Violations of complementarity enable beyond-quantum nonlocality, distinguishability and cloning

Marcos C. de Oliveira
University of Campinas - SP, Brazil

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Quantum Information Theory

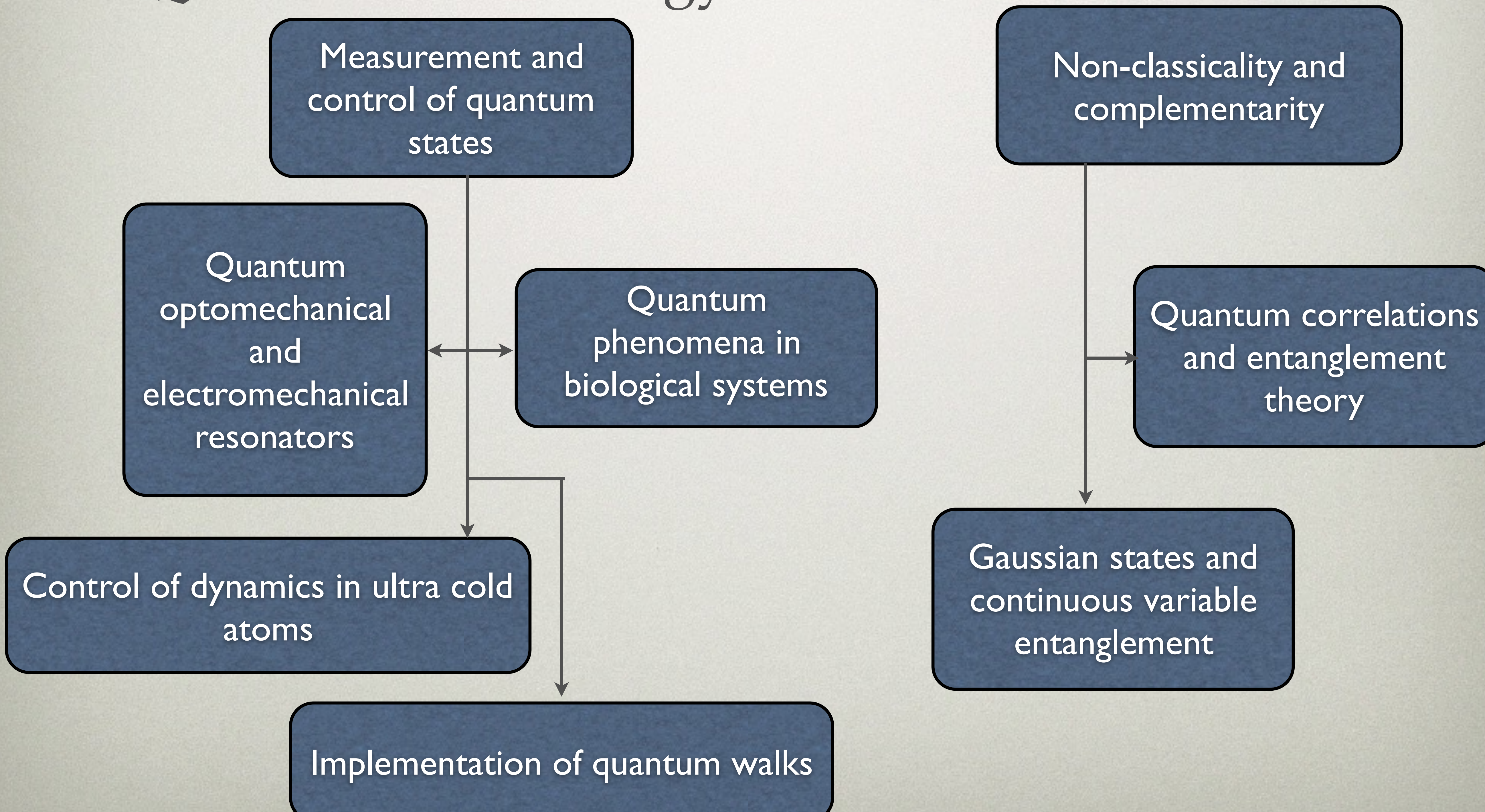
Marcos C. de Oliveira

- PostDoc:
  Jalil Khatibi Moqadam
- Students:
  - Alejandro Carrillo
  - Marina Vasques Moreira
  - João Fernando Doriguello Diniz
  - Leandro Raffhael da Silva Mendes

Quantum Information Theory

Quantum Technology

Foundations



Outline

- Complementarity
- Violation of Complementarity in two-level systems
- Implications

F. E. S. Steinhoff, MCO, *Violations of complementarity enable beyond-quantum nonlocality, distinguishability and cloning*, arXiv:1406.1710

Complementarity

Two observable are complementary if certainty in the measurement of one of them precludes certainty in the measurement of a complementary one.

- All possible measurement results of the second observable are equiprobable.

- Dual wave-particle behavior.

Bohr (1927):

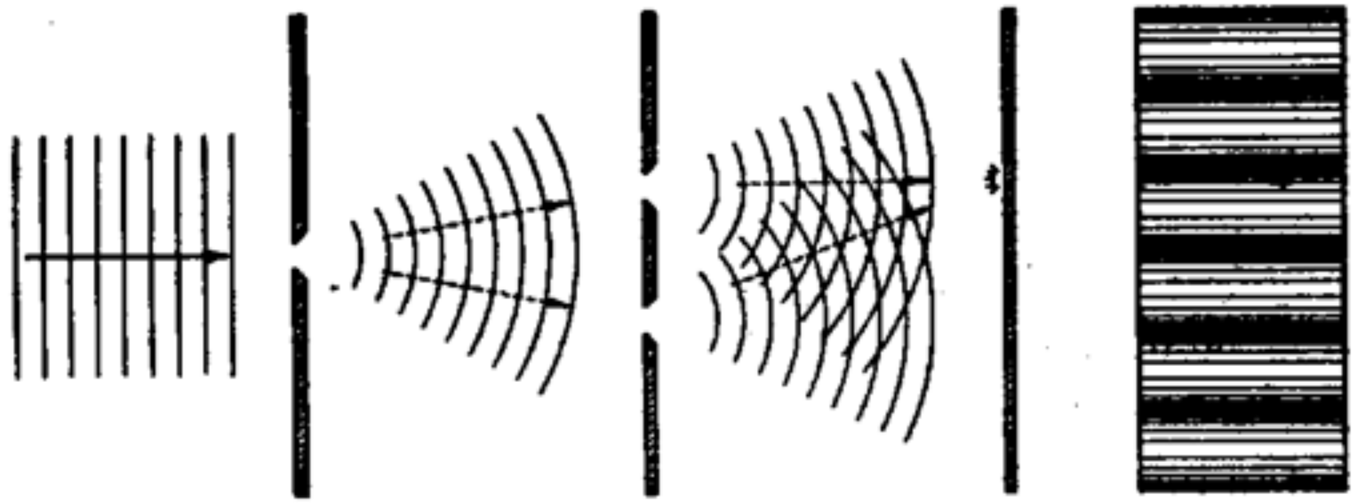


FIG. 3

Niels Bohr (1949). "Discussions with Einstein on Epistemological Problems in Atomic Physics". In P. Schilpp. Albert Einstein: Philosopher-Scientist. Open Court.

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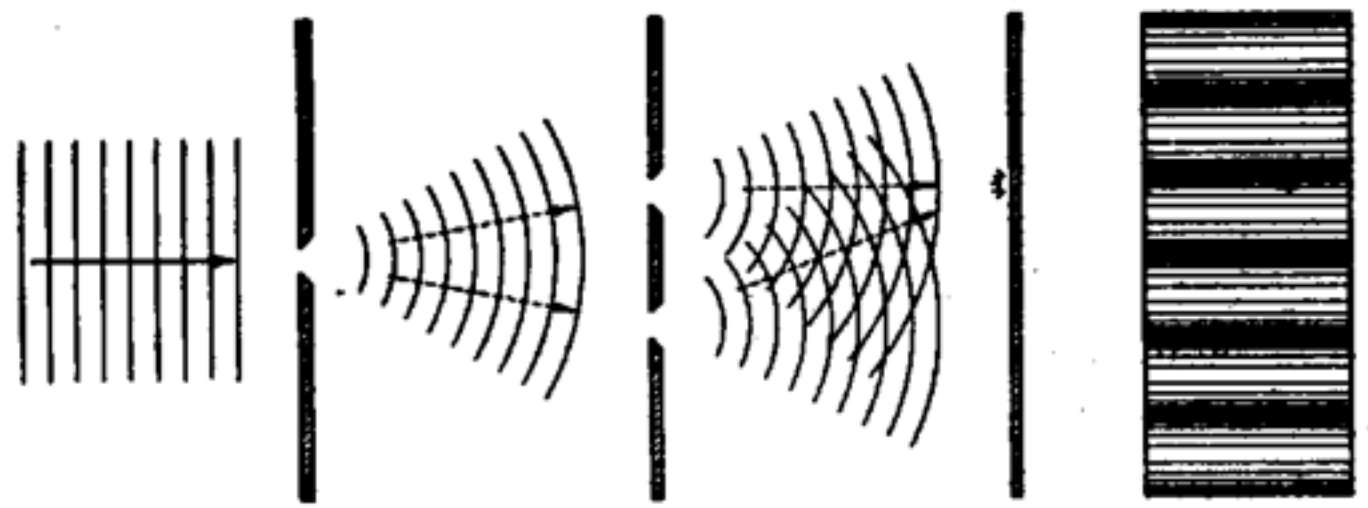

FIG. 3

Heisenberg (1927):

Bohr has brought to my attention [that] the uncertainty in our observation does not arise exclusively from the occurrence of discontinuities, but is tied directly to the demand that we ascribe equal validity to the quite different experiments which show up in the [particulate] theory on one hand, and in the wave theory on the other hand.

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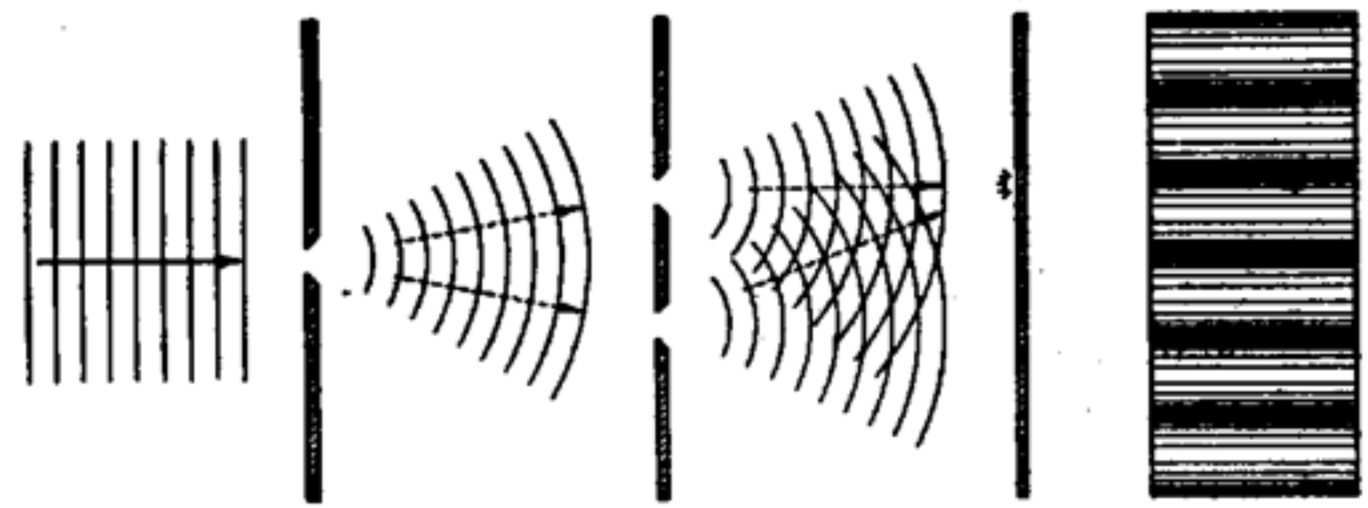

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Examples

- Position and momentum of a particle

If $\Delta p = 0$ $\Rightarrow$ $\Delta x = \infty$

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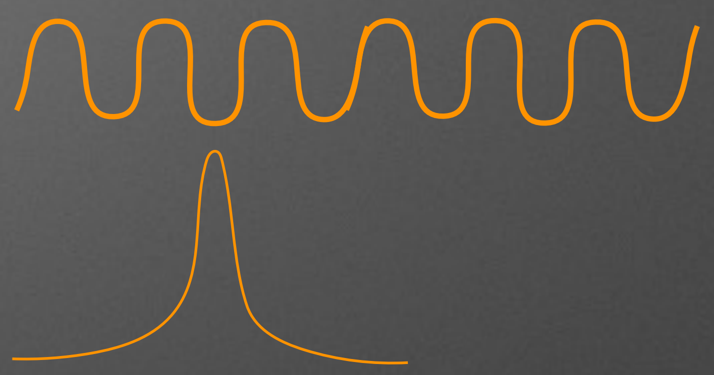


- **Spin 1/2:** $\{|\uparrow\rangle, |\downarrow\rangle\}$

If $|\psi\rangle = |\uparrow\rangle$ $\Rightarrow$ $\Delta S_v = 0$

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$$ **50%**

$$|\leftarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$$ **50%**

Englert–Greenberger duality relation

$$\boxed{D^2 + V^2 \leq 1}$$

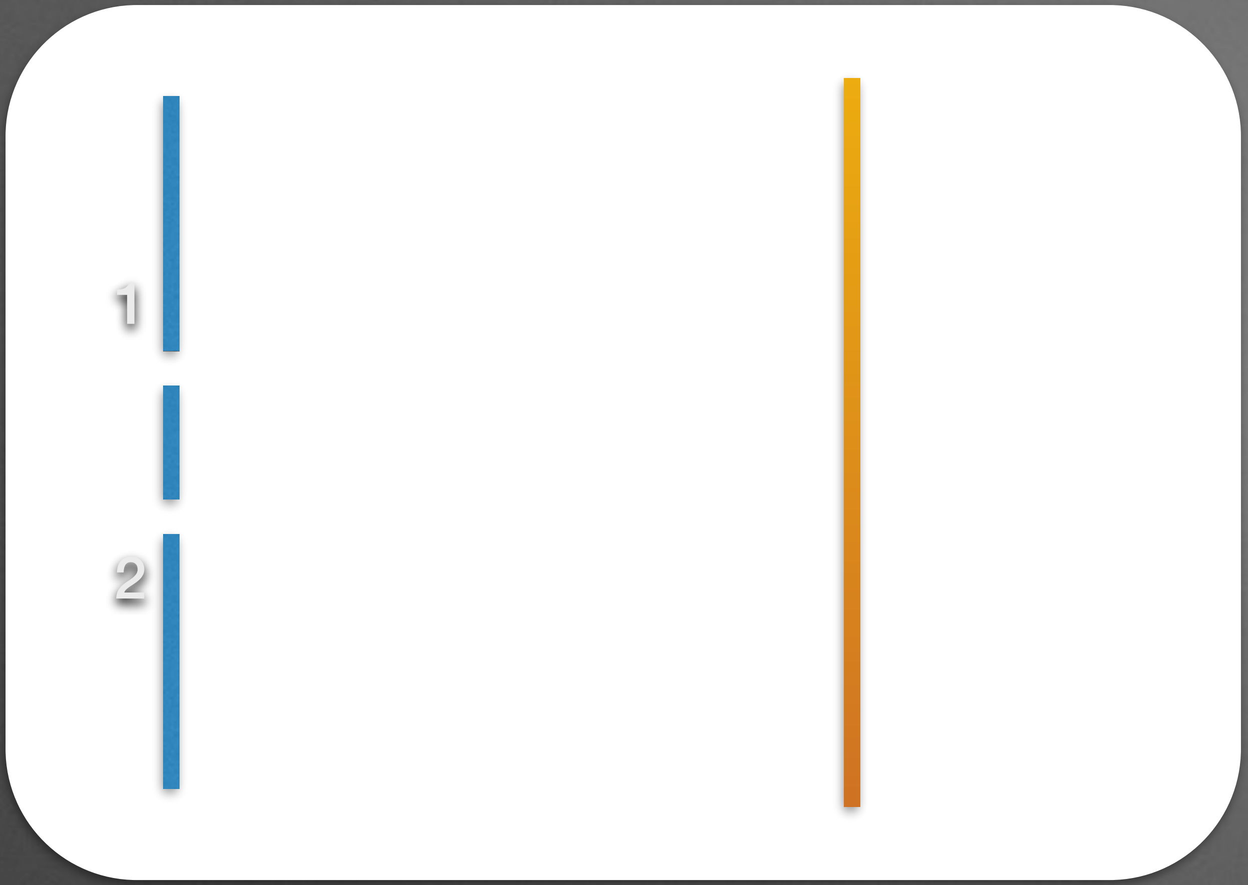
V : visibility

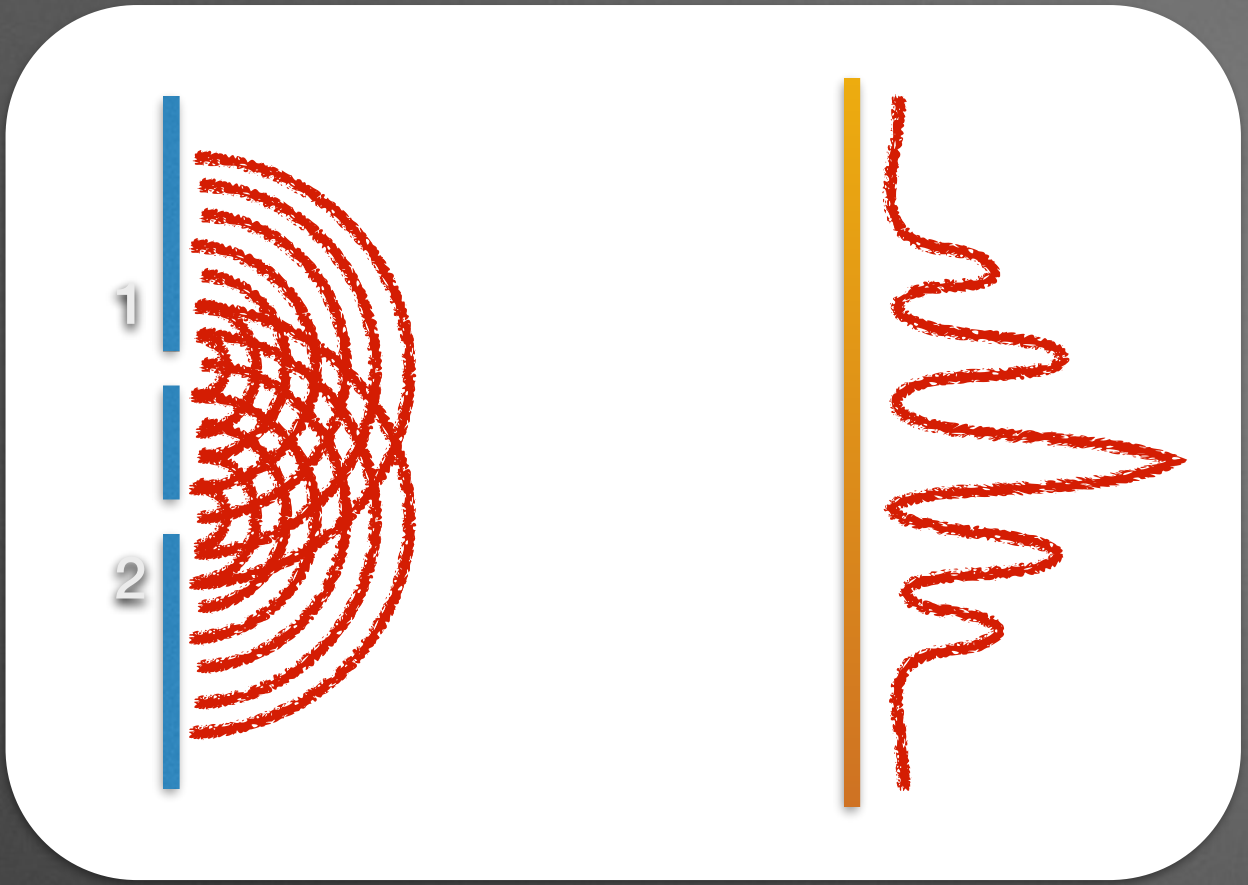
D : distinguishability (predictability)

D. M. Greenberger, A. Yassin, Phys. Lett. A 128, 391 (1988);

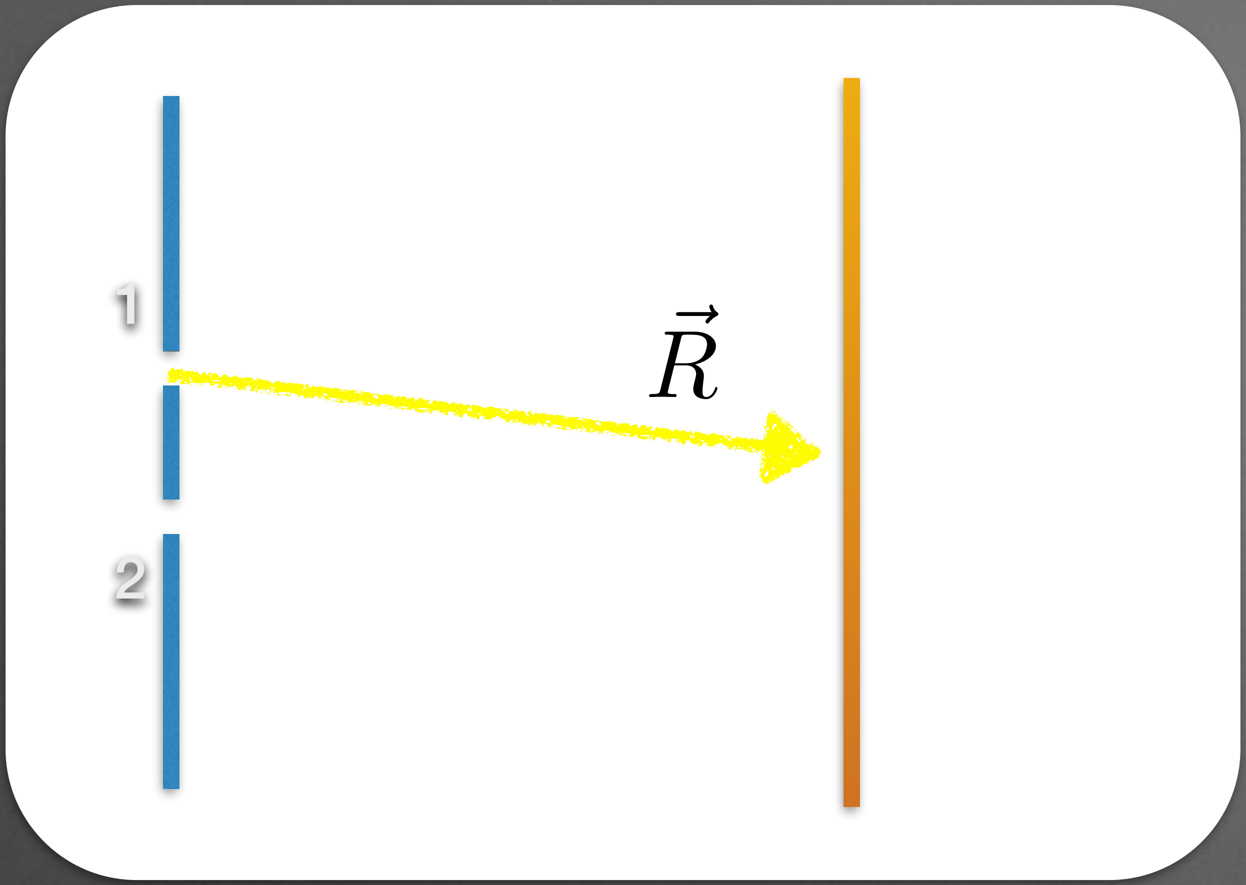
G. Jaeger, A Shimony, L. Vaidman, Phys. Rev. A 51, 54 (1995);

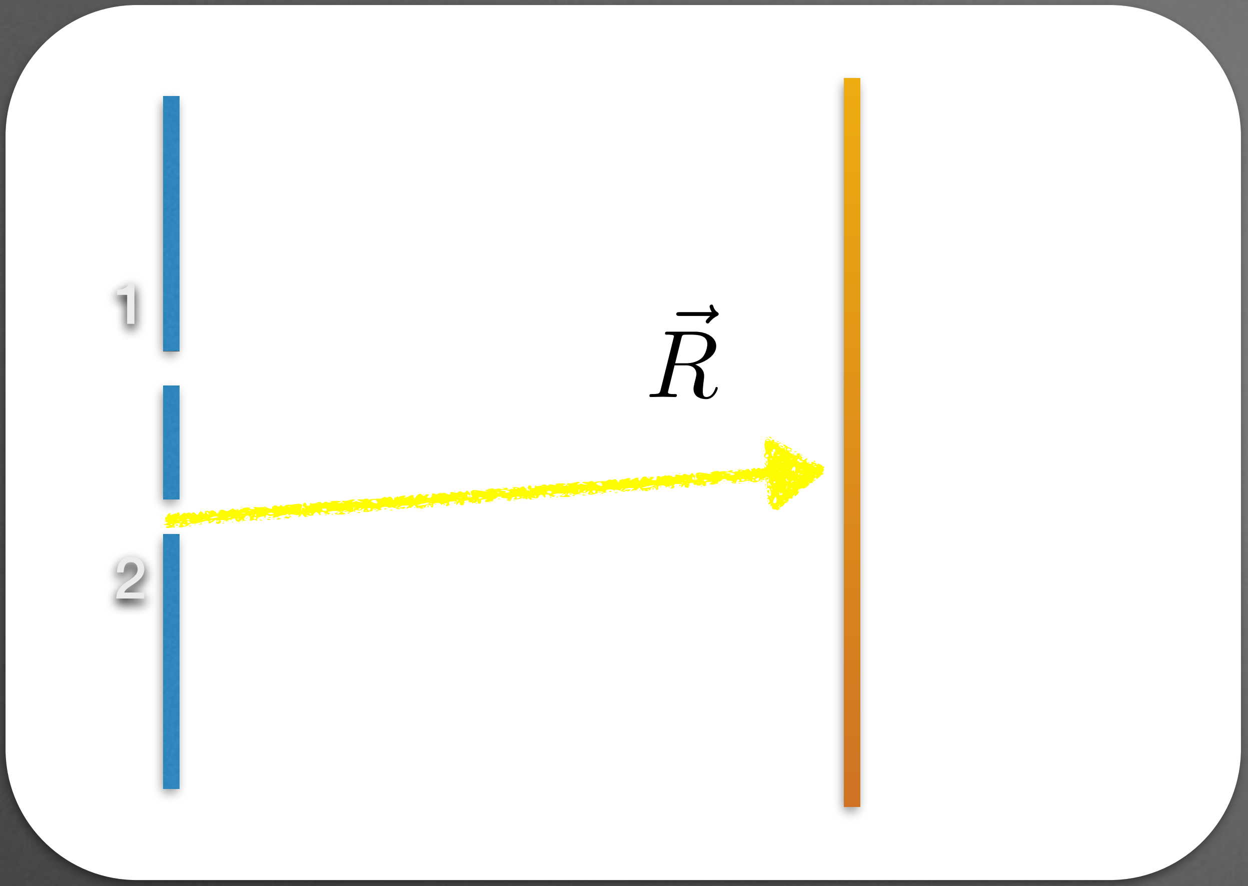
B.-G. Englert Phys. Rev. Lett. 77, 2154 (1996).


1
2



1
2


1
2
$\vec{R}$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2
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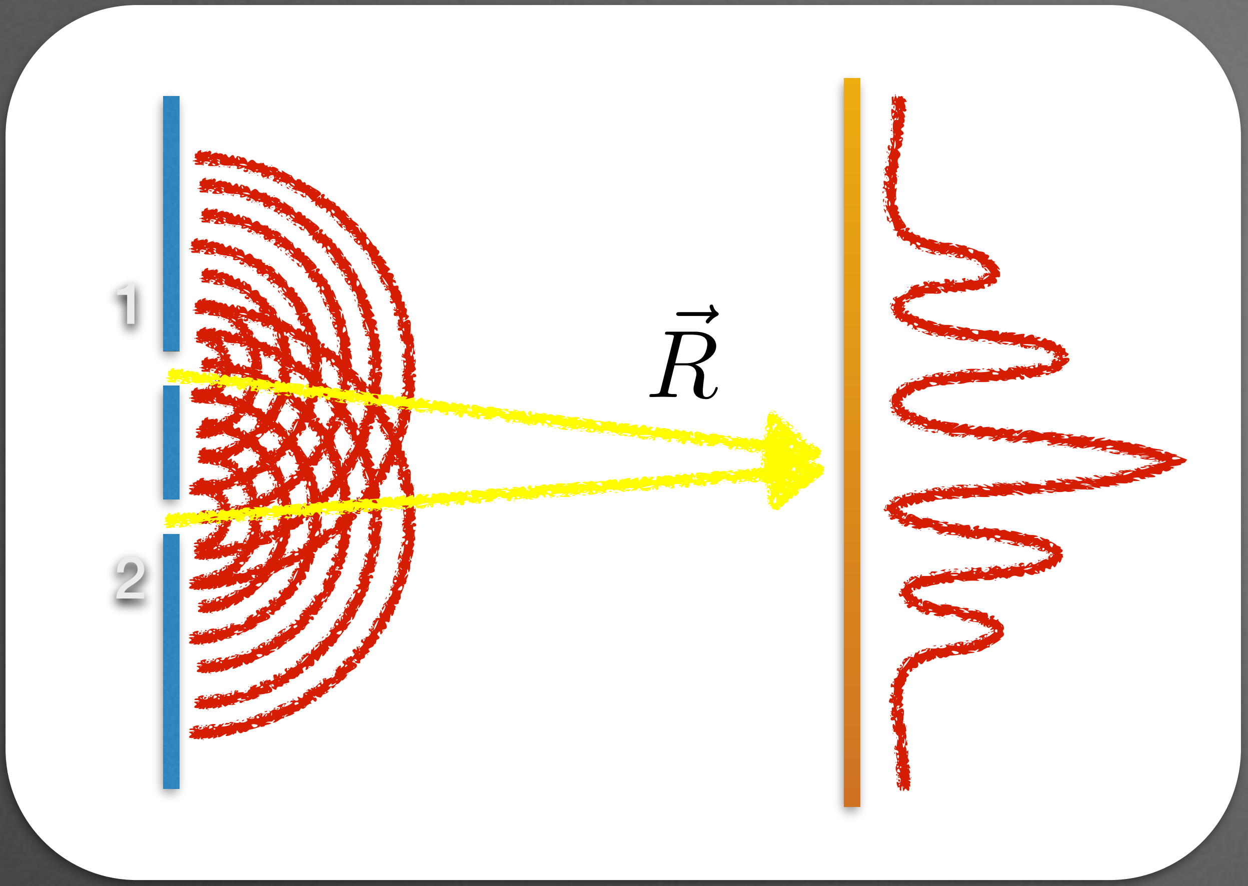
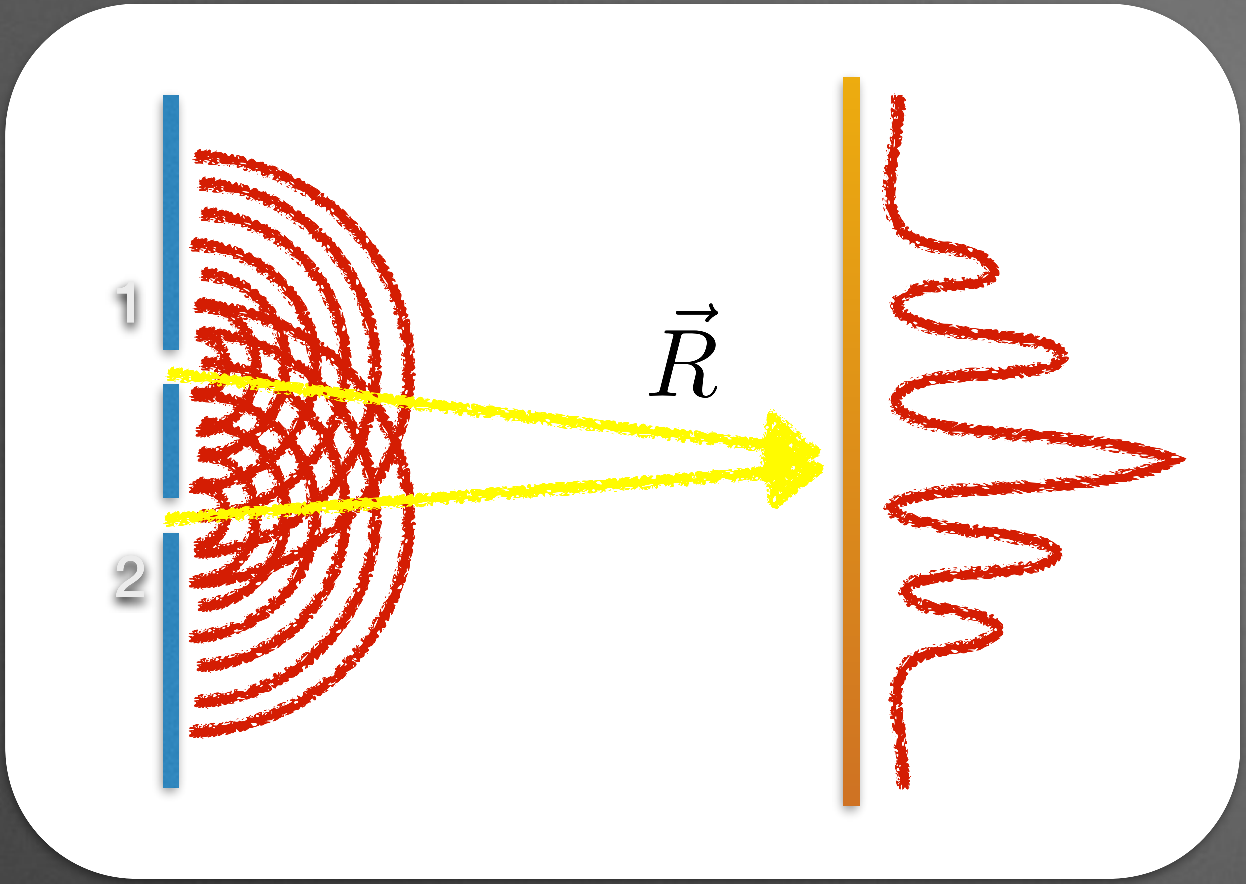


$$\psi(\vec{R}) = \psi_1(\vec{R}) + \psi_2(\vec{R})$$

$$\psi_i(\vec{R}) = C_i \psi_1(\vec{r} - \vec{r}_i)$$

$$\psi(\vec{r}) \propto \frac{e^{i\vec{p}\cdot\vec{r}/\hbar}}{|\vec{r}|}$$



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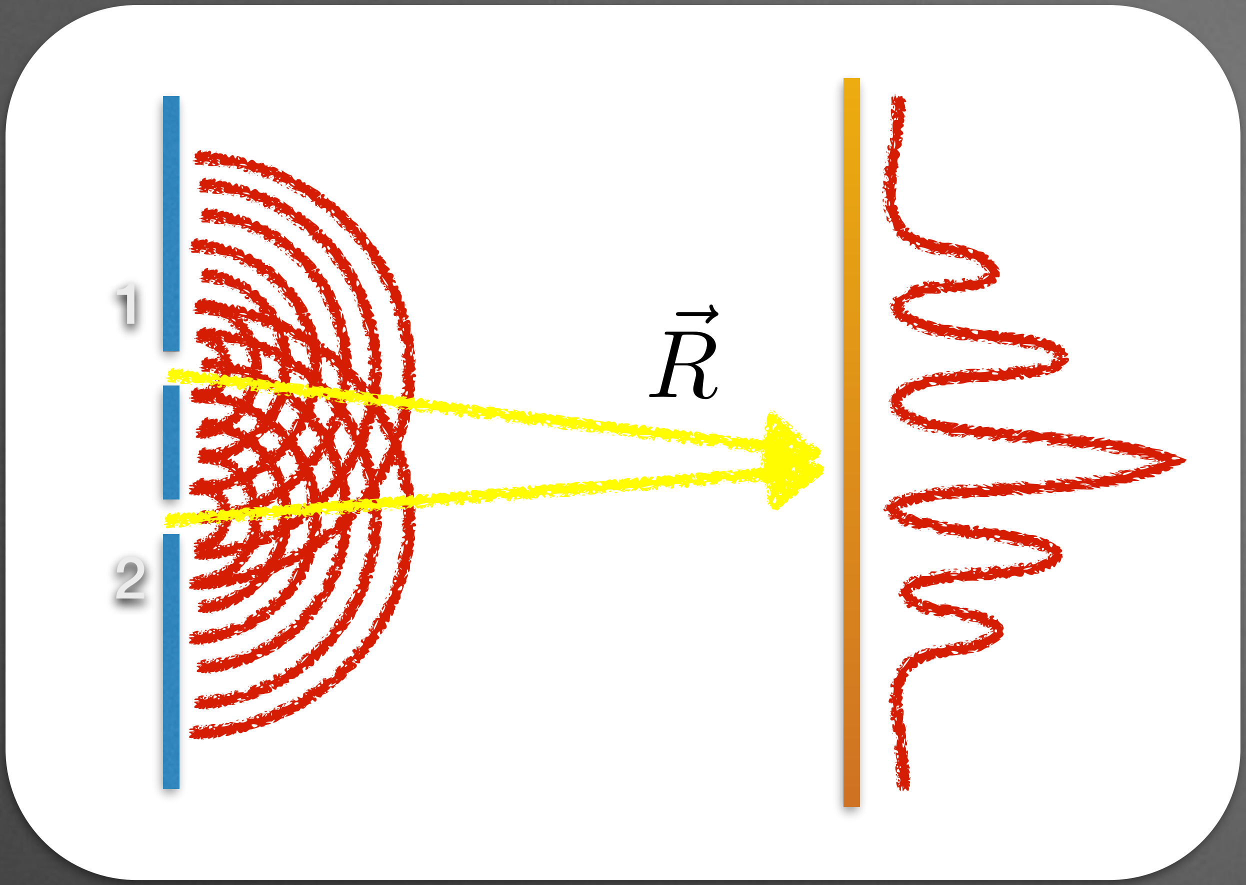
$$P_T = P_1 + P_2$$

$$P_1 = \frac{|C_1|^2}{|C_1|^2 + |C_2|^2} \quad P_2 = \frac{|C_2|^2}{|C_1|^2 + |C_2|^2}$$



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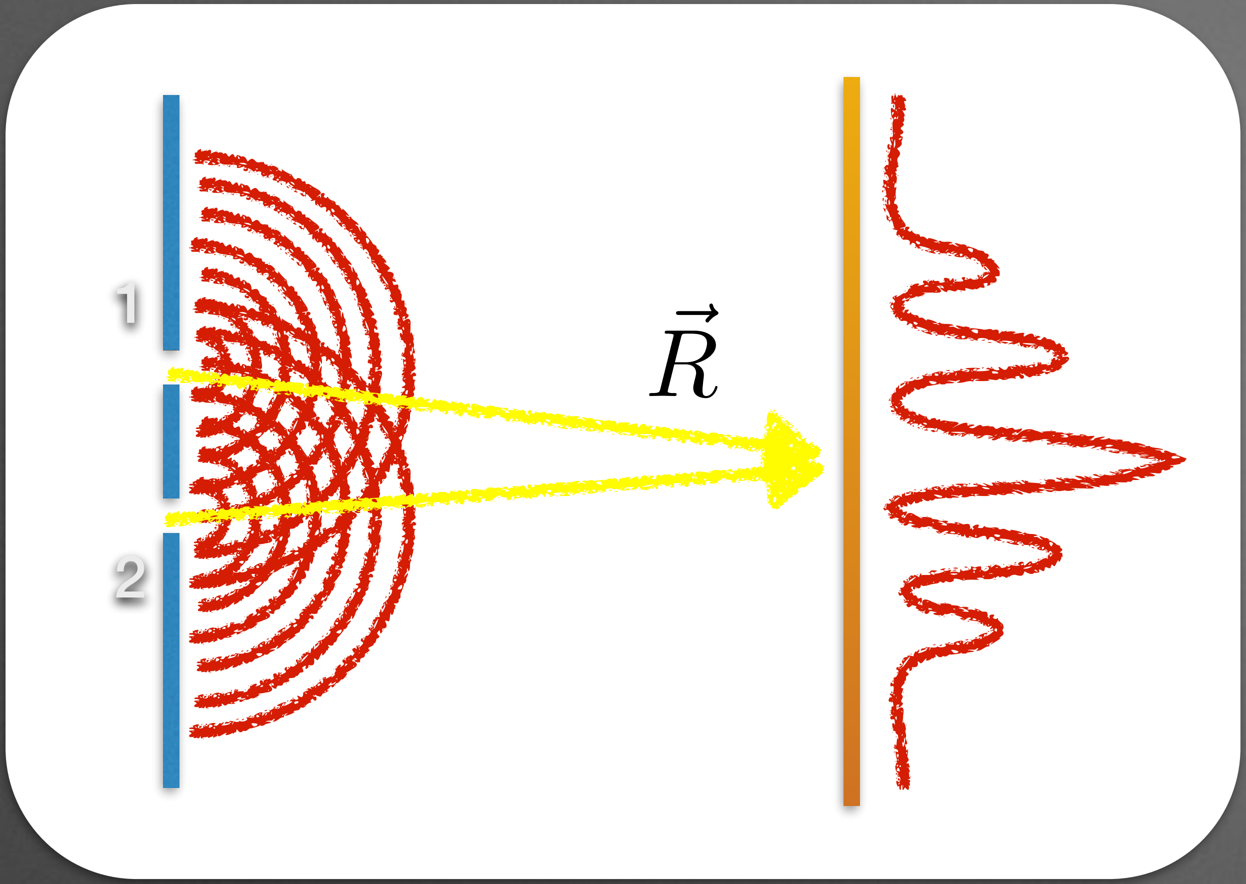
Distinguishability:



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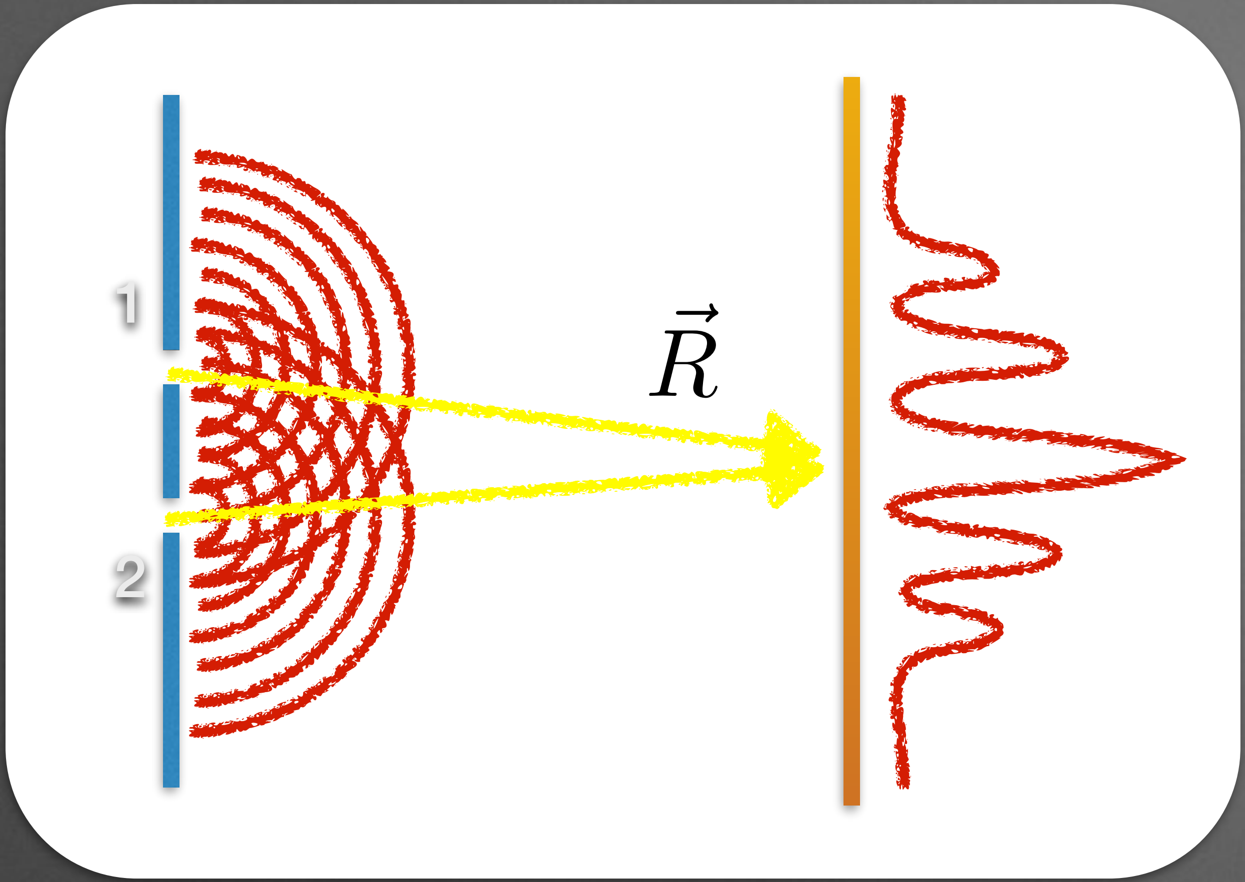

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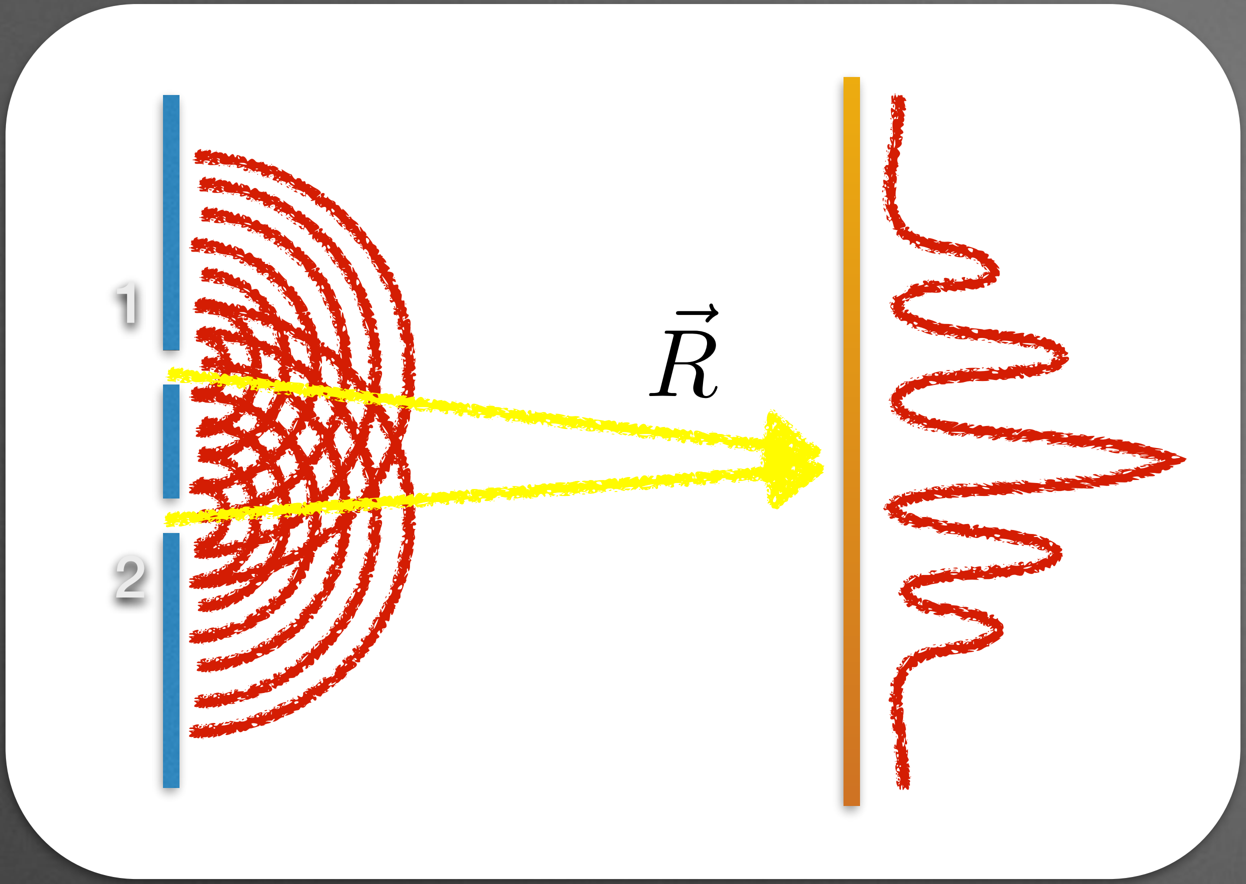
Visibility:

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = 2\frac{|C_1C_2^*|}{|C_1|^2 + |C_2|^2}$$

$$I_{max} \propto ||C_1| + |C_2||^2$$

$$I_{min} \propto ||C_1| - |C_2||^2$$



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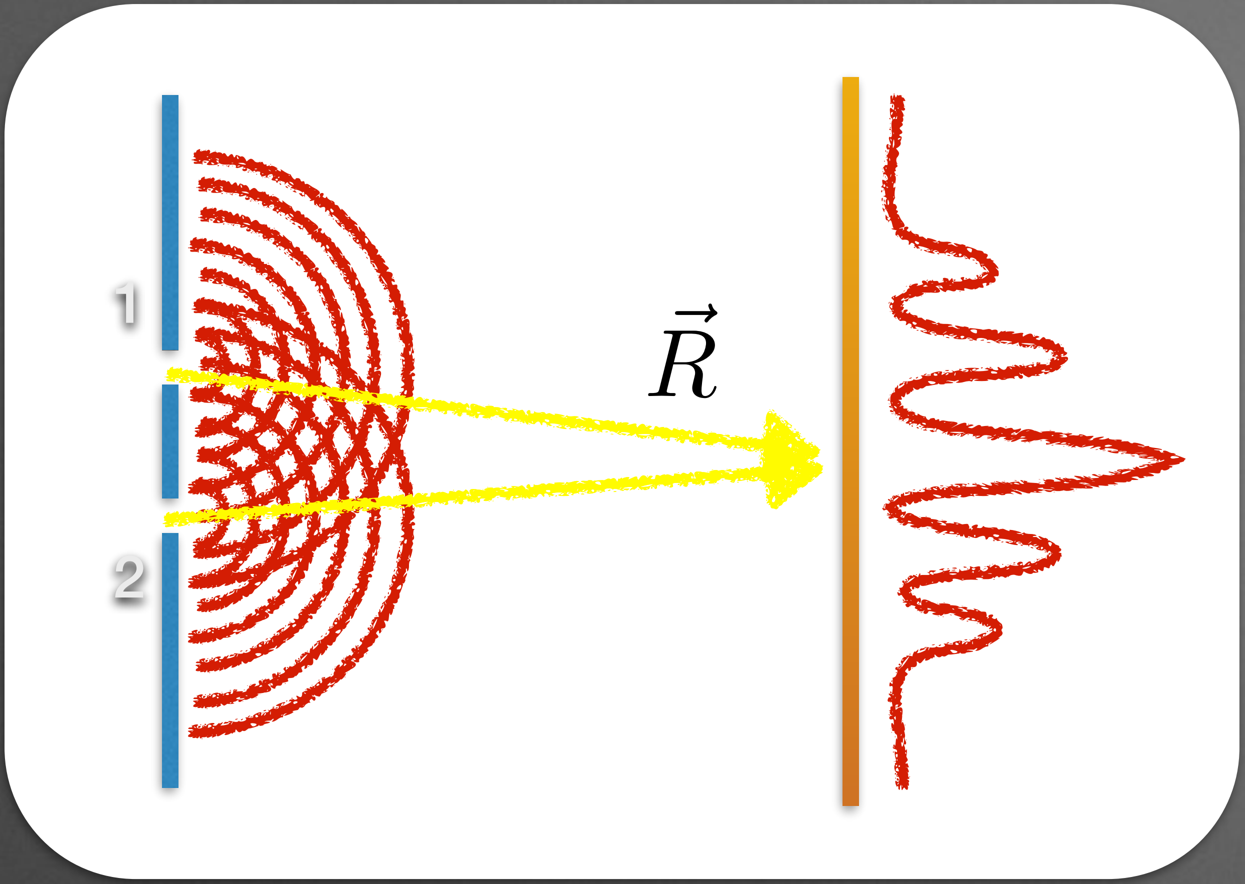
$$D^2 + V^2 = 1$$



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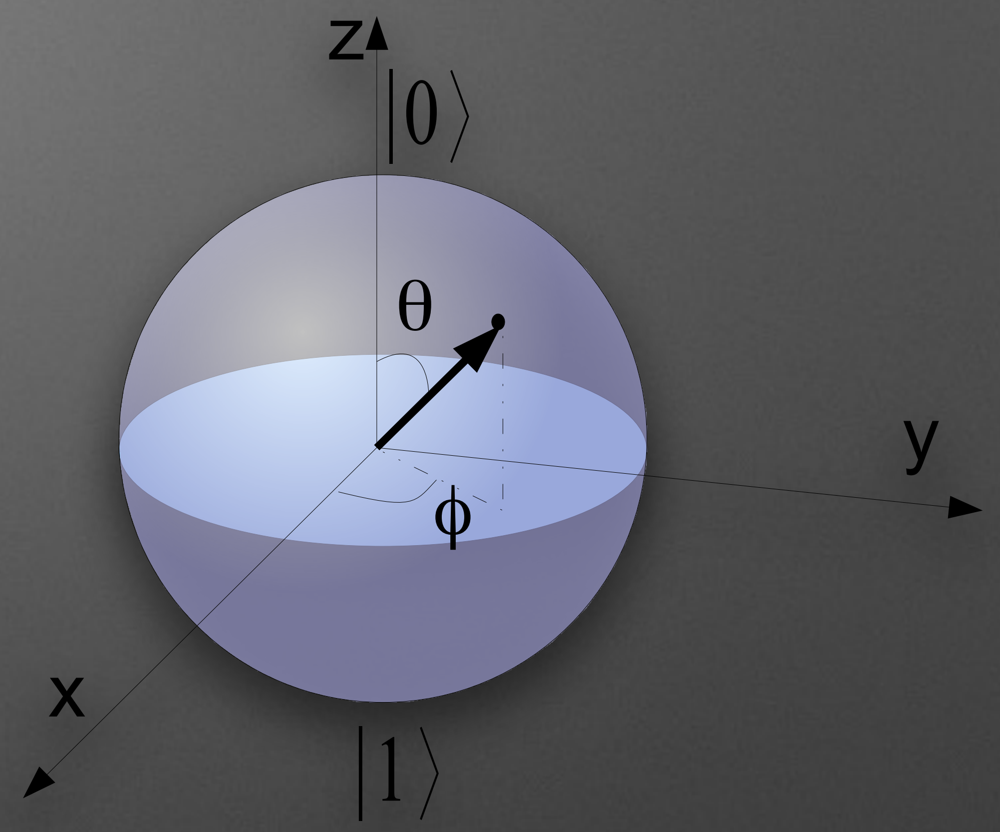
$$D^2 + V^2 \leq 1$$

Two-level systems

$$\rho = \frac{1}{2}(I + \vec{r} \, . \, \vec{\sigma})$$



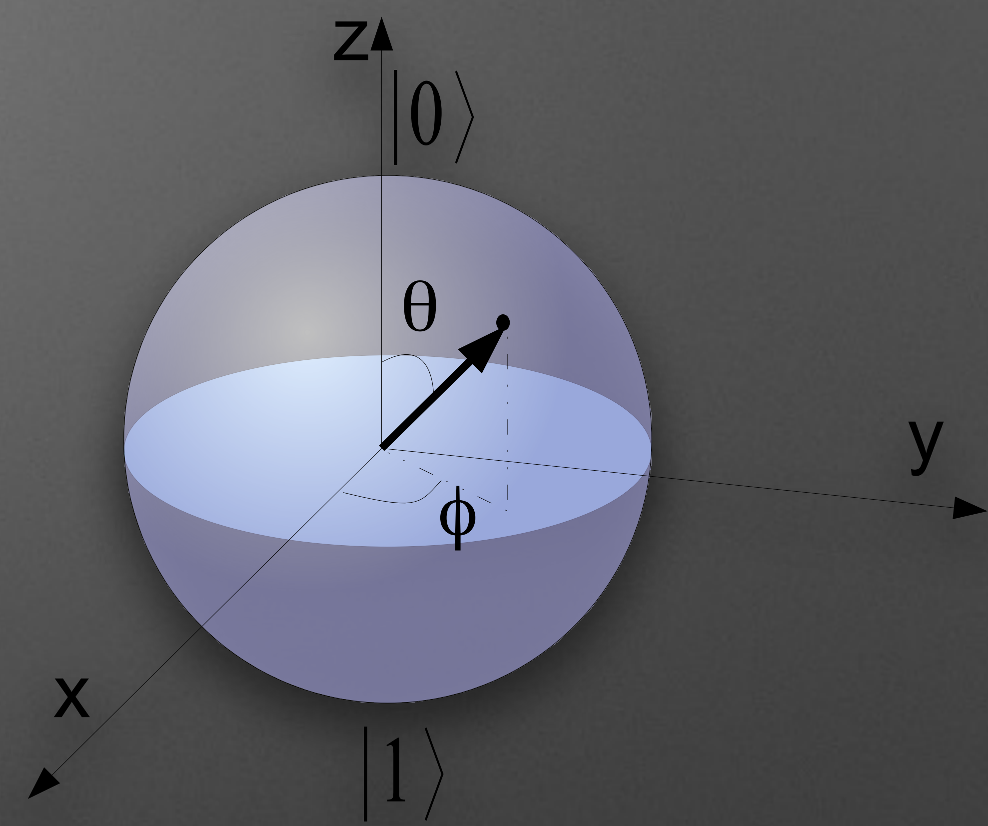
Measurements: {+1.-1}

$$p(\pm 1|\mathbf{r}, \sigma_{\hat{\mathbf{n}}}) = \frac{1}{2}(1 \pm \mathbf{r} \cdot \hat{\mathbf{n}})$$

Mean value in direction $\hat{\mathbf{n}}$:

$$\langle \sigma_{\hat{\mathbf{n}}} \rangle = (1)p(+1|\mathbf{r}, \sigma_{\hat{\mathbf{n}}}) + (-1)p(-1|\mathbf{r}, \sigma_{\hat{\mathbf{n}}}) = \mathbf{r} \cdot \hat{\mathbf{n}}$$

Complementarity for a 2-level system



$$r = ||\mathbf{r}|| \leq 1$$

$$\langle\sigma_{\hat{\mathbf{n}}_1}\rangle^2 + \langle\sigma_{\hat{\mathbf{n}}_2}\rangle^2 + \langle\sigma_{\hat{\mathbf{n}}_3}\rangle^2 \leq 1$$

$$p(\pm 1|\mathbf{r}, \sigma_{\hat{\mathbf{n}}}) = \frac{1}{2}(1 \pm \mathbf{r}\cdot\hat{\mathbf{n}})$$

$$= \frac{1}{2}(1 \pm r\hat{\mathbf{r}}\cdot\hat{\mathbf{n}}) = 1 \quad \rightarrow \quad r = (\hat{\mathbf{r}}\cdot\hat{\mathbf{n}})^{-1}$$

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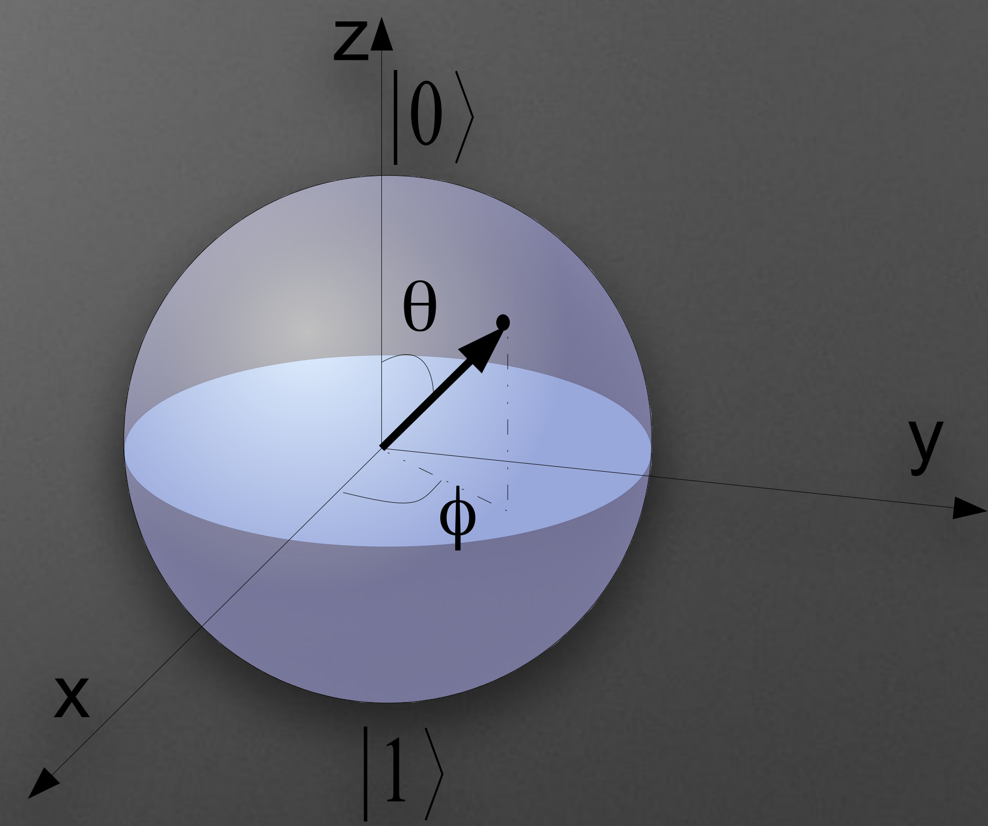

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$$r > 1?$$

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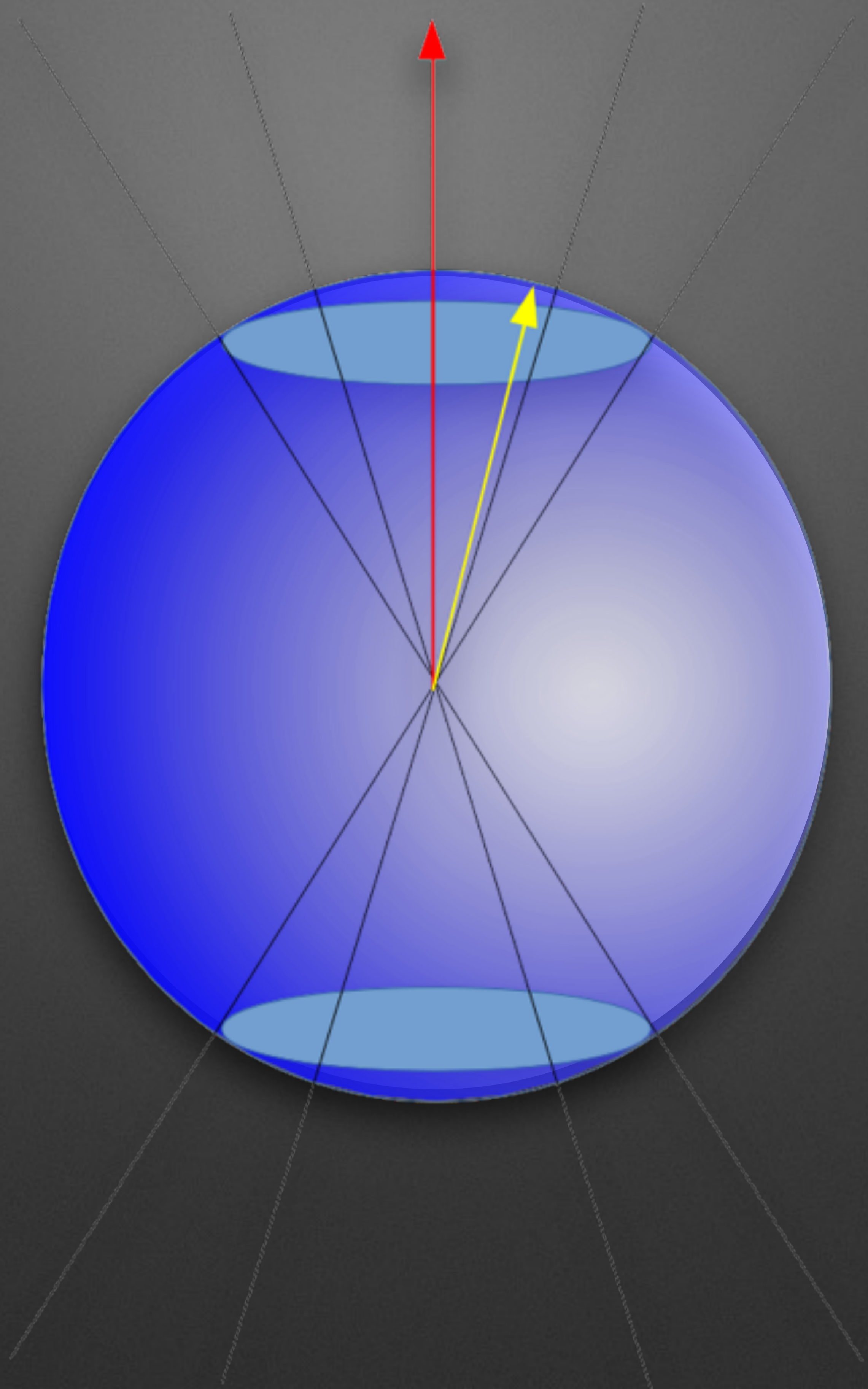


Operational Theory

Mathematically models a physical experiment in terms of primitive notions: preparations, measurements, outcomes and systems.

$$\{\mathcal{P}, \mathcal{M}, \mathcal{K}, p(k|P, M)\}$$

$\mathcal{P}:$ *set of mutuallly exclusive preparations*

$\mathcal{M}:$ *set of mutually exclusive measurements*

$\mathcal{K}:$ *set of mutually exclusive (and exhaustive) outcomes*

EX: QM is an OT: $\left\{\rho, O, p_k = Tr\{E_k^\dagger E_k \rho\}\right\}$

Extension of Q. theory

Projector: $\Pi_{\hat{\mathbf{n}}} := (1/2)(I + \hat{\mathbf{n}} \cdot \sigma)$

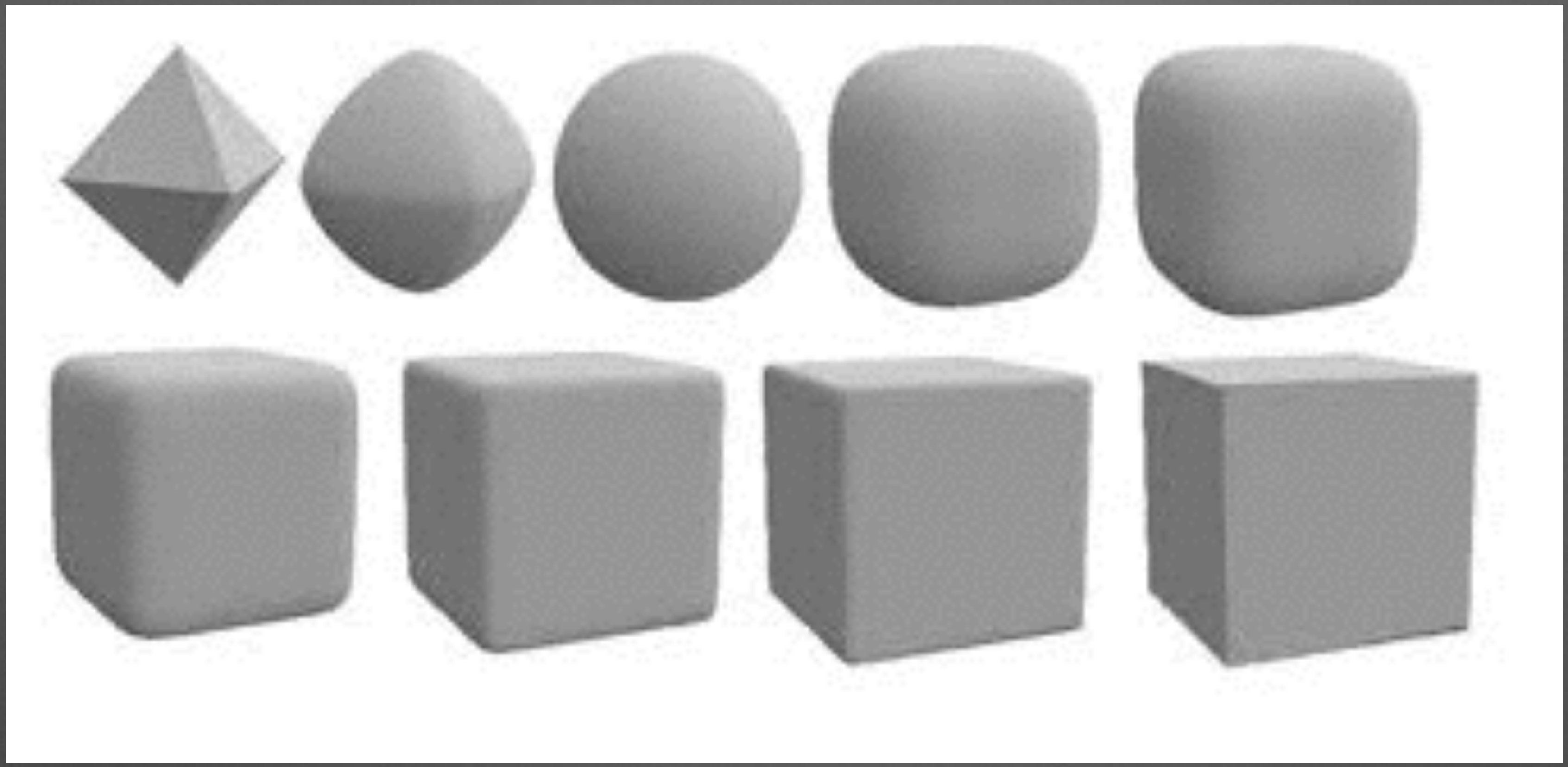
$$\Pi_{\hat{\mathbf{n}}} + \Pi_{-\hat{\mathbf{n}}} = I \qquad \sigma \cdot \hat{\mathbf{n}} = \Pi_{\hat{\mathbf{n}}} - \Pi_{-\hat{\mathbf{n}}}$$

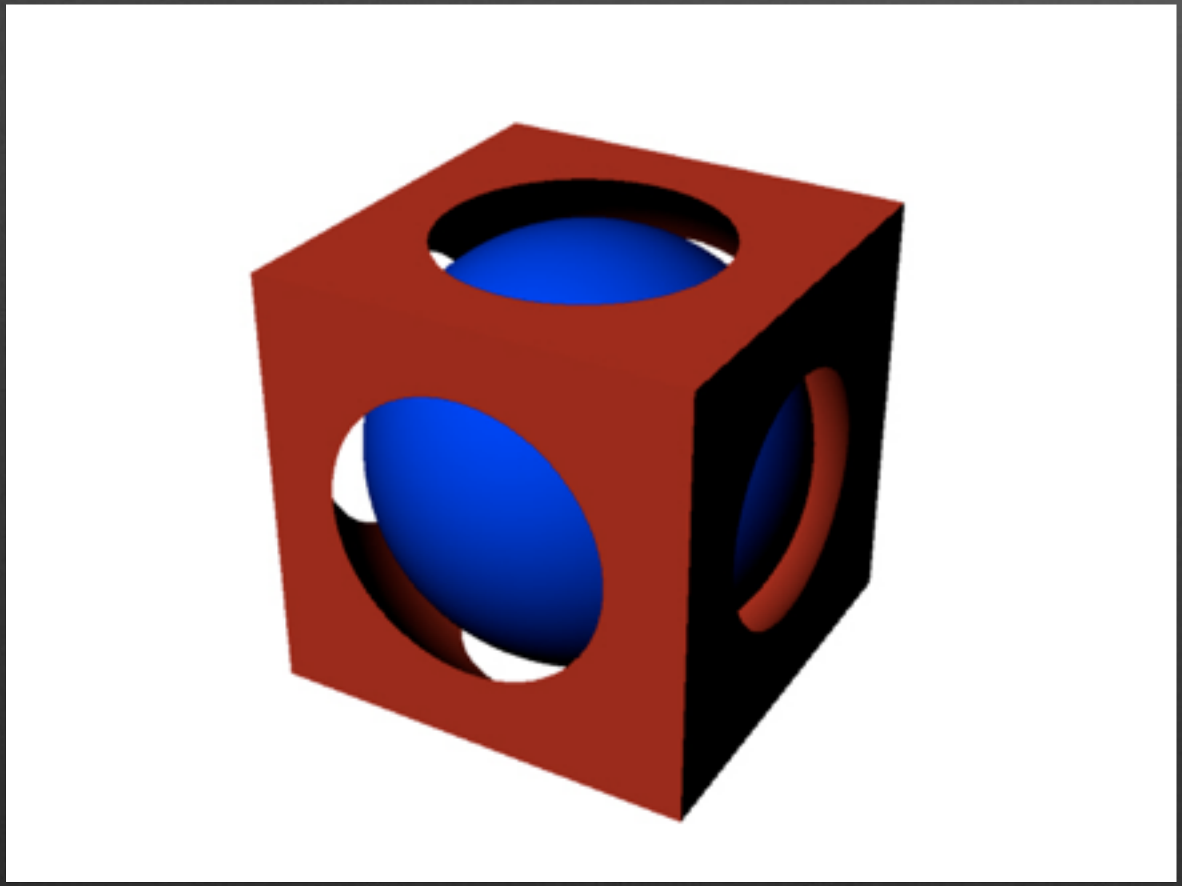
$$p(\pm 1|\mathbf{r}, \sigma_{\hat{\mathbf{n}}}) = Tr[\Pi_{\pm\hat{\mathbf{n}}}\rho(\mathbf{r})]$$

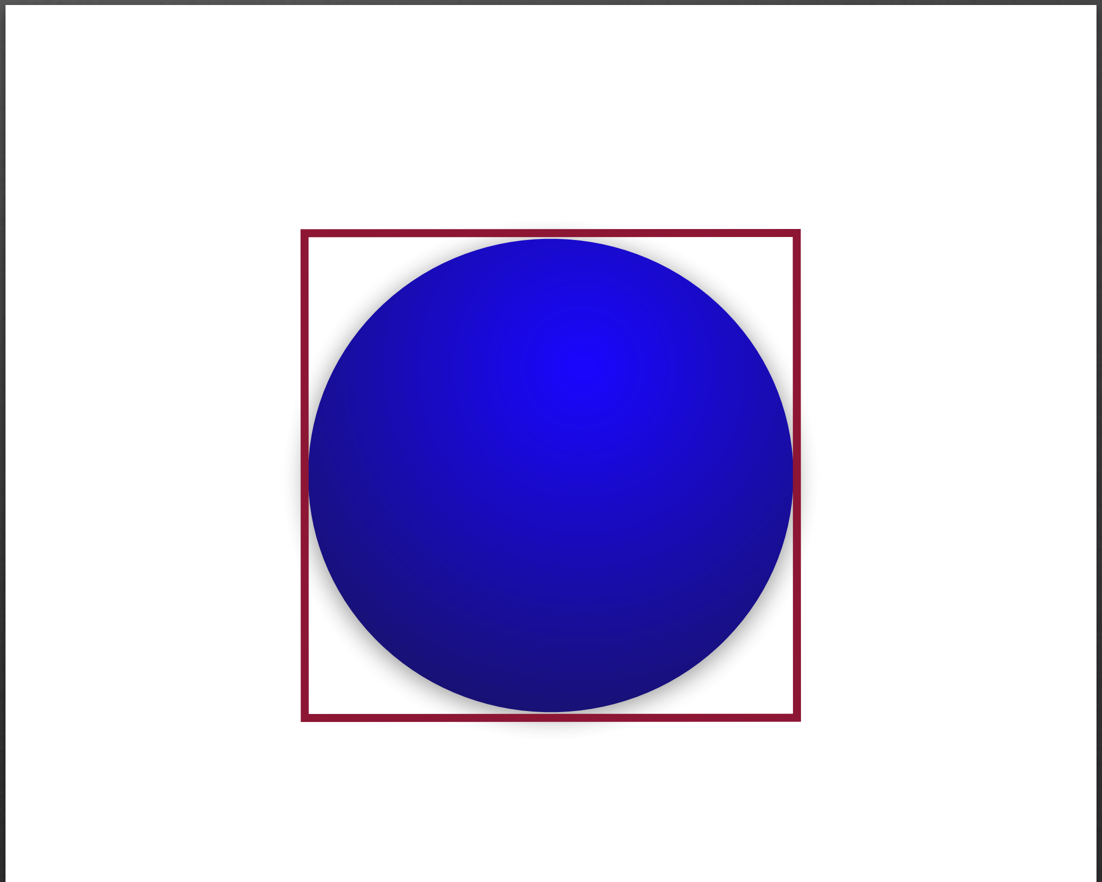
→ *Principle of Complementarity is equivalent to imposing the postulate of positive operators*

PC violation → *abdication of this postulate and a legitimate use of non-positive operators to represent preparations.*

similar extensions: nonlocal boxes, toy models of q. theory and ``boxworld''







Implications

Non-Local Boxes creation

Two observers measure dichotomic observables

$A_1 \ A_2$ (first observer)

$B_1 \ B_2$ (second observer)

$$\mathcal{B} := A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2$$

Clauser-Horne-Shimony-Holt (CHSH): $|\langle\mathcal{B}\rangle| \leq 2$

Maximal violation attainable by quantum states: $|\langle\mathcal{B}\rangle| \leq 2\sqrt{2}$
(Tsirelson's bound)

Popescu and Rohrlich non-signalling probability distributions: $|\langle\mathcal{B}\rangle| \leq 4$

Theorem:

For a two-level system, any preparation violating the principle of complementarity enables the deterministic generation of a bipartite preparation that violates Tsirelson's bound.

A preparation violates CP iff $r > 1$

$$\rho(\mathbf{r}) = \frac{1}{2}[(1+r)|\xi\rangle\langle\xi| + (1-r)|\xi^\perp\rangle\langle\xi^\perp|]$$ (eigenstates of ρ)

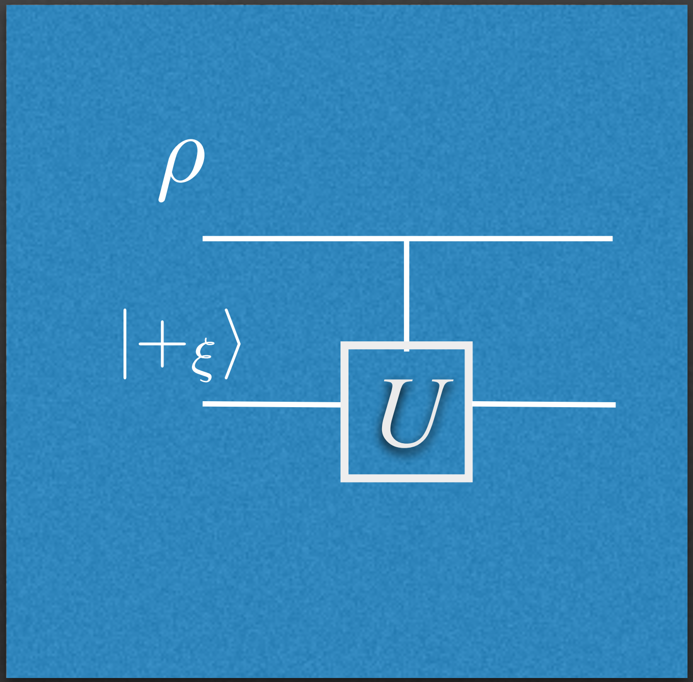
Defining: $|\pm_\xi\rangle := (1/\sqrt{2})(|\xi\rangle \pm |\xi^\perp\rangle)$

$$X_\xi := |\xi\rangle\langle\xi| - |\xi^\perp\rangle\langle\xi^\perp|$$

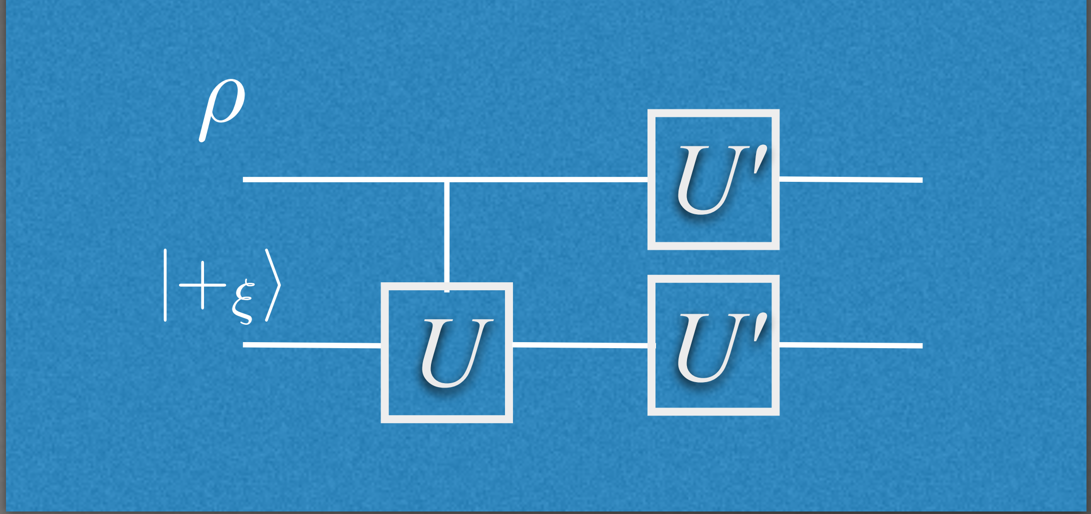
$$U = |+_\xi\rangle\langle+_\xi| \otimes I + |-_\xi\rangle\langle-_\xi| \otimes X_\xi$$

$$P = U[\rho(\mathbf{r}) \otimes |+_\xi\rangle\langle+_\xi|]U^\dagger$$
$$= \frac{1}{2}[(1+r)|\phi_+\rangle\langle\phi_+| + (1-r)|\phi_-\rangle\langle\phi_-|]$$

$$|\phi_\pm\rangle = (1/\sqrt{2})(|+_\xi +_\xi\rangle \pm |-_\xi -_\xi\rangle)$$



$$U' = |0\rangle\langle +_\xi| + |1\rangle\langle -_\xi|$$



$$U' \otimes U' \quad \Rightarrow \quad P' = \frac{1}{2}[(1+r)|\phi'_+\rangle\langle\phi'_+| + (1-r)|\phi'_-\rangle\langle\phi'_-|]$$

$$|\phi'_\pm\rangle = (1/\sqrt{2})(|00\rangle \pm |11\rangle)$$

$$r \leq \sqrt{2}$$

$$A_1 = (\sigma_x + \sigma_y)/\sqrt{2},\ A_2 = (\sigma_x - \sigma_y)/\sqrt{2},$$
$$B_1 = \sigma_x,\ B_2 = -\sigma_y$$

$$\Rightarrow \quad \langle \mathcal{B} \rangle = Tr(\mathcal{B}P') = 2\sqrt{2}r$$

violation of Tsirelson's bound

$$1 < r \leq \sqrt{2}$$

$r > \sqrt{2}$

$$A_1 = (\sigma_x + \sigma_y)/\sqrt{2},\ A_2 = (\sigma_x - \sigma_y)/\sqrt{2},$$

$$B_1 = (\frac{\sqrt{2}}{r})\sigma_x + (\frac{\sqrt{r^2-2}}{r})\sigma_y,\ B_2 = (\frac{\sqrt{r^2-2}}{r})\sigma_y - (\frac{\sqrt{2}}{r})\sigma_y$$

$$\langle \mathcal{B} \rangle = Tr(\mathcal{B}P') = 4, \forall r$$

Does not violate non-signaling

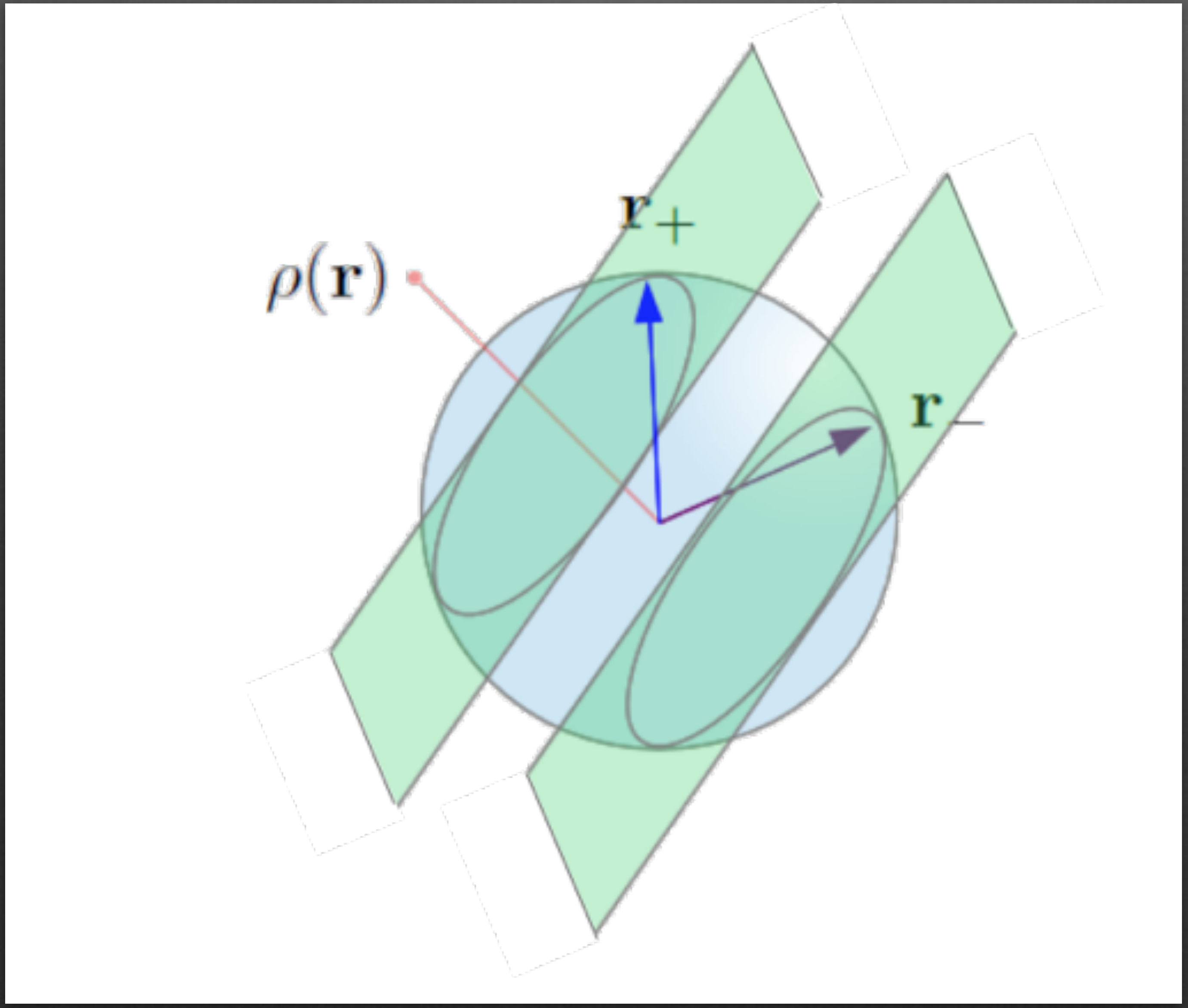
Theorem:

For a two-level system, any preparation violating the principle of complementarity enables the deterministic generation of a bipartite preparation that violates Tsirelson's bound.

Cloning

Theorem:

Two preparations with Bloch vectors $\mathbf{r}$ and $\mathbf{r}'$ are jointly-clonable only if $\mathbf{r} \cdot \mathbf{r}' = \pm 1$



Let two preparations with Bloch vectors $\mathbf{r}$ and $\mathbf{r}'$ be joint-clonable.

There exists an unitary U such that

$$U(\rho \otimes |e_0\rangle\langle e_0|)U^\dagger = \rho \otimes \rho$$
$$U(\rho' \otimes |e_0\rangle\langle e_0|)U^\dagger = \rho' \otimes \rho$$

We then have

$$Tr[(\rho \otimes \rho)(\rho' \otimes \rho')] = Tr[U(\rho \otimes |e_0\rangle\langle e_0|)U^\dagger U(\rho' \otimes |e_0\rangle\langle e_0|)U^\dagger]$$

$$= Tr(\rho\rho') \ = [Tr(\rho\rho')]^2$$

Since $Tr(A \otimes B) = Tr(A)Tr(B)$

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$\Leftrightarrow$ $Tr(\rho\rho') = 0$ or $Tr(\rho\rho') = 1$ $\Rightarrow$ $\mathbf{r} \cdot \mathbf{r}' = \pm 1$

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$\Longleftrightarrow$ $Tr(\rho\rho') = 0$ or $Tr(\rho\rho') = 1$ $\Rightarrow$ $\boxed{\mathbf{r} \cdot \mathbf{r}' = \pm 1}$

Conclusions

- simple and operational formulation of the principle of complementarity in terms of the empirical unpredictability of fully incompatible measurements.

For two-level systems violation of complementarity is equivalent to:

(i) Creation of nonlocal preparations that violate Tsirelson's bound without violating non-signalling, with deterministic operations;

(ii) Distinguishability and cloning of a plethora of states via deterministic protocols.

- Extension for higher-dimensional systems
- Complementarity is a major physical principle and we believe it is, if not the main reason, one strong argument ruling out superquantum phenomema in nature.

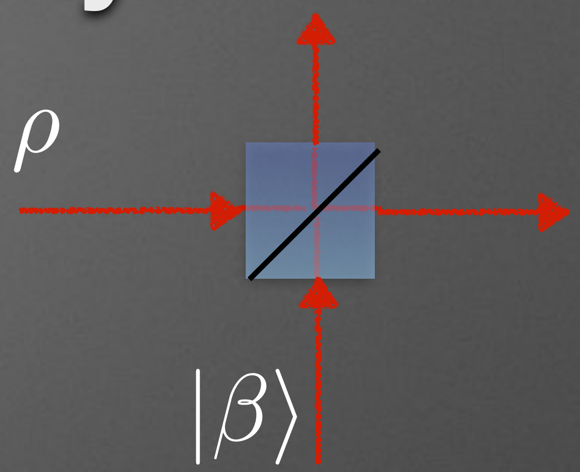
Non-classicality

Non-classicality

$$\rho = \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|$$

Non-classicality

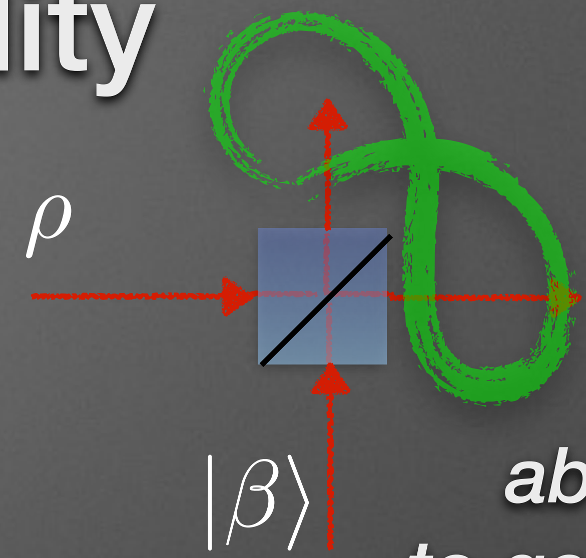
$$\rho = \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|$$



Non-classicality

$$\rho = \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|$$

non-classical

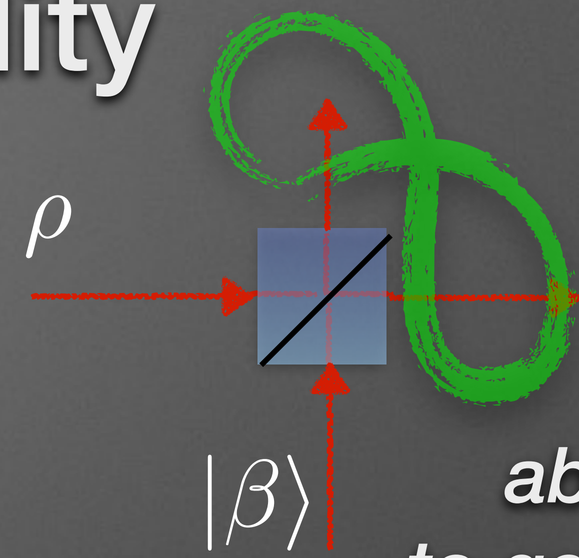


ability to generate entanglement

Non-classicality

$$\rho = \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|$$

non-classical



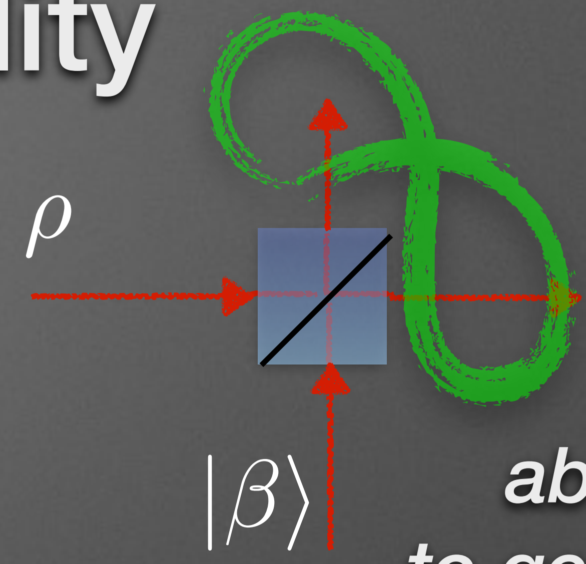
ability to generate entanglement

$$\rho = \sum_{k=1}^{2} \sum_{i=x,y,z} P_i^{(k)} |\sigma_i^{(k)}\rangle\langle\sigma_i^{(k)}|$$

Non-classicality

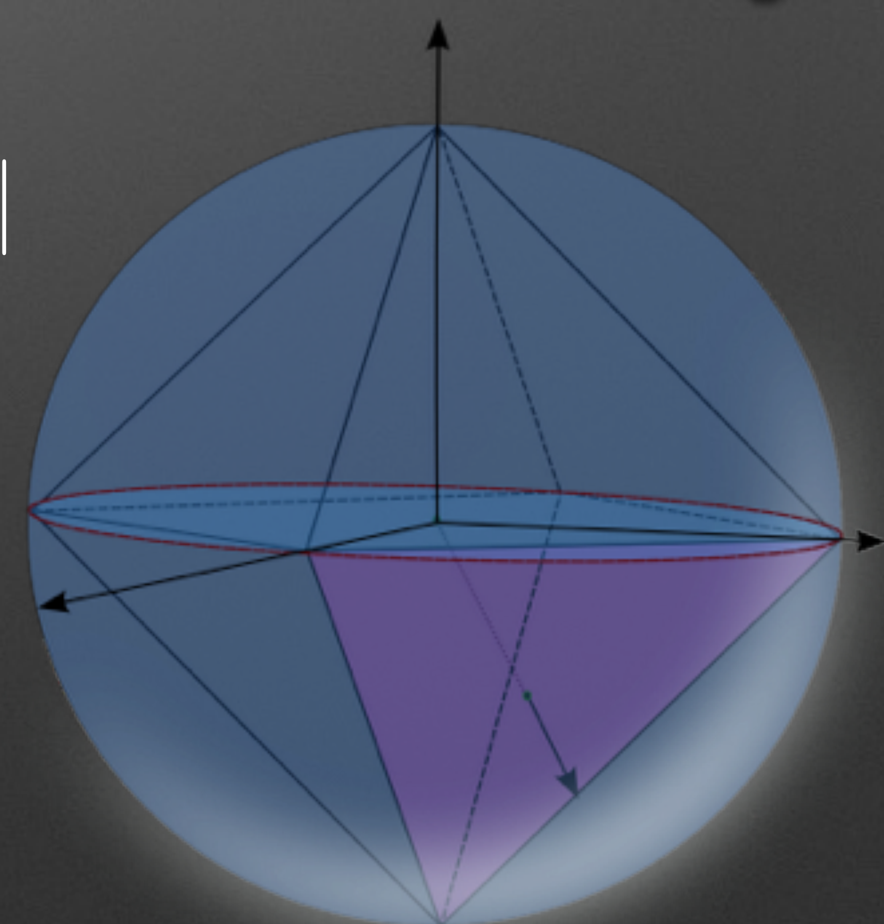
$$\rho = \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|$$

non-classical



ability to generate entanglement

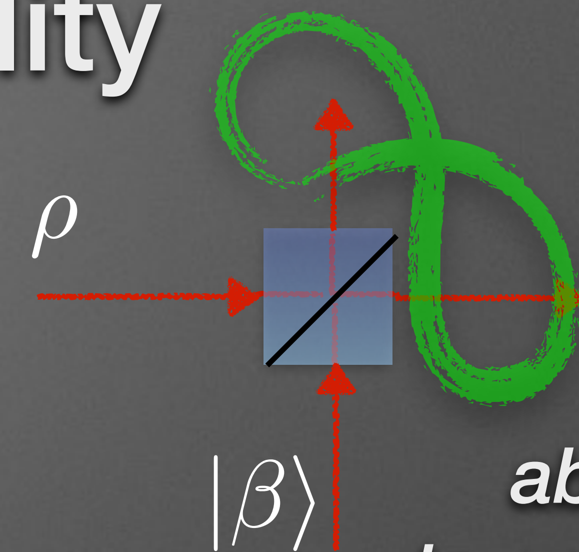
$$\rho = \sum_{k=1}^{2} \sum_{i=x,y,z} P_i^{(k)} |\sigma_i^{(k)}\rangle\langle\sigma_i^{(k)}|$$



Non-classicality

$$\rho = \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|$$

non-classical

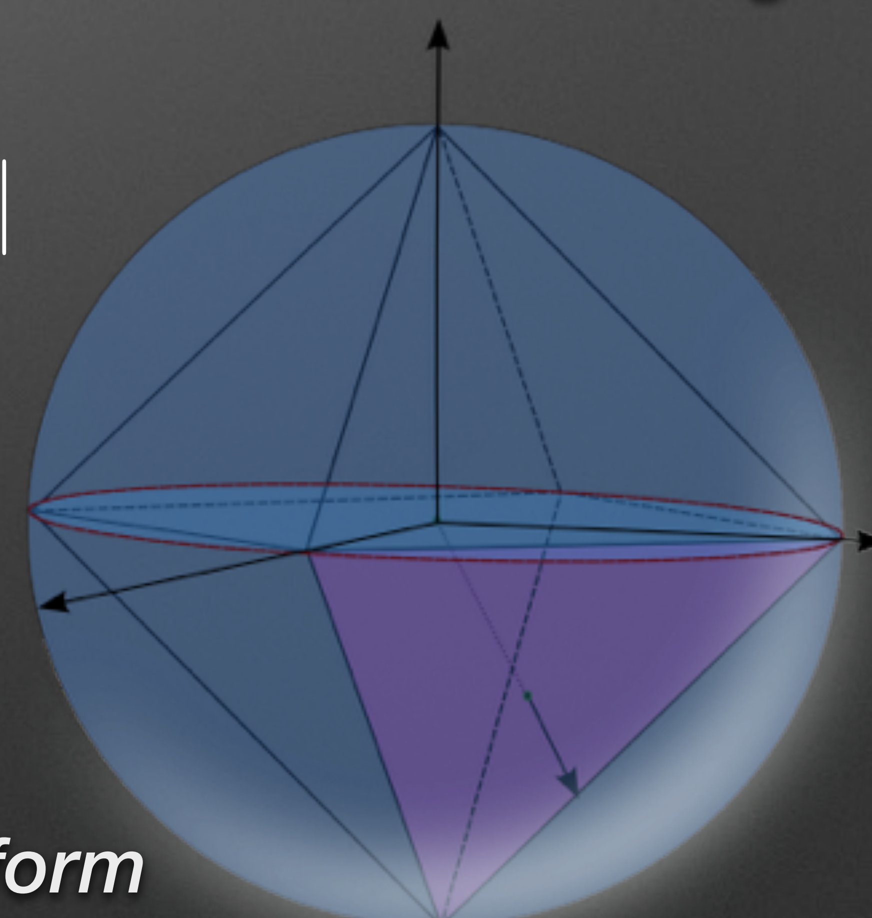


ability
to generate
entanglement

$$\rho = \sum_{k=1}^{2} \sum_{i=x,y,z} P_i^{(k)} |\sigma_i^{(k)}\rangle\langle\sigma_i^{(k)}|$$

non-classical

ability
of quantum computation to outperform
classical computation



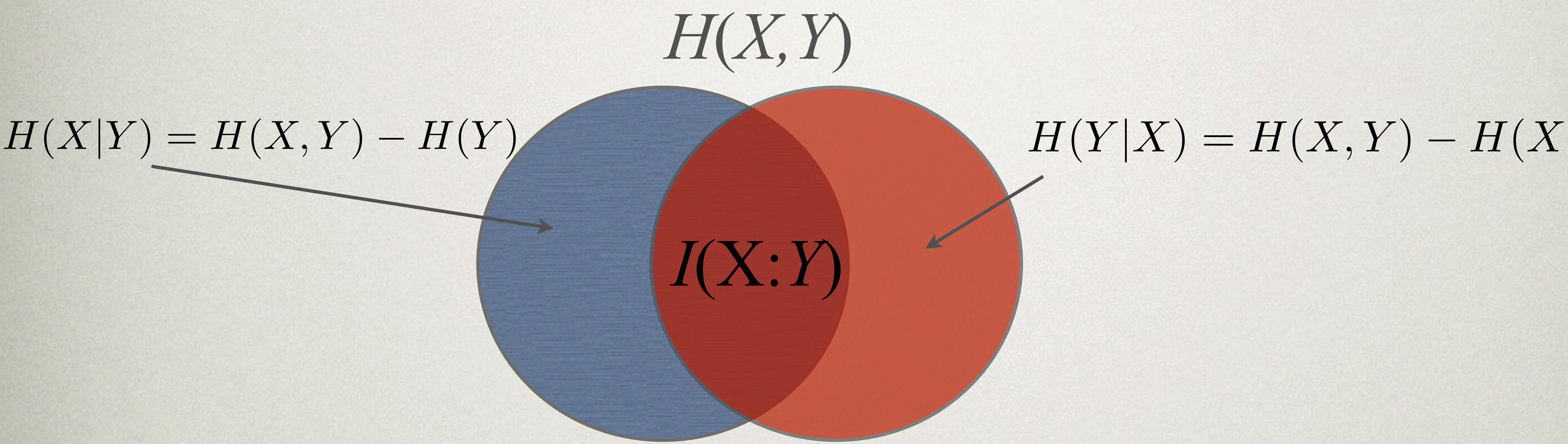
Nonzero Classical Discord

Vlad Gheorghiu, Marcos C. de Oliveira, and Barry C. Sanders, PRL 115, 030403 (2015)

Outline

- Mutual information
- Quantum discord
- Imperfect measurement
- Classical discord

Mutual Information (Classical)



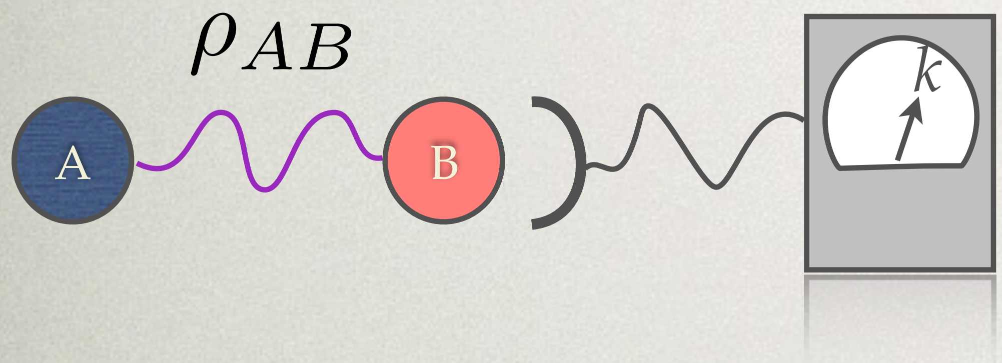
$$H(X) = -\sum_j p(x_j)\log_2 p(x_j) \qquad H(Y) = -\sum_k p(y_k)\log_2 p(y_k)$$

$$H(X,Y) = -\sum_{j,k} p(x_j, y_k)\log_2 p(x_j, y_k)$$

$$I(X:Y) \equiv H(X:Y) = H(X) + H(Y) - H(X,Y)$$

Post and pre-selected states

Measurement on B with outcome k



$$\rho_{AB}^k = \frac{\Pi_k \rho_{AB} \Pi_k}{p_k} \qquad \Pi_k = |\phi_k\rangle\langle\phi_k|_B$$

$$p_k = Tr\{\Pi_k \rho_{AB}\}$$

$$\rho_A^k = \frac{Tr_B\{\Pi_k \rho_{AB} \Pi_k\}}{p_k}$$

Post-selected state

$$\rho_A = \sum_k p_k \rho_A^k = \sum_k Tr_B\{\Pi_k \rho_{AB} \Pi_k\}$$

Pre-selected state

(QUANTUM) MUTUAL INFORMATION

$$S(A:B) = S_A - S(A|B)$$

$$J_{A|B} = S(\rho_A) - \sum_j p_j S(\rho_A^j)$$

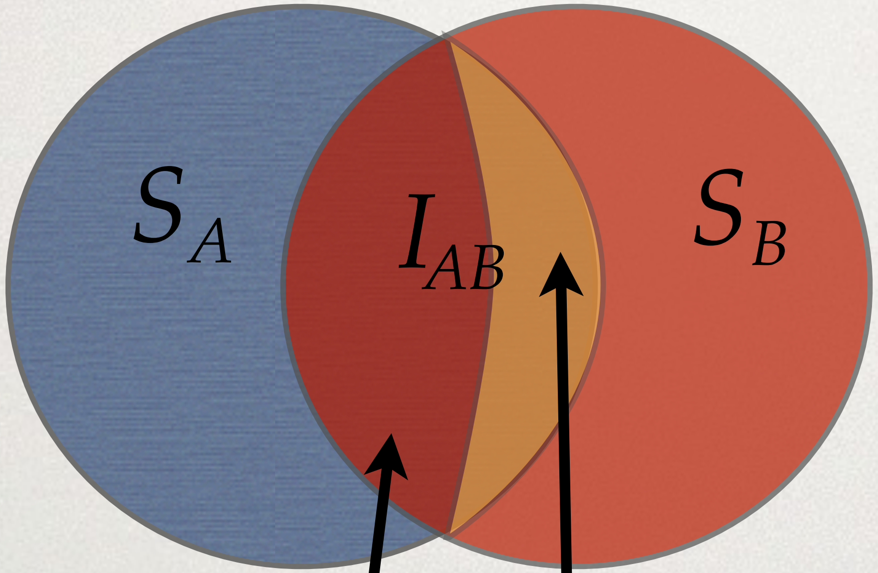
$$p_j = \mathrm{Tr}_{AB}\left\{\Pi_j^B \rho_{AB} \Pi_j^B\right\}, \quad \rho_A^j = \frac{\mathrm{Tr}_B\{\Pi_j^B \rho_{AB} \Pi_j^B\}}{p_j}$$

$$J_{AB}^{\leftarrow} = \max_{\{\Pi_j^B\}} \left[S(\rho_A) - \sum_j p_j S(\rho_A^j) \right]$$

Classical Correlation

L. Henderson and V. Vedral, J. Phys. A 34, 6899 (2001)

Local accessible and Inaccessible information



$$I_{AB} = S_A + S_B - S_{AB}$$

$$J^{\leftarrow}_{AB} = \max_{\{\Pi_k\}} \left[S_A - \sum_k p_k S_{A|k} \right],$$

$$\delta^{\leftarrow}_{AB} = I_{AB} - J^{\leftarrow}_{AB}$$ (Quantum Discord)

H. Ollivier and W. H. Zurek, Phys. Rev. Lett. 88, 017901 (2001)

$$E_{AB} = 0 \Leftrightarrow \rho_{AB} = \sum_i p_i \rho_i^A \otimes \rho_i^B$$

$$\delta_{AB}^{\leftarrow} = 0 \Leftrightarrow \rho_{AB} = \sum_i p_i \rho_i^A \otimes \Pi_i^B$$

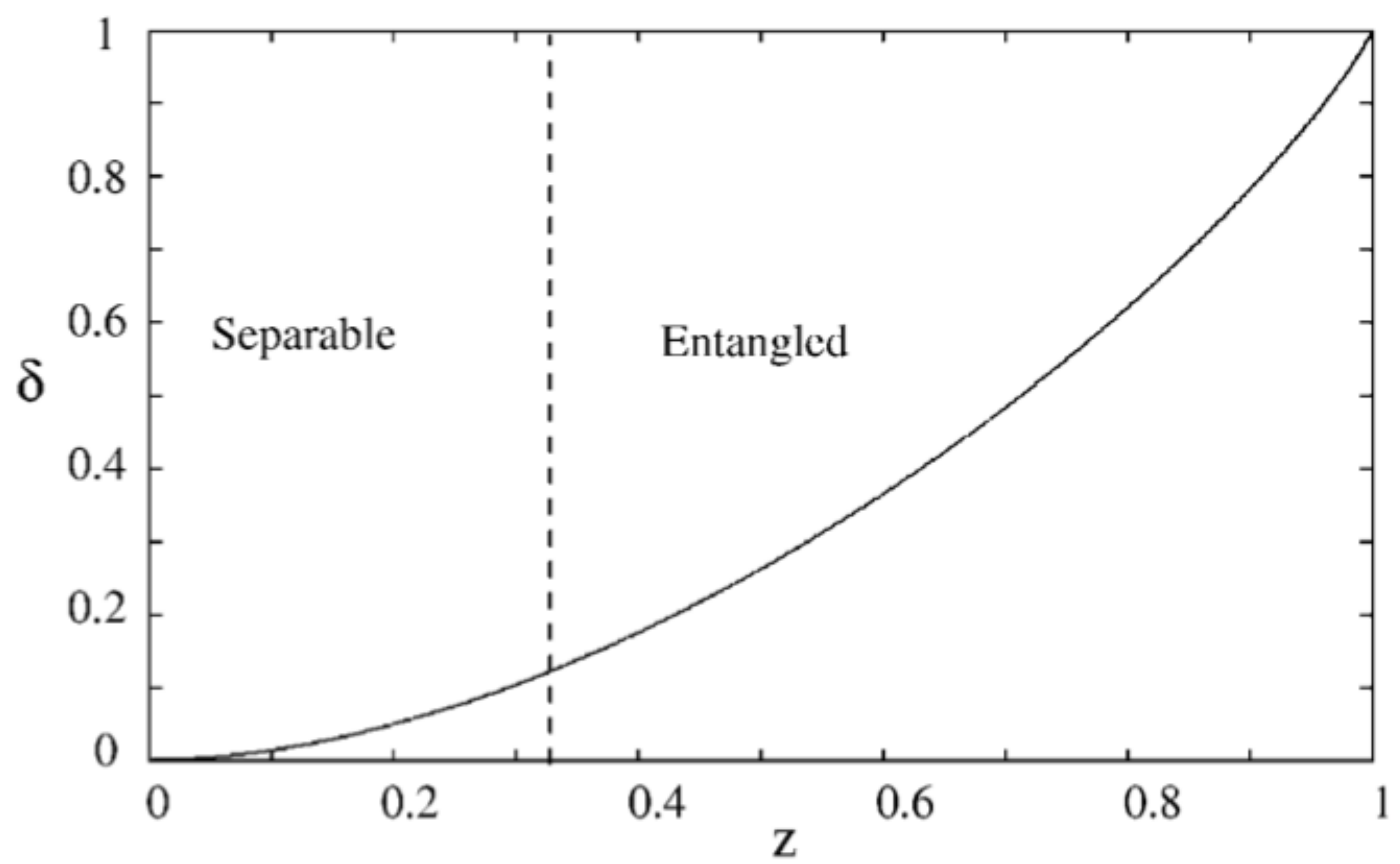


FIG. 2. Value of the discord for Werner states $\frac{1-z}{4}\mathbf{1} + z|\psi\rangle\langle\psi|$, with $|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. Discord does not depend on the basis of measurement in this case because both $\mathbf{1}$ and $|\psi\rangle$ are invariant under local rotations.

H. Ollivier and W. H. Zurek, Phys. Rev. Lett. 88, 017901 (2001)

Some previous results

PHYSICAL REVIEW A **84**, 012313 (2011)

Conservation law for distributed entanglement of formation and quantum discord

Felipe F. Fanchini,[1,*] Marcio F. Cornelio,[2] Marcos C. de Oliveira,[2] and Amir O. Caldeira[2]
[1]*Departamento de Física, Universidade Federal de Ouro Preto, CEP 35400-000, Ouro Preto, Minas Gerais, Brazil*
[2]*Instituto de Física Gleb Wataghin, Universidade Estadual de Campinas, P.O. Box 6165, CEP 13083-970, Campinas, São Paulo, Brazil*
(Received 30 August 2010; revised manuscript received 3 May 2011; published 13 July 2011)

PHYSICAL REVIEW A **87**, 032317 (2013)

Why entanglement of formation is not generally monogamous

F. F. Fanchini,[1,*] M. C. de Oliveira,[2,3,†] L. K. Castelano,[4] and M. F. Cornelio[5]
[1]*Faculdade de Ciências, UNESP–Universidade Estadual Paulista, Código de Endereçamento Postal 17033-360, Bauru, São Paulo, Brazil*
[2]*Instituto de Física Gleb Wataghin, Universidade Estadual de Campinas, Código de Endereçamento Postal 13083-859, Campinas, São Paulo, Brazil*
[3]*Institute for Quantum Information Science, University of Calgary, Alberta, Canada T2N 1N4*
[4]*Departamento de Física, Universidade Federal de São Carlos, Código de Endereçamento Postal 13565-905, São Carlos, São Paulo, Brazil*
[5]*Instituto de Física, Universidade Federal de Mato Grosso, Código de Endereçamento Postal 78060-900, Cuiabá, Mato Grosso, Brazil*

PRL **107,** 020502 (2011) PHYSICAL REVIEW LETTERS week ending 8 JULY 2011

Entanglement Irreversibility from Quantum Discord and Quantum Deficit

Marcio F. Cornelio,[1,*] Marcos C. de Oliveira,[1,†] and Felipe F. Fanchini[1,2]
[1]*Instituto de Física Gleb Wataghin, Universidade Estadual de Campinas, CEP 13083-859, Campinas, São Paulo, Brazil*
[2]*Departamento de Física, Universidade Federal de Ouro Preto, CEP 35400-000, Ouro Preto, MG, Brazil*
(Received 1 July 2010; revised manuscript received 28 April 2011; published 5 July 2011)

PRL **109,** 190402 (2012) PHYSICAL REVIEW LETTERS week ending 9 NOVEMBER 2012

Emergence of the Pointer Basis through the Dynamics of Correlations

M. F. Cornelio,[1,2] O. Jiménez Farías,[3,4] F. F. Fanchini,[5] I. Frerot,[6] G. H. Aguilar,[3] M. O. Hor-Meyll,[3] M. C. de Oliveira,[2,7] S. P. Walborn,[3] A. O. Caldeira,[2] and P. H. Souto Ribeiro[3]
[1]*Instituto de Física, Universidade Federal de Mato Grosso, 78060-900, Cuiabá, Mato Grosso, Brazil*
[2]*Instituto de Física Gleb Wataghin, Universidade Estadual de Campinas, 13083-859, Campinas, São Paulo, Brazil*
[3]*Instituto de Física, Universidade Federal de Rio de Janeiro, Caixa Postal 68528, Rio de Janeiro, RJ 21941-972, Brazil*
[4]*Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Apartado Postal 70-543, México 04510 D.F., Mexico*
[5]*Departamento de Física, Faculdade de Ciências, Universidade Estadual Paulista, Bauru, São Paulo, CEP 17033-360, Brazil*
[6]*Département de Physique, Ecole Normale Supérieure, 24, rue Lhomond. F 75231 PARIS Cedex 05*
[7]*Institute for Quantum Information Science, University of Calgary, Alberta, Canada T2N 1N4*

New Journal of Physics
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Locally inaccessible information as a fundamental ingredient to quantum information

F F Fanchini[1,5]**, L K Castelano**[2]**, M F Cornelio**[3] **and M C de Oliveira**[3,4,5]
[1] Departamento de Física, Universidade Federal de Ouro Preto, CEP 35400-000, Ouro Preto, MG, Brazil
[2] Departamento de Física, Universidade Federal de São Carlos, CEP 13565-905, São Carlos, SP, Brazil
[3] Instituto de Física Gleb Wataghin, Universidade Estadual de Campinas, CEP 13083-859, Campinas, SP, Brazil
[4] Institute for Quantum Information Science, University of Calgary, Alberta, T2N 1N4, Canada
E-mail: fanchini@iceb.ufop.br and marcos@ifi.unicamp.br

PRL **112,** 210402 (2014) PHYSICAL REVIEW LETTERS week ending 30 MAY 2014

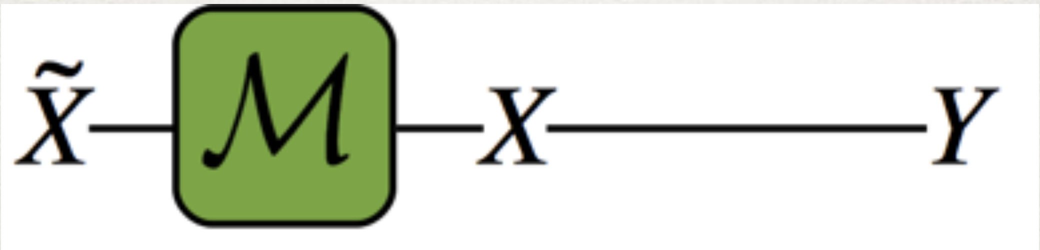
Non-Markovianity through Accessible Information

F. F. Fanchini,[1,*] G. Karpat,[1] B. Çakmak,[2] L. K. Castelano,[3,†] G. H. Aguilar,[4] O. Jiménez Farías,[4] S. P. Walborn,[4] P. H. Souto Ribeiro,[4,‡] and M. C. de Oliveira[5,§]
[1]*Faculdade de Ciências, UNESP—Universidade Estadual Paulista, Bauru, SP 17033-360, Brazil*
[2]*Faculty of Engineering and Natural Sciences, Sabanci University, Tuzla, Istanbul 34956, Turkey*
[3]*Departamento de Física, Universidade Federal de São Carlos, 13565-905 São Carlos, SP, Brazil*
[4]*Instituto de Física, Universidade Federal do Rio de Janeiro, CP 68528, 21941-972 Rio de Janeiro, RJ, Brazil*
[5]*Instituto de Física Gleb Wataghin, Universidade Estadual de Campinas, 13083-859 Campinas, SP, Brazil*
(Received 21 March 2014; published 29 May 2014)

What are the requirements for the existence of a similar classical stochastic theory?

Imperfect (Classical) Measurements

- Accessing X through a noisy channel



$$p(X=x,\widetilde{X}=\tilde{x}) \equiv p(x,\tilde{x}) = p(\widetilde{x}|x)p(x)$$

$$\widetilde{X} = \mathcal{M}(X,\Xi) \qquad\qquad \Xi = \mathcal{M}^{-1}(X,\widetilde{X})$$

- Back-action: Probability that a measurement on $\widetilde{X}$ causes a transition from $X=x'$ to $X=x$: $T_{\tilde{x}}(x|x')$

$$T_{\tilde{x}}(x|x') \geq 0, \;\; \sum_x T_{\tilde{x}}(x|x') = 1$$

Noise+ back-action

$$M_{\tilde{x}}(x|x') \equiv T_{\tilde{x}}(x|x')p(\tilde{x}|x')$$

Noise+ back-action

$$M_{\tilde{x}}(x|x') \equiv T_{\tilde{x}}(x|x')p(\tilde{x}|x')$$

$$p(x,\tilde{x}) = p(\tilde{x}|x)p(x)$$

Noise+ back-action

$$M_{\tilde{x}}(x|x') \equiv T_{\tilde{x}}(x|x')p(\tilde{x}|x')$$

$$p'(x|\tilde{x}) = \frac{p(\tilde{x}|x)p(x)}{p(\tilde{x})}$$

Bayes posterior state

Noise+ back-action

$$M_{\tilde{x}}(x|x') \equiv T_{\tilde{x}}(x|x')p(\tilde{x}|x')$$

$$p'(x|\tilde{x}) = \frac{\sum_{x'} M_{\tilde{x}}(x|x')}{p(\tilde{x})} p(x')$$

Bayes
posterior
state

Noise+ back-action

$$M_{\tilde{x}}(x|x') \equiv T_{\tilde{x}}(x|x')p(\tilde{x}|x')$$

$$p'(x|\tilde{x}) = \frac{\sum_{x'} M_{\tilde{x}}(x|x')}{p(\tilde{x})}p(x')$$

Bayes
posterior
state

$$\rho_A^k = \frac{Tr_B\{\Pi_k \rho_{AB} \Pi_k\}}{p_k}$$

Post-selected state

Classical Discord

$$H_i(Y|X) = -\sum_x p(x) H_i(Y|x)$$

$$H_i(Y|x) = -\sum_y p'(y|x) \log p'(y|x)$$

$$H(Y|X) = H(X,Y) - H(X).$$

$$\begin{aligned}\mathcal{D}_{Y|X} &= H_i(Y|X) - H(Y|X)\\ &= I(X:Y) - J_i(Y|X)\end{aligned}$$

Classical Discord

$$H_i(Y|X) = -\sum_x p(x) H_i(Y|x)$$

$$H_i(Y|x) = -\sum_y p'(y|x) \log p'(y|x)$$

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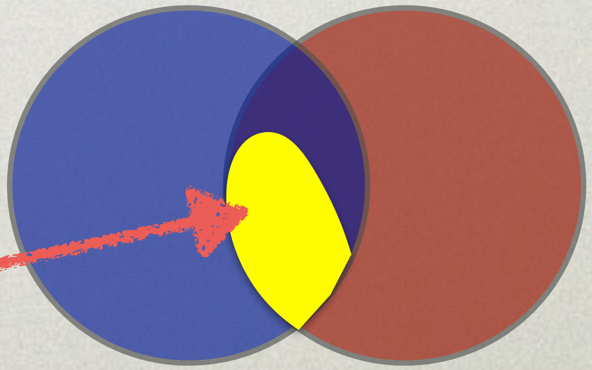

Conclusions

- Discord manifests when there is some stochasticity affecting the acquisition of information
- Quantum discord: natural stochasticity due to non-orthogonal basis
- Discord can be understood classically as a stochastic information figure of merit

(except when entanglement is present)

- State merging, thermodynamical aspects…
- Relevance for inference…