Weak Measurements and NonClassical Correlations

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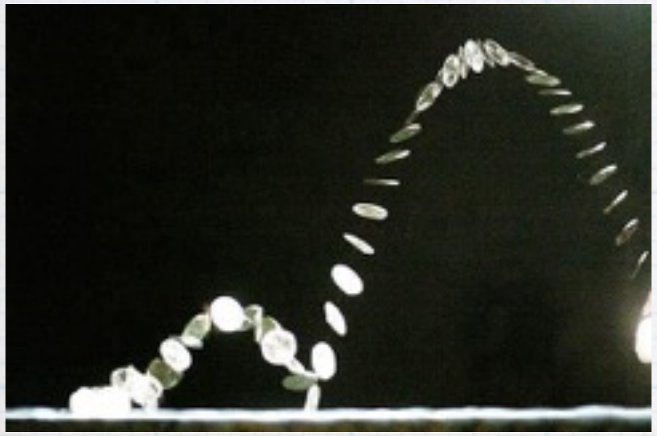
Work done with

Lekshmi S

N. Shaji

arXiv:1511.09224

Correlations



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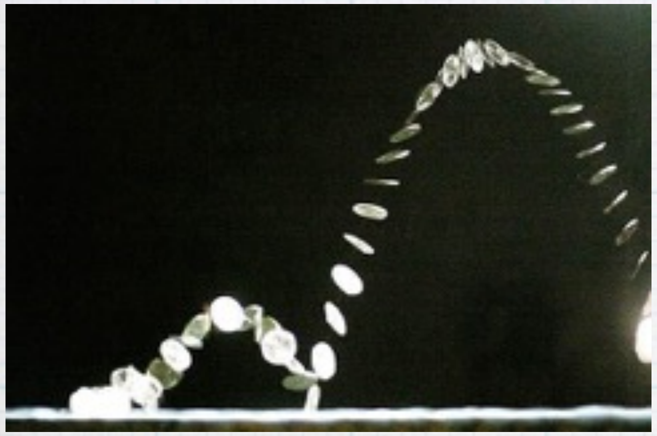

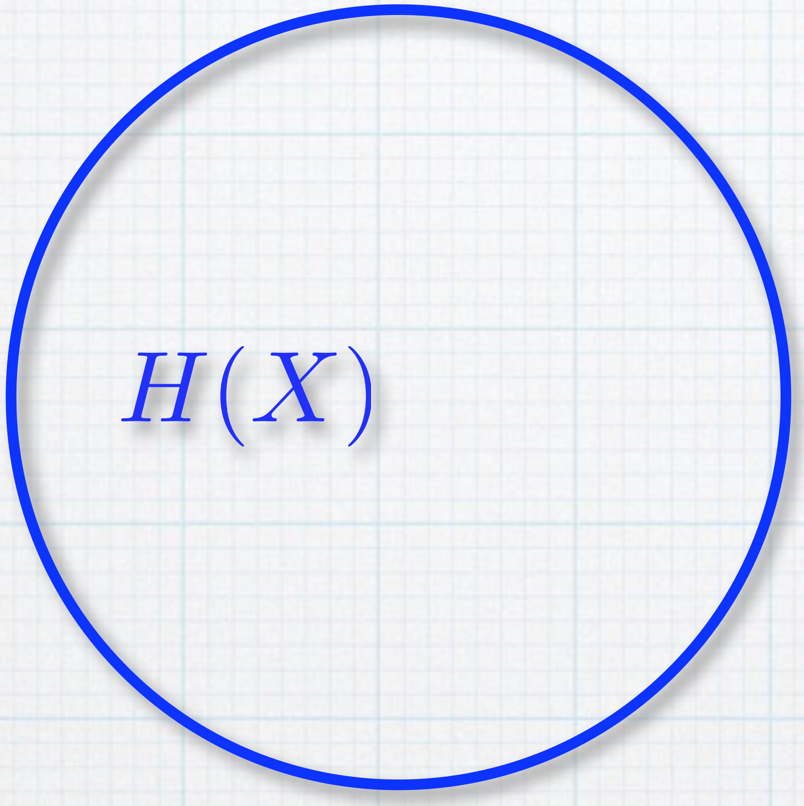

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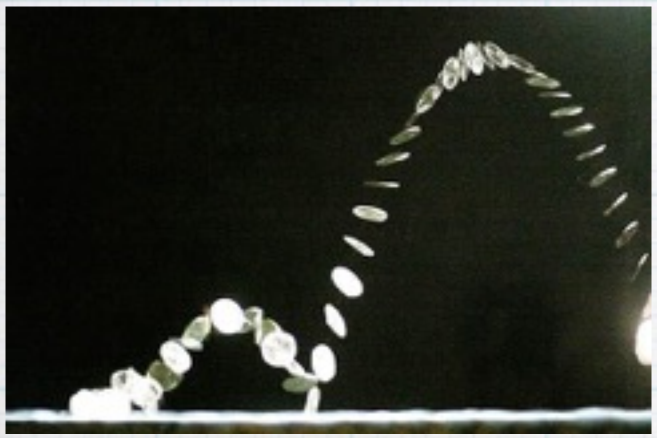
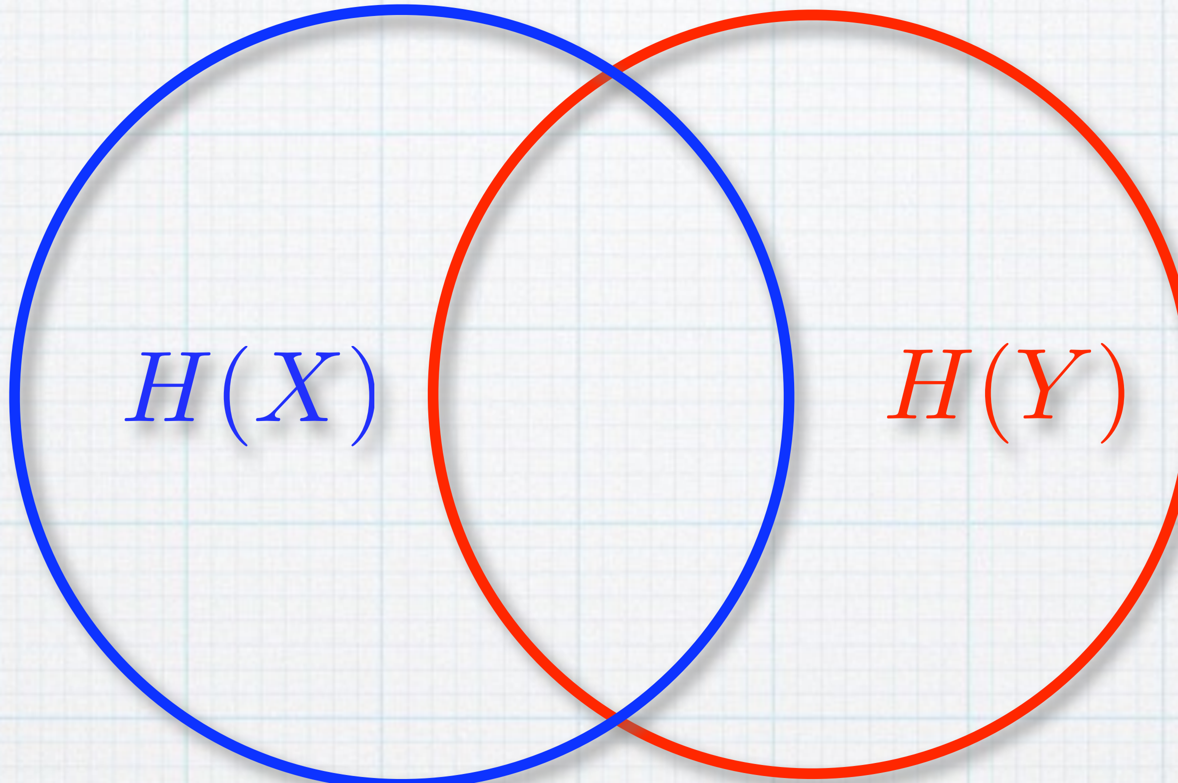
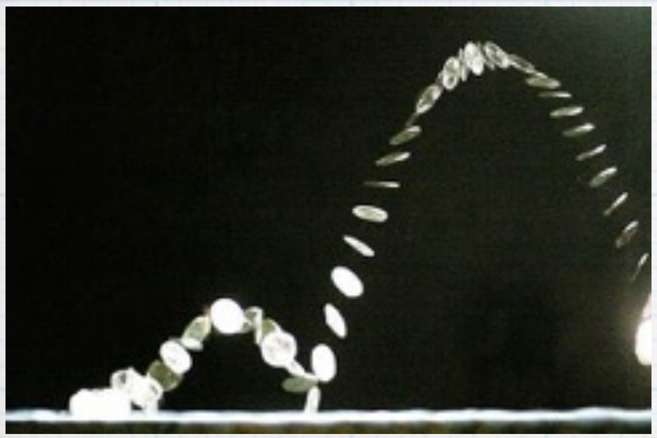
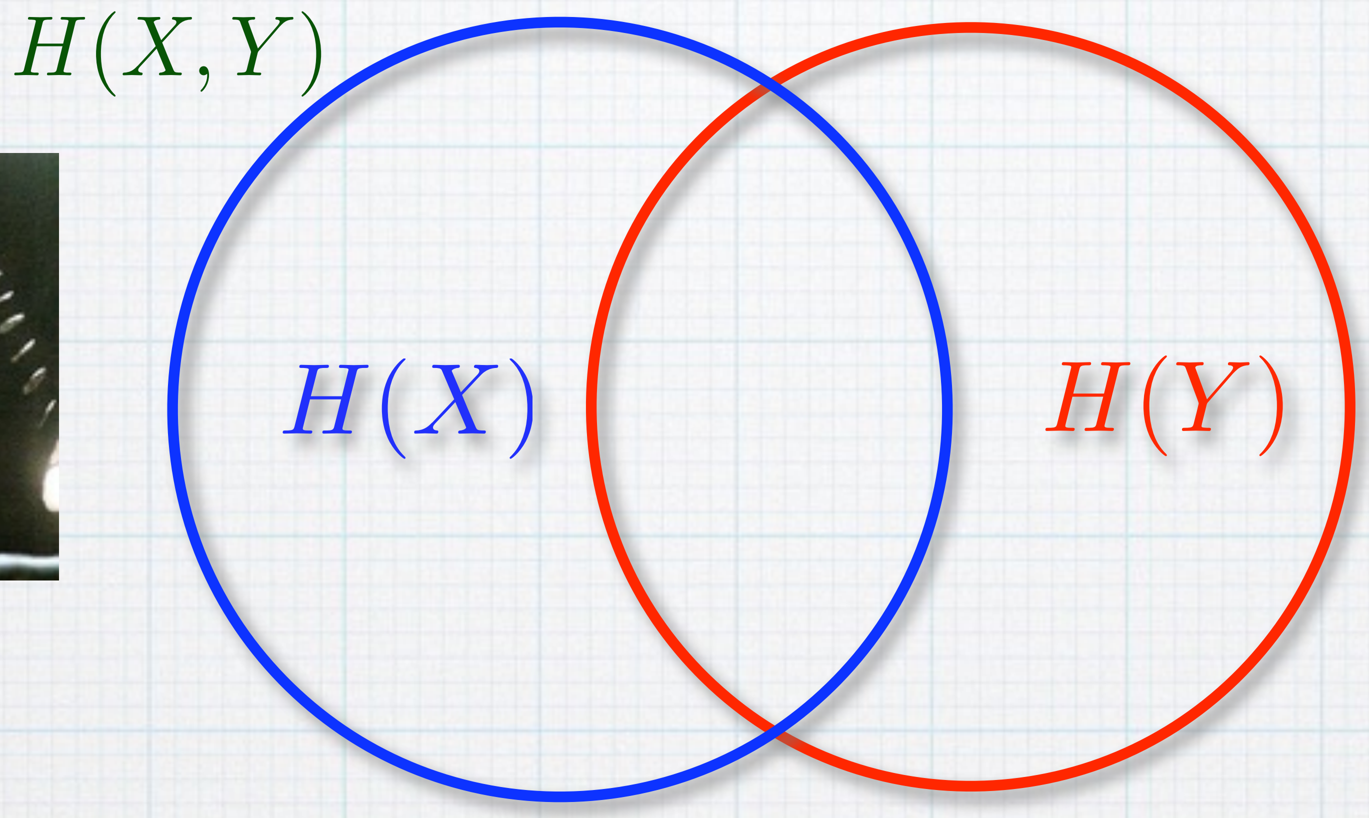




$$H(X) = -\sum_{x} p(x) \log p(x)$$

Correlations





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$$H(X:Y) = H(X) + H(Y) - H(X,Y)$$

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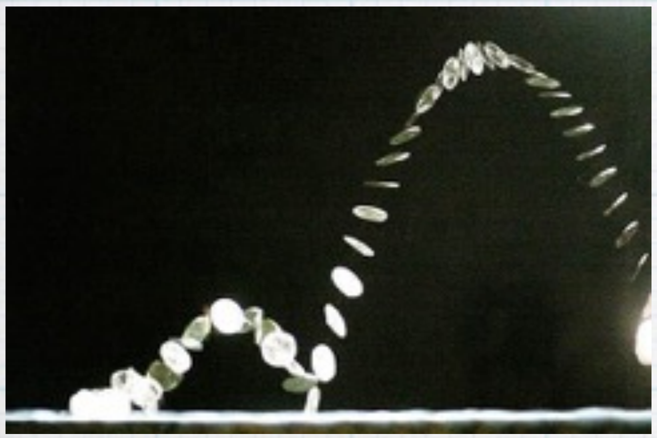

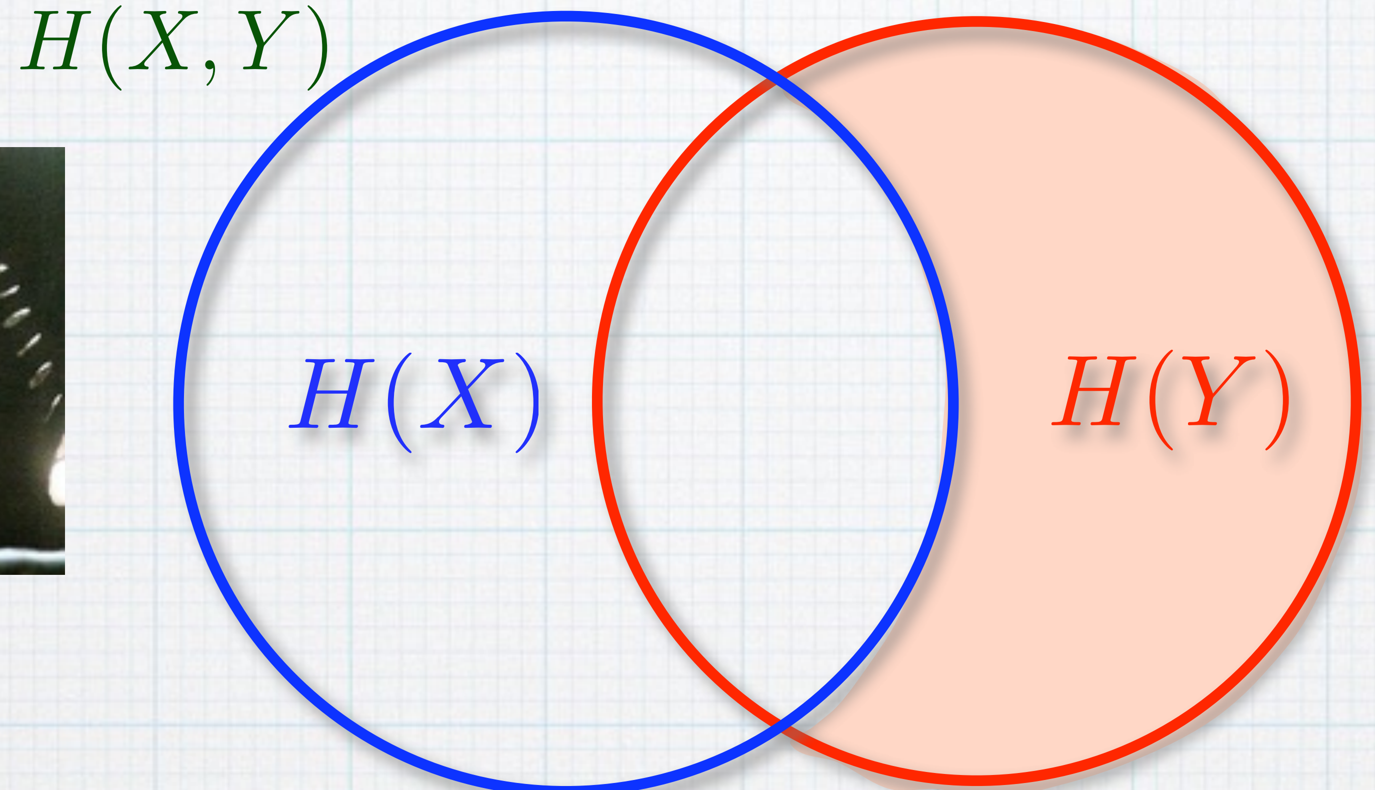

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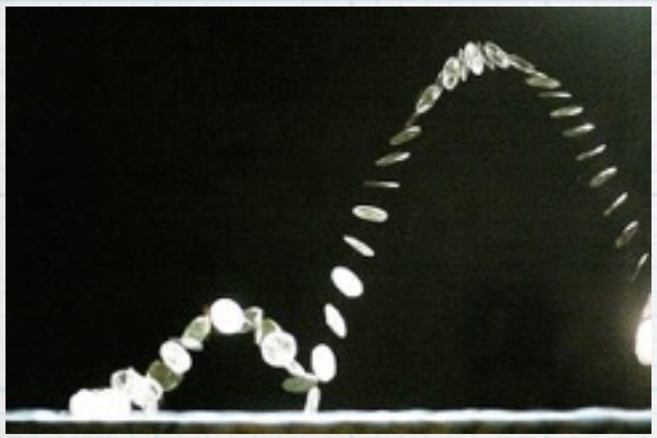

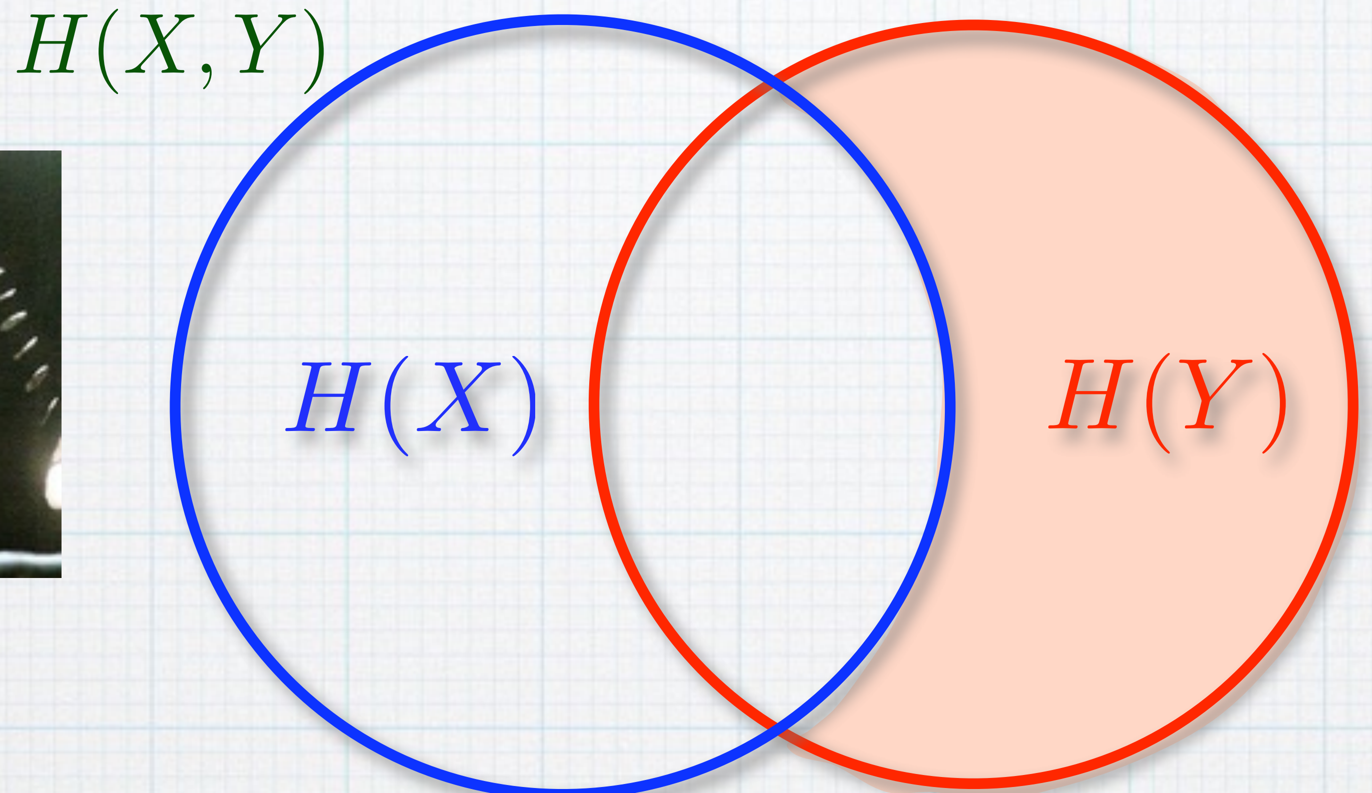

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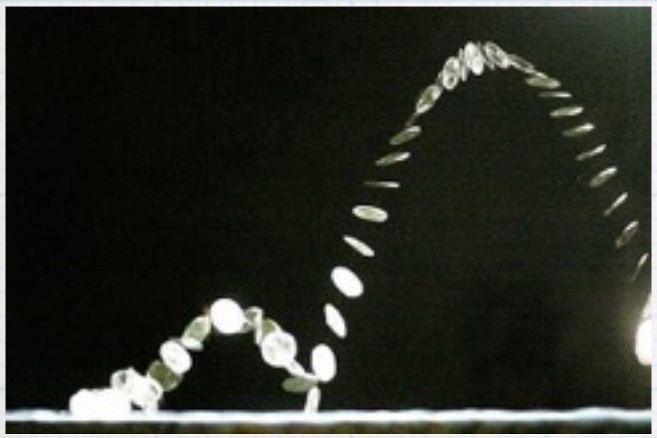

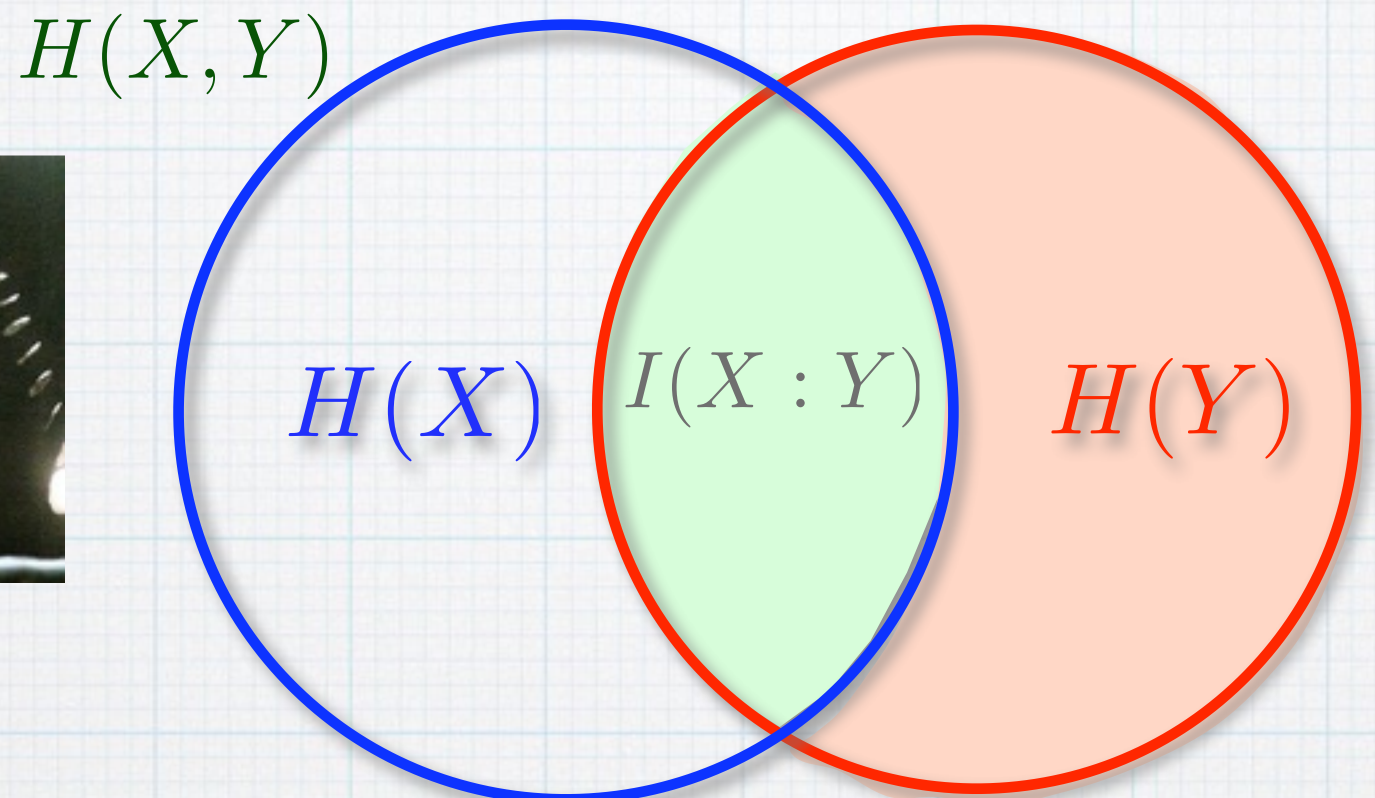

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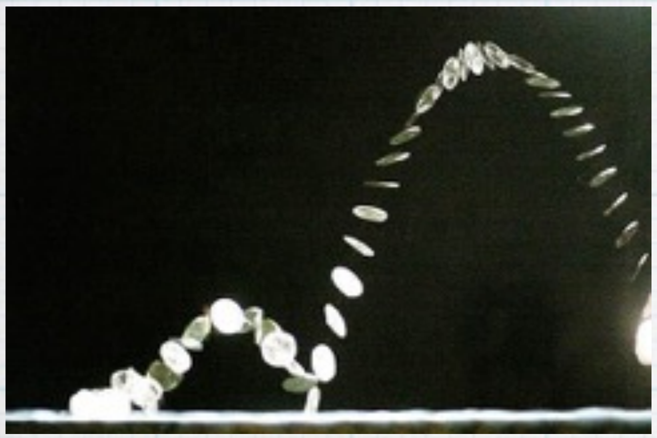

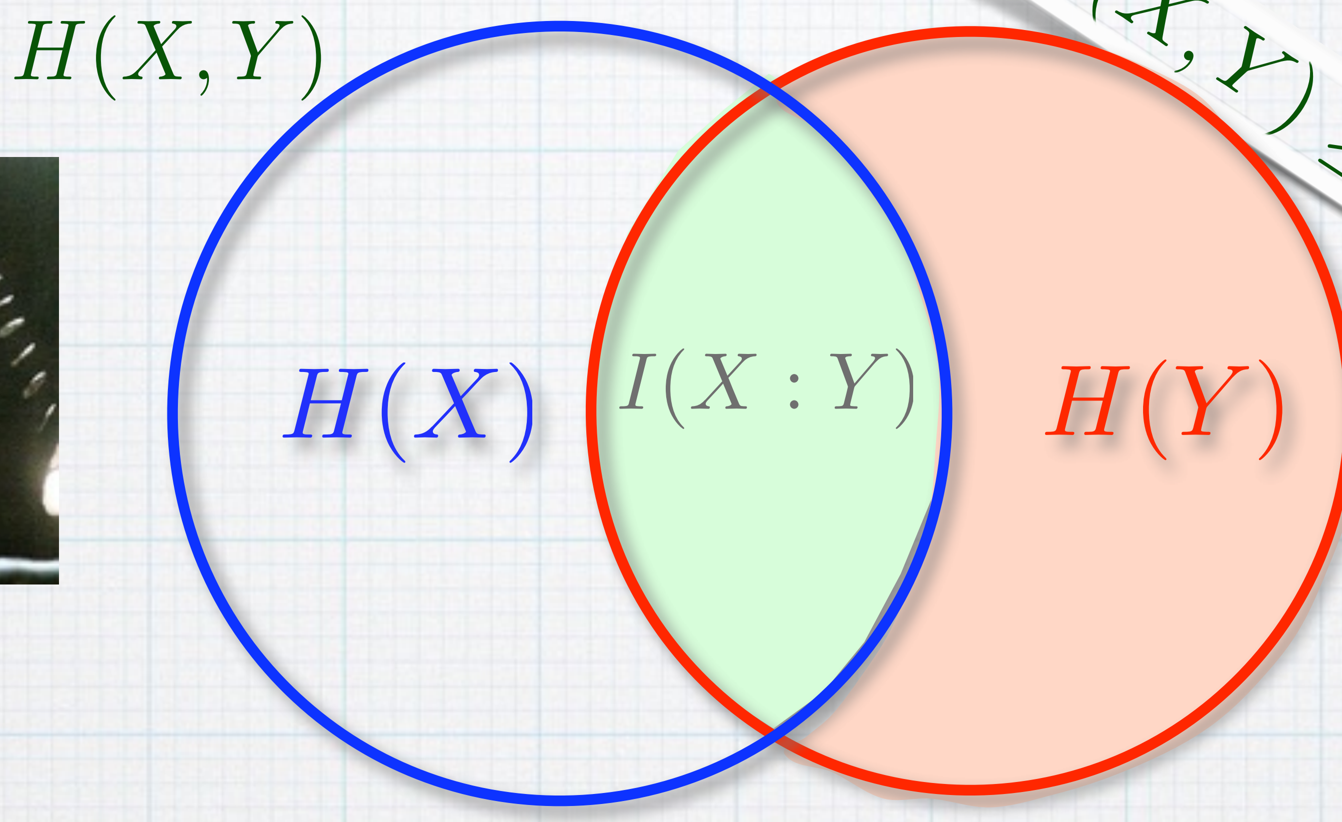

$$H(X,Y) \geq \max\{H(X), H(Y)\}$$

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All correlations between quantum systems

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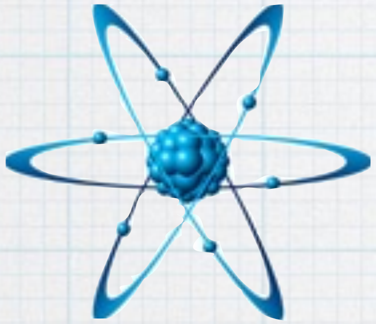

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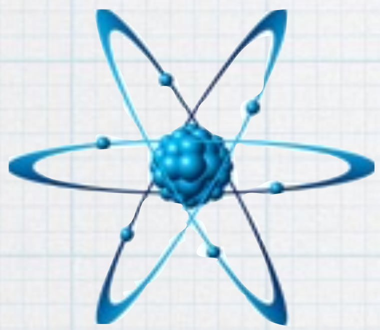

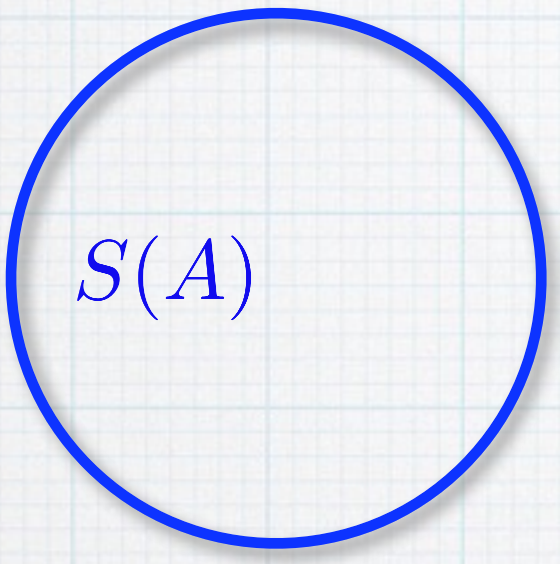

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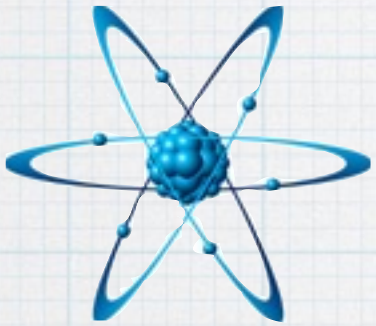

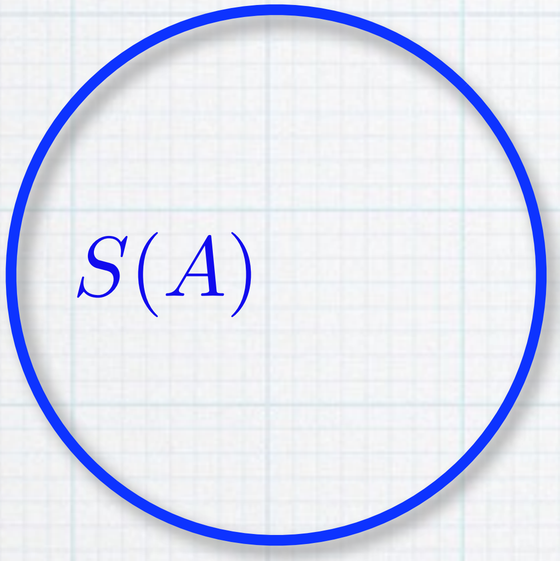

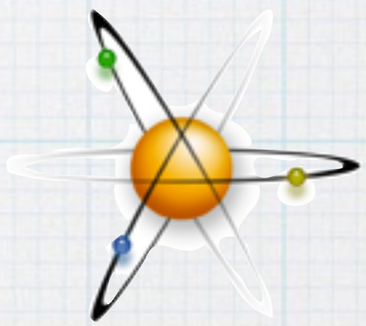

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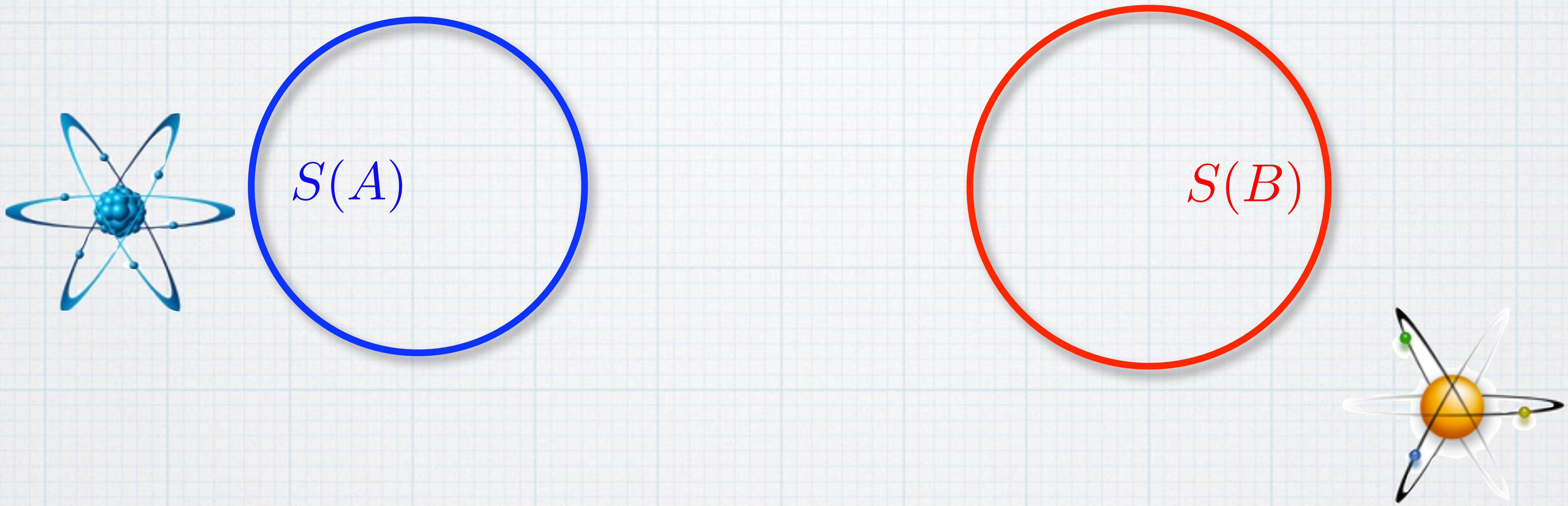

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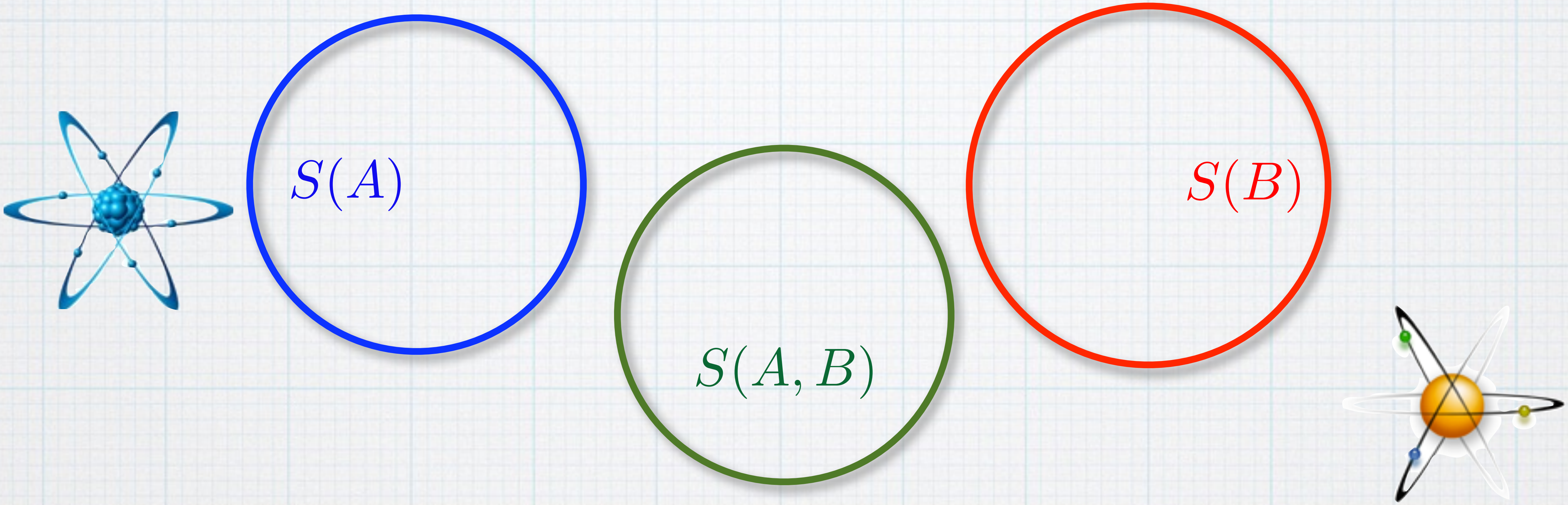

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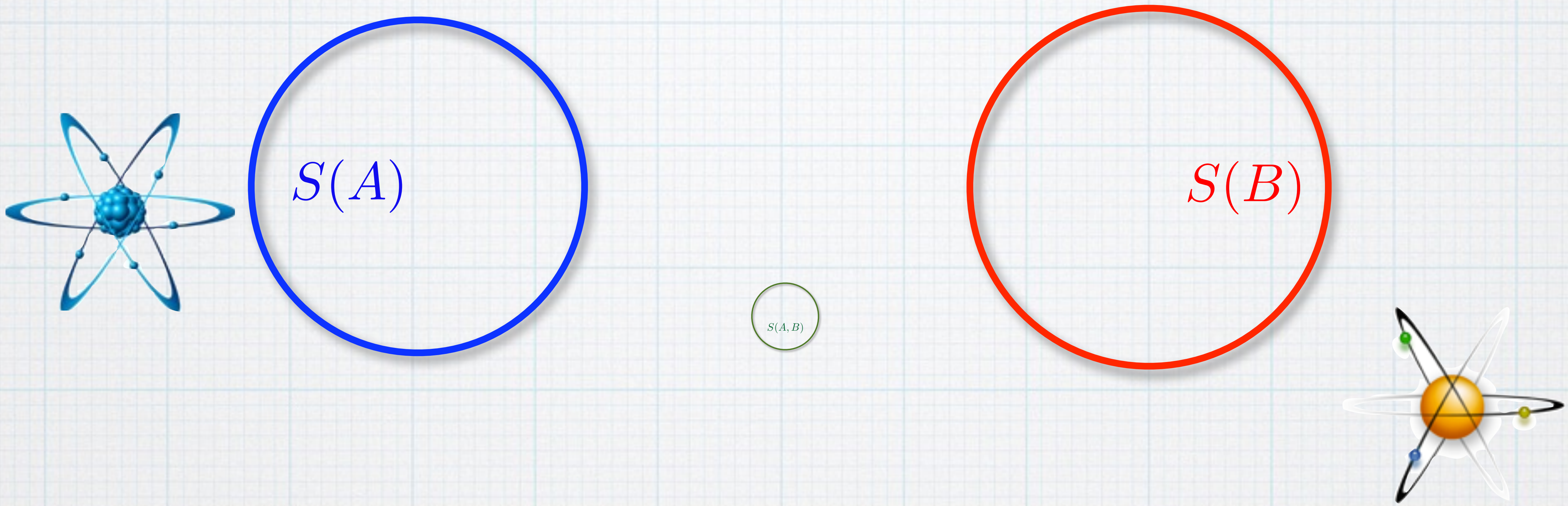

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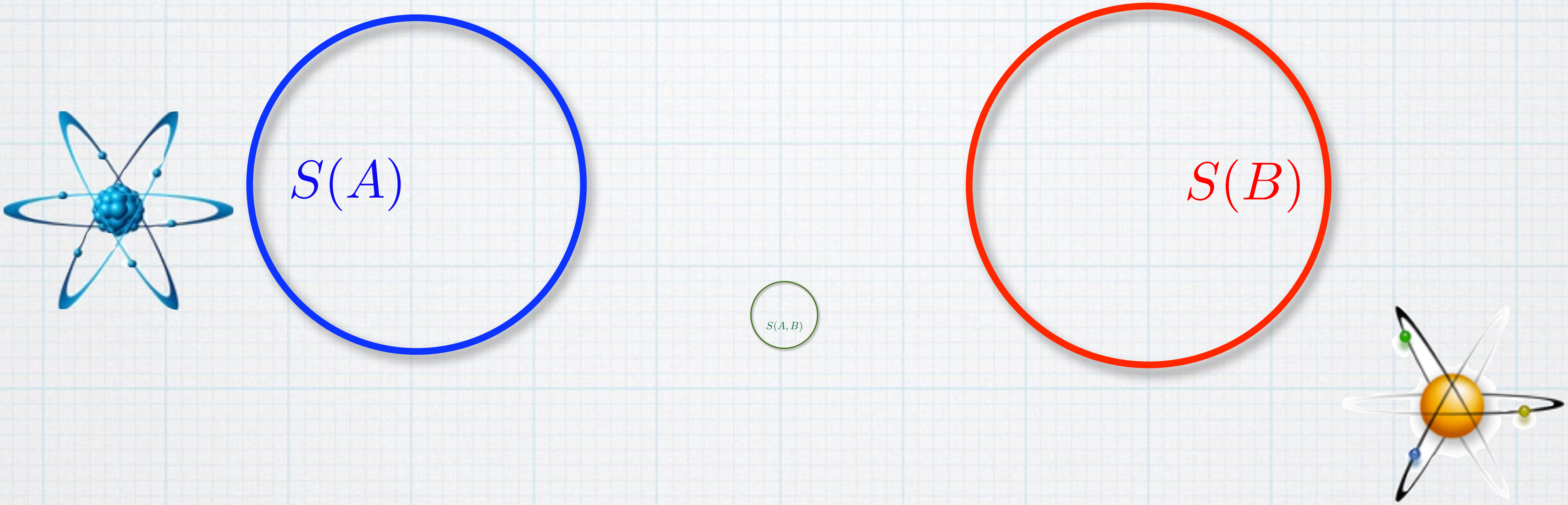

For maximally entangled pure states ignorance about the subsystems may be maximal while the global state is perfectly known

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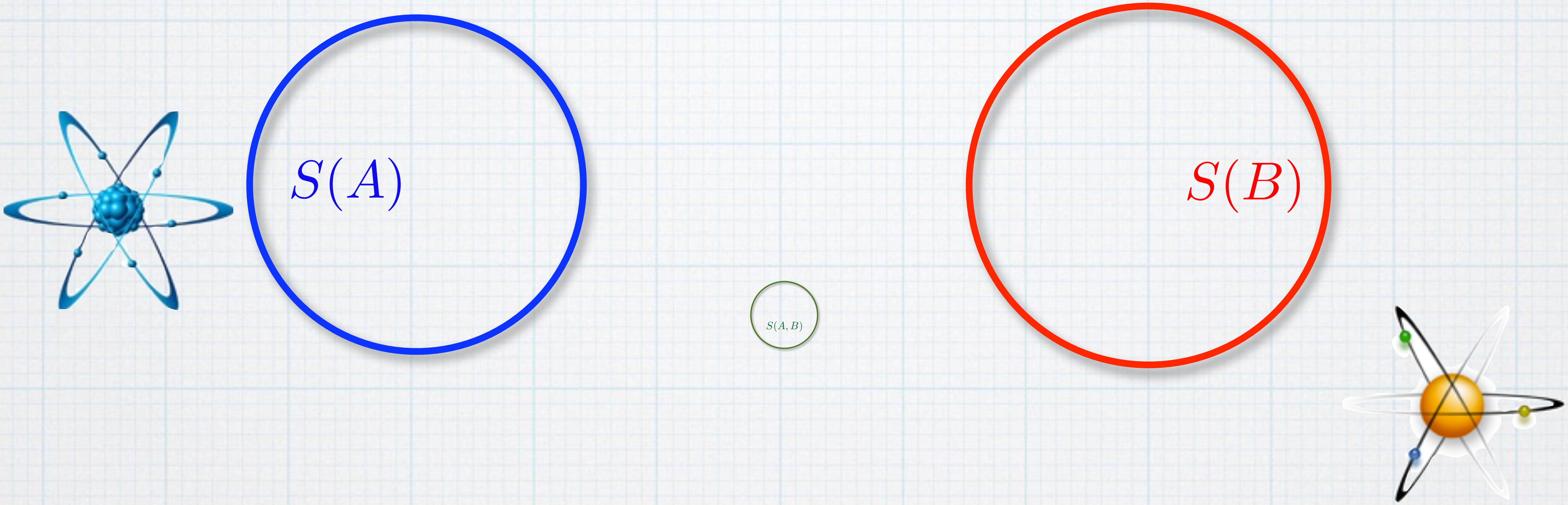


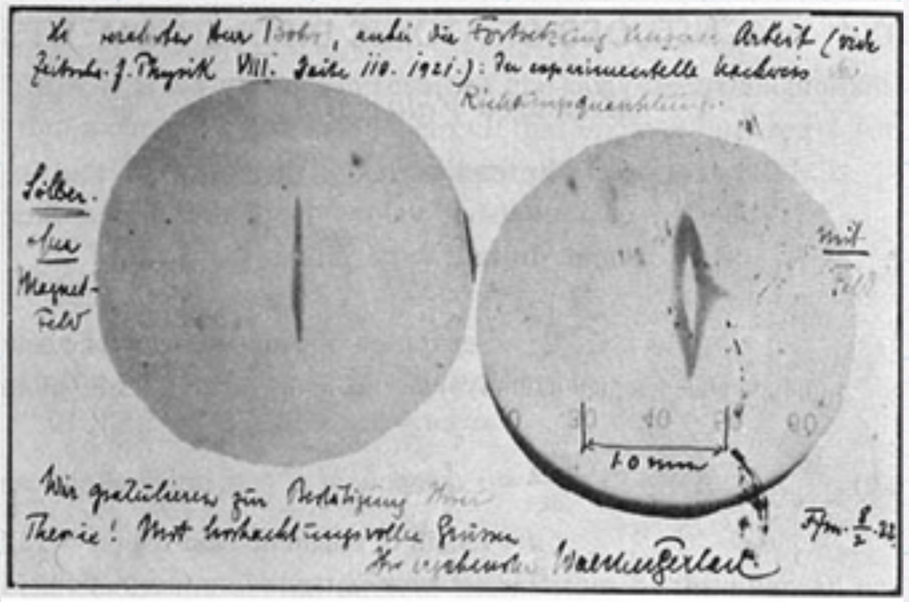
For maximally entangled pure states ignorance about the subsystems may be maximal while the global state is perfectly known

$$S(B:A) = S(\rho_B) + S(\rho_A) - S(\rho_{AB}), \quad S(\rho) = -\mathrm{Tr}\left[\rho \log \rho\right]$$

$$S(B|A) = S(\rho_{AB}) - S(\rho_A)$$

Measurements and Discord

To "know" a quantum system one has to do measurements and we start by thinking of projective measurements.



Gerlach's postcard, dated 8 February 1922, to Niels Bohr. It shows a photograph of the beam splitting, with the message, in translation: "Attached [is] the experimental proof of directional quantization. We congratulate [you] on the confirmation of your theory." (Physics Today December 2003)

$$\rho_{B|\Pi_k^A} = \frac{\Pi_k^A \rho_{AB} \Pi_k^A}{p_k}, \quad p_k = \mathrm{Tr}\left[\Pi_k^A \rho_{AB}\right]$$

$$\widetilde{J}(B:A) = S(\rho_B) - \sum_k p_k S(\rho_B|\Pi_k^A)$$

$$\mathcal{I}(B:A) \neq \widetilde{J}(B:A) \text{ in general}$$

$$\begin{aligned} \mathcal{D} &\equiv \mathcal{I}(B:A) - \mathcal{J}(B:A) \\ &= S(\rho_A) - S(\rho_{AB}) + \min_{\{\Pi_k^A\}} \sum_k p_k S(\rho_B|\Pi_k^A) \end{aligned}$$

H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001)

Zero Discord States

$$\rho_{AB} = \sum_k \rho_k^B \otimes \Pi_k^A$$

* Measurements (projective) can be done on subsystem A without disturbing the state of B
* The measurements have been generalized to POVMs and other interpretations of discord proposed
* Difference between the total correlations and classical correlations
* Classical correlations being the ones that can be 'extracted' via measurements.

H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001)

Pointers and measurements

* Our emphasis is on the disturbance to the measured system
* A system and a pointer in the initial state:

$$|\psi_i\rangle|\Phi\rangle$$

* An interaction of the form

$$H = g\mathcal{O} \otimes P$$

* The pointer is described by the canonical variables Q and P. After the joint evolution, the system-pointer state is

$$\sum_k \psi_{ik}|a_k\rangle|\Phi(Q - ga_k)\rangle, \qquad \mathcal{O} = \sum_k a_k|a_k\rangle\langle a_k|$$

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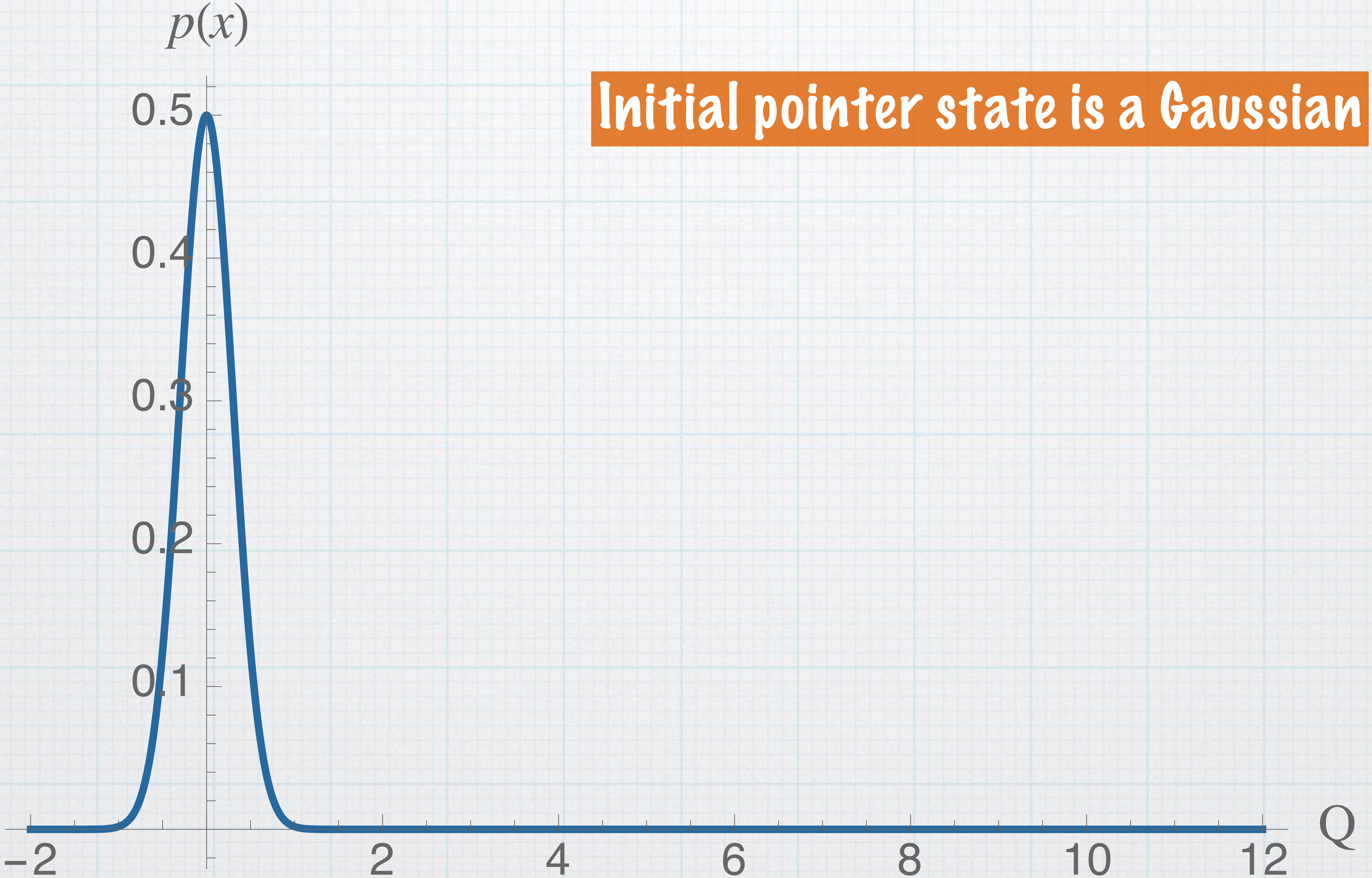
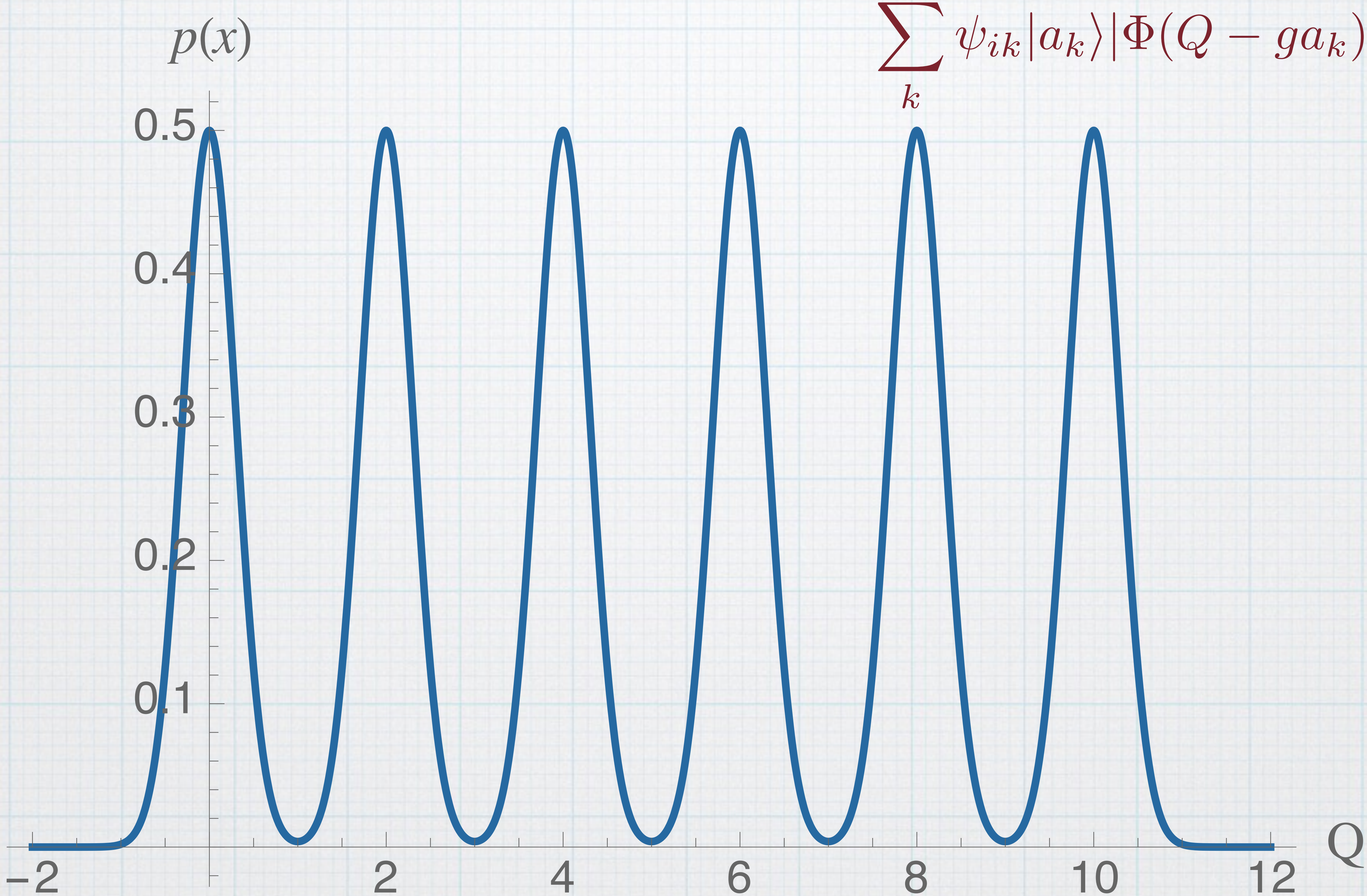
Projective measurements



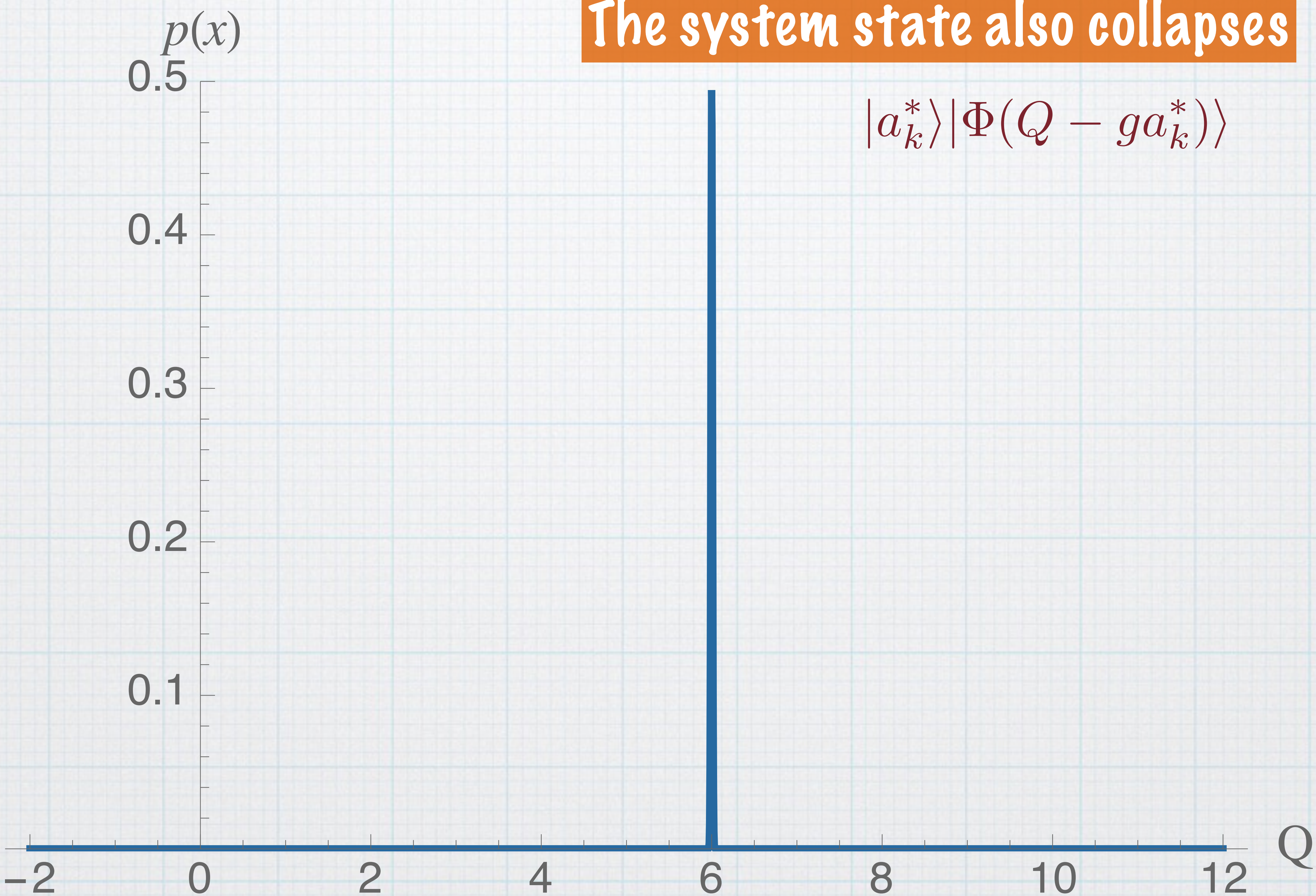
Initial pointer state is a Gaussian

Projective measurements

$$\sum_k \psi_{ik}|a_k\rangle|\Phi(Q - ga_k)\rangle$$



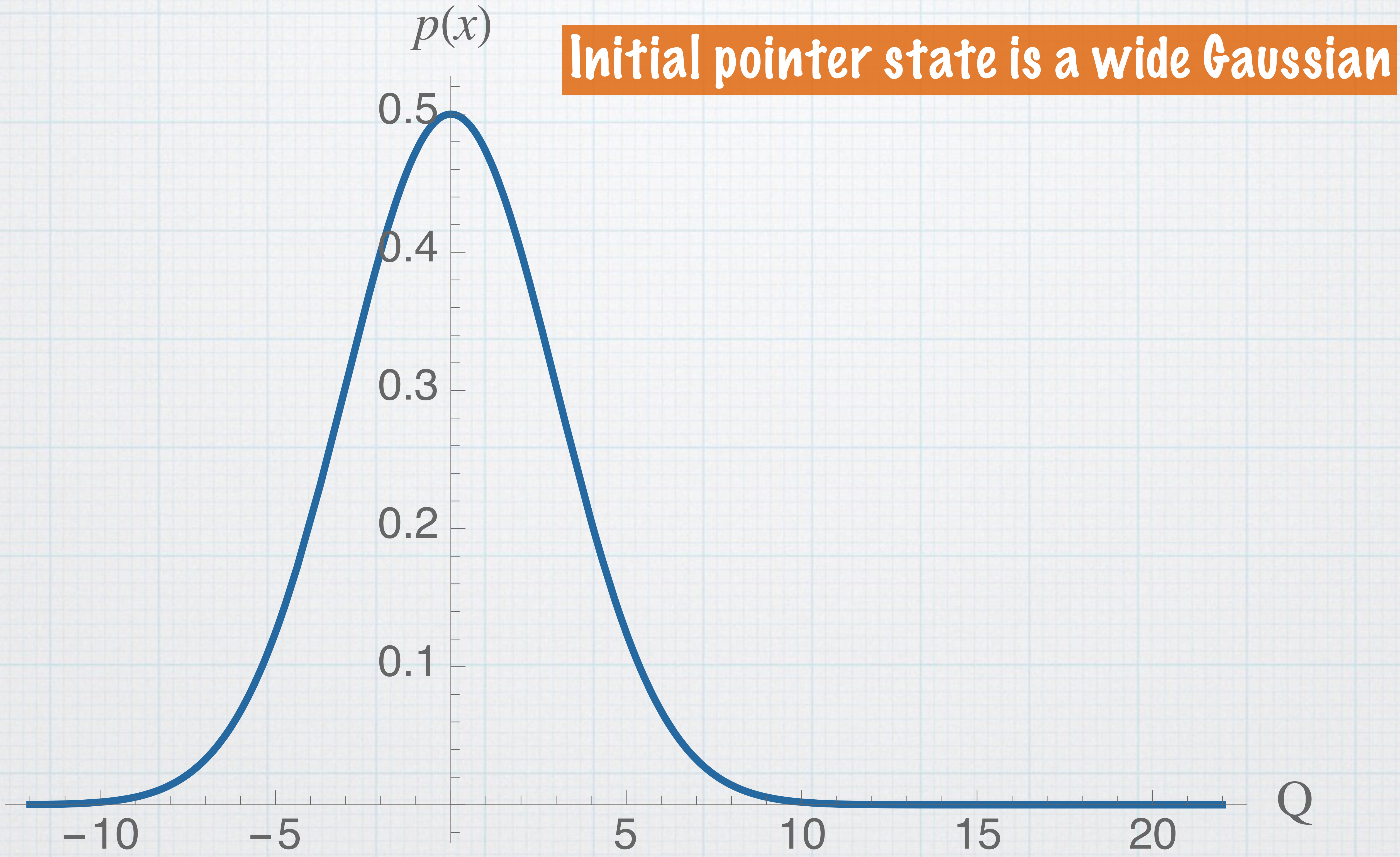
Projective measurements



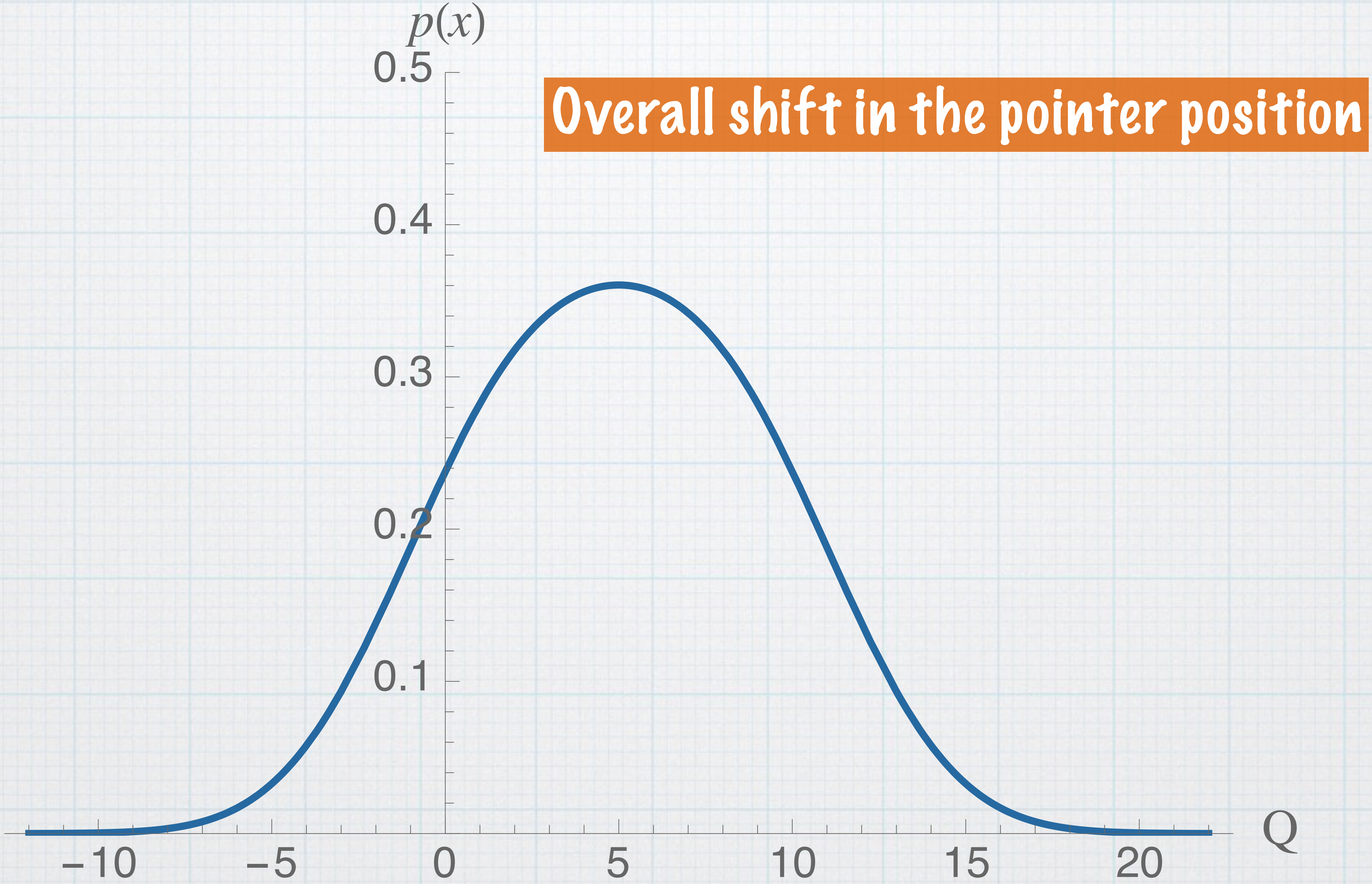
The system state also collapses

$$|a_k^*\rangle|\Phi(Q - g a_k^*)\rangle$$

Weak measurements



Weak measurements



Weak value

* What can one measure?

* The weak value, post selected on a particular final state of the system:

$$\langle O\rangle_w = \frac{\langle\psi_f|\mathcal{O}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}.$$

* The weak measurement condition:

$$g\Delta P = \frac{g}{2\Delta} \ll \frac{|\langle\psi_f|\psi_i\rangle|}{|\langle\psi_f|\mathcal{O}^n|\psi_i\rangle|^{1/n}}, \quad \text{for} \quad n = 1, 2, \ldots.$$

* If the initial state is mixed and post selection is on to a positive operator (POVM element)

$$\langle O\rangle_w = \frac{\mathrm{tr}(P_f\mathcal{O}\rho_i)}{\mathrm{tr}(P_f\rho_i)}$$

Y. Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev. Lett. **60**, 1351 (1988)
I. M. Duck, P. M. Stevenson, and E. C. G. Sudarshan, Phys. Rev. D **40**, 2112 (1989)
J. Dressel and A. N. Jordan, Physical Review A **85**, 012107 (2012)

Weak quantum discord

$$\mathcal{D}_w = S(\rho_A) - S(\rho_{AB}) + \sum_j p_k^w S(\rho_B | a_k)$$

$$\mathcal{D} = S(\rho_A) - S(\rho_{AB}) + \min_{\Pi_k^A} \sum_k p_k S(\rho_B | a_k)$$

* The probabilities p_k gives the measurement statistics corresponding to a complete set of orthogonal measurements on subsystem A
* The probabilities p_k^w are estimates of p_k obtained from weak measurements on A

Observable

* We want to find

$$p_k = \mathrm{tr}[(\Pi_k^A \otimes \mathbb{I}_{d_B})\rho_{AB}]$$

* Consider the observable:

$$\mathcal{O} = \sum_{k=1}^{d_A} a_k \Pi_k^A \otimes \mathcal{I}_{d_B}$$

* The eigenvalues a_k are chosen so that the following set of equations are invertible:

$$\langle \mathcal{O}^n \rangle = \sum_{k=1}^{d_A} a_k^n p_k, \qquad n = 0, \ldots, d_A - 1$$

* To get the probabilities p_k^w we replace the expectation values with the corresponding weak values:

$$\langle \mathcal{O}^n \rangle \leftrightarrow \langle \mathcal{O}^n \rangle_w$$

Post selection

* For obtaining the "weak" version of Discord we choose the post selection operator

$$P_f = (1-\alpha)\rho_{AB} + \alpha \sum_{k=1}^{d_A} \Pi_k^A \otimes \mathbb{I}_{d_B} = (1-\alpha)\rho_{AB} + \alpha \mathbb{I}_{d_{AB}}$$

$$P_f' = (1-\alpha)(\mathbb{I}_{d_{AB}} - \rho_{AB})$$

* The projectors on to subsystem A are assumed to be the ones that maximize normal discord.

* For $\alpha=0$ the state of subsystem A is not disturbed by the measurement

* For $\alpha=1$ corresponds to projective measurements on A

Measurements on Qubits

$$\mathcal{O} = (\Pi_+^A - \Pi_-^A) \otimes \mathbb{I}_B$$

$$p_\pm^w = \frac{1 \pm \langle \mathcal{O} \rangle_w}{2}$$

$$\langle \mathcal{O} \rangle_w = \frac{(1-\alpha)\mathrm{tr}(\mathcal{O}\rho_{AB}^2) + \alpha \langle \mathcal{O} \rangle}{(1-\alpha)\mathrm{tr}(\rho_{AB}^2) + \alpha}$$

$$\mathrm{tr}(\mathcal{O}\rho_{AB}^2) = \langle \mathcal{O} \rangle \mathrm{tr}(\rho_{AB}^2) \quad \Rightarrow \quad \langle \mathcal{O} \rangle = \langle \mathcal{O} \rangle_w$$

For several families of states, the weak and normal discords coincide

Bell-Diagonal States

$$\rho_{AB} = \frac{1}{4}\left(\mathbb{I}\otimes\mathbb{I} + \sum_{j=1}^{3} c_j \sigma_j^A \otimes \sigma_j^B\right)$$

$$\Pi_\pm^A = \frac{1}{2}(\mathbb{I} \pm \sigma_s), \quad c_s \equiv \max c_j$$

$$\mathcal{O} = \sigma_s^A \otimes \mathbb{I}_B$$

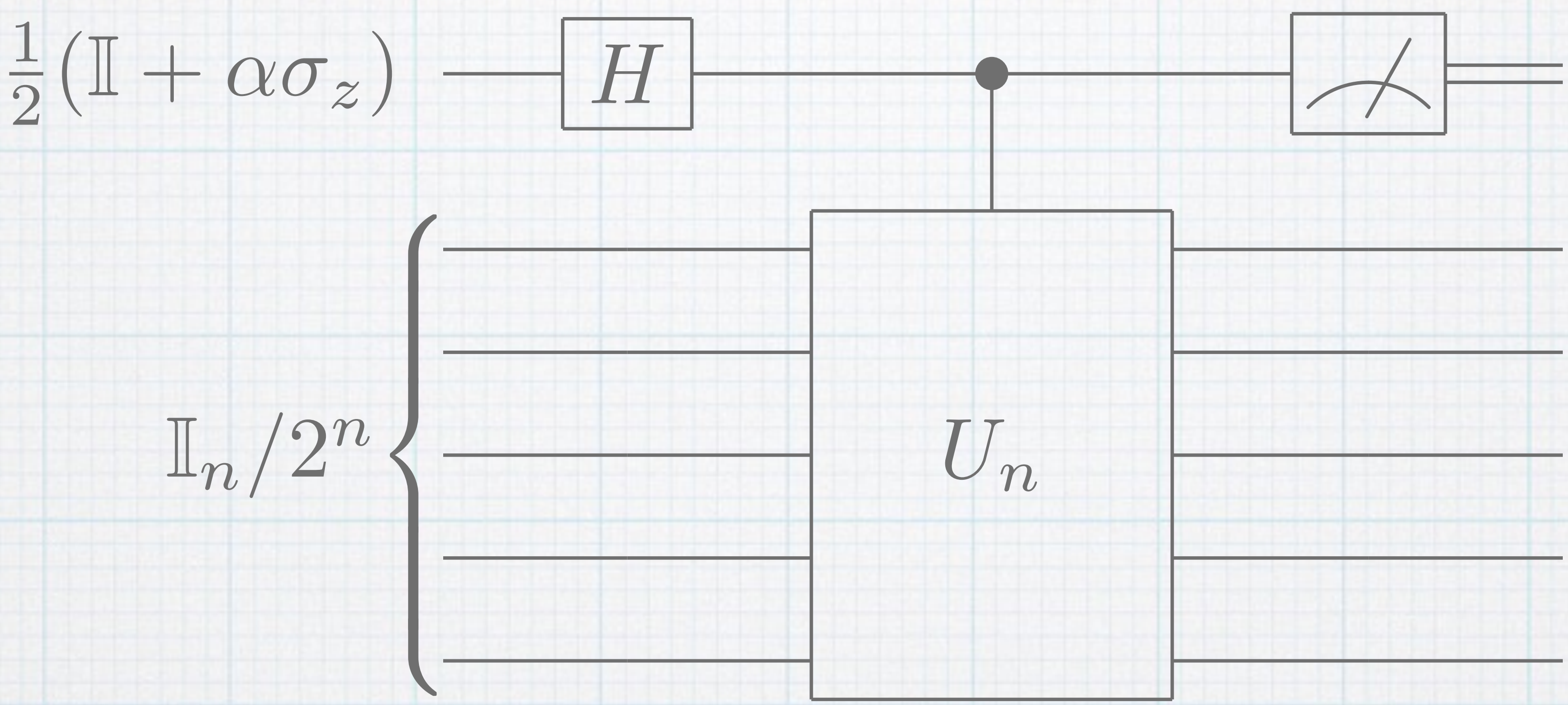
$$\langle\mathcal{O}\rangle = \mathrm{tr}(\mathcal{O}\rho_{AB}^2) = 0$$

$$\langle\mathcal{O}\rangle_w = \langle\mathcal{O}\rangle = 0, \quad p_\pm^w = p_\pm = \frac{1}{2}$$

$$\mathcal{D} = \mathcal{D}_w$$

Werner states are a special case of Bell-Diagonal States

DQC1 state

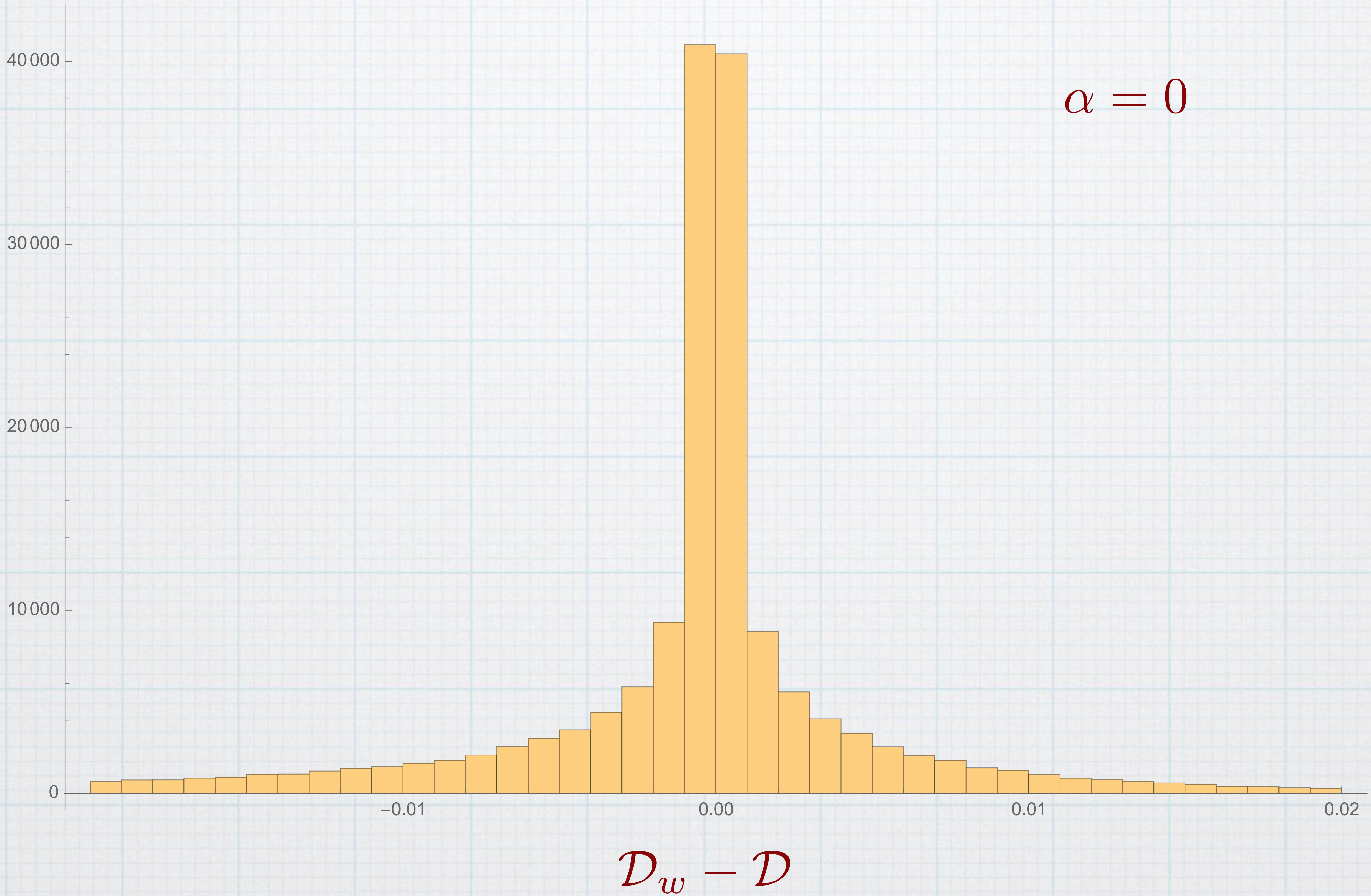


$$\rho_{AB} = \frac{1}{2^{n+1}}\Big(|0\rangle\langle 0| \otimes \mathbb{I}_{d_n} + |1\rangle\langle 1| \otimes \mathbb{I}_{d_n} + |0\rangle\langle 1| \otimes U^\dagger + |1\rangle\langle 0| \otimes U\Big)$$

$$\rho_{AB}^2 = \frac{1}{2^n}\rho_{AB}$$

$$\mathcal{D} = \mathcal{D}_w$$

Randomly Generated Two Qubit States



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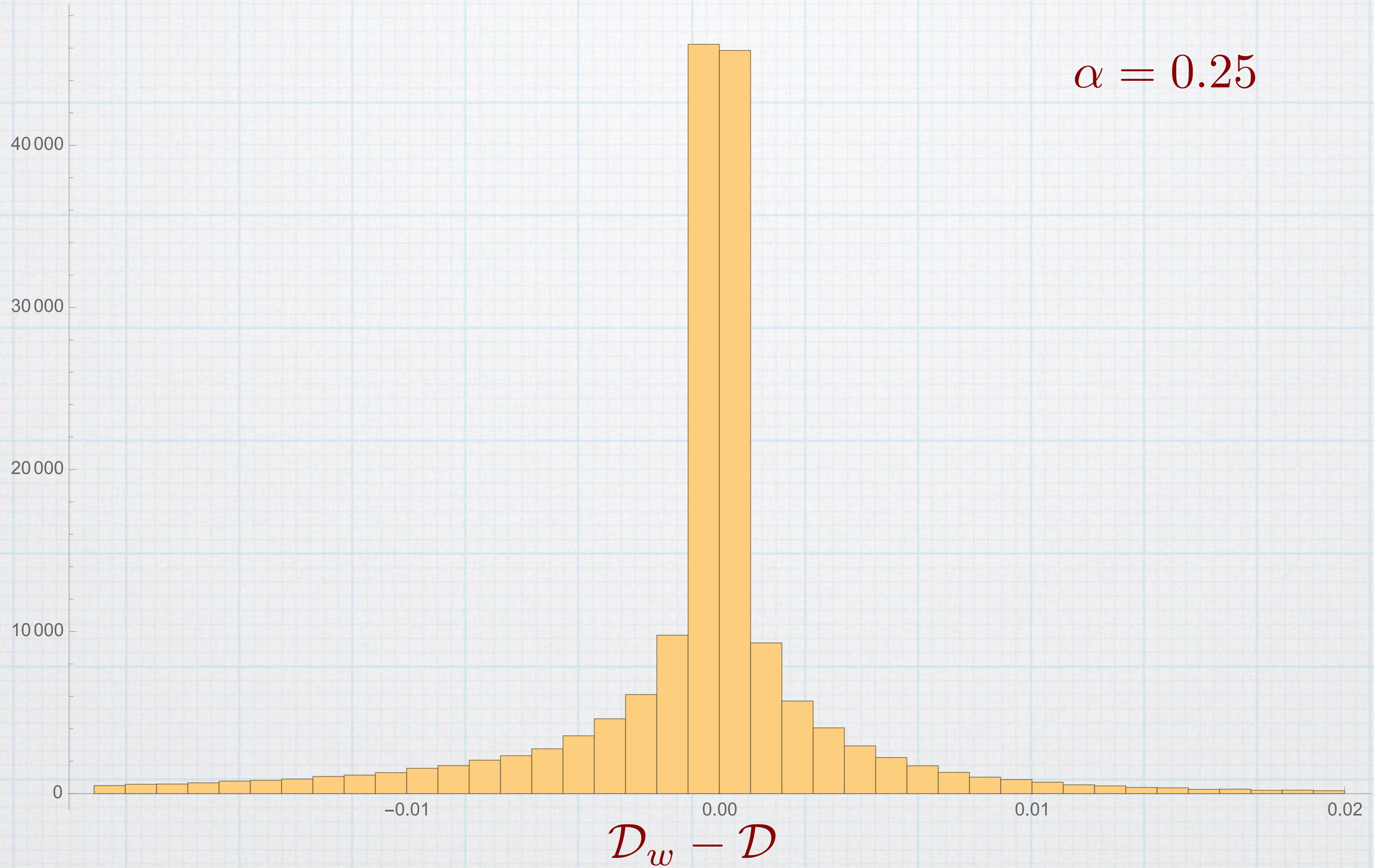

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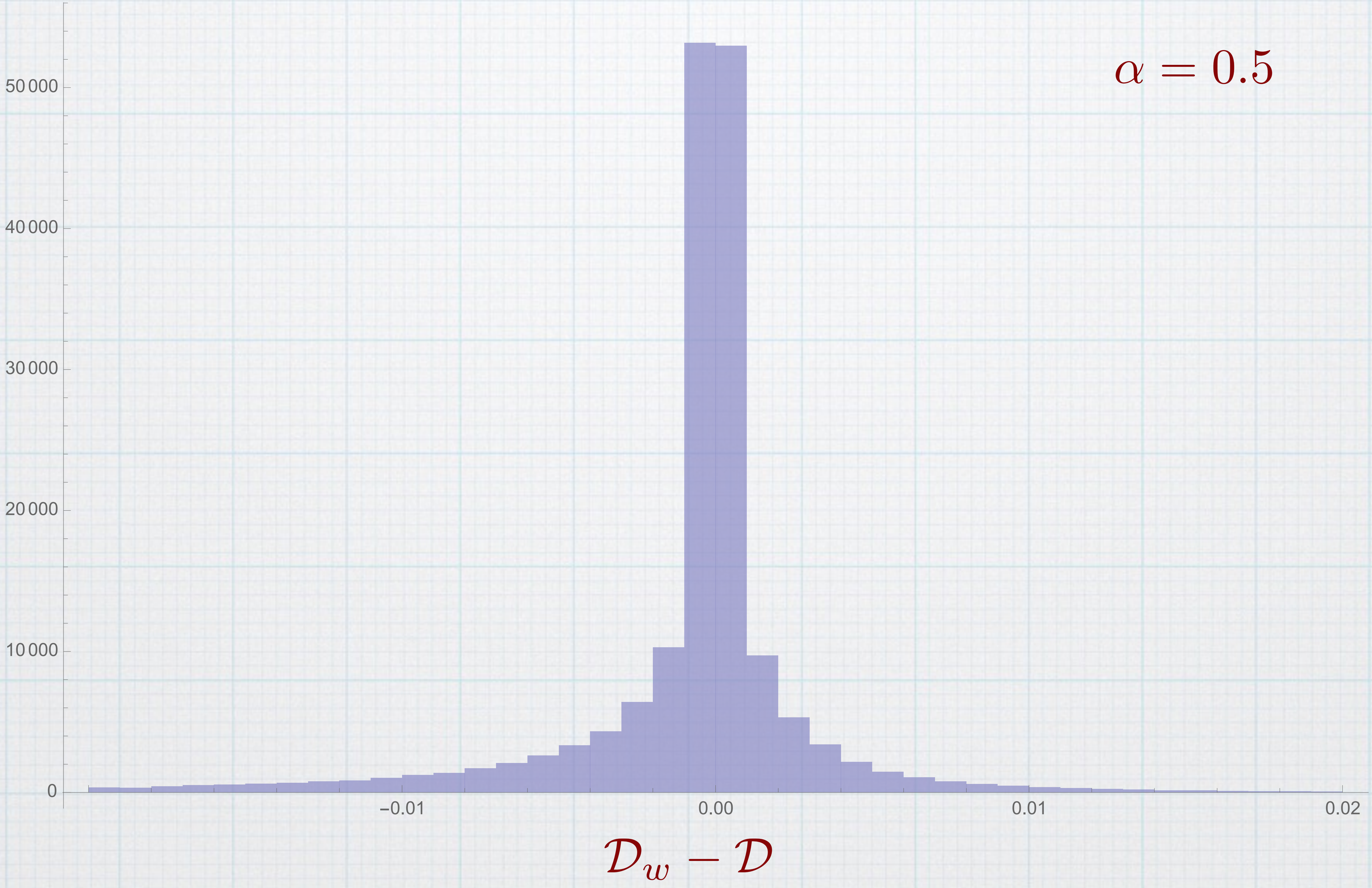

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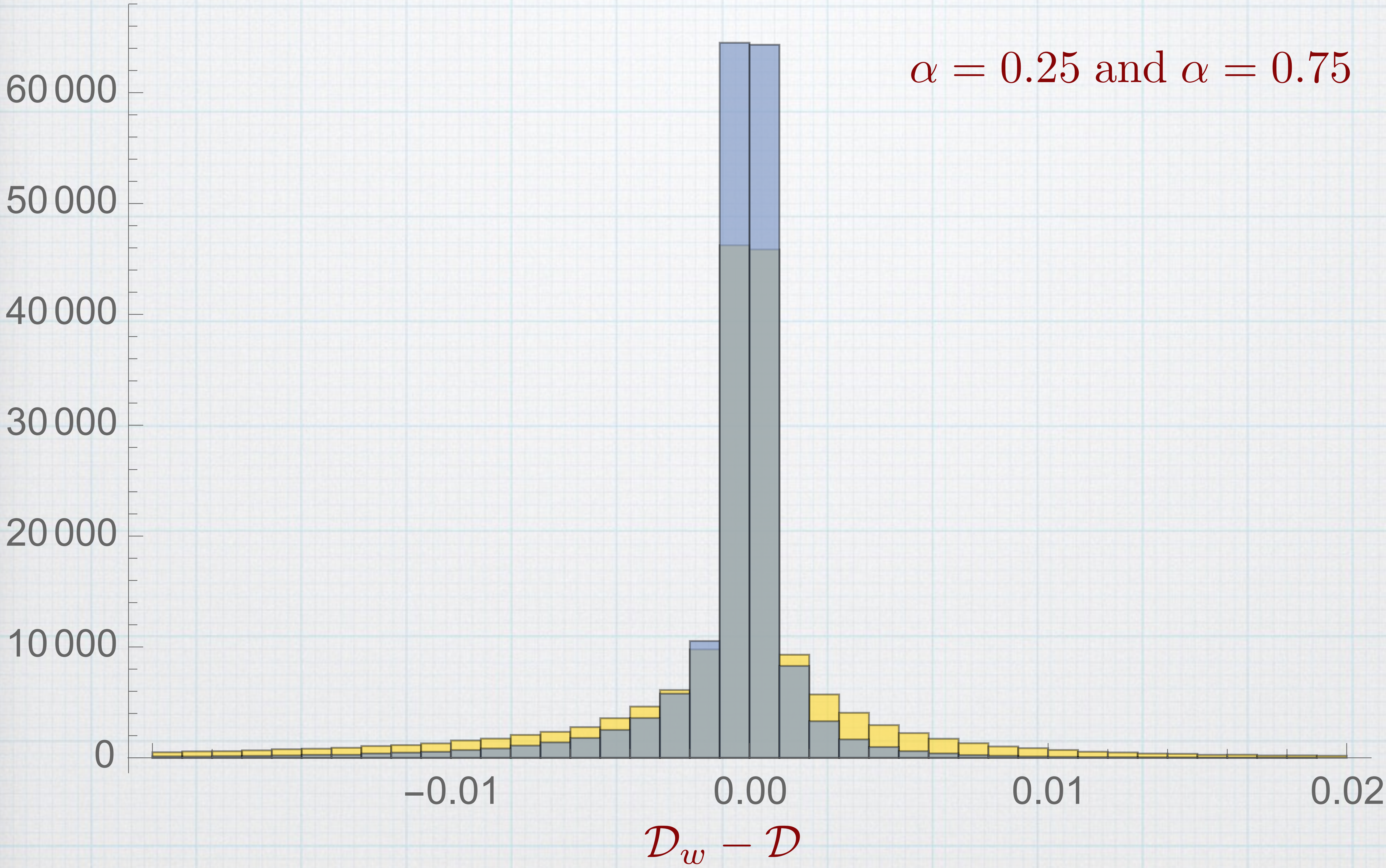

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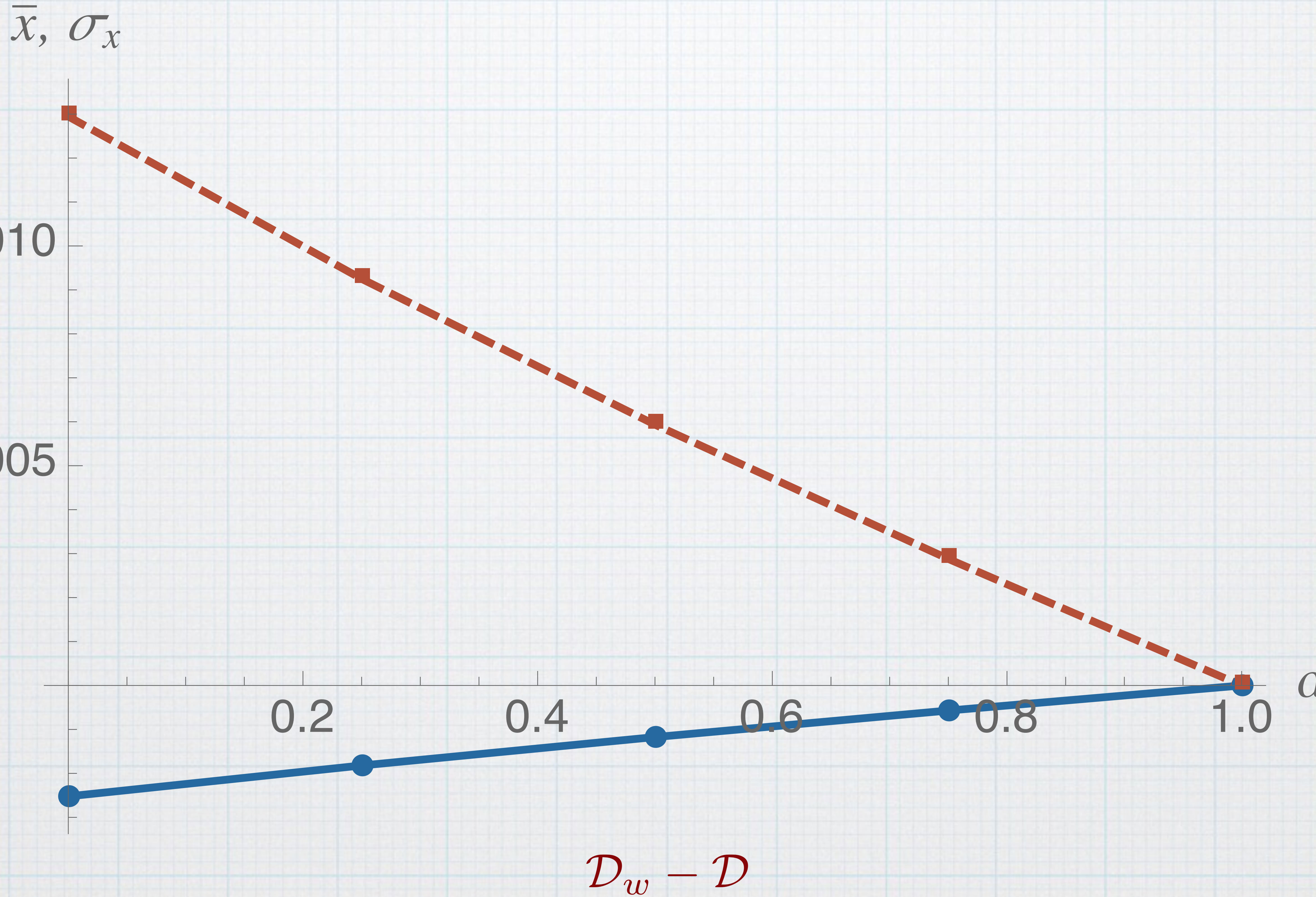


Disturbance on system B

* Olliver and Zurek's original interpretation of discord as the disturbance on B due to measurement on A

* If measuring on A is taken to mean, estimating p_k then weak measurements can do the same with small disturbance on A, B and AB

* The disturbance due to the weak measurements is characterized by

$$p_f = \mathrm{tr}(P_f \rho_{AB} P_f^\dagger)$$

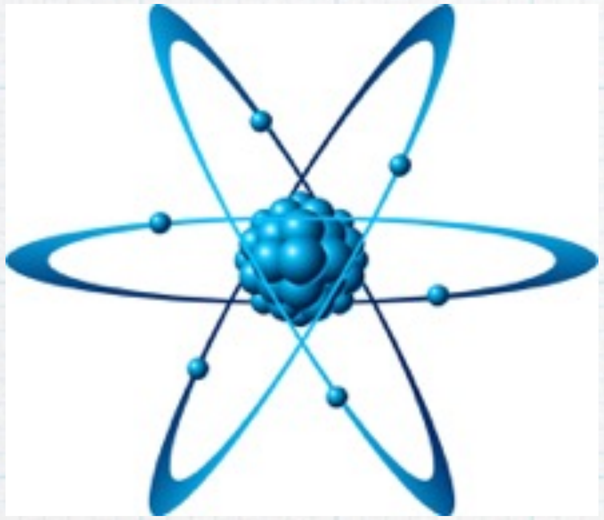
$$\rho_{AB}^2 = k\rho_{AB} \;\Rightarrow\; \rho'_{AB} = \frac{P_f \rho_{AB} P_f^\dagger}{\mathrm{tr}(P_f \rho_{AB} P_f^\dagger)} = \rho_{AB}$$

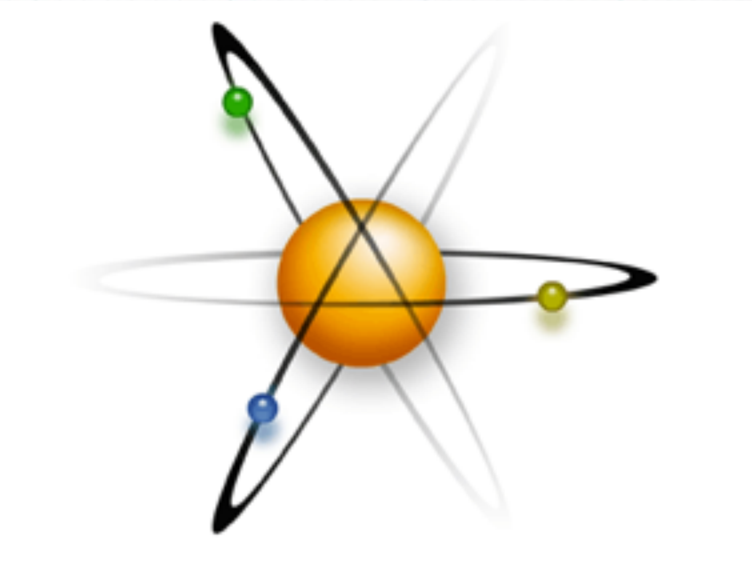
* The disturbance can be made arbitrarily small

$$p_f = \left[k(1-\alpha)+\alpha\right]^2$$

H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001)

Demon based interpretations





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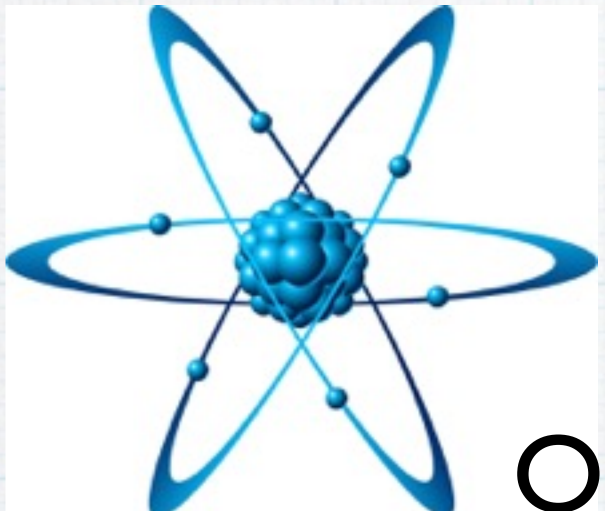
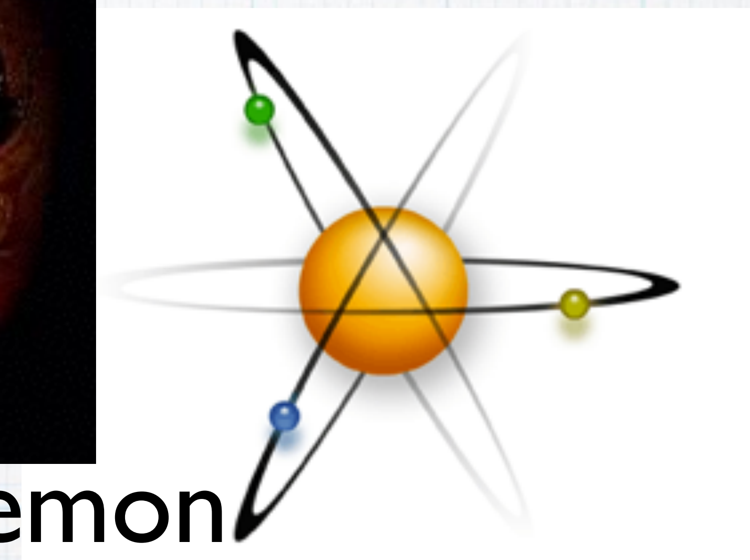
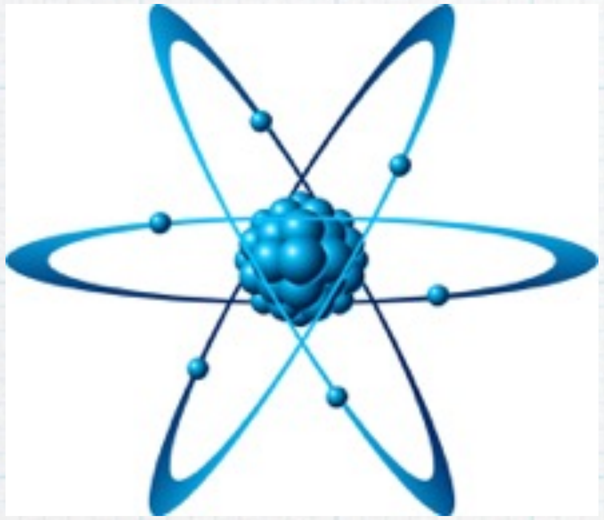
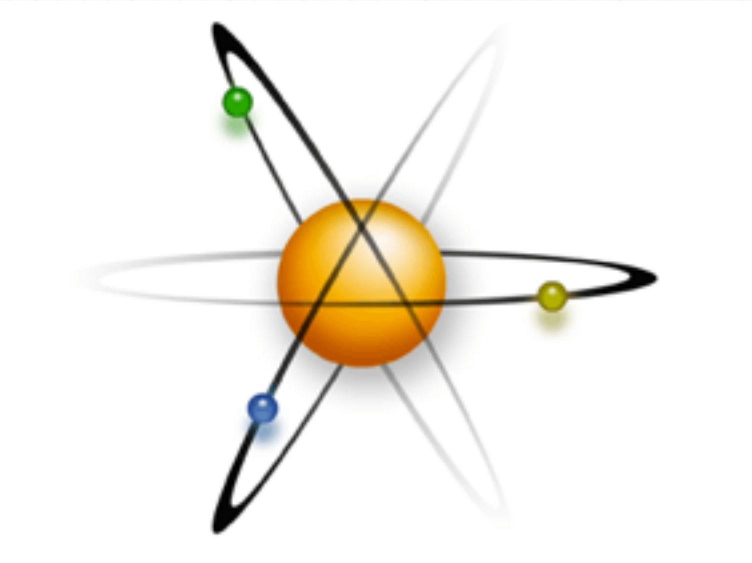




Quantum demon

Demon based interpretations





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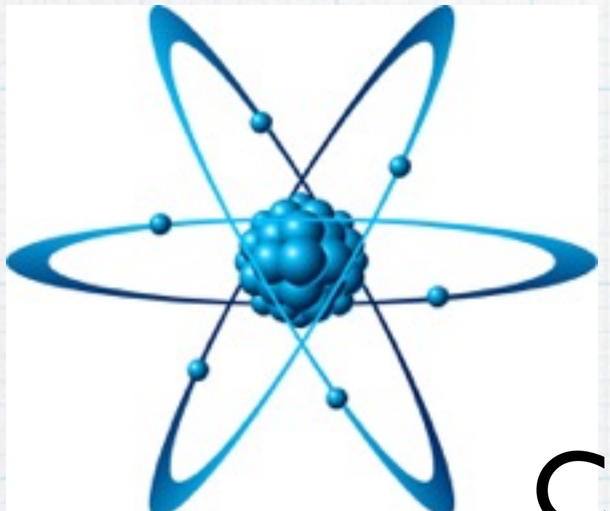
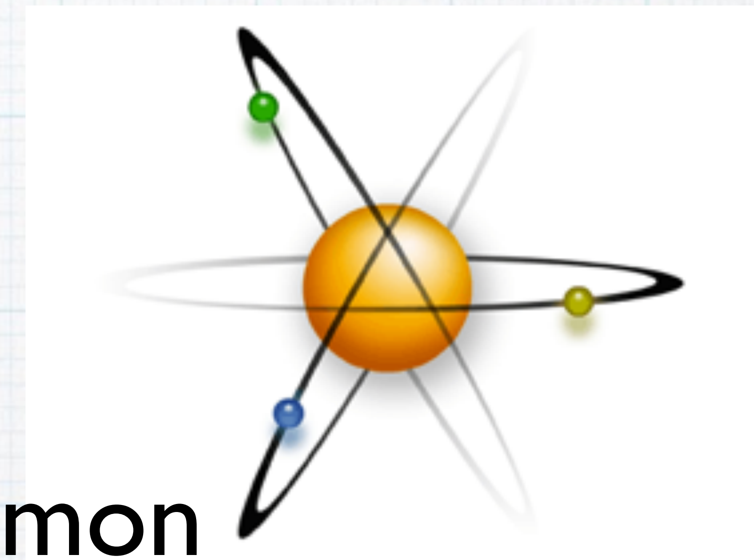
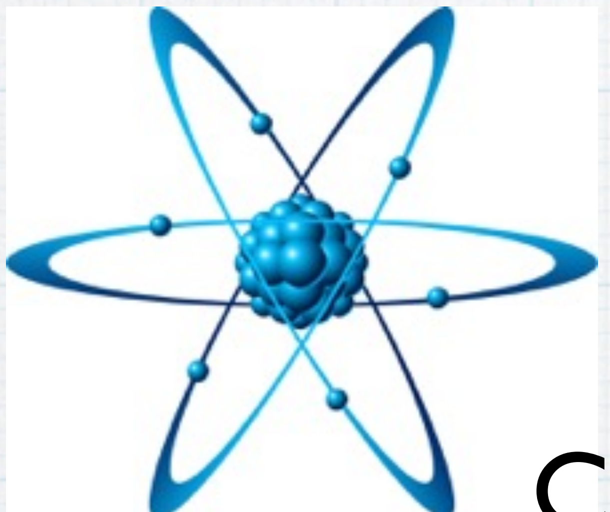


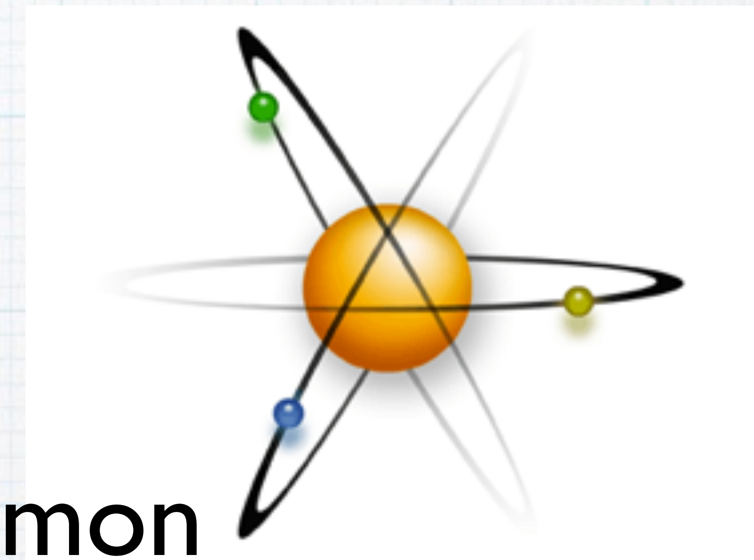


Classical demon

Demon based interpretations



Classical demon



* The demon(s) job is to extract maximum possible work out of the quantum system
* Quantum demons can see the whole quantum state of a bipartite system
* The classical demon has to employ one of its friends and measure the quantum system to know anything about it.
* Communication may or may not be allowed between the classical demons (Erasure with or without communication)

W. H. Zurek, Physical Review A **67**, 012320 (2003).
Oppenheim et al., Phys. Rev. Lett. **89**, 180402 (2002)

Demon based interpretations

* Work extracted from B by classical demon knowing A:

$$W_c^+ = \log d_B - S(B|\{\Pi_k^A\})$$

* Cost of resetting the demon's memory

$$W_c^- = \log d_A - S(A)$$

* Total work extracted by Classical demon

$$W_c = \log d_{AB} - [S(A) + S(B|\Pi_k^A)]$$

* Total work extracted by Quantum demon

$$W_q = \log d_{AB} - S(A, B)$$

* Discord is the difference - the interpretation goes through relatively unchanged.

Operational Interpretation(s)

* Operational interpretations based on state merging
* One of the two interpretations identifies quantum discord as the markup in the quantum communication needed from B to A to do state merging in case A chooses to measure her state before state merging.
* If the measurement is taken in the sense that the probabilities of various outcomes are estimated, then this operational interpretation does not apply in the case where weak measurements.

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D. Cavalcanti, L. Aolita, S. Boixo, K. Modi, M. Piani, and A. Winter, Physical Review A **83**, 032324 (2011)
V. Madhok and A. Datta, Physical Review A **83**, 032323 (2011)

Thank you