Complete framework for efficient characterisation of non-Markovian processes

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dynamics

quantum to classical

Non-Markovian

coherence

environmental effects

memory kernel

heat

spectroscopy

system-bath correlations

memory effects

entanglement

Redfield

transport

density matrix

evolution

strong coupling

temperature

decoherence

energy exchange

master equation

weak coupling

dissipation

dephasing

Hierarchical Equation of Motion

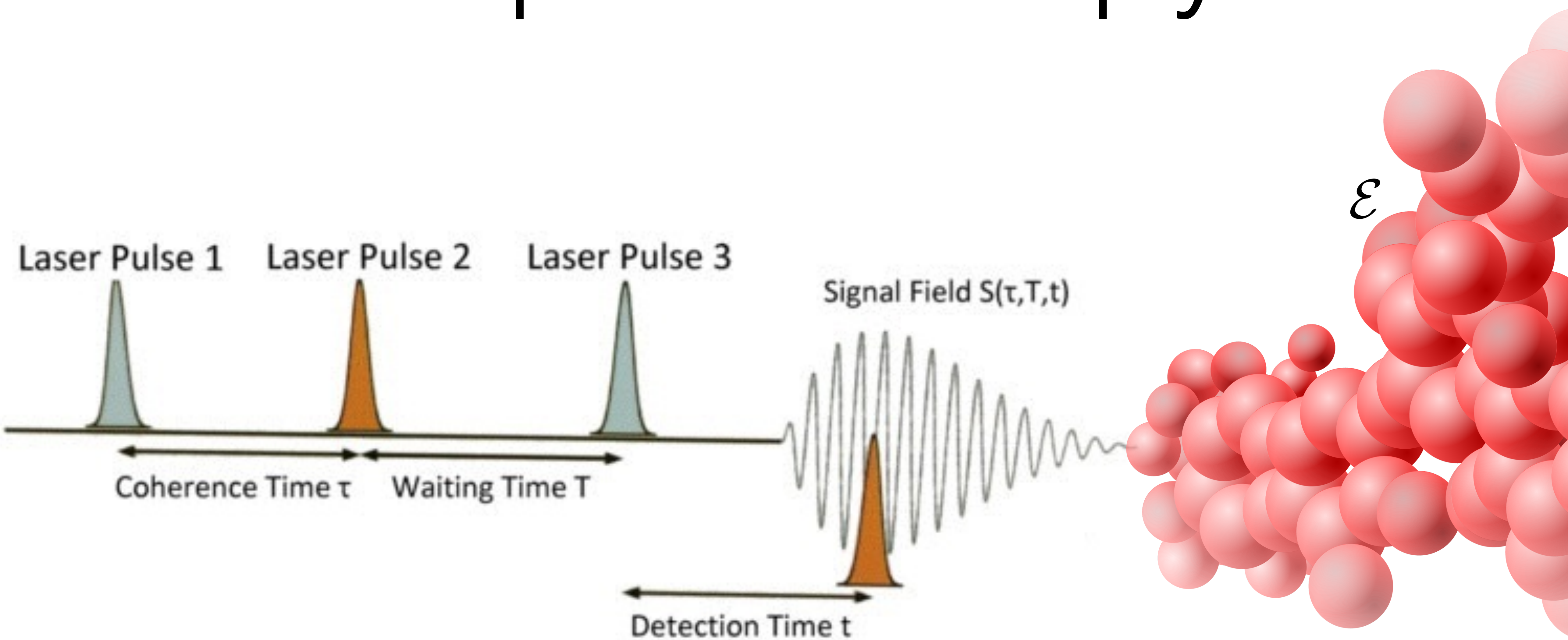
positivity

spin-boson model

Markovian

Lindblad

2D spectroscopy



http://gaigroup.chem.upenn.edu/images/2DIR_pulse_sequence_small.jpg

Why is this important?

- How can we characterise non-Markovian processes?

- Can we do this without making assumptions?

- Can have our cake and eat it?
  Keep positivity & linearity

- Can we do this practically?

Toy model

Dynamical map

We can use the system to characterise the Blackbox.

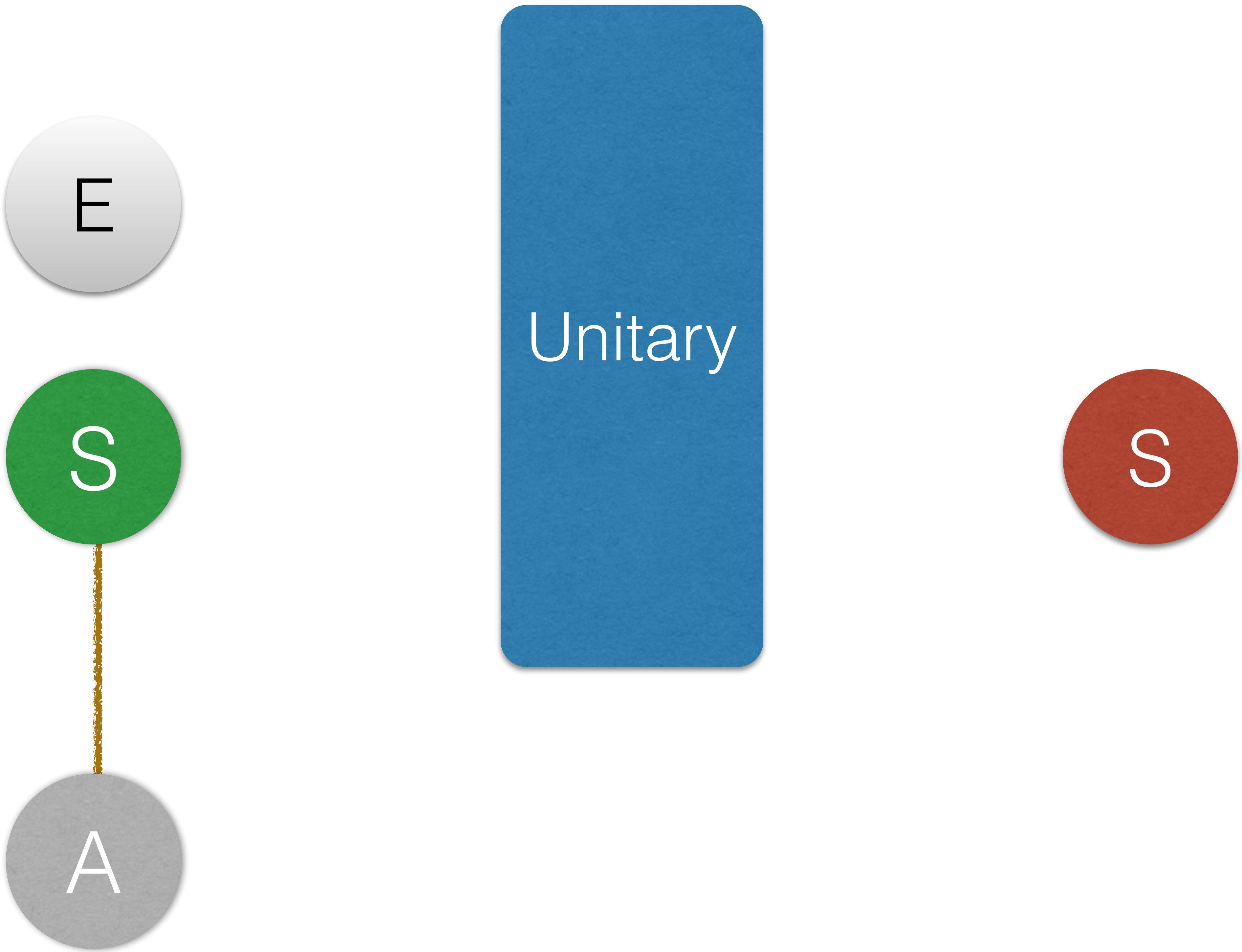
Stinespring dialation

Dynamical maps = Reduced unitary dynamics

Properties of map

- Linear
- Completely positive
- Sequence of dynamical maps=Markovian process

Complete positivity



Initially correlated SE

Pechukas: We must give up **complete positivity** or **linearity**

This is the simplest non-Markovian case study.

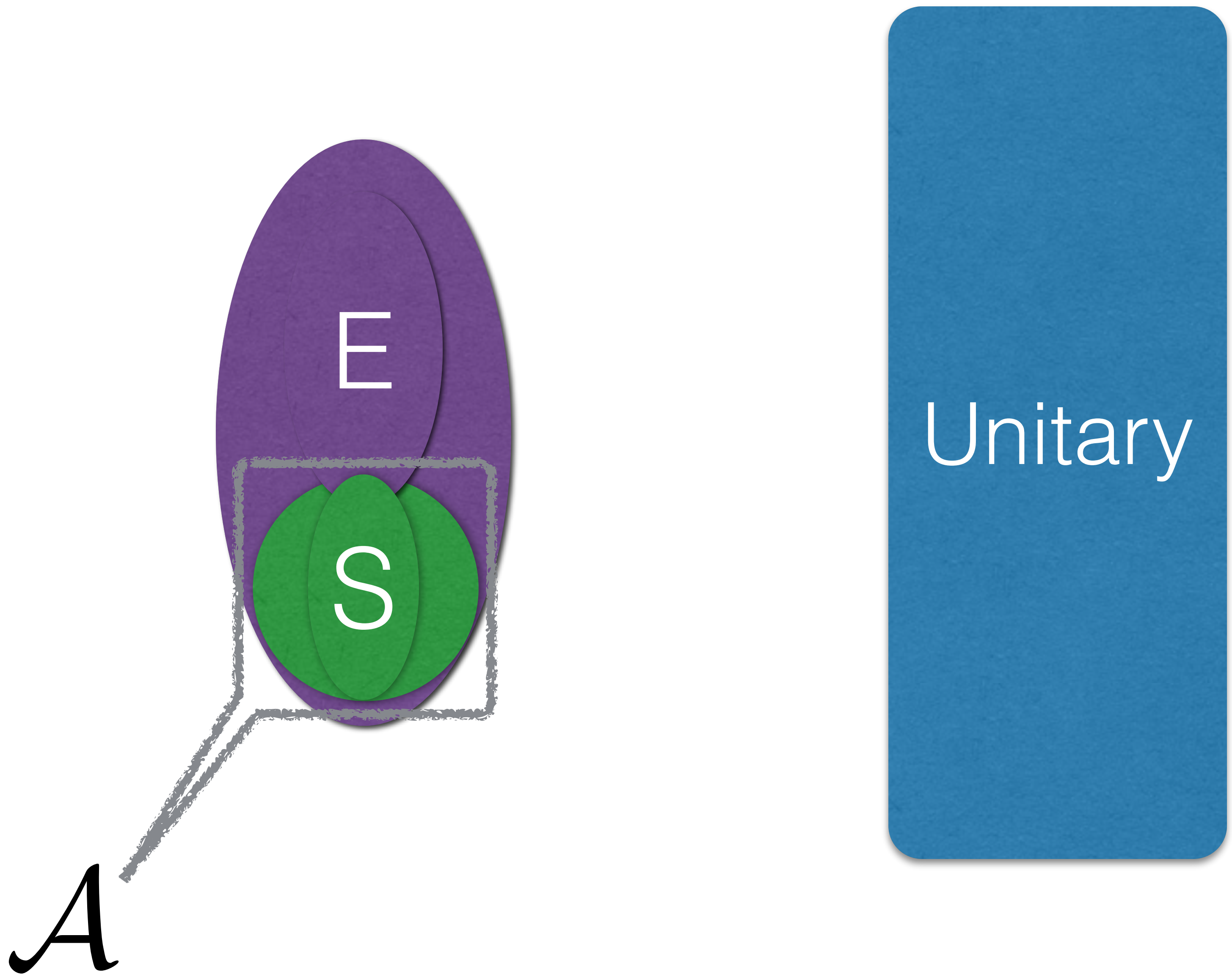
Giving up CP?

Holevo bound [Masillo, Scolarici, Solombrino, JMP 52, 012101 (2011)]

Data processing inequality [Buscemi, PRL. 113, 140502 (2014)]

Entropy production [Argentieri, Benatti, Floreanini,… EPL. 107, 50007 (2014)]

Initially correlated SE

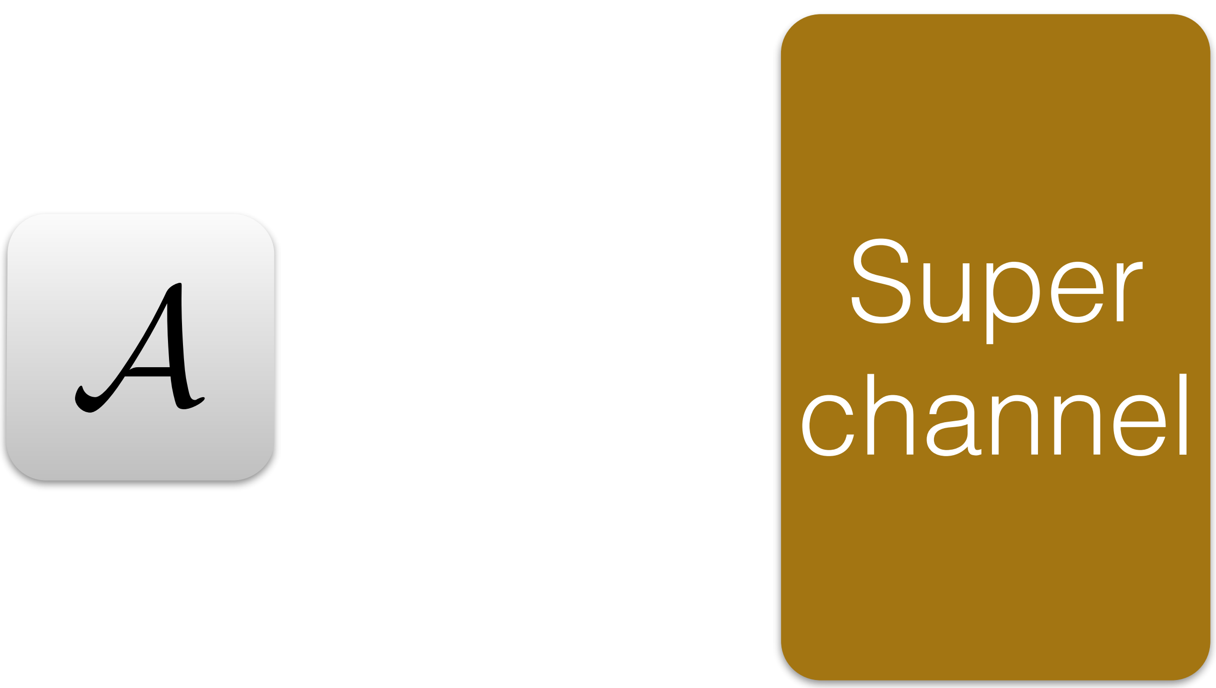


Grad student presses buttons

We can give up states as inputs

Superchannel

Completely positive and linear



$$\mathcal{M}[\mathcal{A}] := \rho$$

KM, Sci. Rep. 2, 581 (2012)   Ringbauer, Wood, KM, Gilchrist, White, Fedrizzi, PRL 114, 090402 (2015)

Using CP

Holevo bound [Masillo, Scolarici, Solombrino, JMP 52, 012101 (2011)]

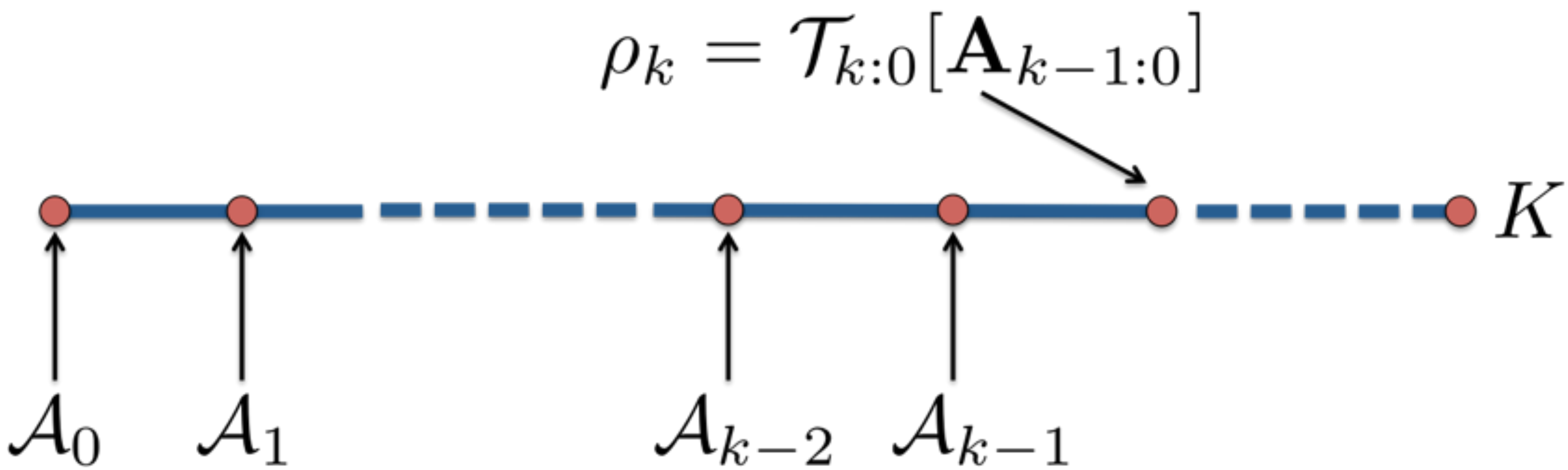
Data processing inequality [Buscemi, PRL. 113, 140502 (2014)]

Entropy production [Argentieri, Benatti, Floreanini,… EPL. 107, 50007 (2014)]

Vinjanampathy, Modi PRA 92, 052310 (2015)

Can we generalise this?

Quantum process



$$\mathbf{A}_{k-1:0} = [\mathcal{A}_{k-1}; \mathcal{A}_{k-2}; \ldots; \mathcal{A}_1; \mathcal{A}_0]$$

These actions are the history of buttons grad student pressed

Open quantum evolution $\Rightarrow \mathcal{T}$

$$\rho_k^{SE} = \mathcal{U}_{k:k-1}\, \mathcal{A}_{k-1}\, \mathcal{U}_{k-1:k-2} \ldots \mathcal{A}_1\, \mathcal{U}_{1:0}\, \mathcal{A}_0\, [\rho_0^{SE}]$$

$$\rho_k = \mathcal{T}_{0:k}[\mathbf{A}_{0:k}] = \sum_l (T_{k:0})_l\, \mathbf{A}_{0:k}\, (T_{0:k})_l^\dagger$$

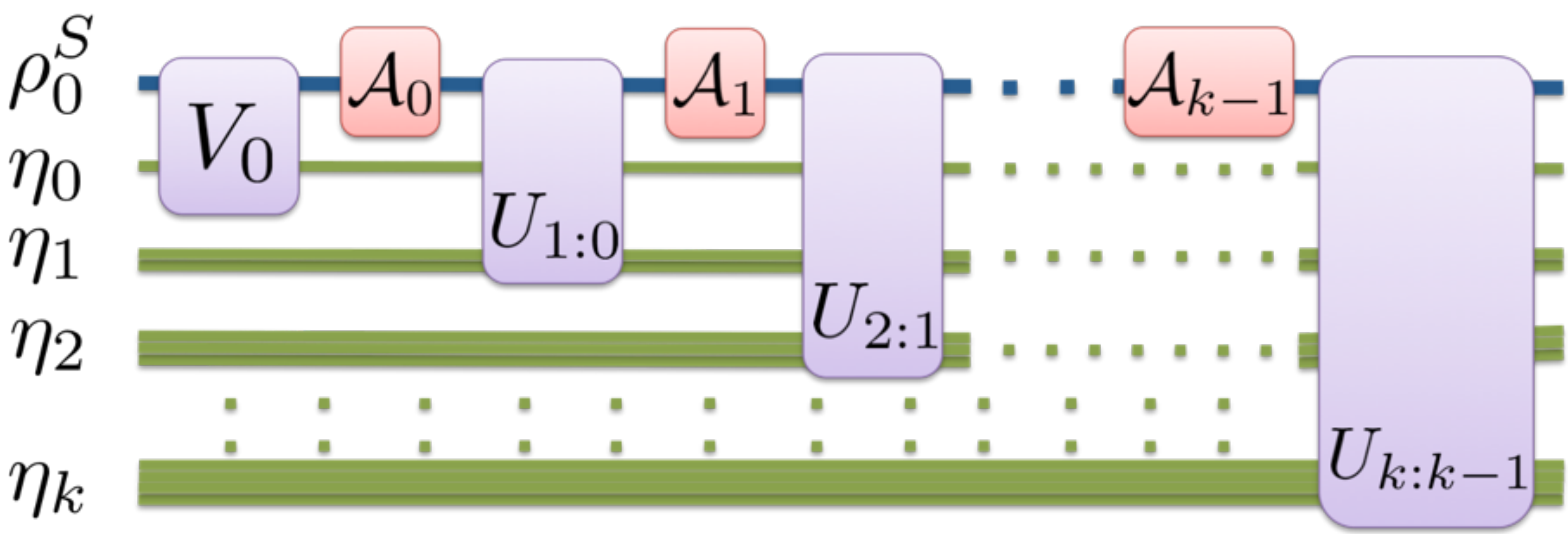
$$\mathbf{A}_{k-1:0} = [\mathcal{A}_{k-1}; \mathcal{A}_{k-2}; \ldots; \mathcal{A}_1; \mathcal{A}_0]$$

Completely positive and linear

$$\mathcal{T}_{k':j'} \subset \mathcal{T}_{k:0}$$

Taking a middle page from the lab notebook

Open quantum evolution $\Leftarrow \mathcal{T}$



Universality

- Process tensor is the most general descriptor of quantum processes

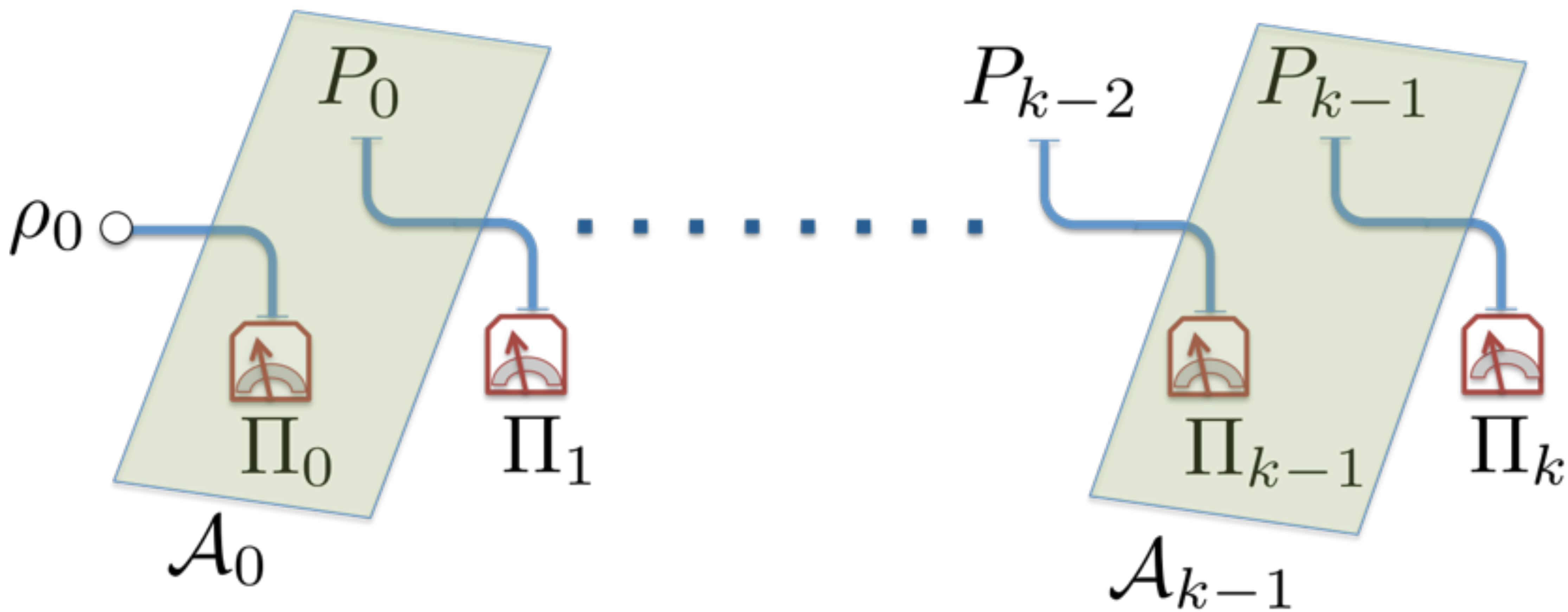
- We have not assumed any model

- It has all of the desired properties:
  linear, complete positivity, containment

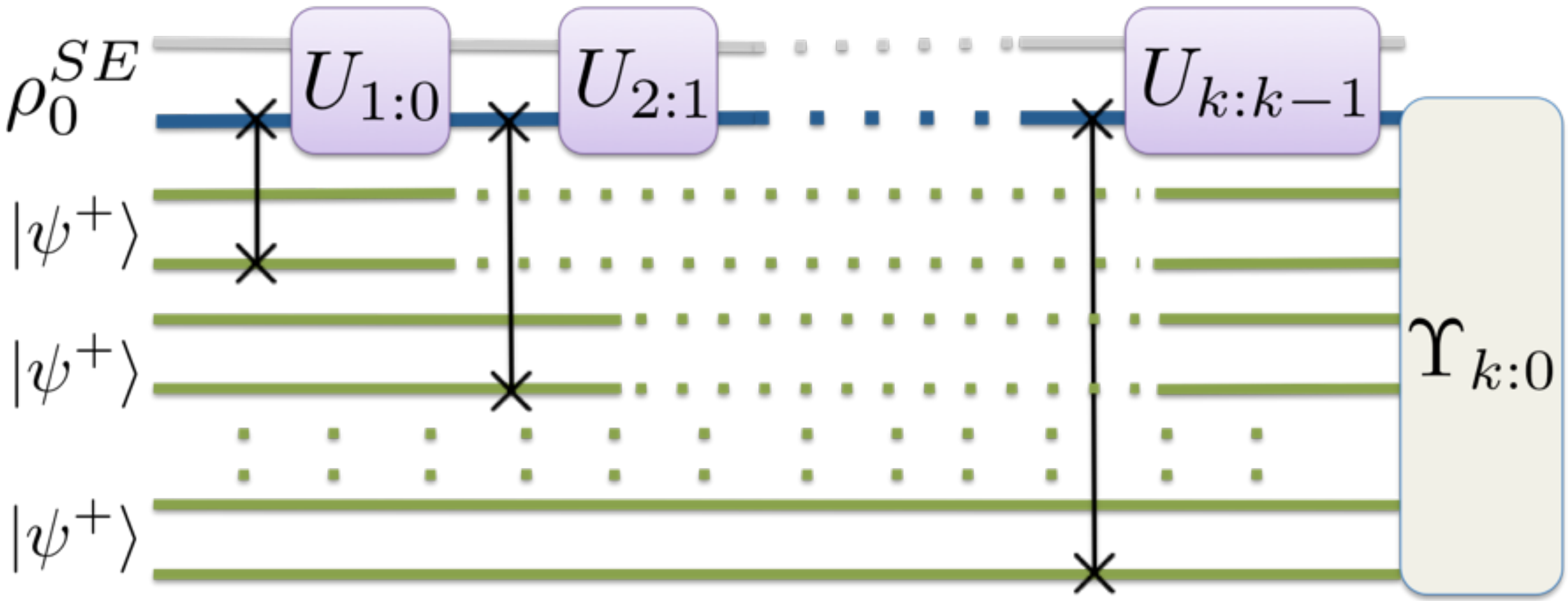
- It’s operationally motivated, meaning it is designed to work hand-in-hand with experiments

How do we find $\mathcal{T}$?

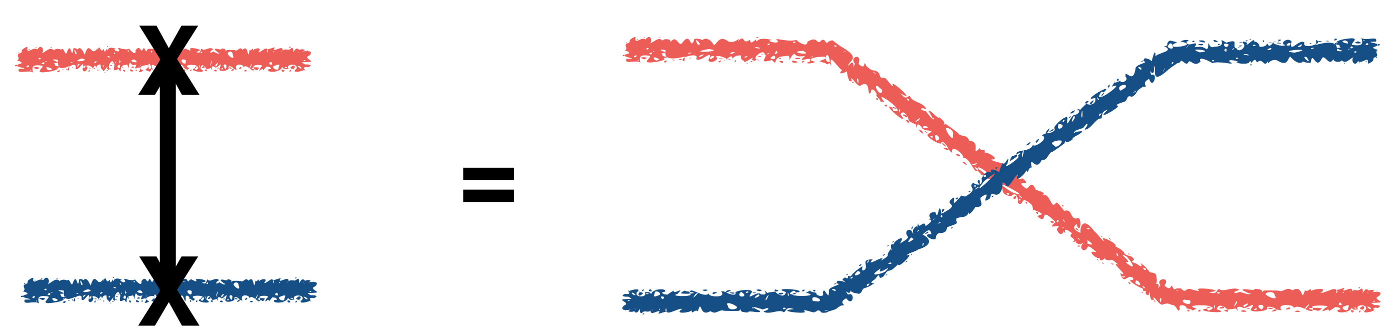
Linear expansion



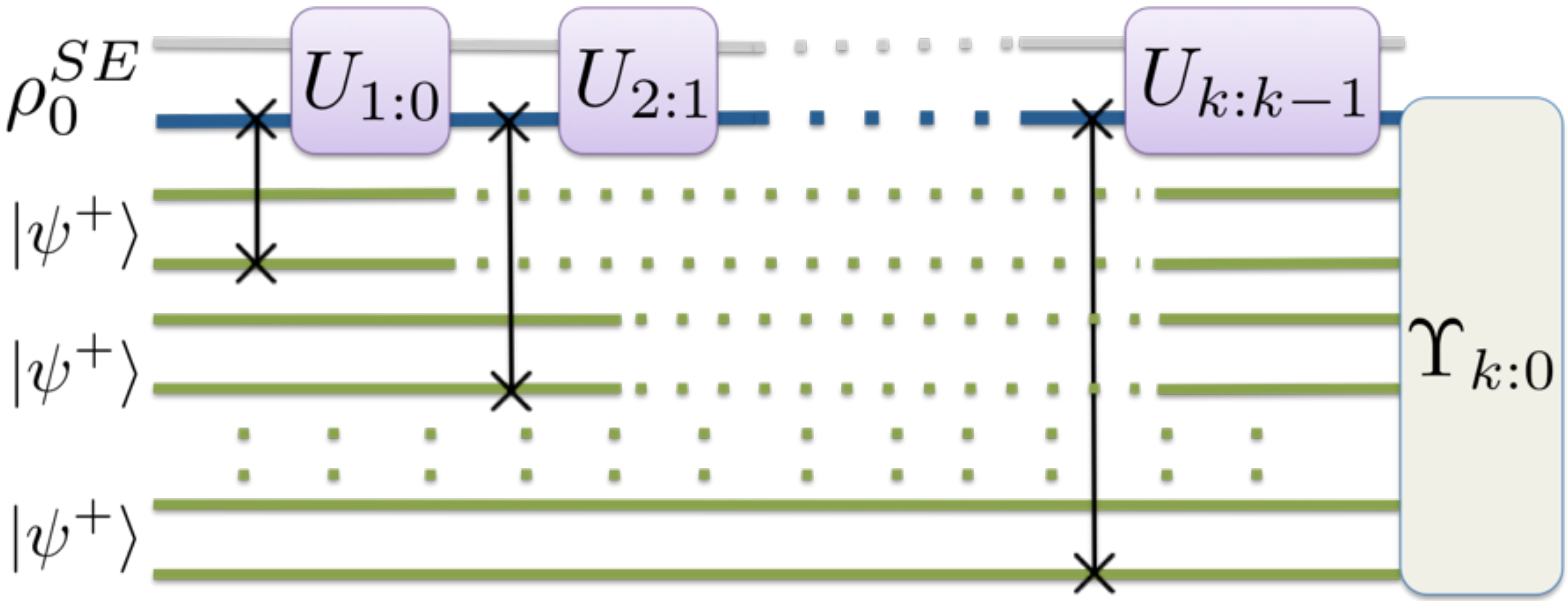
Encode into state



$$|\psi^+\rangle = \frac{1}{\sqrt{d}} \sum_x |xx\rangle$$



Encode into state



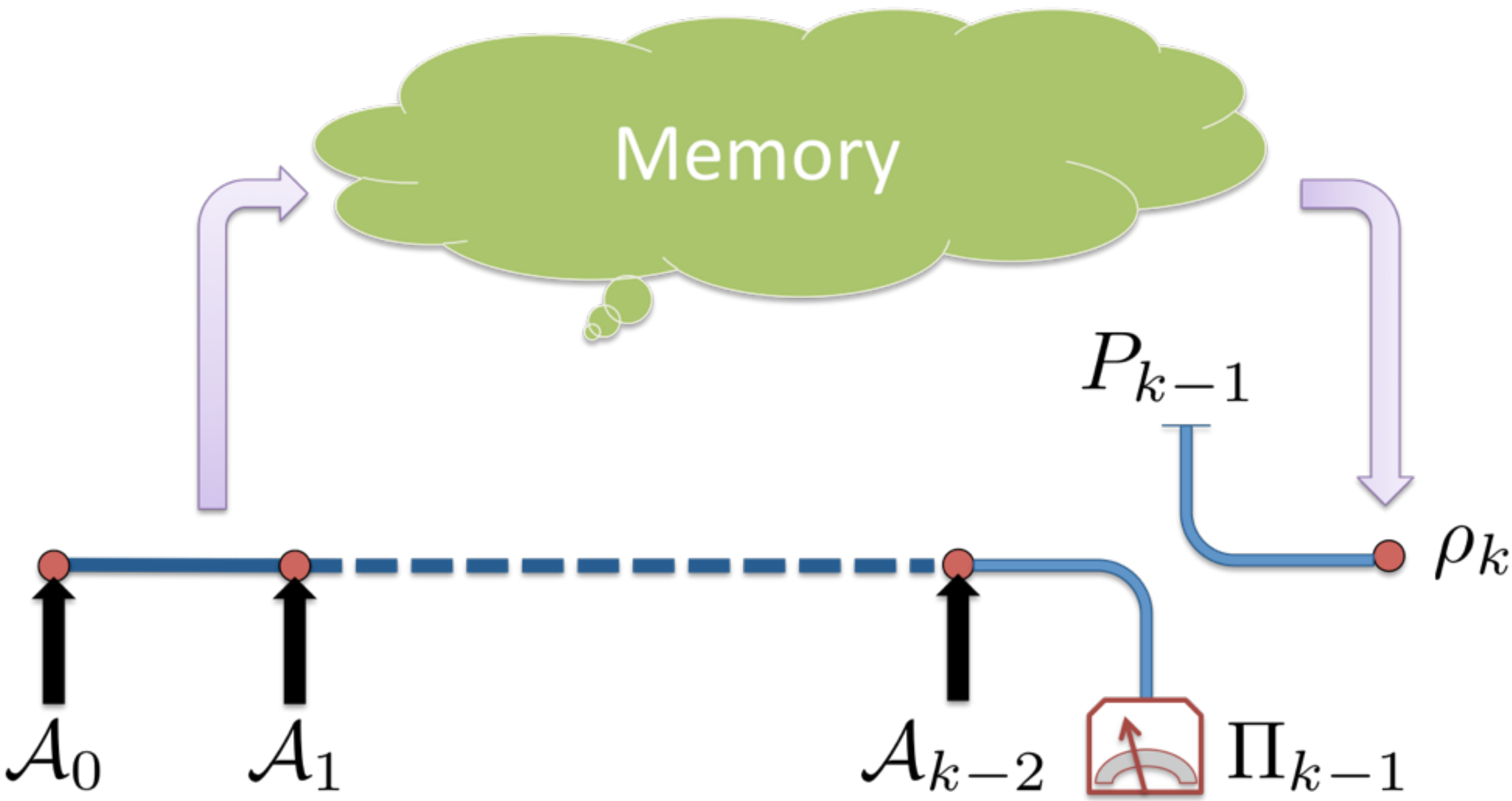
$$\mathcal{T}_{k:0} \longleftrightarrow \Upsilon_{k:0}$$

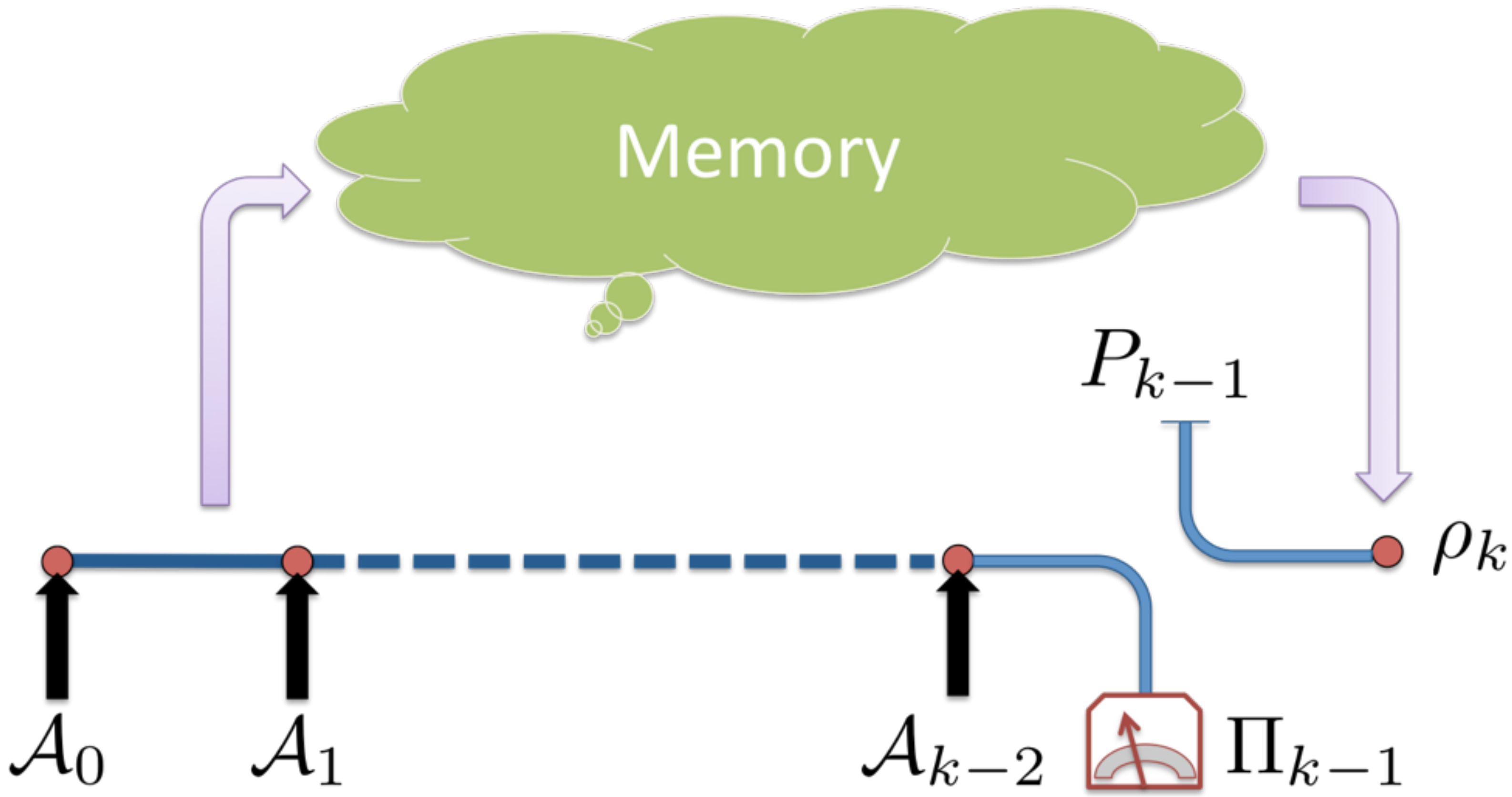
$$\mathcal{T}_{k':j'} \longleftrightarrow \Upsilon_{k':j'}$$

What is Markovian?

Shift switch of grad student

Causal break





$$\rho_k(P_{k-1}|\Pi_{k-1};\mathbf{A}_{k-2:0}) = \rho_k(P_{k-1})$$

$$\rho_k(P_{k-1}|\Pi_{k-1};\mathbf{A}_{k-2;0}) \neq \rho_k(P_{k-1}|\Pi'_{k-1};\mathbf{A}'_{k-2;0})$$

Markovian $\Rightarrow$ Divisible

$\not\Leftarrow$

Semigroup

Measuring non-Markovianity with Relative entropy

$\Upsilon_{7:0}$ S A1B1 A2B2 A3B3 A4B4 A5B5 A6B6 A7B7

$\Upsilon_{7:0}^{\mathrm{Markov}}$ S | A1B1 | A2B2 | A3B3 | A4B4 | A5B5 | A6B6 | A7B7

$$\mathcal{N} = R(\Upsilon_{7:0} \| \Upsilon_{7:0}^{\mathrm{Markov}})$$

$$\text{Confusion probability} = e^{-n\mathcal{N}}$$

You have a Markovian model to describe a non-Markov process.
How surprised are you when model gives wrong answer

What now?

- 2D spectroscopy
- Bounds on information transfer and transport
- Typicality of non-Markovian process
- Quantum information theory / error correction
- Causal structures

Conclusions

arXiv:1512.00589

- We have a universal descriptor for arbitrary quantum processes
- We can encode the process into a many-body state, leading to an efficient characterisation
- Operational definition of non-Markovianity

MonQIS

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