Simulations with optical time-multiplexed quantum walks

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Experiments: Silberhorn group, UNIVERSITÄT PADERBORN, Die Universität der Informationsgesellschaft

Theory: Jex group, CTU Prague

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Allahabad

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Outline

- Quantum Walks
  - Simple time evolution: experimentally attractive
  - Quantum algorithms (e.g. search)
  - Computational universality
  - Applications to quantum simulation
- Optical time-multiplexed implementation
  - Basic principle
  - Features: compact, versatile, well-controlled
- Review experiments with time-multiplexing (Ch. Silberhorn)
  - Dynamical localization
  - Interacting DTQW
  - Recently: percolation

Discrete time quantum walks (DTQW)

Walk on a graph

- position space: $\mathcal{H}^V$ – discrete basis
- quantum coin: C
- evolution: discrete time step $U = SC$, $\quad \mathcal{H}^C \otimes \mathcal{H}^V$

  C: coin "flip" operator

  S: propagator (graph edges)

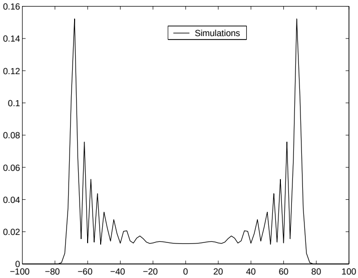
Coin operator

- balanced

  unbalanced

- n-regular graph

  homogenous coin: $C = C_0 \otimes \mathbb{1}$

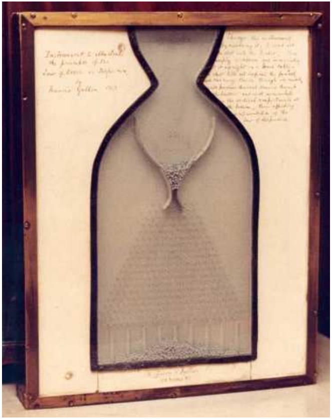


$t = 100$ steps, balanced coin

spreading: $\sim t \quad$ (not $\sim \sqrt{t}$)

Quantum optical implementation by analogy

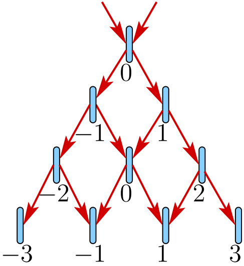
Galton board



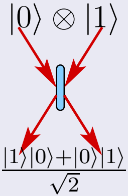
Binomial distribution

$\downarrow N = \infty$

Gaussian



Beam-splitter



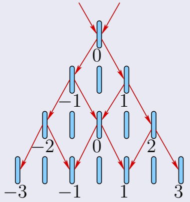
Coin

$$|1\rangle\,|0\rangle \equiv |L\rangle$$
$$|0\rangle\,|1\rangle \equiv |R\rangle$$

Position: place of BS

Time multiplexing

Naïve solution

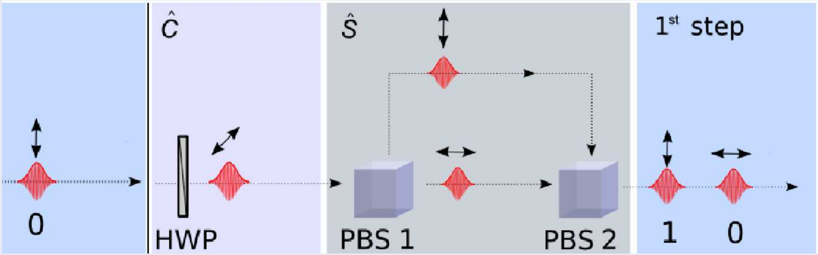


"Optical Galton board"
Unfavorable: cost, stability $\sim n^2$

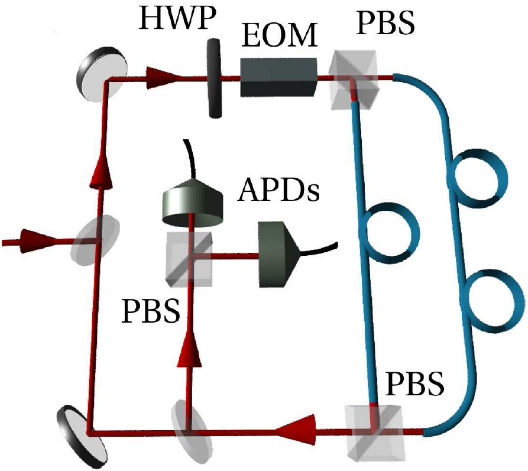
More compact solution

Time-bin encoding of position
PRL **104**, 050502 (2010)

Time delay (polarization dependent) ~ 5ns



Time multiplexing and feedback loop



Electro-optical modulator

relative phase shift
modulation of polarization

Time dependent coin

- Compact, low budget
- Great degree of control

Next

Review simulation experiments

Dynamical localization

Anderson localization

Well-known in solid-state physics:
metal-insulator transition

Phase disorder

$$C = \sum_x F(\varphi_x) C_0 \otimes |x\rangle\langle x|$$

Position and time dependent phase shift:

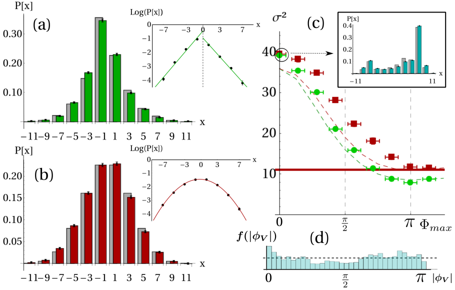
$$F(\varphi_{xt}) = \begin{pmatrix} e^{i\varphi^L_{xt}} & 0 \\ 0 & e^{-i\varphi^R_{xt}} \end{pmatrix}$$

[PRL 106 180403 (2011)]

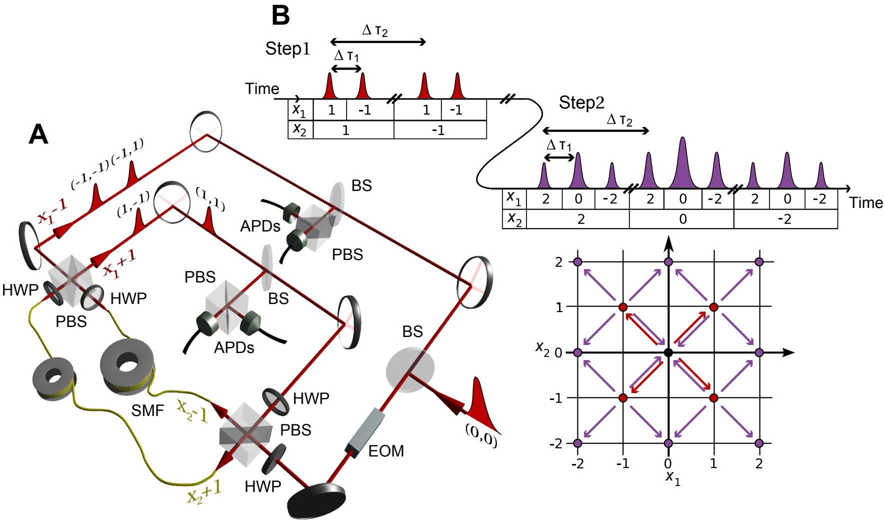
Quentched disorder: $\varphi_{xt} = \varphi_x$

Exponential localization in d dimensions
Joye: Quant. Inf. Proc. **11** 1251 (2012)

Experimental results

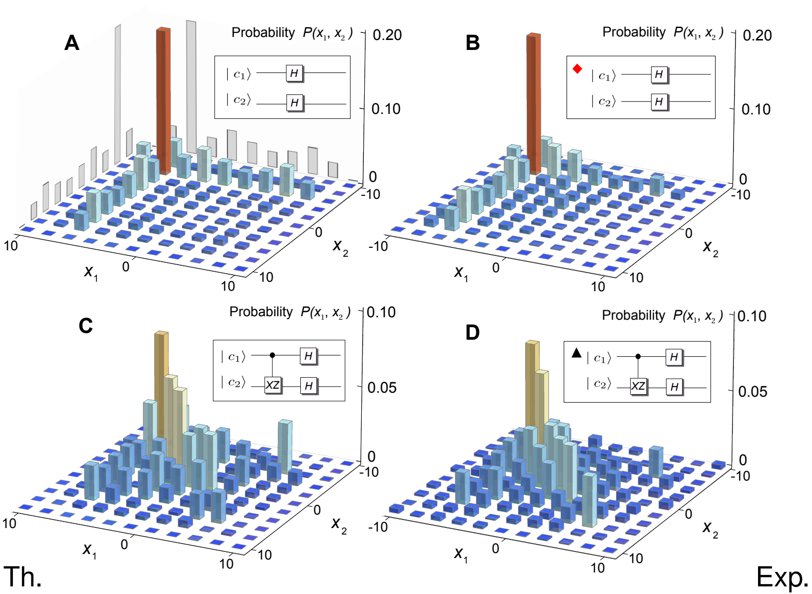


Two interacting walkers: “2D setup”



Carefully chosen delays [*Science* **336**, 55 (2012)]

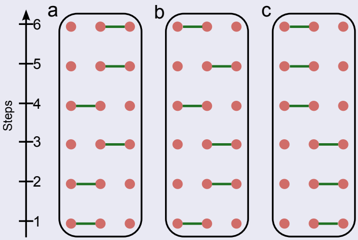
Two interacting walkers: measurement results



Percolation Quantum Walk

Dynamical percolation graph

- Percolation: edge present with probability p
- Dynamical percolation: at each instant a new percolation graph



RUM (random unitary map)

$$\hat{\rho}(n) = \sum_{\kappa \in \mathcal{K}} p(\kappa, \mathfrak{p}) \left(\hat{S}_\kappa \hat{C}\right) \hat{\rho}(n-1) \left(\hat{S}_\kappa \hat{C}\right)^\dagger$$

Attractors

$$\left(\hat{S}_\kappa \hat{C}\right) \hat{X}_n \left(\hat{S}_\kappa \hat{C}\right)^\dagger = \lambda_n \hat{X}_n, \forall \kappa \in \mathcal{K}.$$

Eigenvalues: $|\lambda| = 1$ survive

Non-trivial asymptotic dynamics

$$\varrho(t \gg 1) = \sum_n \lambda^t \, \mathrm{Tr}\{\hat{X}_n \varrho_0\} \hat{X}_n,$$

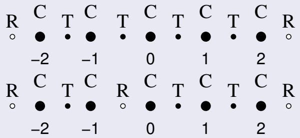
Non-trivial: $\lambda \neq 1$ (oscillations)

Implementation by quantum walk primitives

QW with operator $\hat{C}$

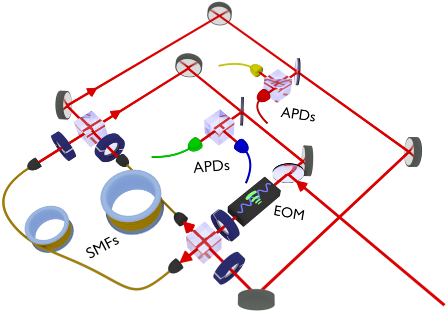
break one step into two

Mid-step coin operators



$$\hat{T} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \hat{R} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

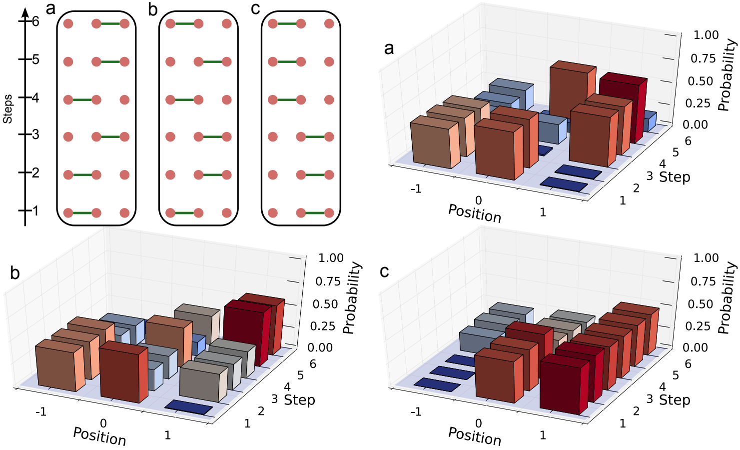
In the experiment:



$$\hat{T} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \hat{R} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}.$$

(limitation of the EOM)

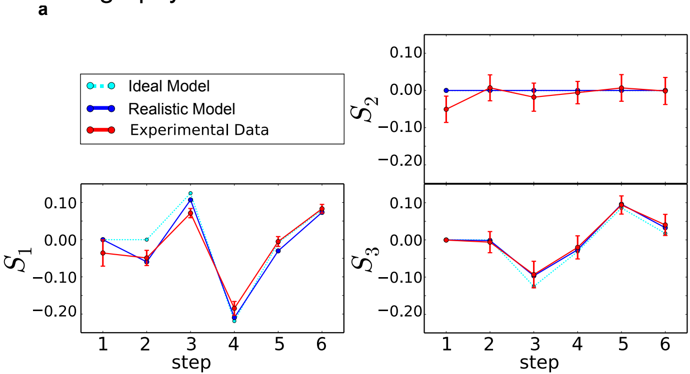
Measurement results: position distribution



Sci. Rep. **5**, 13495 (2015)

Measurement results: coin state

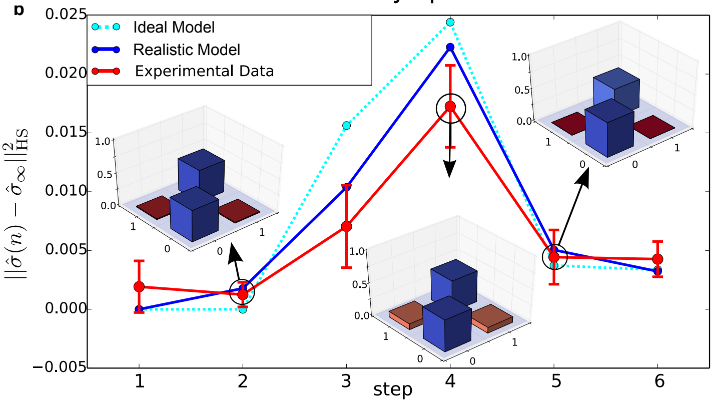
Full tomography on the coin state



Reduced density operator

$$\hat{\sigma} = \mathrm{Tr}_C\, \hat{\varrho} = \frac{\mathbb{1} + \vec{S}\hat{\vec{\sigma}}}{2}$$

Hilbert–Schmidt distance from asymptotic



Non-Markovian evolution; $\sigma_\infty \sim \mathbb{1}$

Summary

- Realization of disorder schemes in QW
  - Coin disorder
  - Graph disorder
- Links to
  - Dynamical localization
  - Transport on random graphs

Prague (theory):

- Igor Jex
- AG
- Jaroslav Novotný

Paderborn (experiments):

UNIVERSITÄT PADERBORN
Die Universität der Informationsgesellschaft

- Christine Silberhorn
- Fabian Elster
- Sonja Barkhofen
- Thomas Nitsche