Multiqubit entangled channels for quantum communication in networks

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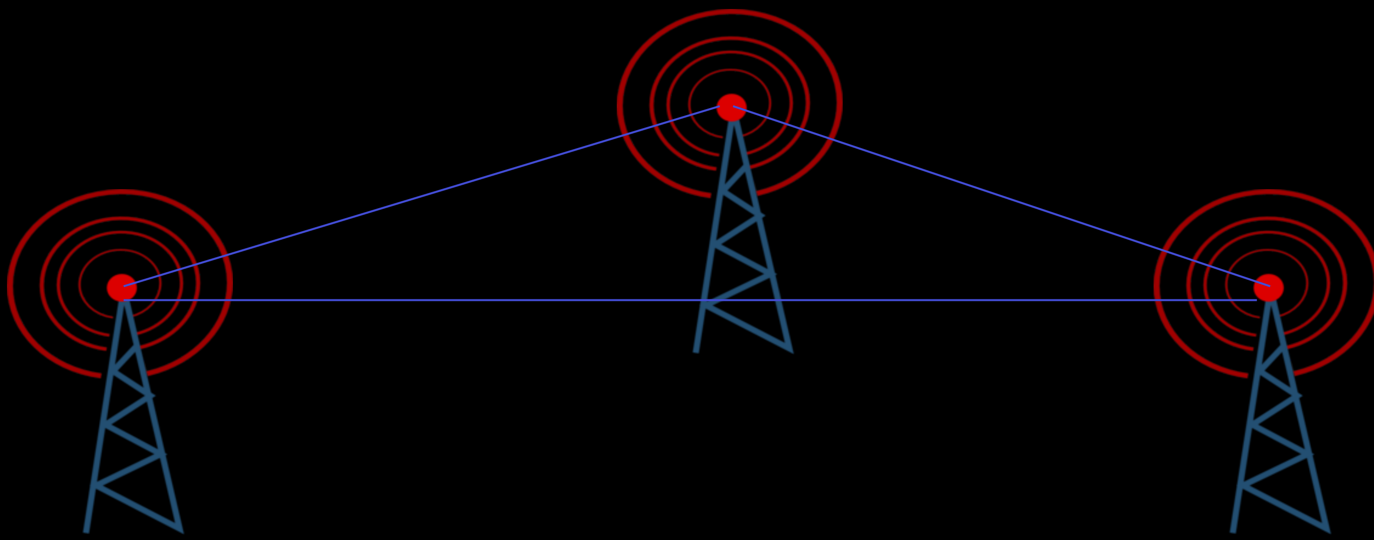

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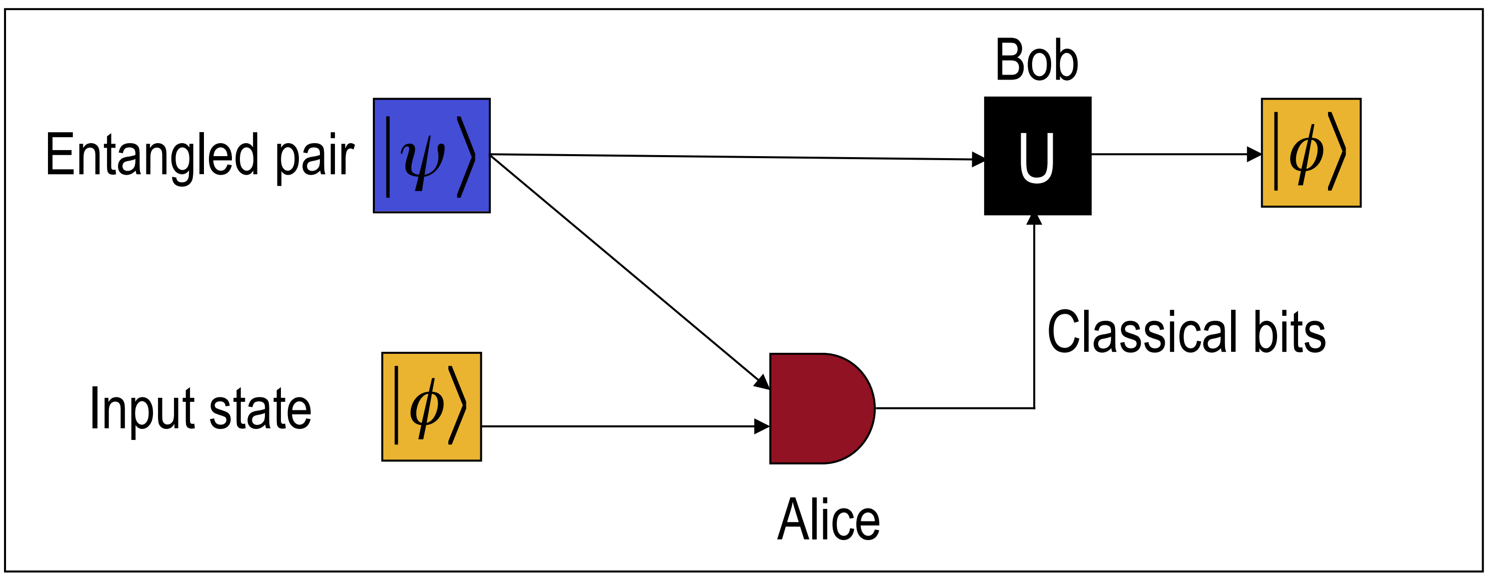
Introduction

2-qubit pure states:

$$|\psi\rangle = \alpha|00\rangle + \beta|11\rangle$$

Maximally entangled Bell states are useful for QIP

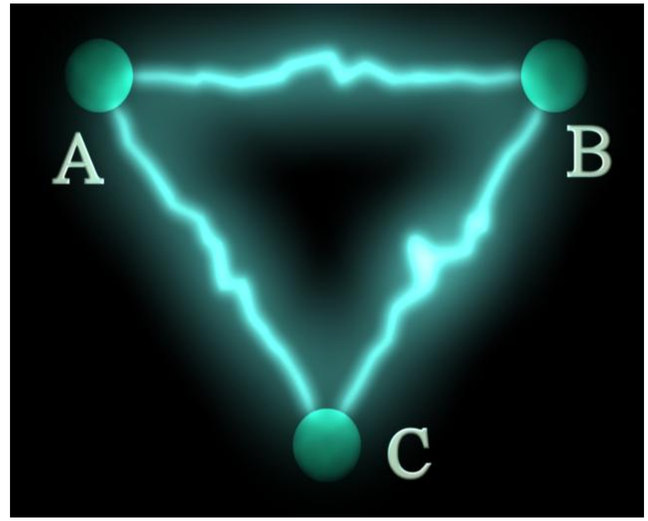
Example: **Teleportation**



Three-qubit entangled states

Generalized GHZ (GGHZ) states

$$|\psi_{GGHZ}\rangle = a|000\rangle + b|111\rangle$$

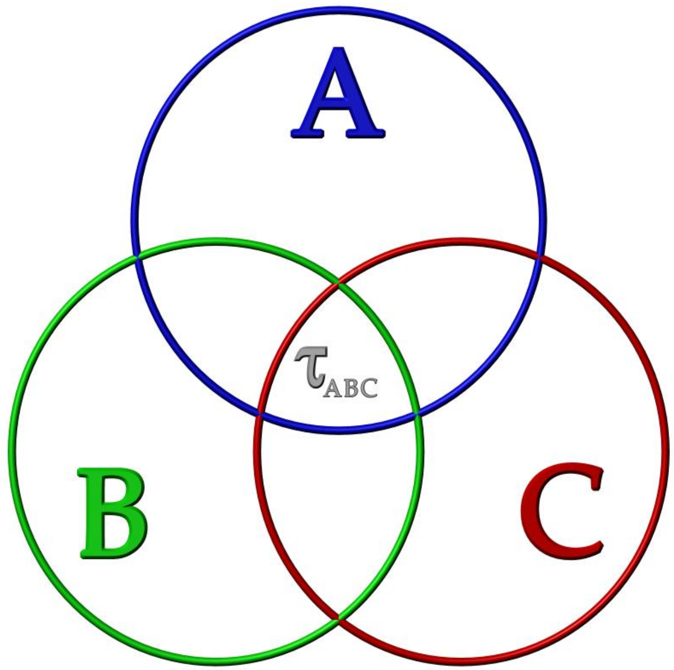


Maximal Slice (MS) States

$$|\psi_{MS}\rangle = \frac{1}{\sqrt{2}}\{|000\rangle + c|111\rangle + d|011\rangle\}$$

Three-qubit entanglement

3 qubits can have bipartite or tripartite entanglement.



Tripartite entanglement measure

3-tangle: $$\tau_{ABC} = \tau_{A(BC)} - \tau_{AB} - \tau_{AC}$$

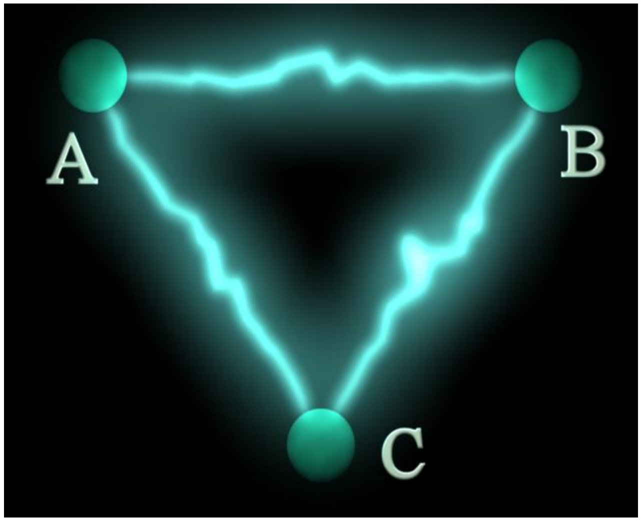
- V. Coffman, J. Kundu, and W. K. Wootters, PRA **61**, 052306 (2000)

Three-qubit entangled states

Generalized GHZ (GGHZ) states

$$\left|\psi_{GGHZ}\right\rangle = a\left|000\right\rangle + b\left|111\right\rangle$$

$$\tau_{ABC} = 4a^2b^2$$



Maximal Slice (MS) States

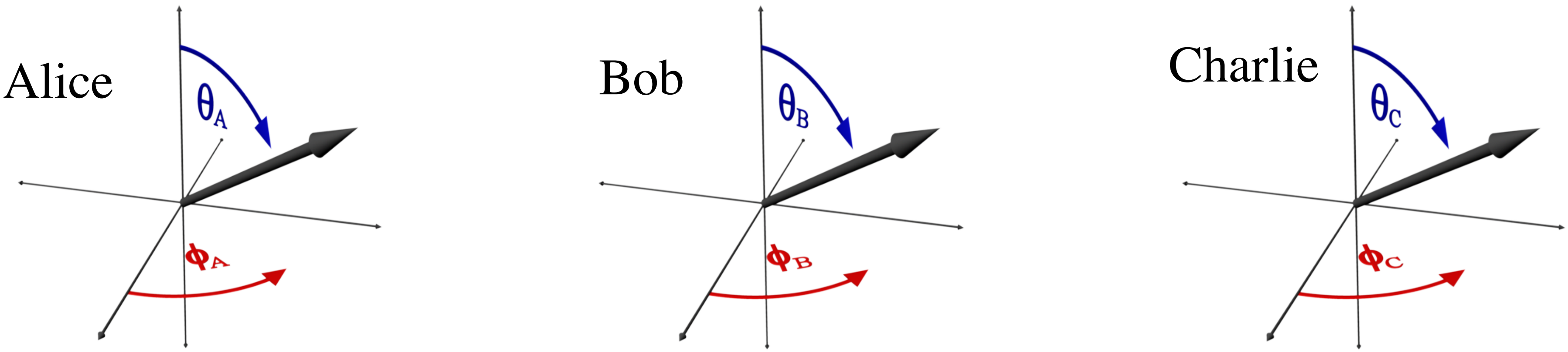
$$\left|\psi_{MS}\right\rangle = \frac{1}{\sqrt{2}}\left\{\left|000\right\rangle + c\left|111\right\rangle + d\left|011\right\rangle\right\}$$

$$\tau_{ABC} = 1 - d^2$$

Three-qubit Bell Inequality

Each qubits measured along one of two spin directions on Bloch sphere

$\boldsymbol{a, a', b, b', c, c'}$



Svetlichny's inequality : If at most two of the qubits are nonlocally correlated,

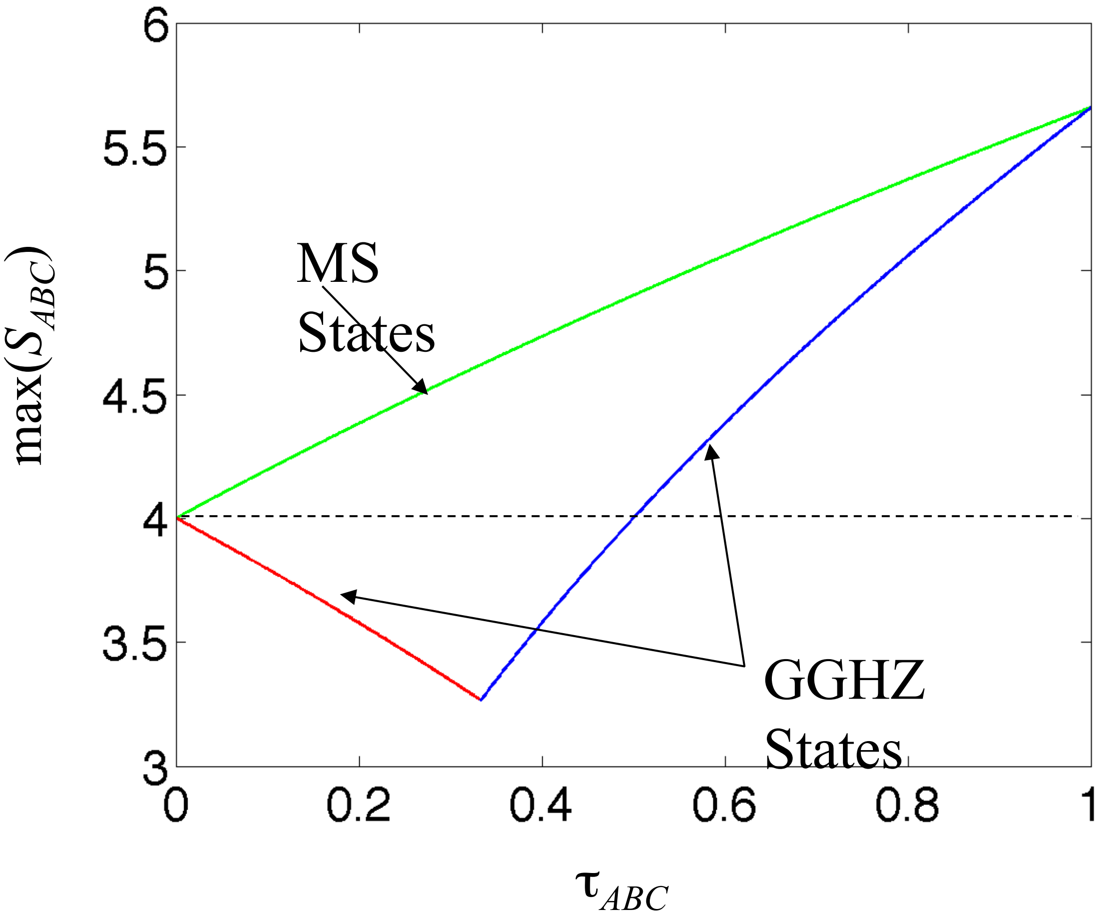
$$S_{ABC} = \left| \left\langle a\left(bk + b'k'\right) + a'\left(bk' - b'k\right) \right\rangle \right| \leq 4$$

$$k = c + c'$$

$$k' = c - c'$$

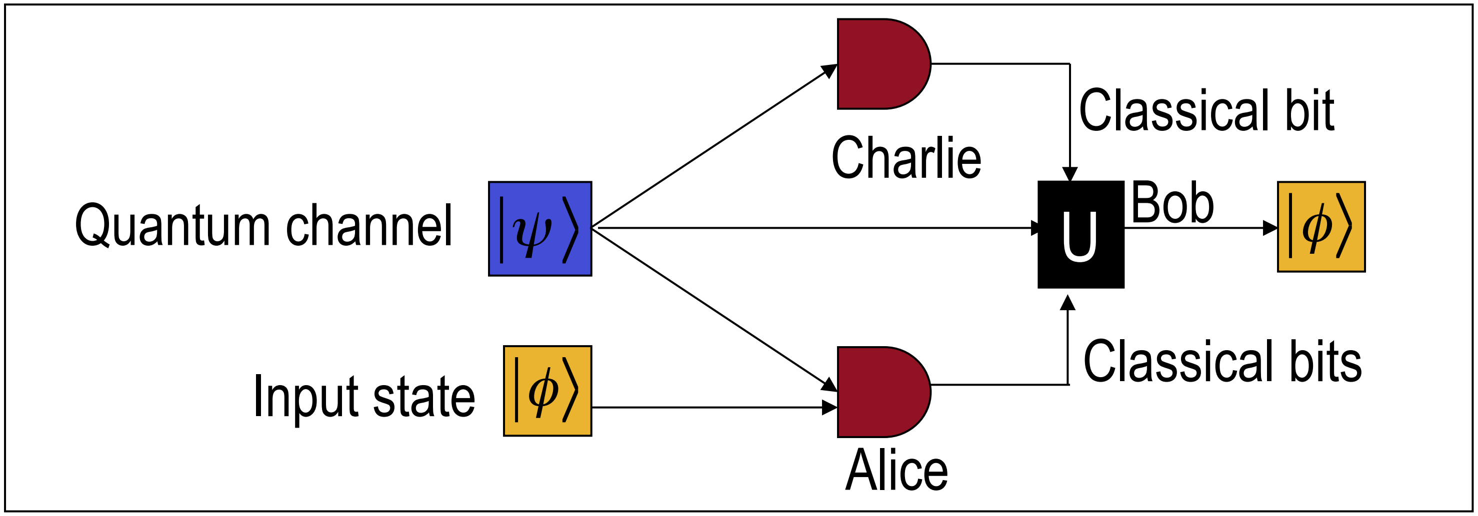
G. Svetlichny, PRD **35**, 3066 (1987)

Three-qubit entanglement versus nonlocality



S. G. *et al.*, PRL **102**, 250404 (2009)

Controlled Teleportation

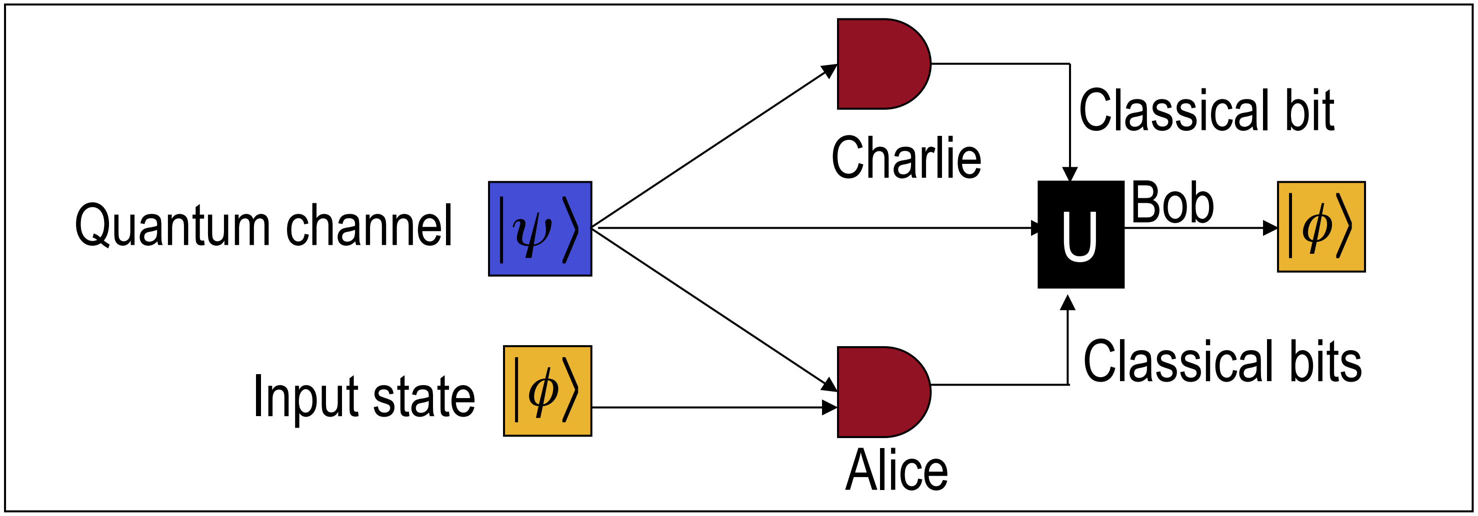


Scenario:

- Charlie controls the teleportation of a qubit from Alice to Bob.
- Bob can only reconstruct the state Alice wants to teleport if Charlie participates in the process.

A Karlsson and M. Bourennane, PRA **58**, 4394, (1998)
M. Hillery, V. Buzek and A. Berthiaume, PRA **59**, 1829 (1999)

Perfect Controlled Teleportation



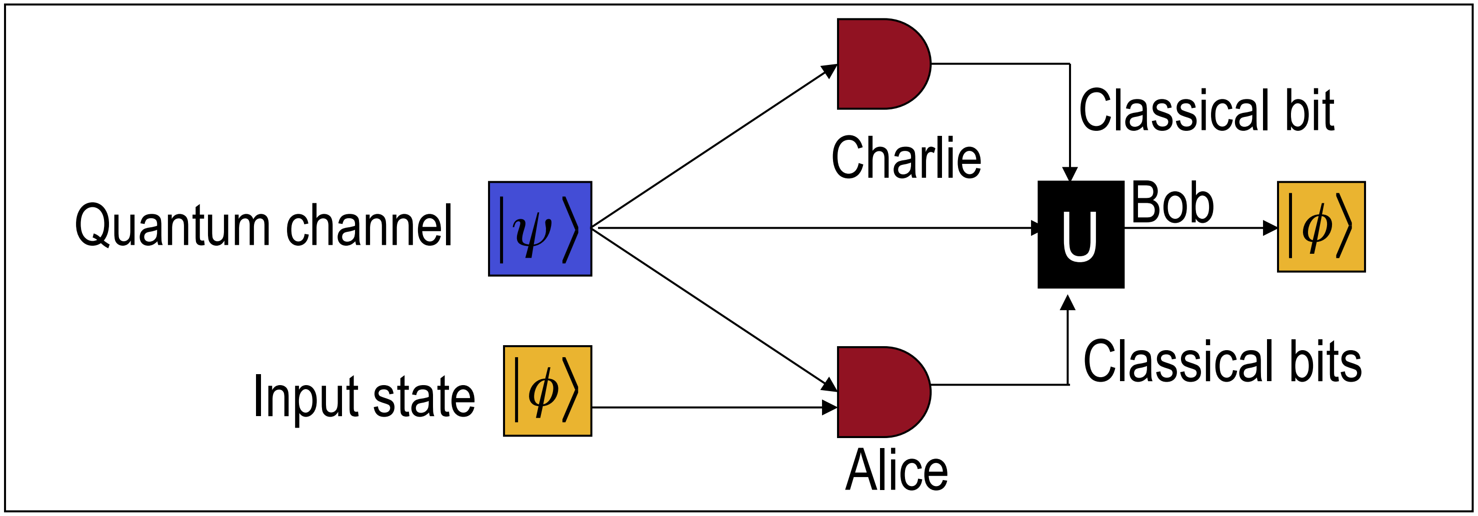
Quantum channel

$$|\psi_{GGHZ}\rangle = a|000\rangle + b|111\rangle$$ ✗

$$|\psi_{MS}\rangle = \frac{1}{\sqrt{2}}\{|000\rangle + c|111\rangle + d|011\rangle\}$$ ✓

T. Gao, F. L. Yan and Y. C. Li, EPL **84**, 50001 (2008)

Perfect Controlled Teleportation



Quantum channel

$$|\psi_{MS}\rangle = \frac{1}{\sqrt{2}}\{|000\rangle + c|111\rangle + d|011\rangle\}$$

$$= \frac{1}{2}\left[(1+d)|0\rangle + c|1\rangle\right]\otimes|\Phi^+\rangle + \frac{1}{2}\left[(1-d)|0\rangle - c|1\rangle\right]\otimes|\Phi^-\rangle$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\{|00\rangle + |11\rangle\} \qquad |\Phi^-\rangle = \frac{1}{\sqrt{2}}\{|00\rangle - |11\rangle\}$$

Control Power

- If Charlie does not participate, Alice can still make Bell measurements.
- Bob's reduced state after Alice's measurement is mixed.
- Non-conditioned fidelity of teleportation:

$$f = \langle \varphi | \rho | \varphi \rangle$$

Control Power

$$C = 1 - \overline{f}$$

Classical fidelity limit

S. Popescu, PRL 74, 1259 (1995)

$$f_{cl} = \frac{2}{3}$$

Lower bound on control power

$$C_{\min} = 1 - f_{cl} = \frac{1}{3}$$

Control Power: MS States

Input state

$$|\phi\rangle = k_0|0\rangle + k_1|1\rangle$$

Quantum channel

$$|\psi_{MS}\rangle = \frac{1}{\sqrt{2}}\{|000\rangle + c|111\rangle + d|011\rangle\}$$

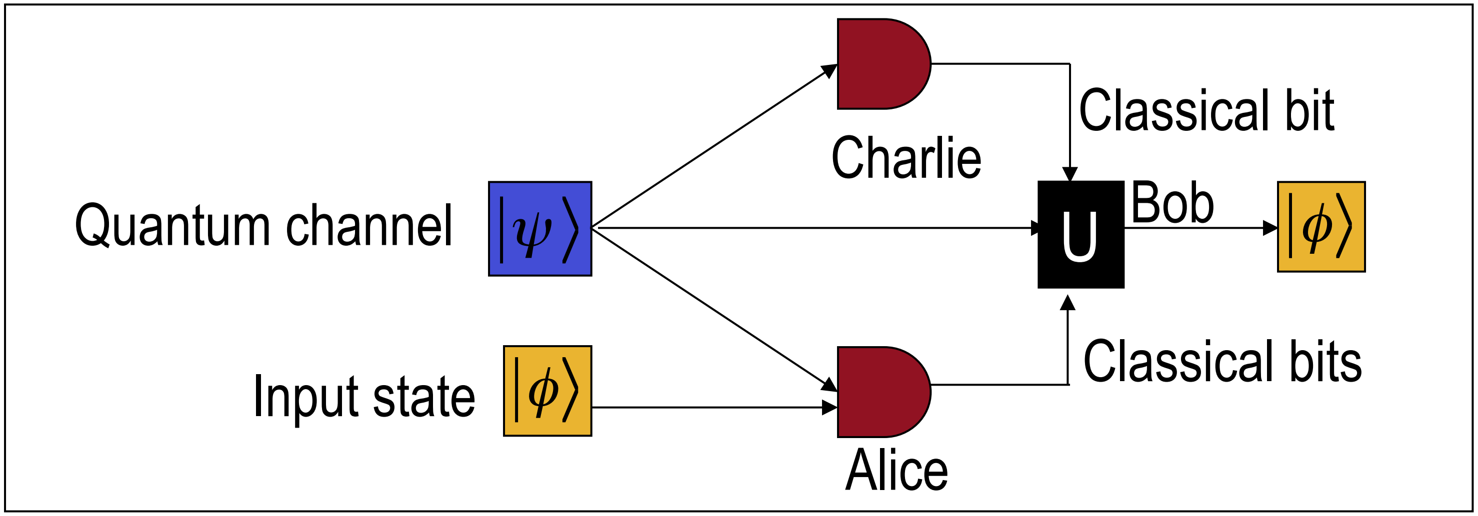
Teleportation fidelity without controller

$$f = |k_0|^4 + |k_1|^4 + 2|d||k_0|^2|k_1|^2 \rightarrow \bar{f} = \frac{2}{3} + \frac{|d|}{3}$$

Control Power

$$C = 1 - \bar{f} = \frac{1}{3} - \frac{|d|}{3} \leq \frac{1}{3}$$

Perfect Controlled Teleportation

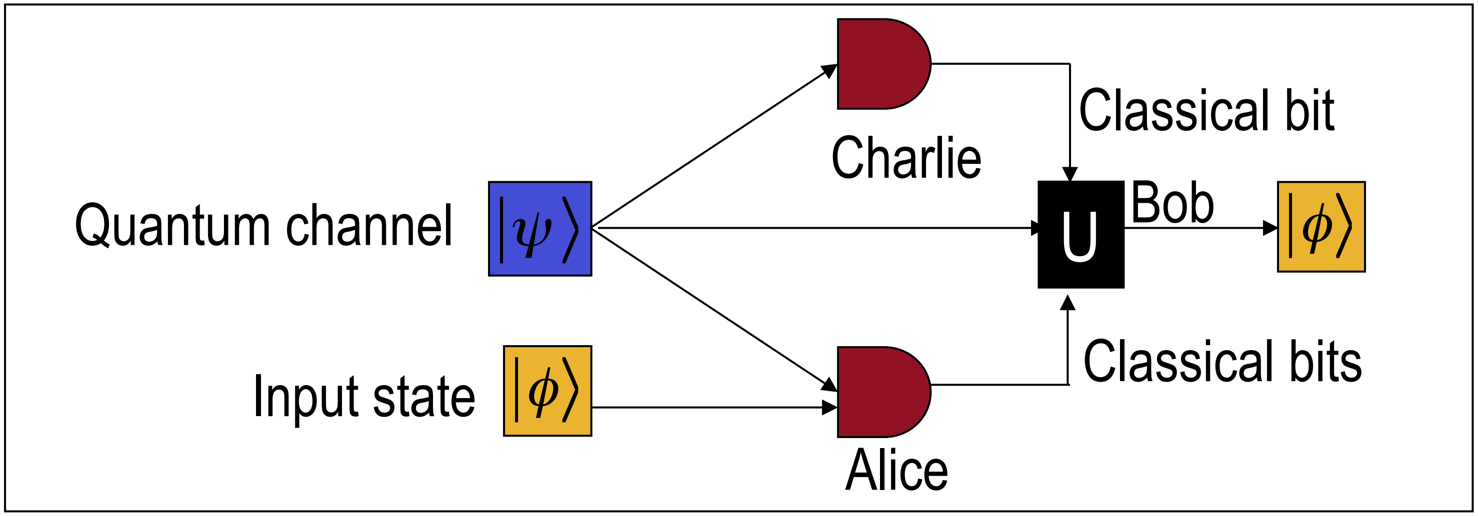


Quantum channel

$$|\psi_{GGHZ}\rangle = a|000\rangle + b|111\rangle$$ ✗

$$|\psi_{MS}\rangle = \frac{1}{\sqrt{2}}\{|000\rangle + c|111\rangle + d|011\rangle\}$$ ✗

Control power for teleporting equatorial states



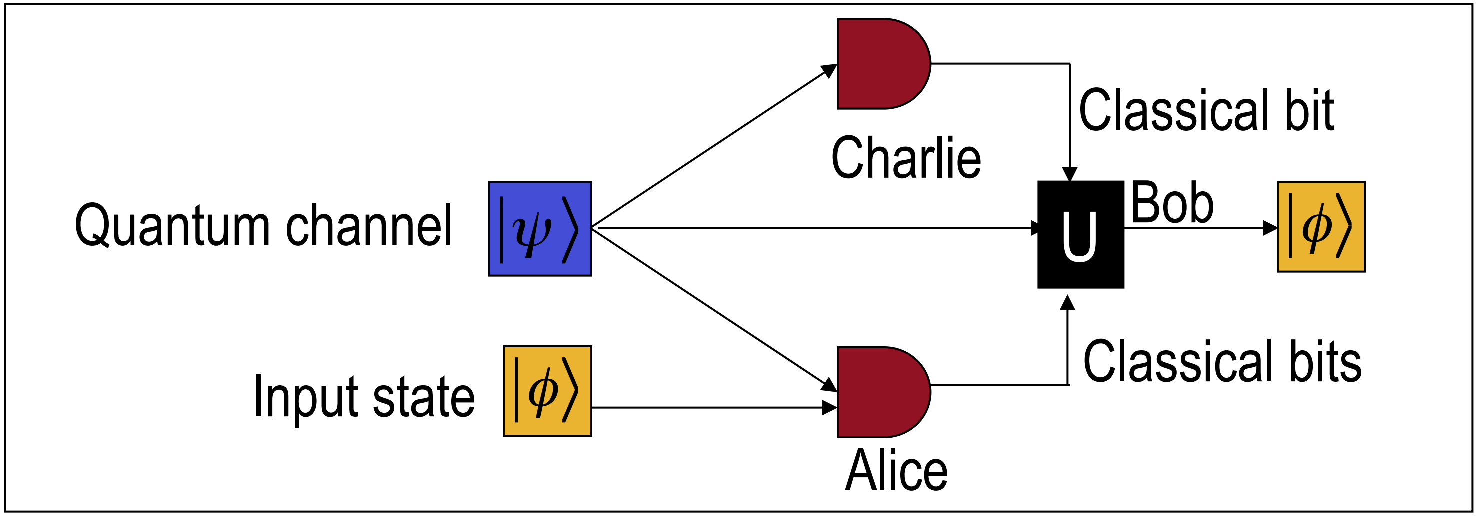
Equatorial states

$$|\phi_{xz}\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}|1\rangle$$

$$|\phi_{xy}\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{e^{i\varphi}}{\sqrt{2}}|1\rangle$$

$$|\phi_{yz}\rangle = \cos\frac{\theta}{2}|0\rangle + i\sin\frac{\theta}{2}|1\rangle$$

Control power for teleporting equatorial states



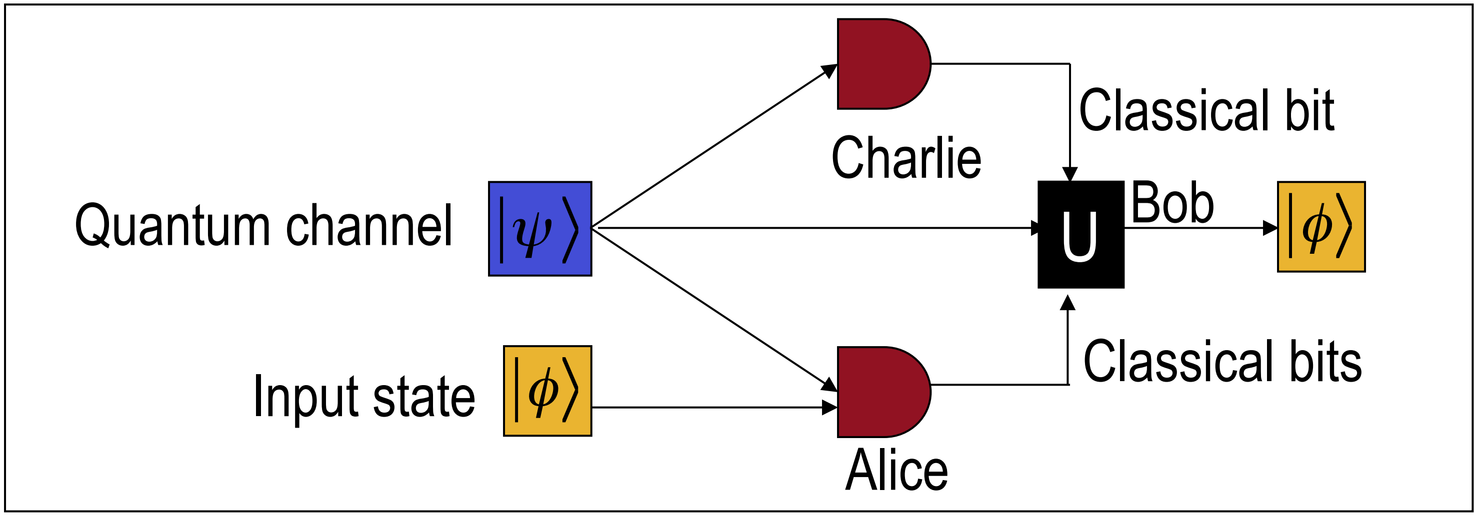
Quantum channels

$$|\Theta_{xz}\rangle = a|0\rangle|\Phi^+\rangle + b|1\rangle|\Psi^-\rangle$$

$$|\Theta_{xy}\rangle = a|0\rangle|\Phi^+\rangle + b|1\rangle|\Phi^-\rangle$$

$$|\Theta_{xy}\rangle = a|0\rangle|\Phi^+\rangle + b|1\rangle|\Psi^+\rangle$$

Control power for teleporting equatorial states



Quantum channels

$$|\Theta_{xz}\rangle = a|0\rangle|\Phi^+\rangle + b|1\rangle|\Psi^-\rangle$$

$$|\Theta_{xy}\rangle = a|0\rangle|\Phi^+\rangle + b|1\rangle|\Phi^-\rangle$$

$$|\Theta_{xy}\rangle = a|0\rangle|\Phi^+\rangle + b|1\rangle|\Psi^+\rangle$$

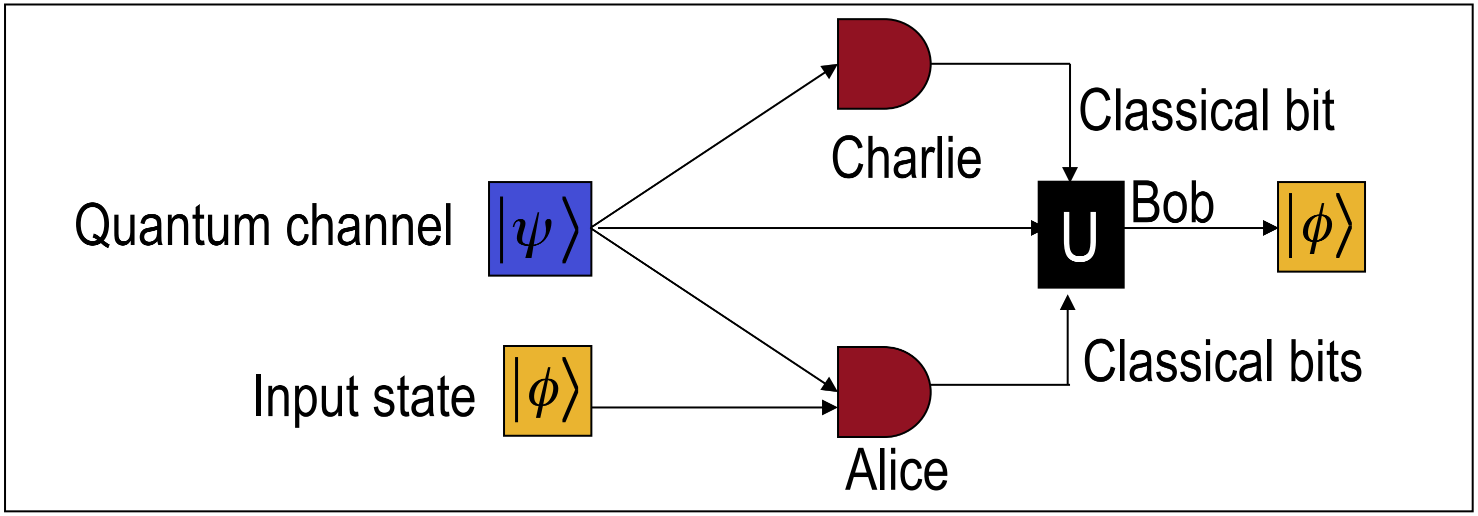
Control Power

$$C = 1 - \max(a^2, b^2)$$

$$\tau = 4a^2b^2 \geq \frac{8}{9} \Rightarrow C \geq \frac{1}{3}$$

X. Li and SG, PRA **90**, 052305 (2014)

Control power in mismatched channels



Equatorial state

$$|\phi_{xz}\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}|1\rangle$$

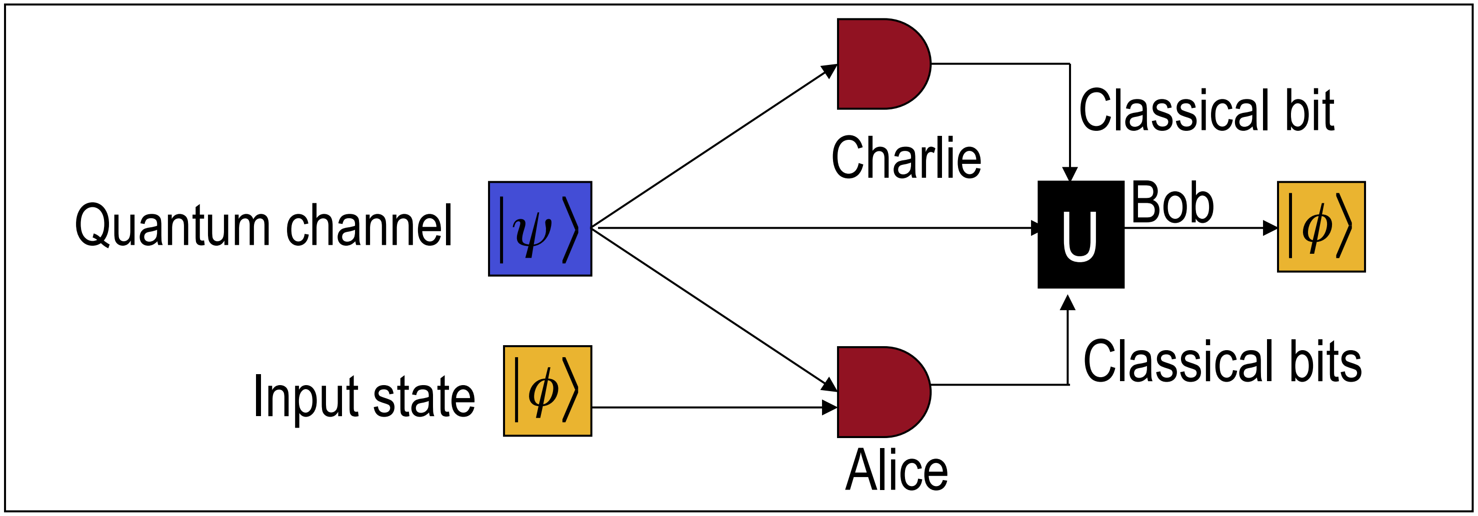
Mismatched channel

$$|\Theta_{xy}\rangle = a|0\rangle|\Phi^+\rangle + b|1\rangle|\Psi^+\rangle$$

Control Power

$$C \leq \frac{1}{3}$$

A partially entangled channel that ensures control



Quantum state

$$|\psi\rangle = a|\Phi^+\rangle_{AB}|00\rangle_C + b|\Phi^-\rangle_{AB}|01\rangle_C + c|\Psi^+\rangle_{AB}|10\rangle_C + d|\Psi^-\rangle_{AB}|11\rangle_C$$

Control Power

$$C = 1 - a^2 - \frac{1}{3}\left(b^2 + c^2 + d^2\right)$$

N-qubit control power

- Alice wants to teleport an N-qubit state to Bob. There are M controllers.
- To compute the mth controller's power, let Alice and other controllers perform their measurements
- Then tracing over the mth controller's state we obtain Bob's reduced state
- Nonconditioned fidelity:

$$f = \langle \varphi | \rho | \varphi \rangle$$

Control power

$$C = 1 - \bar{f}$$

Classical fidelity limit

$$f_{cl} = \frac{2}{2^N + 1}$$

Lower bound on control power of mth controller

$$C_{\min} = 1 - f_{cl} = \frac{2^N - 1}{2^N + 1}$$

P. Badziag et al., PRA 62, 012311 (2000)

Assessing N-qubit control schemes

2-GHZ scheme

F. G. Deng, et al., PRA **72**, 022338 (2005)

- Input: 2-qubit state
- Quantum channel: 2 GHZ states
- Alice performs 2 Bell measurements
- Charlie performs 1 Bell measurement

Control power

$$C = 1 - \bar{f} = \frac{3}{5}$$

$$C_{\min} = \frac{2^N - 1}{2^N + 1} = \frac{3}{5}$$

Assessing N-qubit control schemes

N-GHZ scheme with m controllers

X. H. Li, et al., J. Phys. B **39**, 1975 (2006)

- Input: N-qubit state
- Quantum channel: N (m+2)-qubit GHZ states. Each controller owns N qubits
- Alice performs N Bell measurements
- Each controller performs single qubit measurements

Control power of each controller

$$C = \frac{2^N - 1}{2^N + 1}$$

$$C_{\min} = \frac{2^N - 1}{2^N + 1}$$

Assessing N-qubit control schemes

Bell-GHZ scheme with m controllers

C. P. Yang and S. Han, Phys. Lett. A **343**, 267 (2005)
C. P. Yang, S. I. Chu, and S. Han, PRA **70**, 022329 (2004).

- Input: N-qubit state
- Quantum channel: Bell-GHZ superpositions. Each controller owns 1 qubit
- Each controller performs single qubit measurements

Control power of each controller

$$C = \frac{2^N - 2^{N-1}}{2^N + 1}$$

$$C_{\min} = \frac{2^N - 1}{2^N + 1}$$

Assessing N-qubit control schemes

Bell-GHZ II scheme with m controllers

Z. X. Man, Y.J. Xia,and N.B. An, PRA **75**, 052306 (2007).
Z. X. Man, Y.J. Xia,and N.B. An, J. Phys. B **40**,1767 (2007).

- Input: N-qubit state
- Quantum channel: Bell pairs, GHZ states. Each controller owns 1 qubit
- Each controller performs single qubit measurements

Control power of each controller

$$C \leq \frac{1}{2}$$

$$C_{\min} = \frac{2^N - 1}{2^N + 1}$$

Resources required for N-qubit control

Suppose a controller, Charlie has N-1 qubits from a maximally entangled channel. Then

$$\rho_B = \frac{1}{2^{N-1}}|\varphi\rangle\langle\varphi| + \frac{1}{2^{N-1}}\sum_{i=1}^{2^{N-1}-1}|\varphi_i\rangle\langle\varphi_i| \rightarrow \bar{f} > \frac{1}{2^{N-1}}$$

Control power of each controller

$$C < \frac{2^{N-1}-1}{2^{N-1}}$$

$$C_{\min} = \frac{2^N-1}{2^N+1}$$

Each controller should have at least N qubits

X. Li and SG, PRA **91**, 012320 (2015)

N-qudit control power

- Alice wants to teleport an N-qudit state to Bob. There are M controllers.
- To compute the mth controller's power, let Alice and other controllers perform their measurements
- Then tracing over the mth controller's state we obtain Bob's reduced state
- Nonconditioned fidelity:

$$f = \langle \varphi | \rho | \varphi \rangle$$

Control power

$$C = 1 - \bar{f}$$

Classical fidelity limit

$$f_{cl} = \frac{2}{d^N + 1}$$

Lower bound on control power of mth controller

$$C_{\min} = 1 - f_{cl} = \frac{d^N - 1}{d^N + 1}$$

X. Li and SG, PRA **91**, 012320 (2015)

Summary

- In controlled teleportation both teleportation and control are important.
- Control power is a quantitative way to measure the controller's authority in controlled communication tasks.
- Certain partially entangled states can provide adequate control power for teleporting single qubits. In specific cases, partially entangled channels outperform maximally entangled channels.
- Control power can be used to assess N-qubit controlled teleportation schemes with m controllers.
- Each controller must possess at least N qubits to retain sufficient control.
- Control power can be generalized to assess N-qudit control schemes.