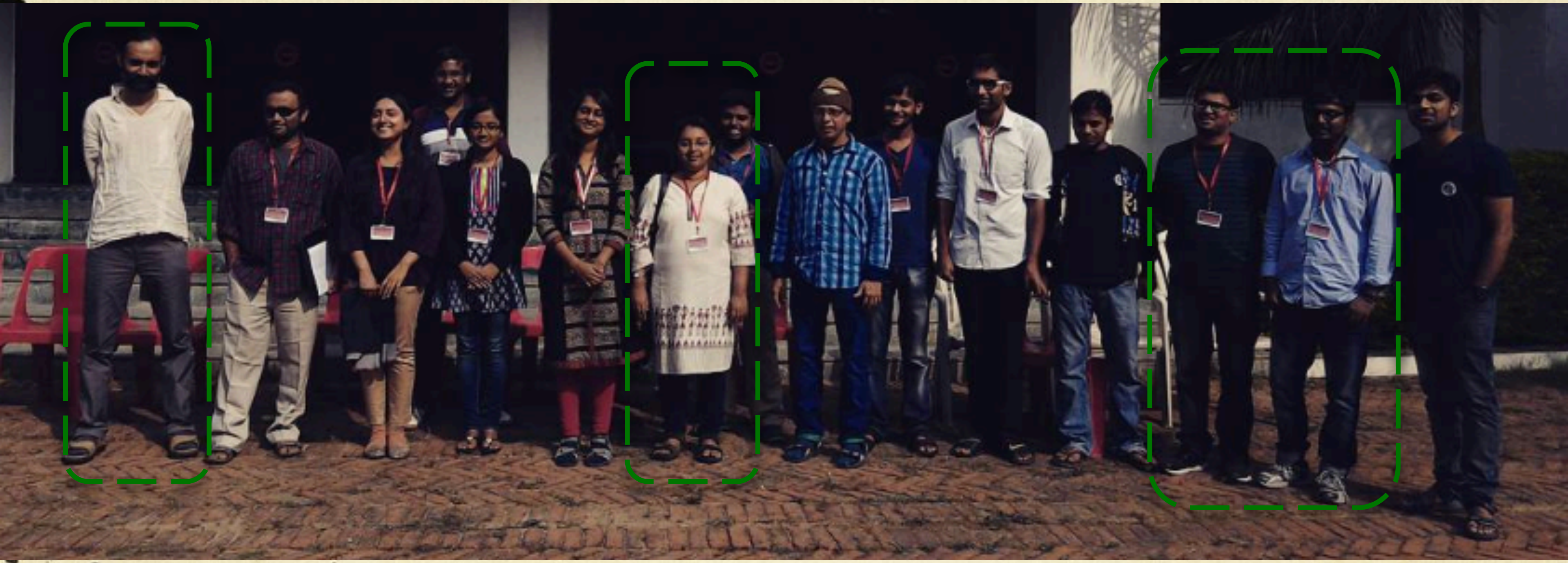
Diverging scaling with converging multisite entanglement in quantum Heisenberg ladders

Debraj Rakshit, Harish-Chandra Research Institute, Allahabad

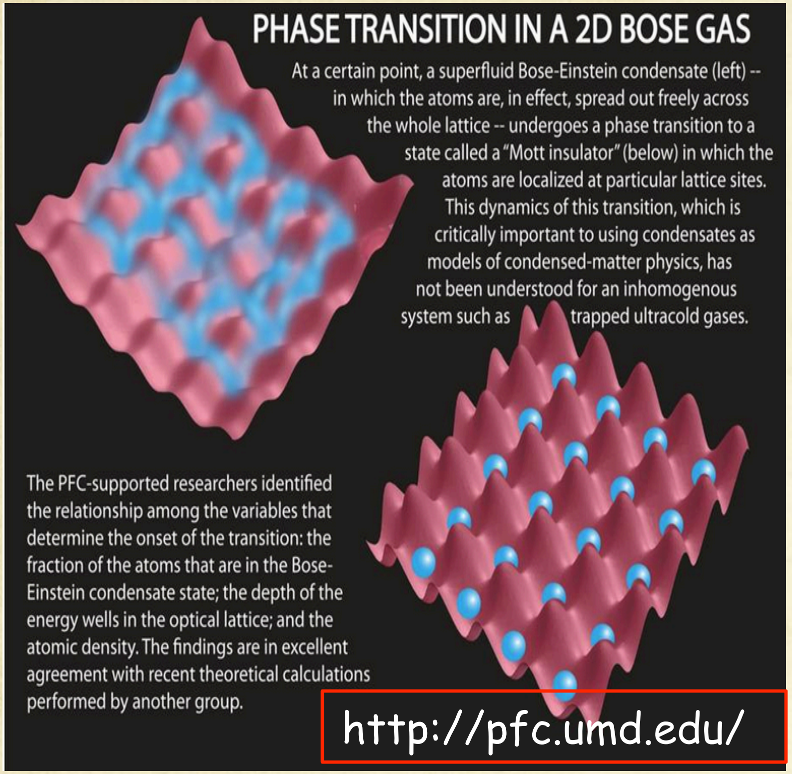


QIPA - 2015

With Sudipto Singha Roy, Himadri S. Dhar, Aditi Sen(De), Ujjwal Sen

arXiv: 1505.06083 (to appear in New. J. Phys.)

Heisenberg model systems



Hamiltonian: $H = H(t,U)$

t = tunneling term

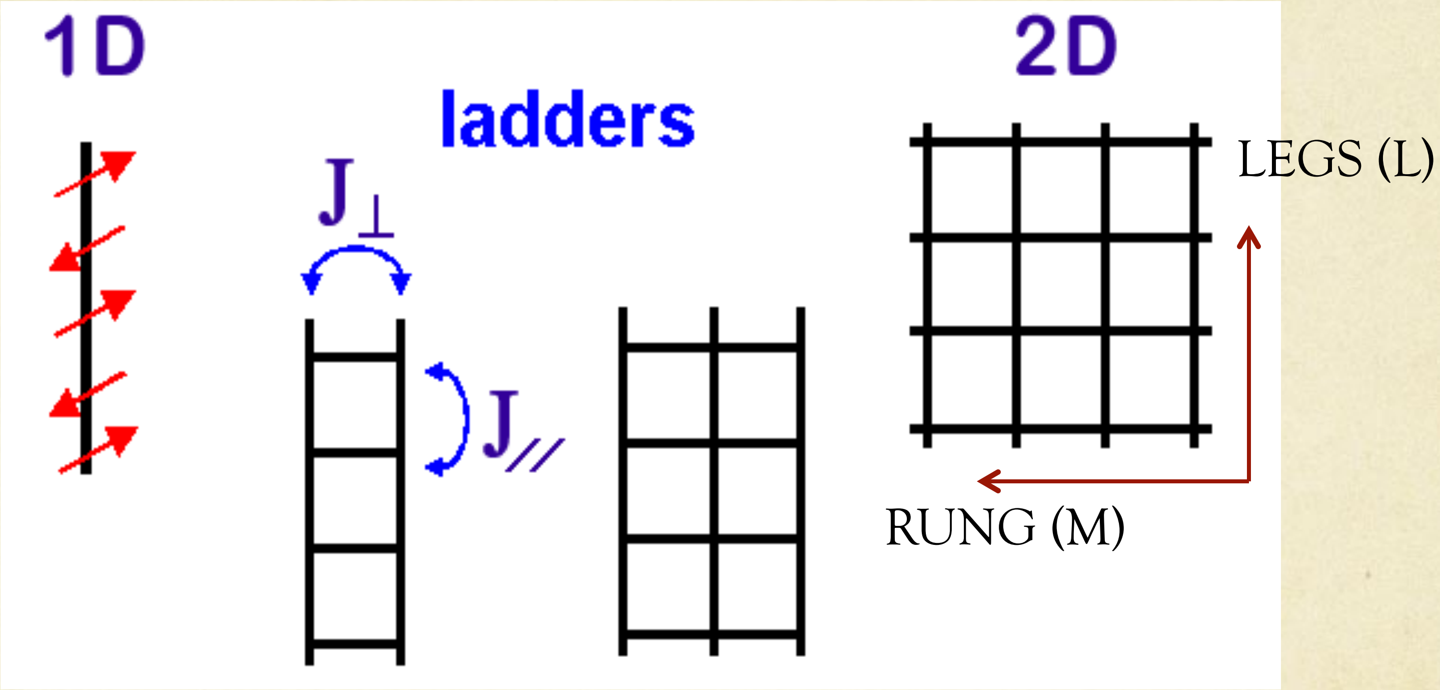
U = On-site interaction term

Suppress tunneling → $t << U$

Effective Hamiltonian: $H = J \sum_{\langle ij \rangle} \vec{S}_i \vec{S}_j$ $\quad J = J(t,U)$

Road from 1D to 2D

$$H = J\sum_i \vec{S}_i \vec{S}_{i+1}$$



- The quantum properties of a ladder **do not extrapolate trivially** from the 1D chain to the 2D isotropic lattice

Road from 1D to 2D: Even - odd dichotomy

$$H = J\sum_i \vec{S}_i \vec{S}_{i+1}$$

The quantum properties of a ladder **do not extrapolate trivially** from the 1D chain to the 2D isotropic lattice

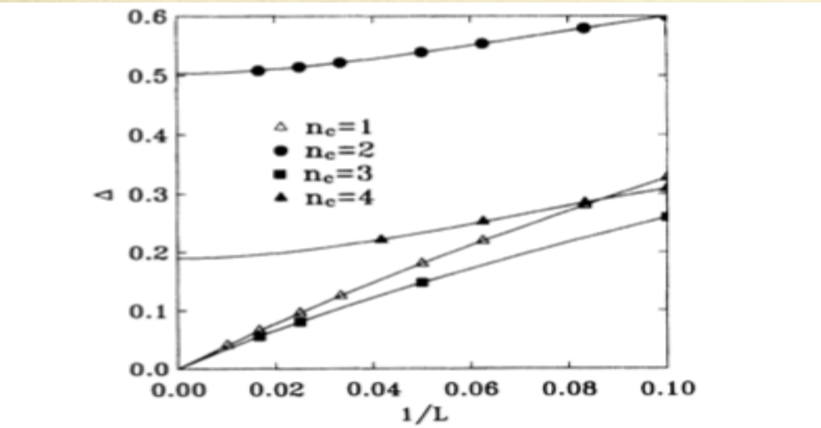


FIG. 1. Spin gaps as a function of system size L for open $L \times n_c$ coupled chain Heisenberg systems.

S. R. White et. al. PRL, 73, 886

Spin gap

- Odd-legged ladders are gapless (power law decay of classical correlation function)

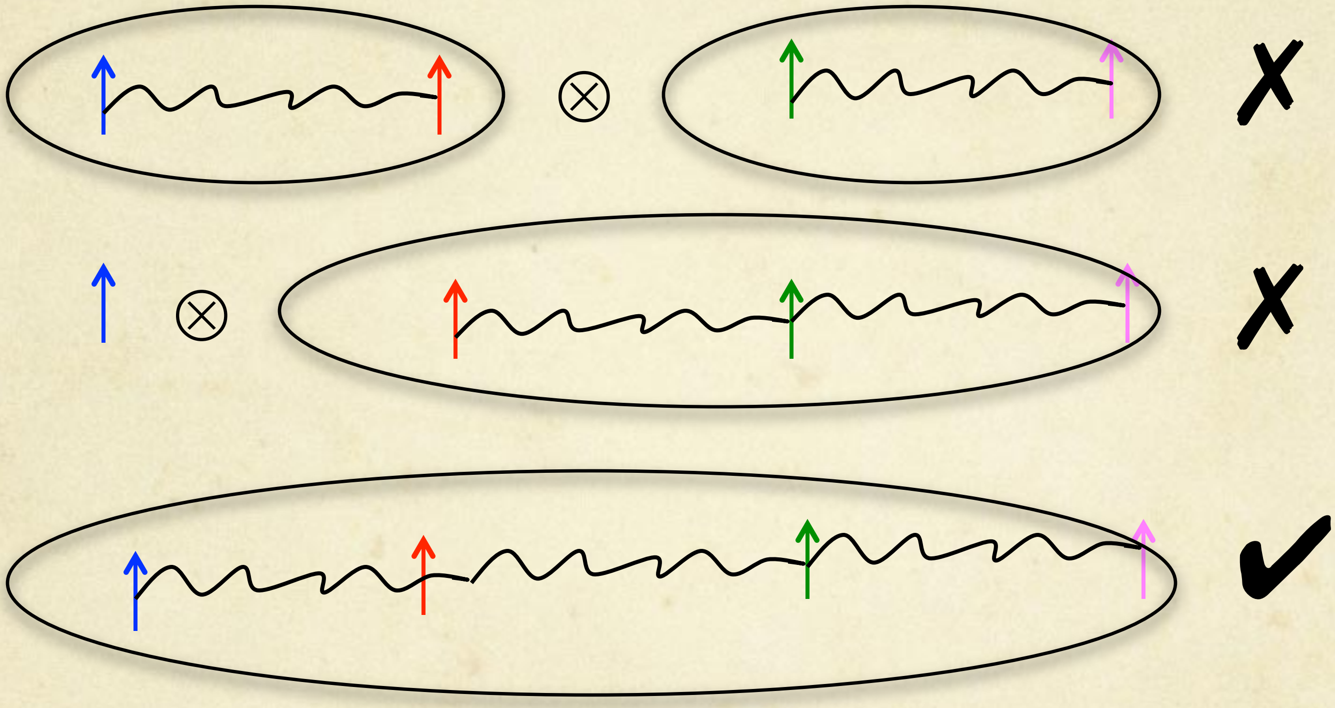
- Even-legged ladders are gapped (exponential decay of classical correlation function)

Generalized geometric measure

A measure for genuine multiparty entanglement

Generalized geometric measure

Consider a four party system



Multiparty pure quantum state is <u>genuinely</u> multiparty entangled if it is entangled across all possible bipartitions

Generalized geometric measure

$|\Psi_N\rangle$ - Quantum state of N-party

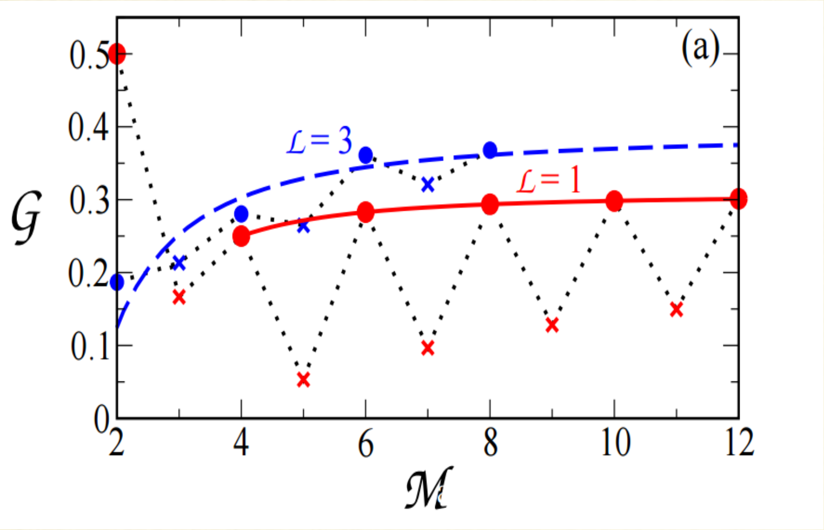
$|\Phi\rangle$ - Pure quantum states that are not genuinely entangled

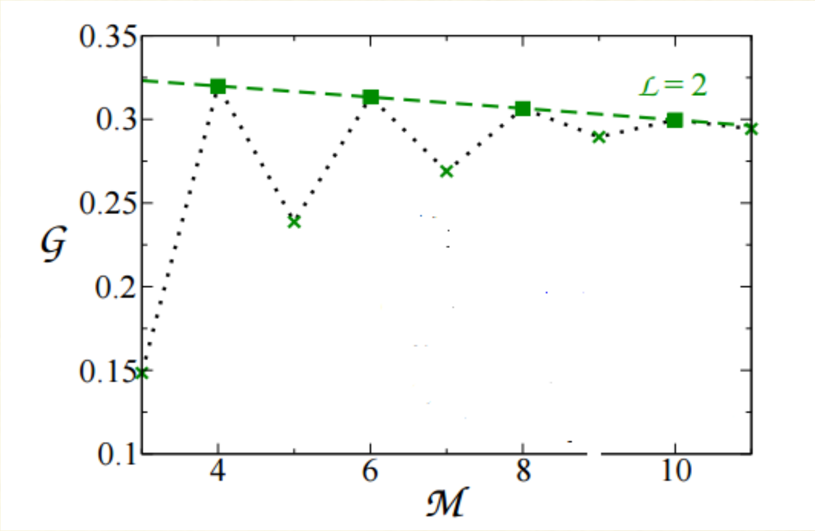
$$\varepsilon = 1 - \underbrace{|\langle\Phi|\Psi_N\rangle|^2}$$

Task is to minimize the distance over all pure quantum states which are not genuinely multiparty entangled

A. Sen(De) and U. Sen, PRA, 2010

Multiparty entanglement in ladders





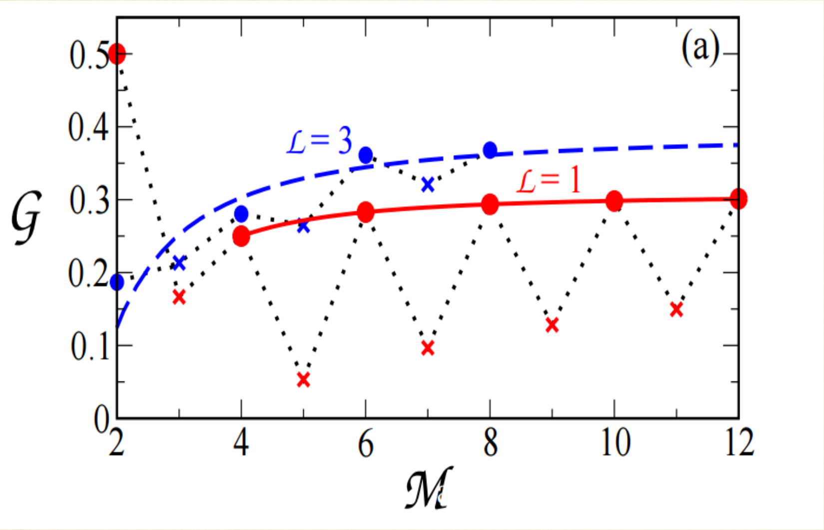
Odd-legged

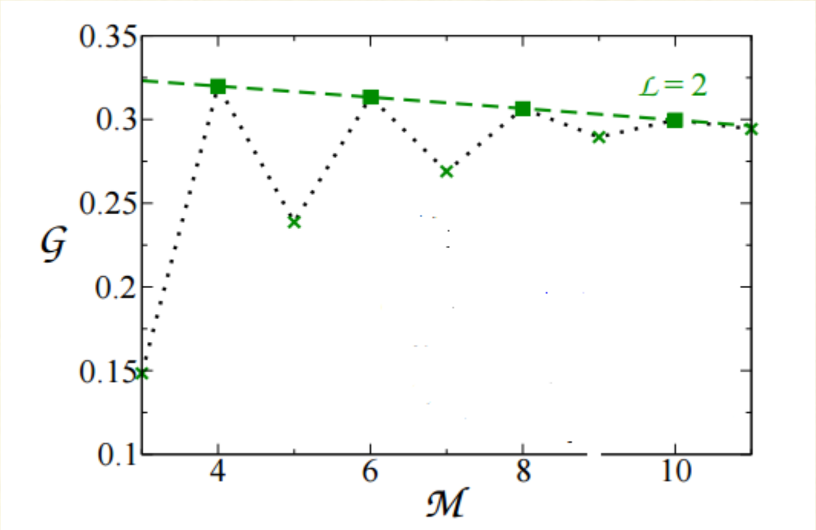
GGM increases

Even-legged

GGM decreases

Multiparty entanglement in ladders



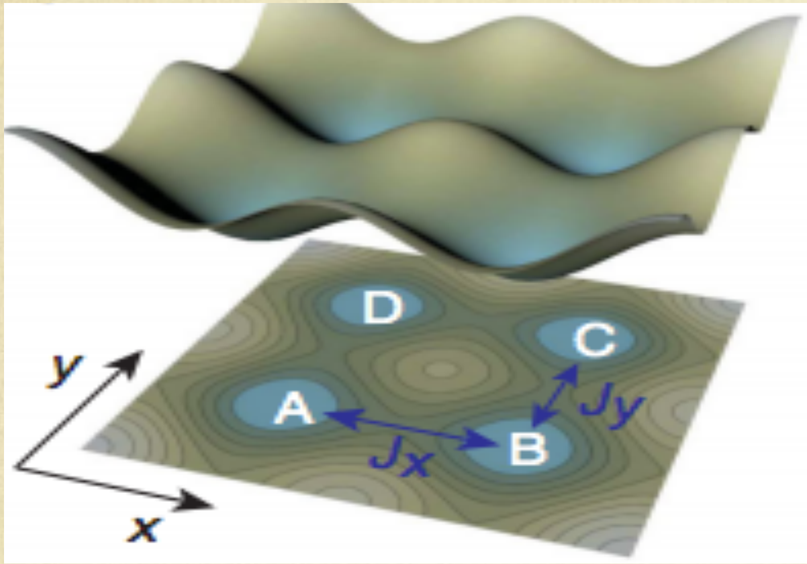


Odd-legged → GGM increases

Even-legged → GGM decreases

- Even-odd dichotomy of genuine multiparty entanglement in Heisenberg ladders

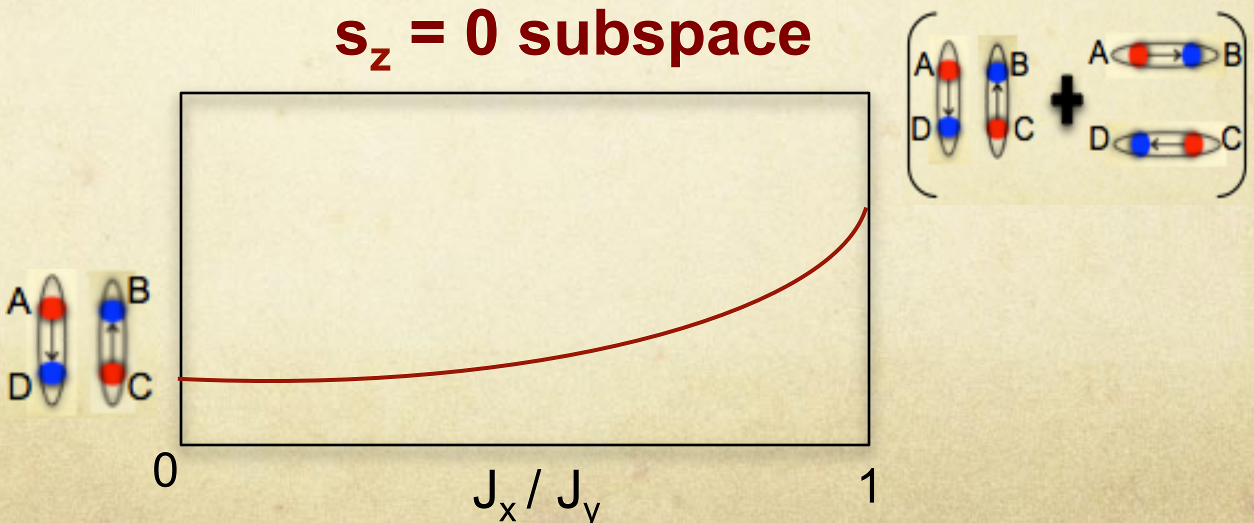
A four-site plaquette: two legs and two rungs



S. Nascimbene, et. al. PRL, 108, 205301 (2012)

$$H = J_x(\vec{\sigma}_A\vec{\sigma}_B+\vec{\sigma}_C\vec{\sigma}_D)+J_y(\vec{\sigma}_B\vec{\sigma}_C+\vec{\sigma}_A\vec{\sigma}_D)$$

s_z = 0 subspace

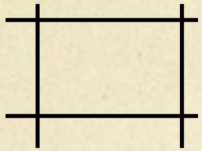


Two legs multi rungs

Ground state of Heisenberg lattice, Ψ

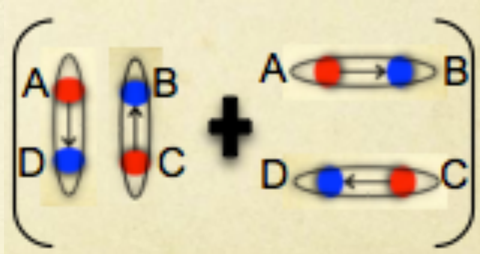
State constructed by all possible dimer coverings, Φ

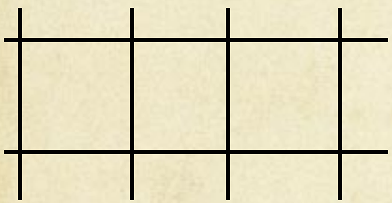
<u>2 legged</u> :



M = 2

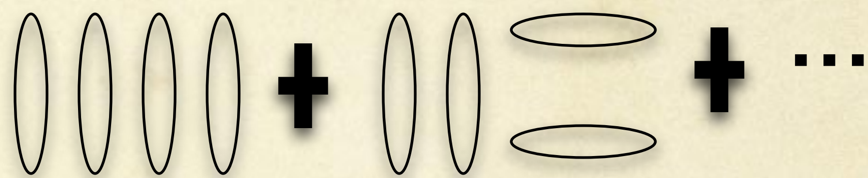
$F(\Psi,\Phi) = 1$





M = 4

$F(\Psi,\Phi) = 0.98$

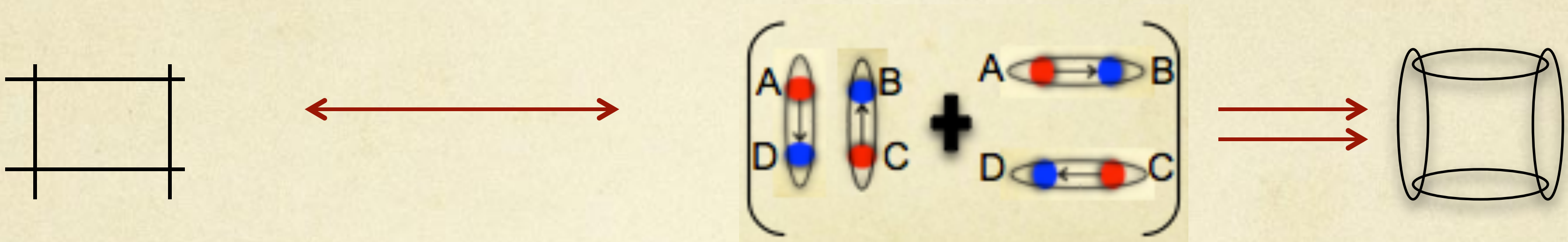


$F(\Psi,\Phi)$ tend to converge towards 0.9 as M increases

Multi legs multi rungs

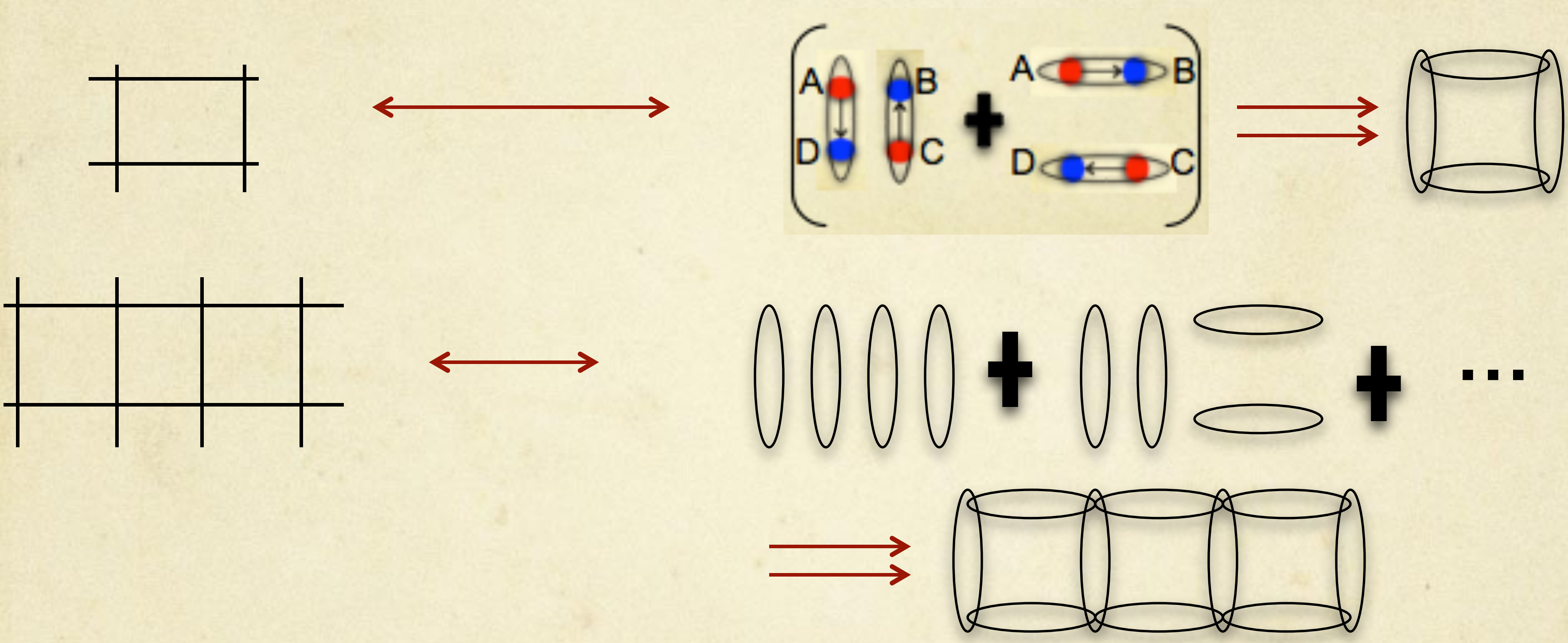
Ground state of Heisenberg lattice

State constructed by all possible dimer coverings



Multi legs multi rungs

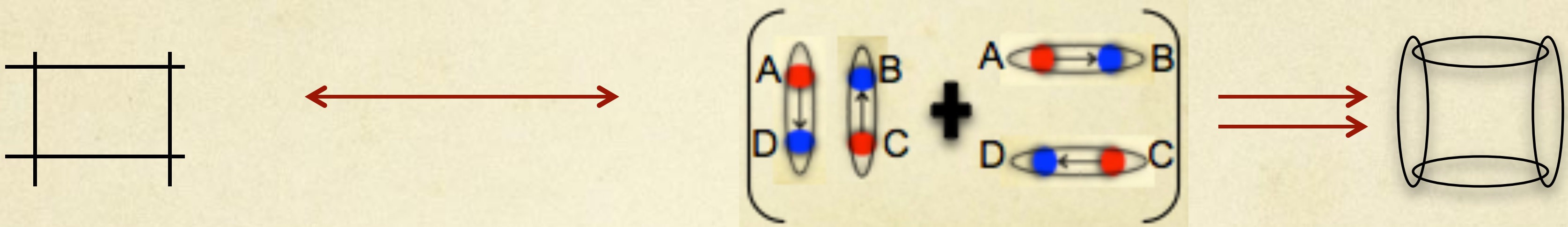
Ground state of Heisenberg lattice	State constructed by all possible dimer coverings

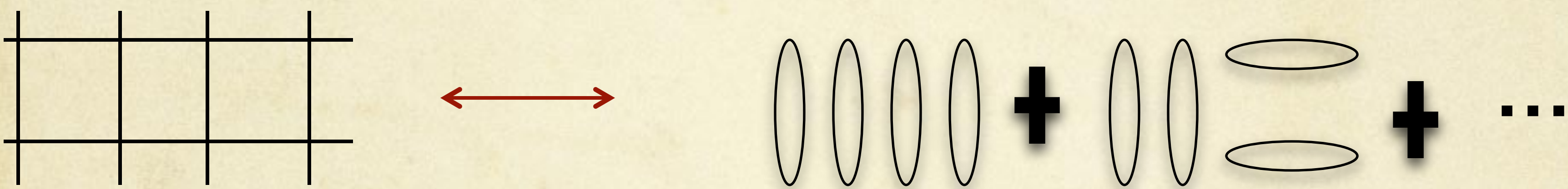


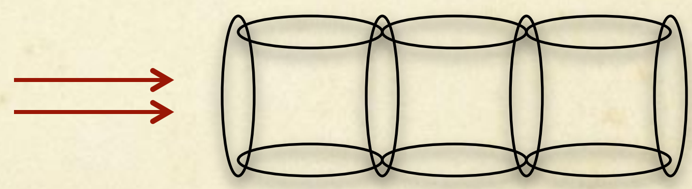
Multi legs multi rungs

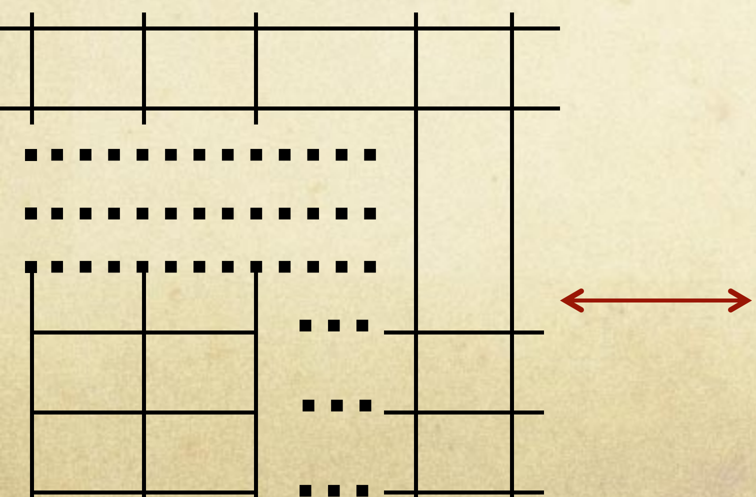
Ground state of Heisenberg lattice

State constructed by all possible dimer coverings







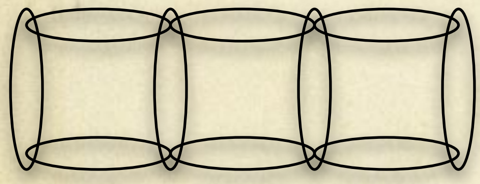


$$|\psi\rangle = \sum_{k} h_k(i_a, i_b) |(a_1, b_1), (a_2, b_2) \ldots (a_N, b_N)\rangle_k$$

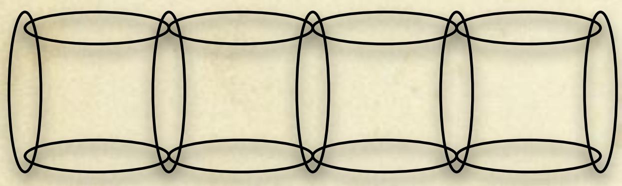
RVB state

Difficulty

RVB state $\longrightarrow$ $|\psi\rangle = \sum_{k} h_k(i_a, i_b) |(a_1, b_1), (a_2, b_2) \ldots (a_N, b_N)\rangle_{k}$



$\longrightarrow$ N=8; 5 coverings, 80 terms

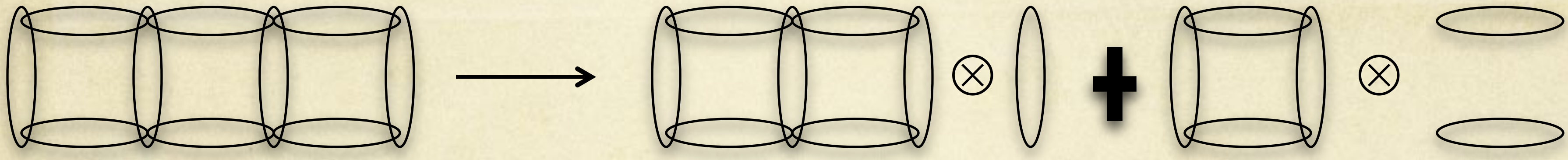


$\longrightarrow$ N=10; 8 coverings, 256 terms

Extensive growth with increasing system size

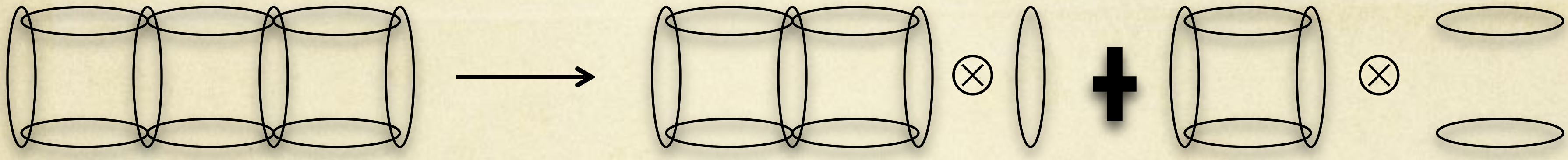
How to handle?

How to handle?



$|4\rangle$ $\quad$ $|3\rangle|1\rangle_4$ $\quad$ $|2\rangle|\bar{2}\rangle_{3,4}$

How to handle?



$|4\rangle$ $\quad$ $|3\rangle|1\rangle_4$ $\quad$ $|2\rangle|\overline{2}\rangle_{3,4}$

For arbitrary *L* - legged with M+2 rungs

$$|\mathcal{M}+2,\mathcal{L}\rangle = |\mathcal{M}+1,\mathcal{L}\rangle|1\rangle_{m+2} + |\mathcal{M},\mathcal{L}\rangle|\bar{2}\rangle_{m+1,m+2}$$

Recursion relation for the state

Reduced density matrix

Reduced density matrix –
Trace out all parties except M+1$^{\text{th}}$ and M+2$^{\text{th}}$ rungs

Recursion relation

$$\rho_{(m+1,m+2)} = \mathcal{N}_{\mathcal{M}}|2\rangle\langle 2|_{(m+1,m+2)} + \mathcal{N}_{\mathcal{M}-1}\bar{\rho}_{m+1} \otimes |1\rangle\langle 1|_{(m+2)} + (|2\rangle_{m+1,m+2}\langle 1|_{m+2}\langle\chi_{\mathcal{M}}|_{m+1} + \text{h.c.})$$

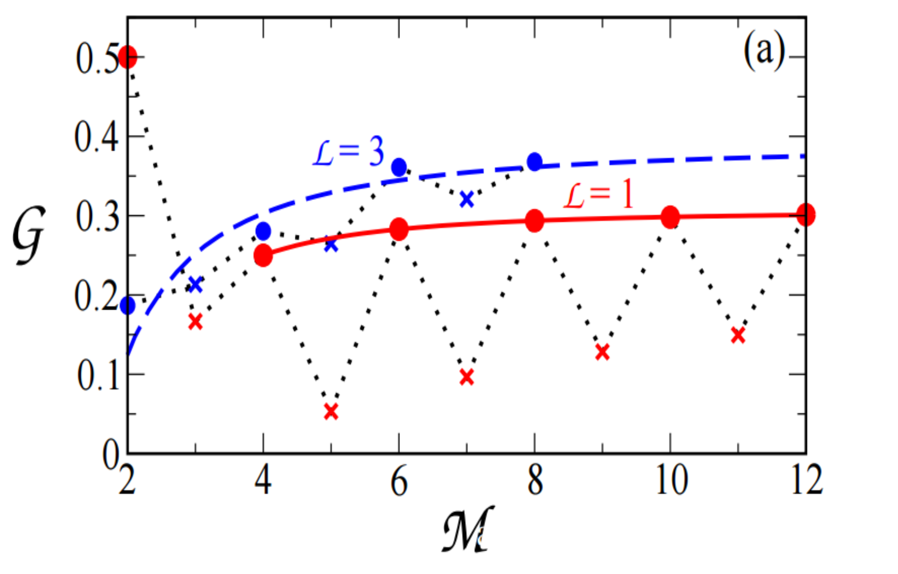
where $\mathcal{N}_{\mathcal{M}} = \langle\mathcal{M}|\mathcal{M}\rangle$ and

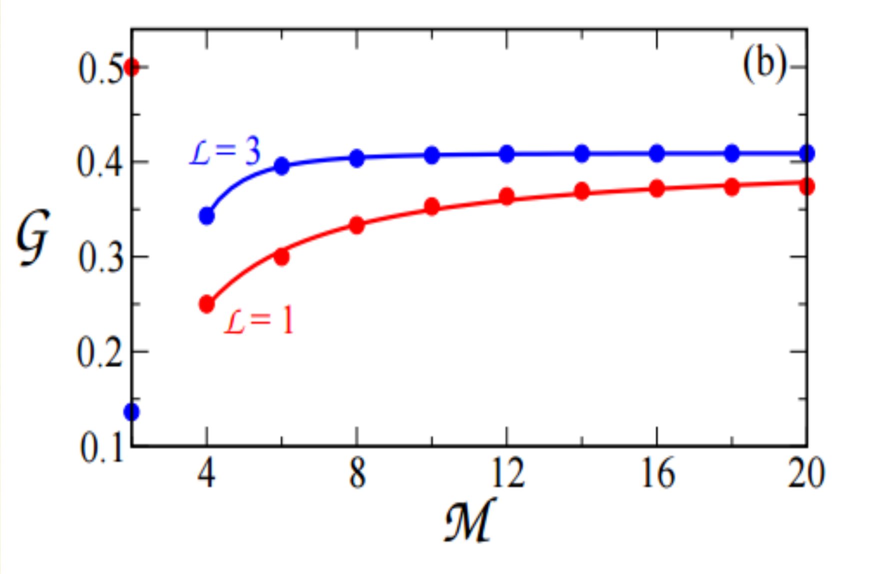
$$\bar{\rho}_{m+1} = \text{tr}_m(|\bar{2}\rangle\langle\bar{2}|_{m,m+1}), \quad \text{and}$$

$$\langle\chi_{\mathcal{M}}|_{m+1} = \langle\bar{2}|_{m,m+1}\langle\mathcal{M}-1|\mathcal{M}\rangle.$$

Himadri, Aditi, Ujjwal, PRL, 2013

Heisenberg vs. RVB (even legged ladders)



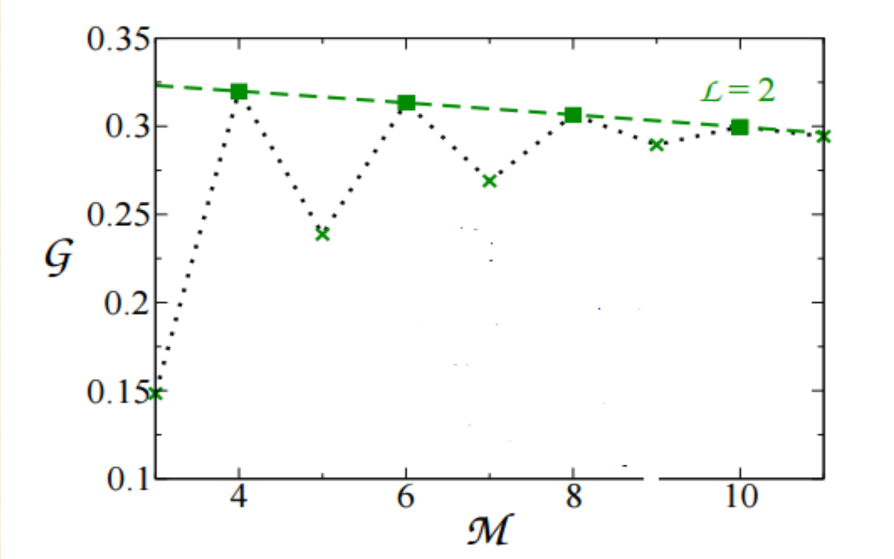


Heisenberg Ground state

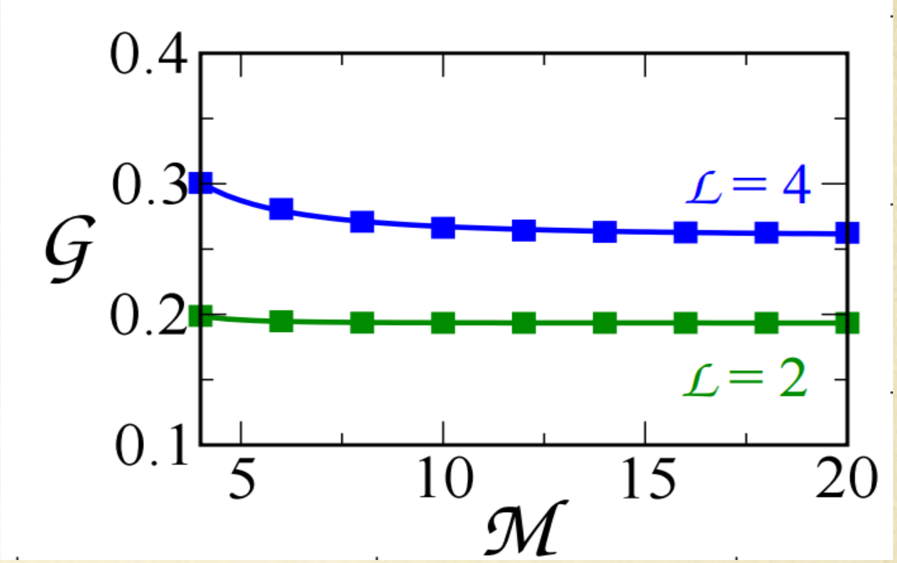
RVB state

For even rungs, GGM increases with increasing rungs

Heisenberg vs. RVB (odd legged ladders)



Heisenberg Ground state



RVB state

For even rungs, GGM increases with decreasing rungs

Asymptotic behavior (Odd vs. even)

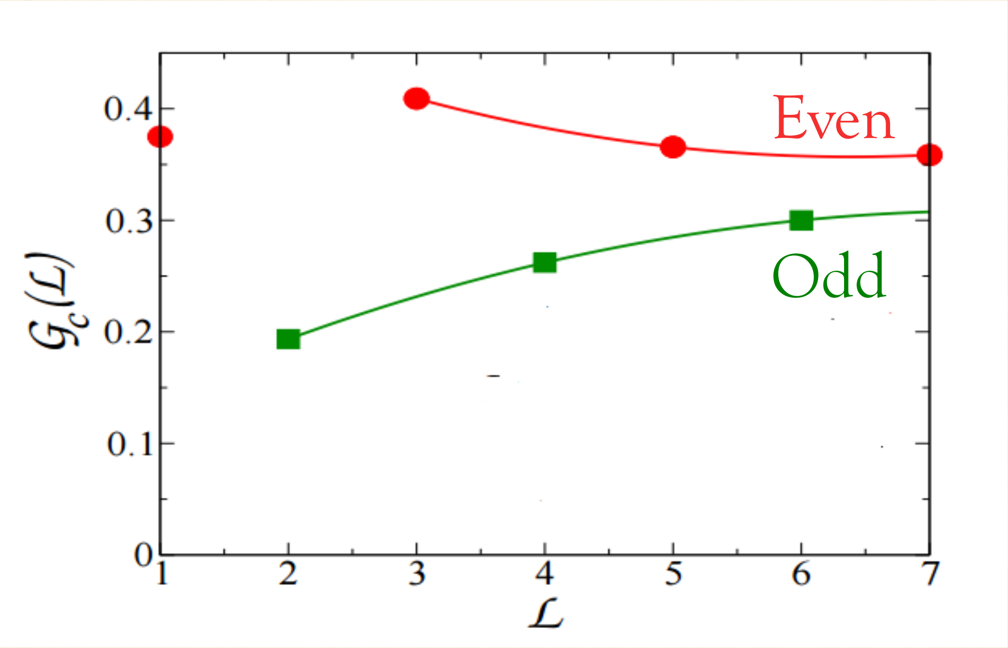


Fig. Asymptotic GGM as a function of number of legs.

Asymptotic behavior (Odd vs. even)

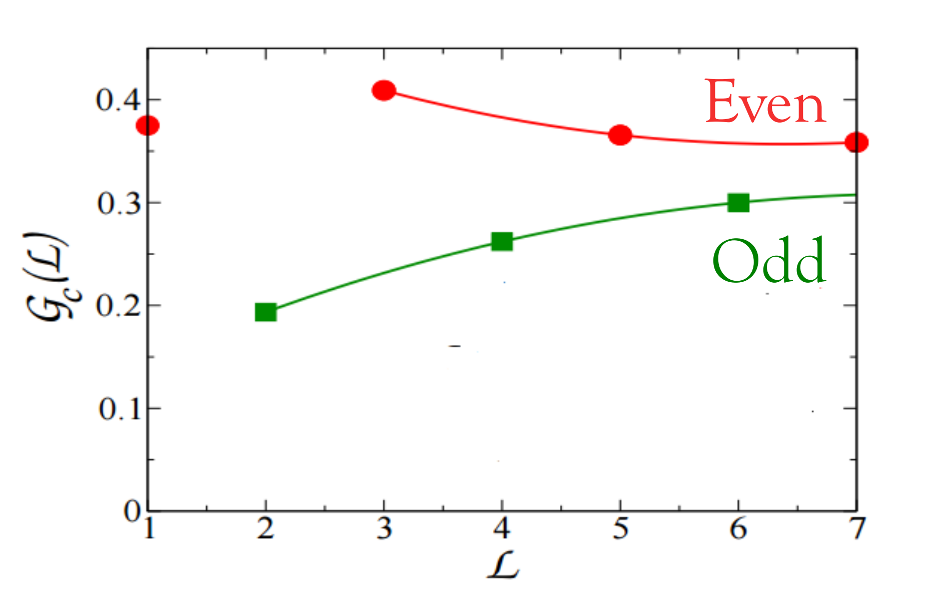


Fig. Asymptotic GGM as a function of number of legs.

- The sequences of the values from even and odd-legged ladders reach same asymptotic value when L → inf.

Asymptotic behavior (Odd vs. even)

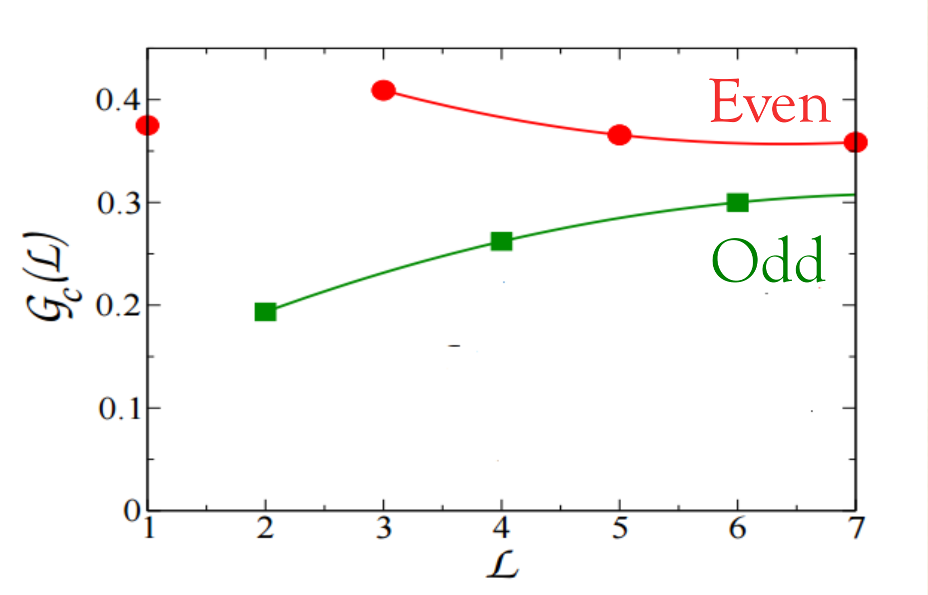
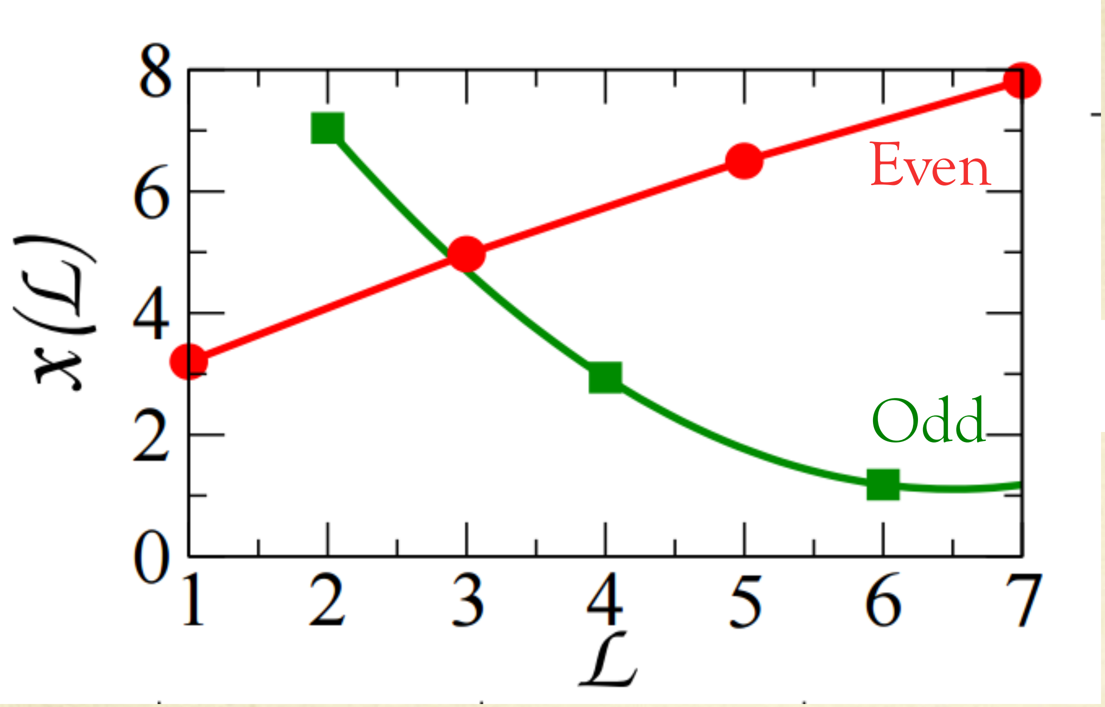


Fig. Asymptotic GGM as a function of number of legs.

- The sequences of the GGM values from even and odd-legged ladders reach same asymptotic value when L → inf.
- However, there a function of GGM, which still distinguishes even from odd at the asymptotic limit !!!

Scaling



$$\mathcal{G}(|\mathcal{L},\mathcal{M}\rangle) \approx \mathcal{G}_c(\mathcal{L}) \pm k n^{-x(\mathcal{L})}$$

- The scaling exponent, x(L), for odd and even ladders, converges to different values with increase of L

- Diverging scaling exponent for odd- and even-legged ladders, even though the corresponding multisite entanglement converge, scaling coefficient can still distinguish even-odd dichotomy

Summary

✓ Even-odd dichotomy of genuine multiparty entanglement, GGM, in Heisenberg ladders

✓ Resonating valance bond ansatz encapsulates qualitative features of GGM in Heisenberg ladders

✓ The scaling exponent of GGM can distinguish the even-odd dichotomy, even when corresponding GGM merge at asymptotic limit

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Thank you