EFFICIENCY OF QUANTUM CONTROLLED NON-MARKOVIAN THERMALIZATION

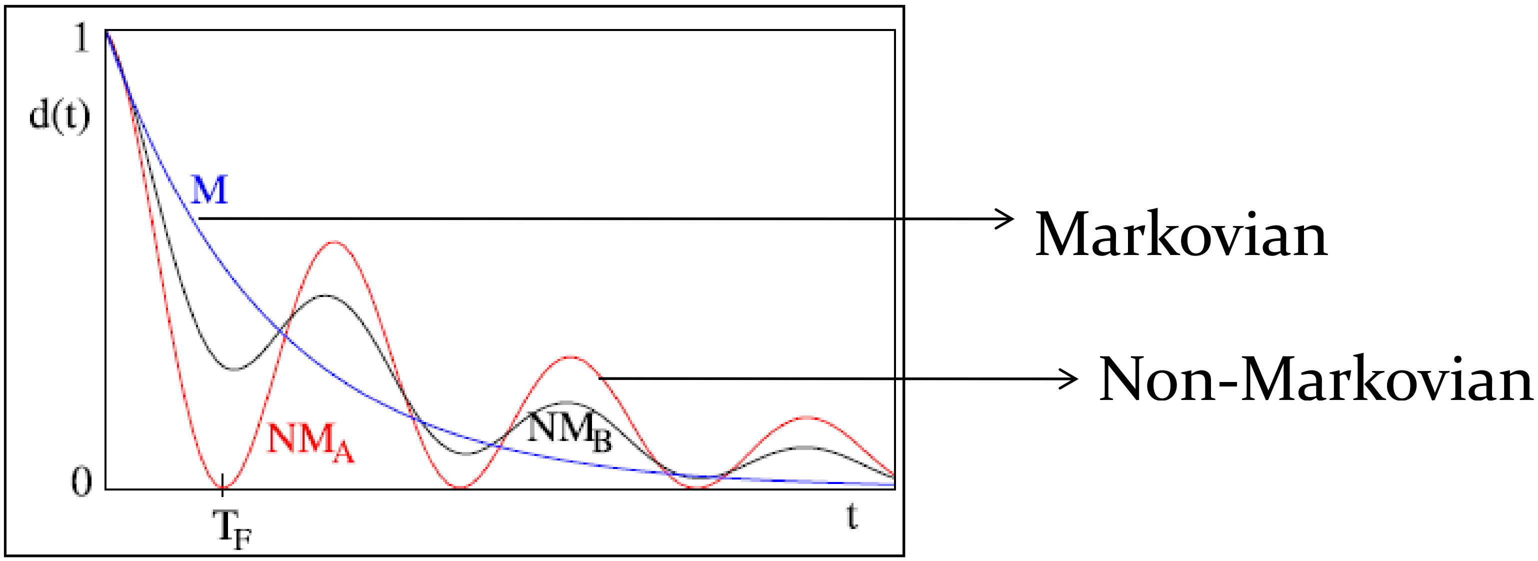
Victor Mukherjee
Weizmann Institute of Science, Rehovot, Israel

Ref: 1) V. Mukherjee, V. Giovannetti, R. Fazio, S. F. Huelga, T. Calarco and S. Montangero, New J. Phys. **17** (2015) 063031 .

2) V. Mukherjee, A. Carlini, A. Mari, T. Caneva, S. Montangero and T. Calarco, Phys. Rev. A **88**, 062326 (2013).

MOTIVATION

➢ *Open question* – Can we exploit the memory effects of NM dynamics to control a system more effectively?



Our focus: Using *information back-flow* from non-Markovian bath to a generic system to our advantage - controlling the relaxation dynamics of the system

Application ⟶ Cooling a quantum system in the minimum possible time

Main Challenges

- Naïve inclusion of arbitrary time dependent unitary control might give us unphysical results, eg. negative probabilities.

- Non-Markovian baths are in general finite size baths - Unitary control applied to a system influences the bath non-trivially.

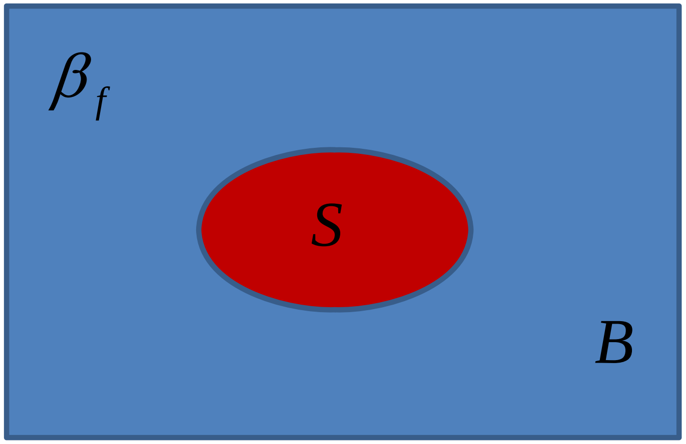
1. D. Chruscinski and A. Kossakowski, Phys. Rev. Lett. **104**, 070406 (2010).
2. A. Rivas, S. F. Huelga, and M. B. Plenio, Rep. Prog. Phys. **77**, 094001 (2014)

OUTLINE

- Markovian and non-Markovian dynamics
- Controlling a quantum system in presence of a Markovian (M) bath
- Controlling a quantum system in presence of a non-Markovian (NM) bath

  A necessary condition to be satisfied so that the effectiveness of optimal control is enhanced in presence of a NM bath subject to suitable unitary controls.
- Conclusion

Quantum system in presence of a dissipative bath



System in contact with a dissipative bath at temperature β_f

$$\rho_i = \rho(t=0) = \begin{pmatrix} \alpha & \zeta \\ \zeta^* & 1-\alpha \end{pmatrix}$$

$$\rho_{fp} = \begin{pmatrix} \dfrac{e^{-\beta_f}}{1+e^{-\beta_f}} & 0 \\ 0 & \dfrac{1}{1+e^{-\beta_f}} \end{pmatrix}$$

Fixed point

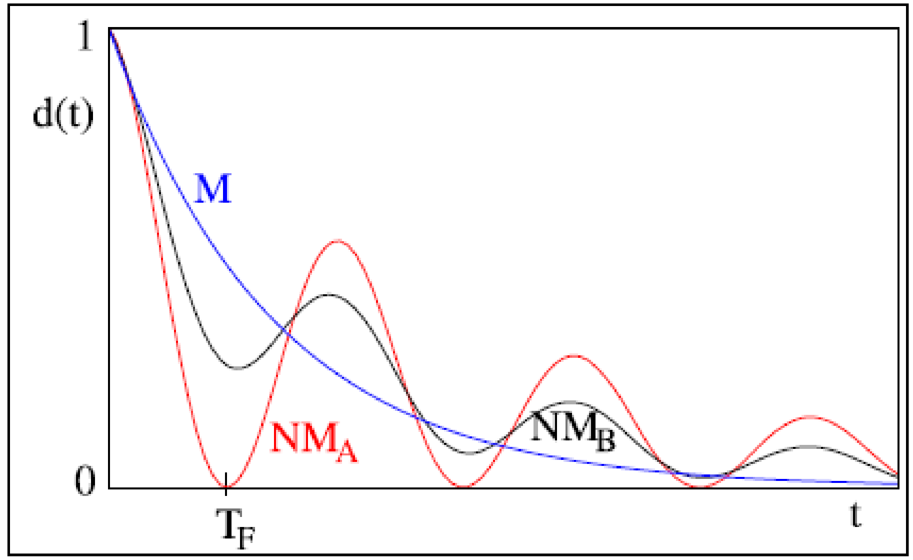
Aim: We want to reach ρ_{fp} in the shortest possible time

→ *Quantum Speed Limit*

$$\dot{\rho}(t) = \gamma_t L(\rho(t))$$

$\gamma_t > 0 \;\; \forall t \longrightarrow$ M

$\gamma_t < 0$ For some time interval $\longrightarrow$ NM



$$\int_0^t \gamma_\tau d\tau \geq 0 \longrightarrow \text{Ensures CP}$$

$$d(t) = \left\| \rho(t) - \rho_{fp} \right\|$$

$$\frac{\partial d(t)}{\partial t} > 0 \longrightarrow \text{Related to measure of non-Markovianity}$$

Optimal protocols to minimize the time to relax to the fixed point

1) H.-P. Breuer, E.-M. Laine and J. Piil, Phys. Rev. Lett. **103**, 210401 (2009).

2) D. Chruscinski and A. Kossakowski, Phys. Rev. Lett. **104**, 070406 (2010).

<u>Controlling a single qubit in presence of a Markovian bath</u>

$$\dot{\rho} = -i[H_s(t), \rho] + \gamma_0 L(\rho)$$

$$L(\rho) = \sum_a \left[L_a \rho L_a^+ - \frac{1}{2}\left(L_a^+ L_a \rho + \rho L_a^+ L_a \right) \right]$$

$$(L_1)_{AD} = \sigma^+, \quad (L_2)_{AD} = e^{\beta/2}\sigma^-, \quad \gamma_0 > 0$$

$$L(\rho_{fp}) = 0 \longrightarrow \textit{Fixed point}$$

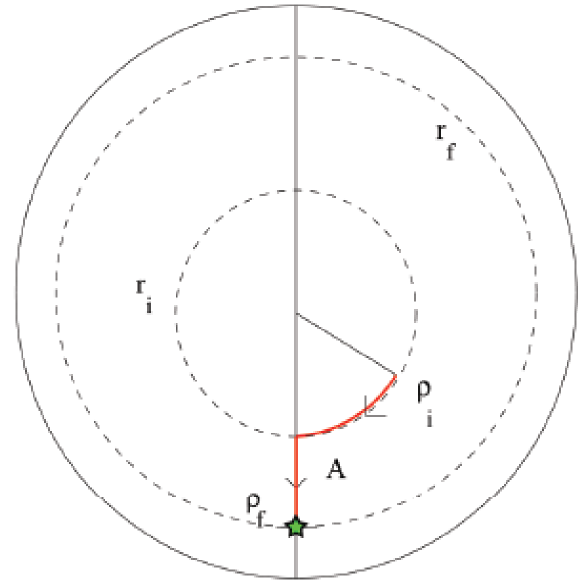
$$\rho = \frac{1}{2}(I + \vec{\sigma}.\vec{r}) \rightarrow \text{Bloch Sphere}$$

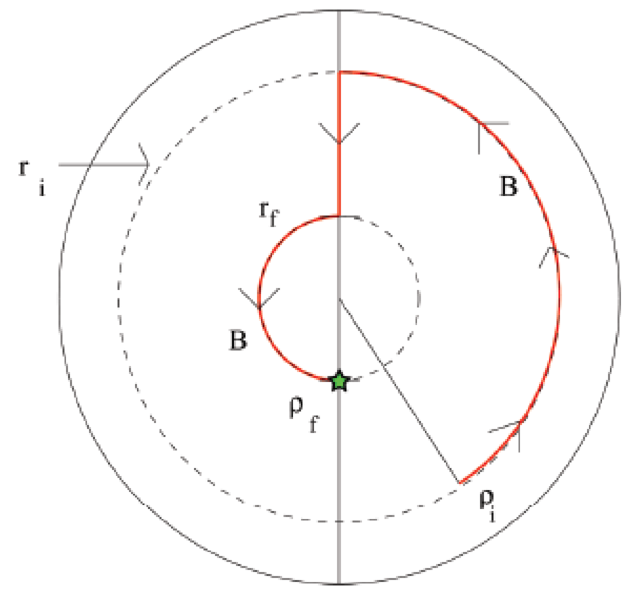
$$P = Tr(\rho^2) = \frac{1}{2}(1 + r^2) \longrightarrow \text{purity}$$

$$v_{AD}(r,\theta) = \frac{\partial P(r)}{\partial t} \longrightarrow \textit{Maximize / minimize this}$$

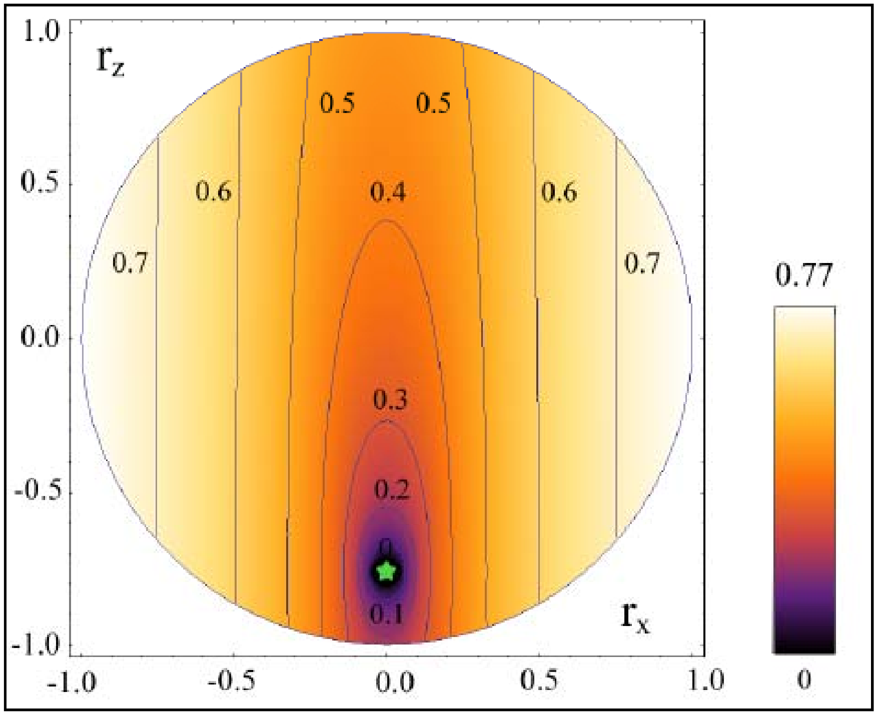
Error tolerance ε

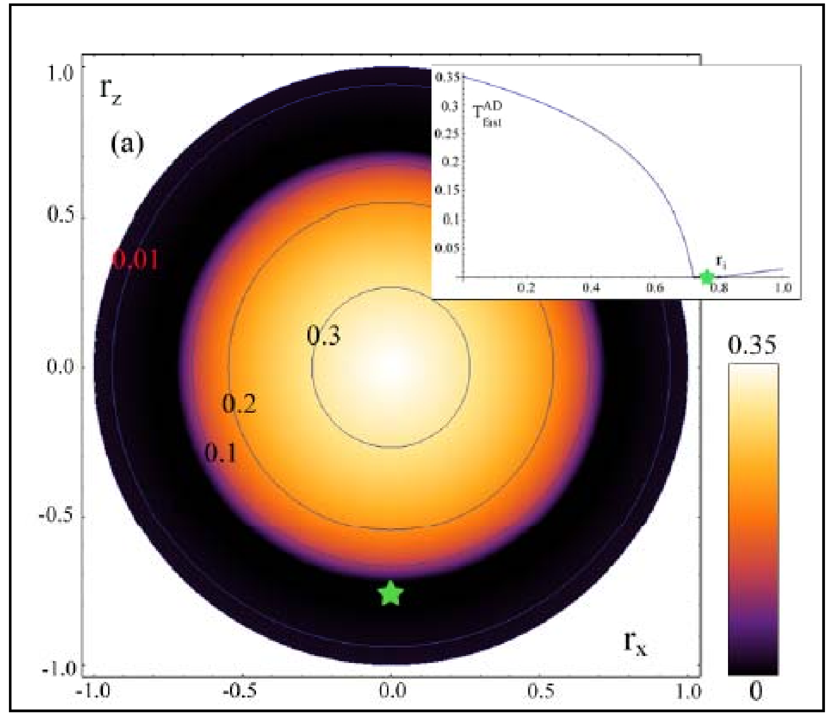
Cooling of a qubit





Heating of a qubit





Time taken to reach the fixed point without optimal control

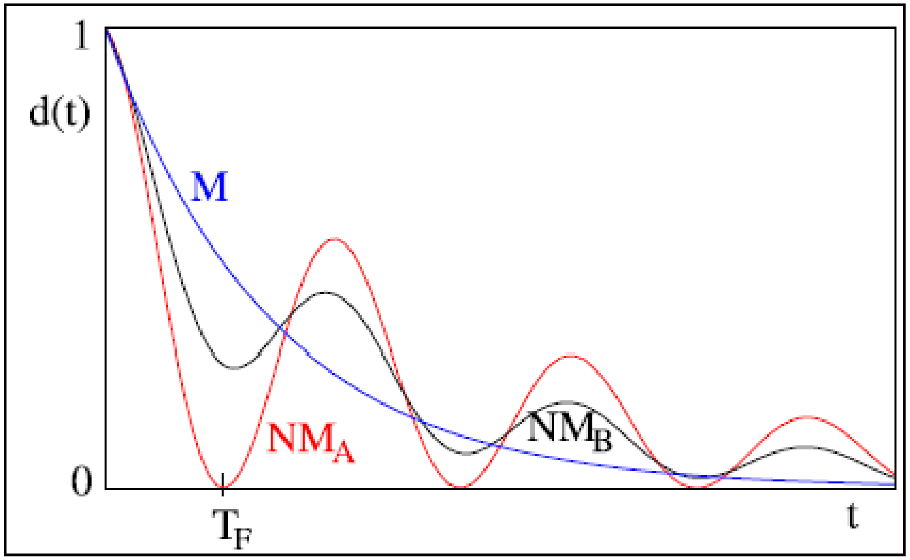
Time taken to reach the fixed point With infinite optimal control

$$\frac{\text{Worst case scenario without optimal control}}{\text{Worst case scenario with optimal control}} = 2$$

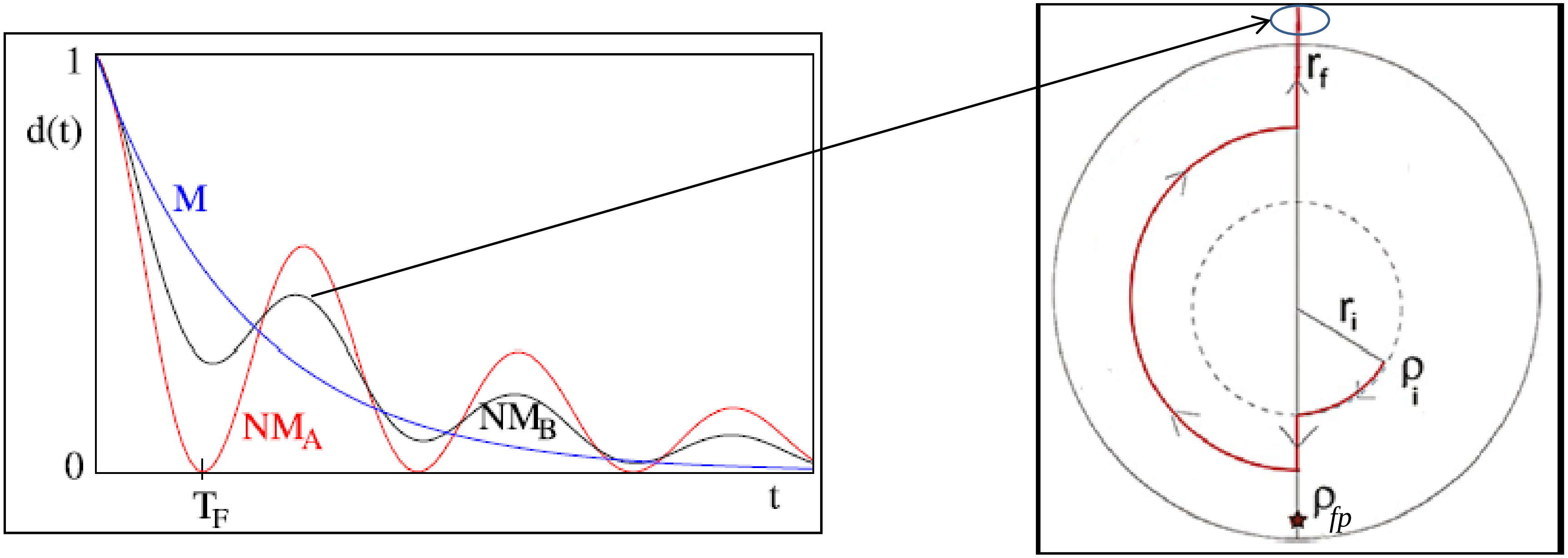
Gain while heating ~ 70 *!!*

V. Mukherjee, M, A. Carlini, A. Mari, T. Caneva, S. Montangero and T. Calarco, Phys. Rev. A **88**, 062326 (2013).

Control in presence of a non-Markovian bath



$$\boxed{\dot{\rho} = -i\left[H_s, \rho\right] + \gamma_t L(\rho)}$$

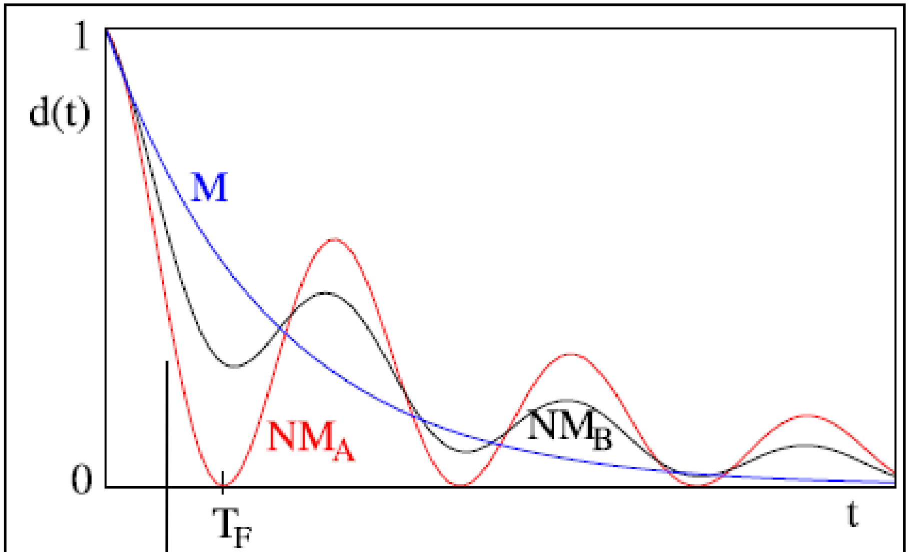


$$\boxed{\dot{\rho} = -i\left[H_s, \rho\right] + \gamma_t L(\rho)}$$

Let us restrict unitary controls at beginning and end of dissipative dynamics

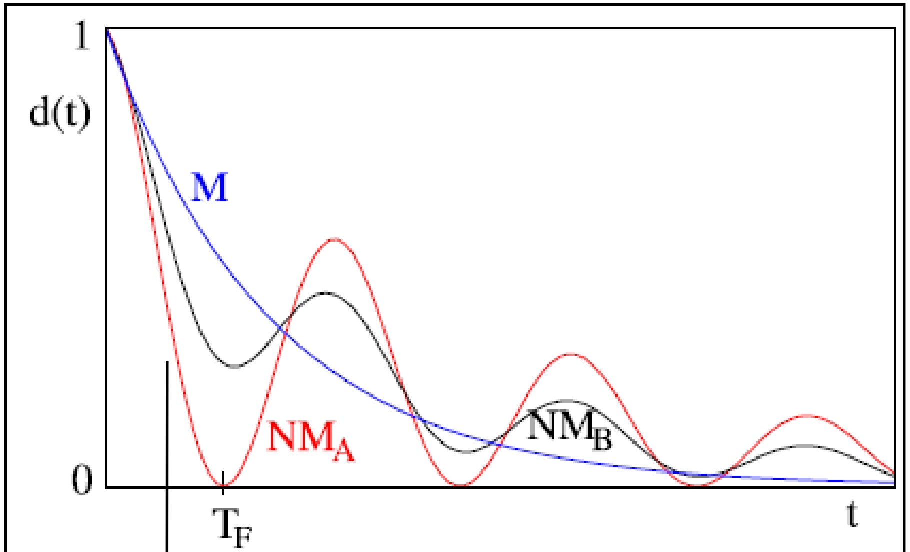
$$\rho(t) = U_{fin} \bullet D \bullet U_{in}\rho(0)$$

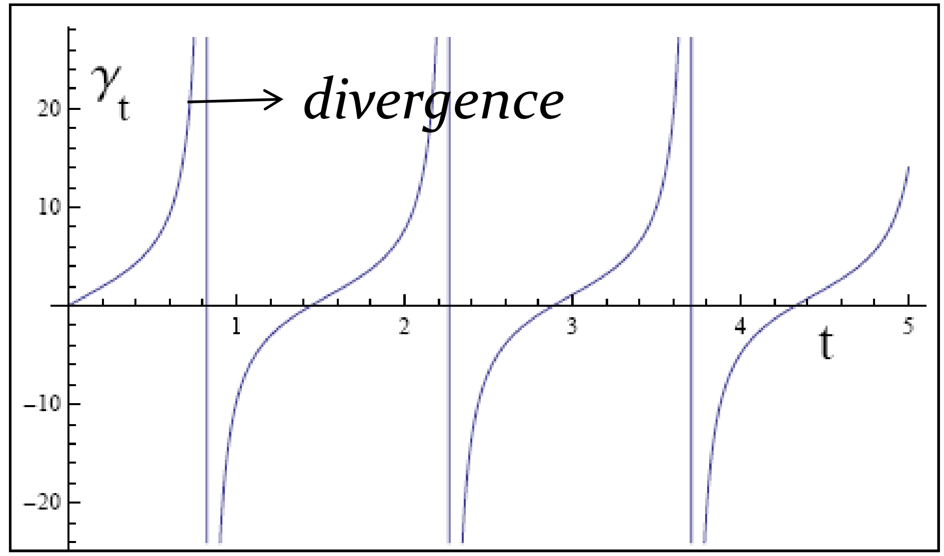
That is all we need for system in presence of a Markovian Bath



$$\dot{\rho} = \gamma_t L(\rho)$$

$$L\left(\rho\left(T_F\right)\right) = L\left(\rho_{fp}\right) = 0, \quad \dot{\rho}\left(t \to T_F^+\right) \neq 0$$



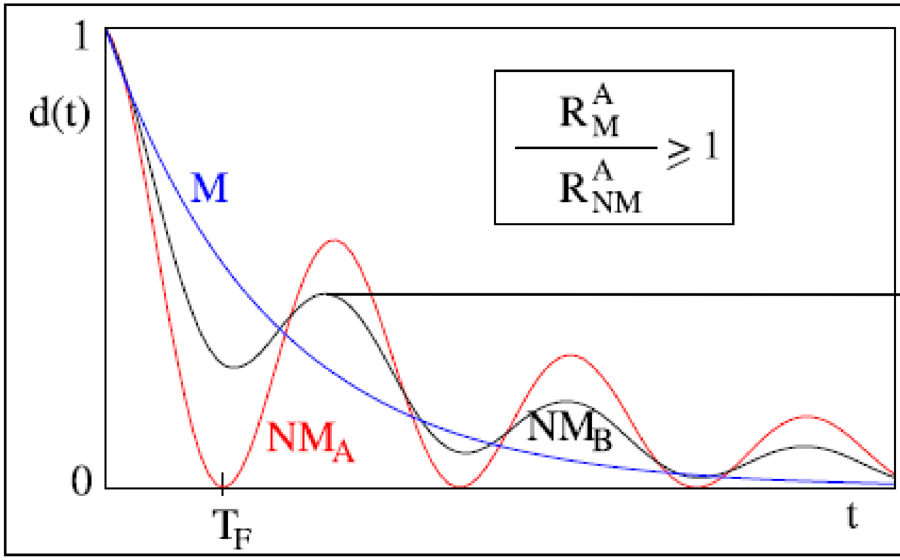


$$\dot{\rho} = \gamma_t L(\rho)$$

$$L(\rho_{fp}) = 0, \quad \dot{\rho} \neq 0 \Rightarrow |\gamma_t(t \to T_F)| \to \infty$$

$T_{QSL} = T_F \longrightarrow$ Depends on
Independent of any unitary transformation of ρ

Unitary control is ineffective for Class A NM dynamics.



γ_t is finite at all times.
Previous arguments do not apply

Class A (divergence): *System reaches fixed point before information backflow* ✗

Class B (no divergence): *Information backflow before system reaches fixed point* ✓

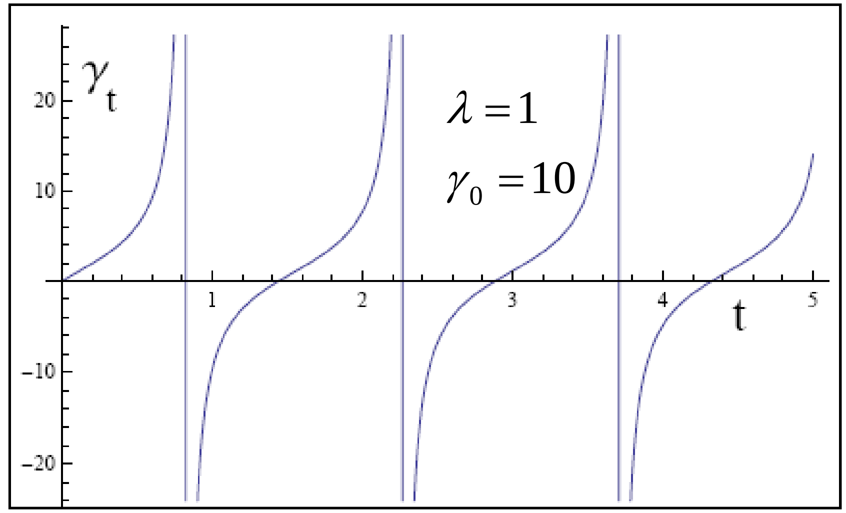
Necessary Condition : *We need class B NM dynamics (black curve) for unitary control to be effective*

SPECIFIC EXAMPLE: TWO LEVEL SYSTEM

$$\dot{\rho} = \gamma_t L(\rho)$$

$$\gamma_t = \frac{2\lambda\gamma_0 \sinh\left(\frac{td}{2}\right)}{d\cosh\left(\frac{td}{2}\right) + \lambda\sinh\left(\frac{td}{2}\right)}$$

$$d = \sqrt{\lambda^2 - 2\gamma_0\lambda}$$



$\lambda^{-1} \longrightarrow$ time scale of bath; $\gamma_0^{-1} \longrightarrow$ time scale of system

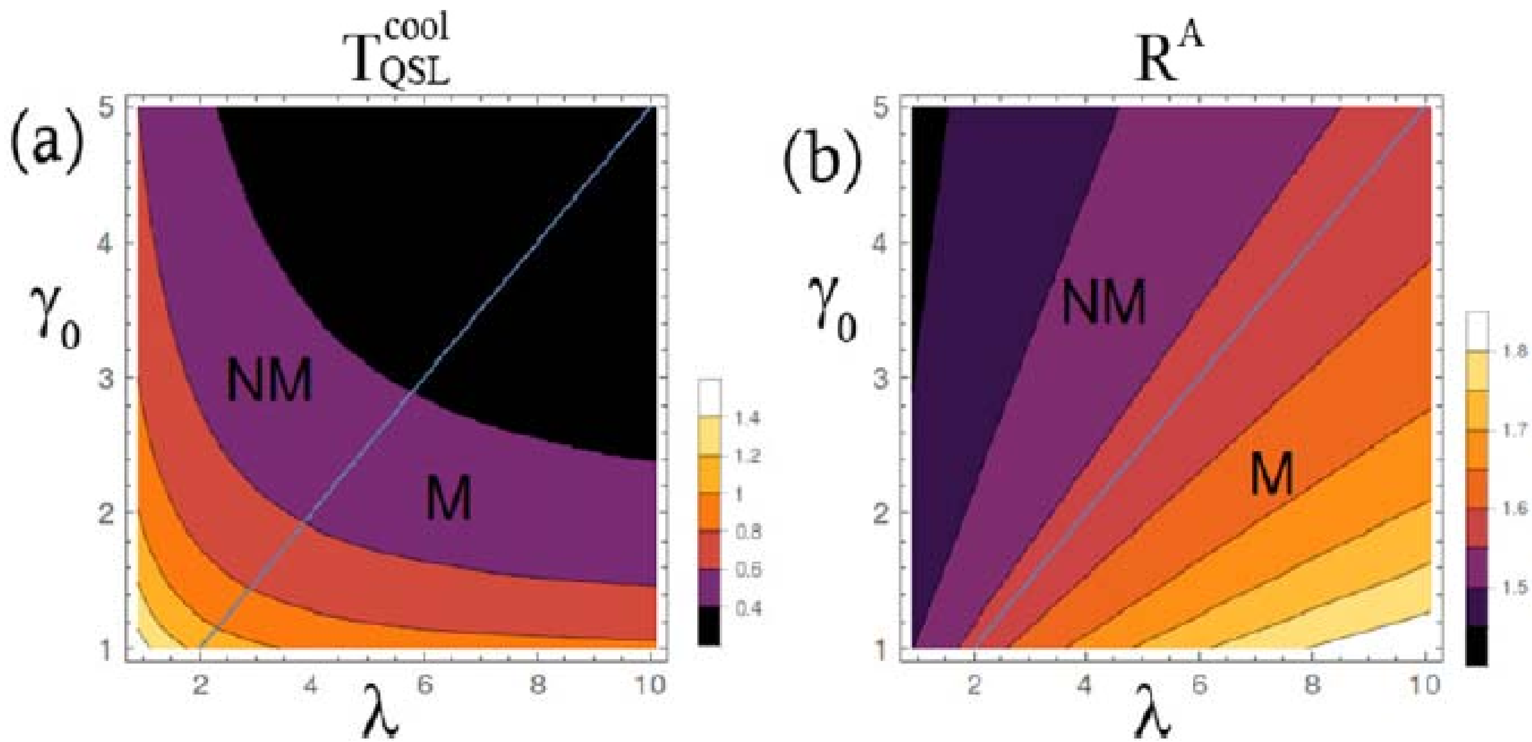
$\lambda / \gamma_0 \to \infty \longrightarrow$ Markovian $\longrightarrow \gamma_t \approx \gamma_0 > 0$

$\lambda / \gamma_0 \to 0 \longrightarrow$ Non-Markovian $\longrightarrow \gamma_t \propto \tan\left(\sqrt{\frac{\lambda\gamma_0}{2}}t\right)$

B. M. Garraway, Phys. Rev. A **55**, 2290 (1997).

H.-P. Breuer, B. Kappler and F. Petruccione, Phys. Rev. **A** 59,1633 (1999).

Quantum speed up ratio for cooling



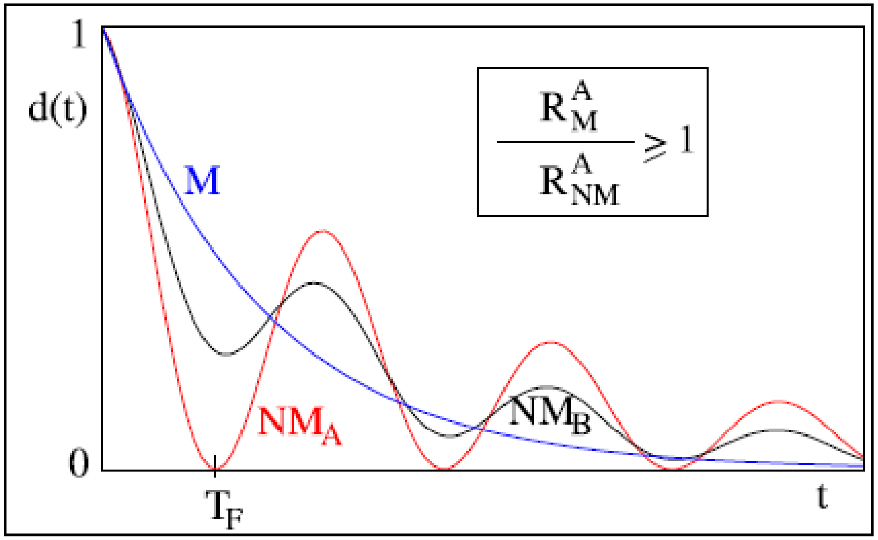
Speedup

$$R = \frac{T_{free}}{T_{QSL}} \geq 1 \longrightarrow$$ *Advantage one gains by optimal control*

$$R_{M,A}^{cool} \to 2; \;\; R_{NM,A}^{cool} \to 1$$

$$\boxed{R_M^A \geq R_{NM}^A}$$

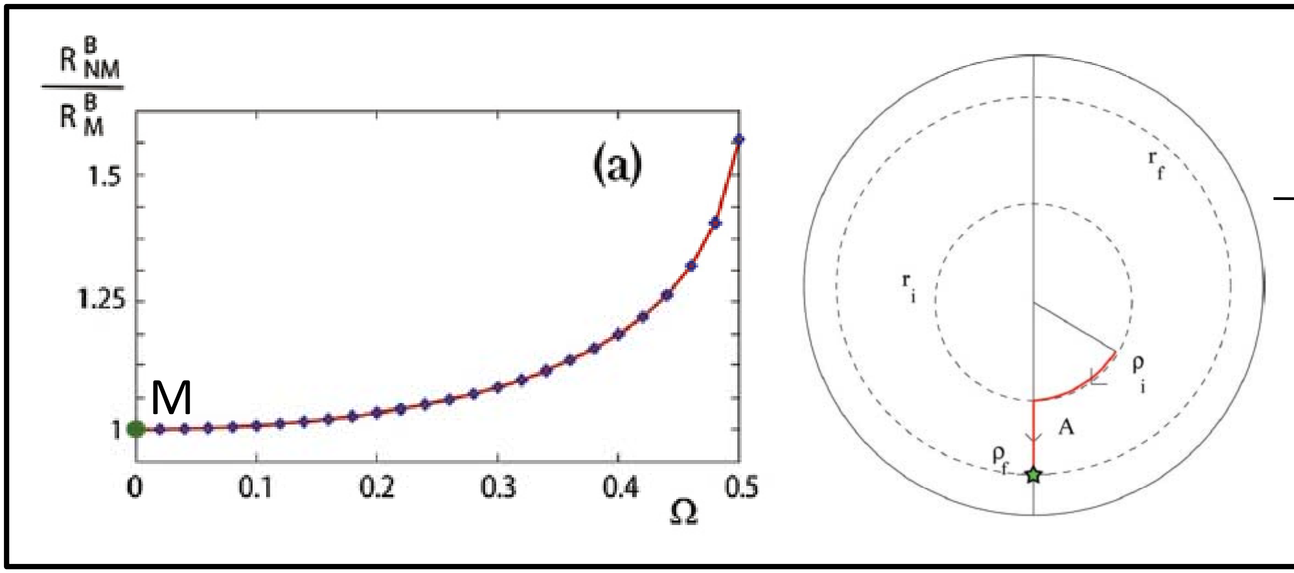
CLASS B NON-MARKOVIAN DYNAMICS *



$$\dot{\rho} = \gamma_t L(\rho)\ ; \quad L(\rho_{fp}) = 0$$
$$\gamma_t = \exp(-\xi t)\cos(\Omega t)$$

$\Omega = 0 \rightarrow M$

$\Omega \neq 0 \rightarrow NM$

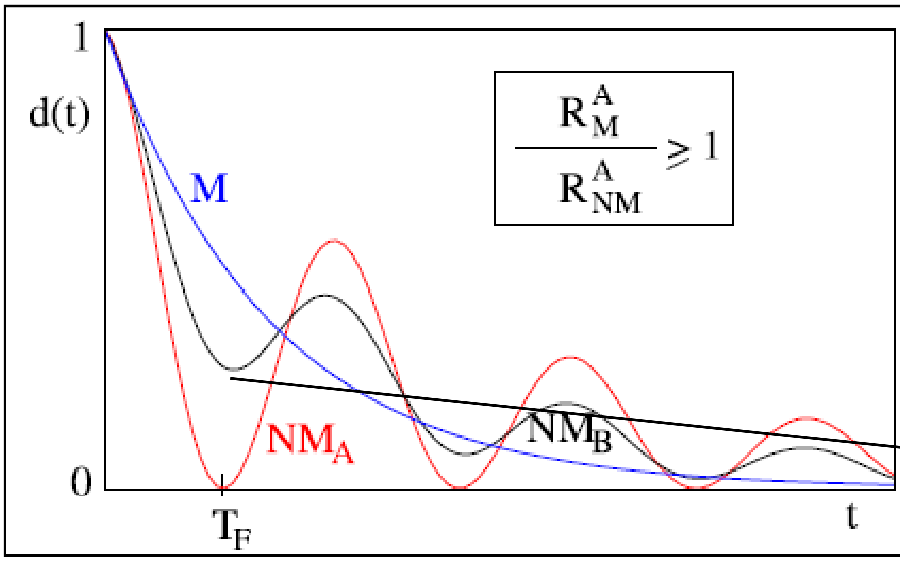


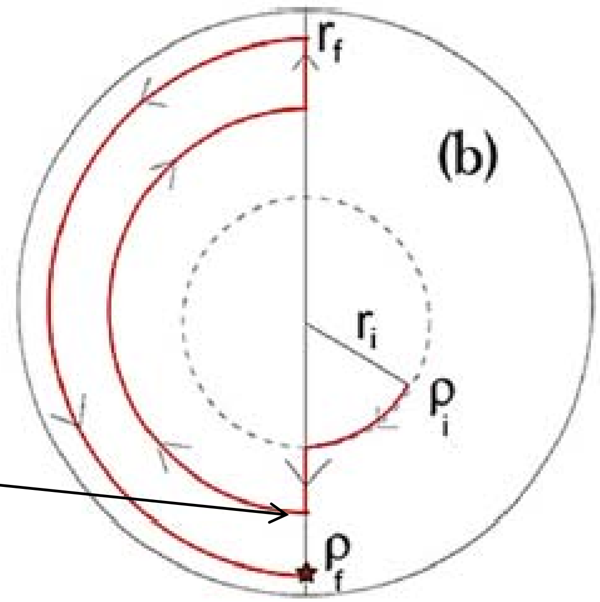
→ *Exploit NM effects to cool faster*

$$\rho(t) = U_{fin} \bullet D \bullet U_{in}\rho(0)$$

* A. M. Souza et. al. arXiv:1308.5761 (2013)

EXPLOITING CLASS B NON-MARKOVIAN DYNAMICS



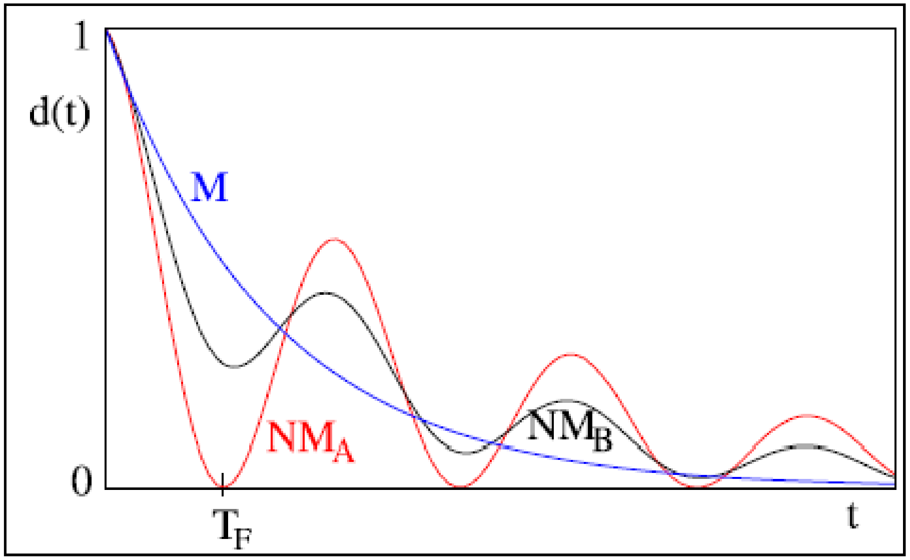


Optimal Path

$$\rho(t) = U_{fin} \bullet D_2 \bullet \tilde{U} \bullet D_1 \bullet U_{in} \rho(0)$$

Follow the maxima of $v_{AD}(r,\theta) = \partial P(t) / \partial t$

Main Result



We can determine if there is a possibility of efficiently controlling the relaxation of a generic system exhibiting non-Markovian dynamics just by looking at the above figure, without solving any complicated master equation !!

Independent of the details of the system, bath : dimensionality, Hamiltonian, explicit form of the dissipative bath, etc.

V. Mukherjee, V. Giovannetti, R. Fazio, S. F. Huelga, T. Calarco and S. Montangero, New J. Phys. **17** (2015) 063031 .

SUMMARY

- *Class A NM dynamics* associated with *divergence* $\longrightarrow$ precludes unitary quantum control

- Necessary Condition : *We need class B NM dynamics for unitary control to be effective*

- *Control protocols can be extended to* N *level system by studying Casimir invariants* $\sim Tr\left(\rho^{j}\right), \quad j = 2,3\ldots, N$

- *Similar results obtained for* $\dot{\rho}(t) = \sum_{k} \gamma_{k}(t) L_{k}(\rho(t))$

- *What are the sufficient conditions?*

Time Scales

$$\boxed{\begin{aligned}&\dot{\rho} = -i[H,\rho] + \gamma_t L(\rho)\\ &\gamma_t = \frac{2\lambda\gamma_0 \sinh\left(\frac{td}{2}\right)}{d\cosh\left(\frac{td}{2}\right) + \lambda \sinh\left(\frac{td}{2}\right)}; \quad d = \sqrt{\lambda^2 - 2\gamma_0\lambda}\end{aligned}}$$

$\lambda / \gamma_0 \to \infty \longrightarrow$ Markovian $\rightarrow \gamma_t \approx \gamma_0$

$\lambda / \gamma_0 \to 0 \longrightarrow$ Non-Markovian $\rightarrow d \approx i\sqrt{2\lambda\gamma_0}$

$$T_M \sim \frac{1}{\gamma_0}$$

$$T_{NM} \sim \frac{1}{\sqrt{\lambda\gamma_0}}$$

Decreases both for increasing Markovianity (λ)
as well as increasing non-Markovianity (γ_0)

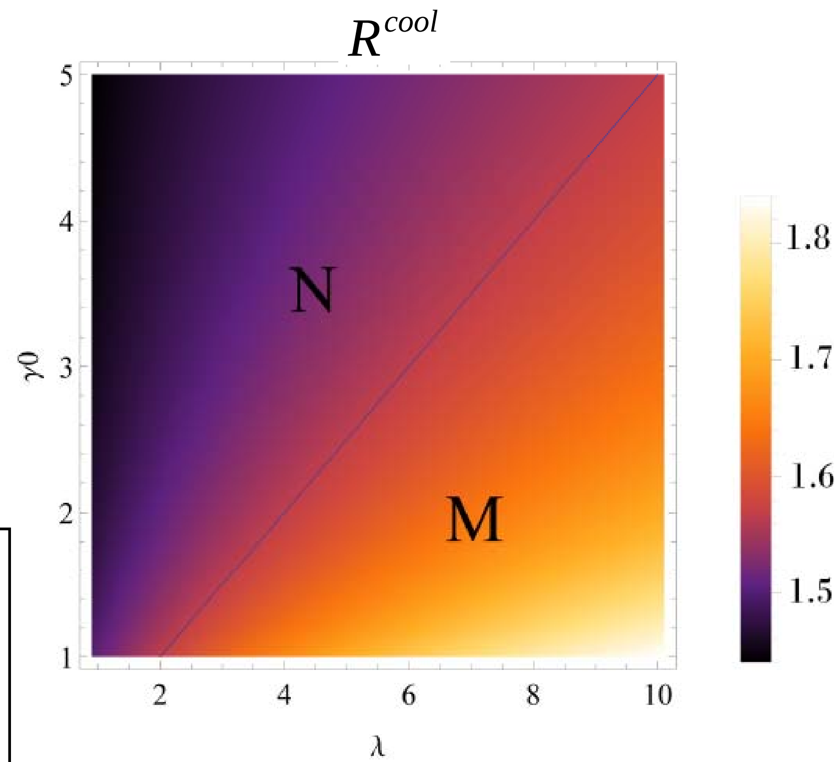
For a generic system in
presence of a generic L

$$\lim_{\varepsilon\to 0} R_M^{cool} = \frac{T_{free,M}}{T_{con,M}^{cool}} \to 2$$

$$R_{NM}^{cool} = \frac{T_{free,NM}}{T_{con,NM}^{cool}} \to 1$$

$$\lim_{\varepsilon\to 0} R_M^{heat} = \frac{T_{free,M}}{T_{con,M}^{heat}} \approx 2\frac{|\ln\varepsilon|}{\ln\left[\dfrac{r_{fp}+r_i}{2r_{fp}}\right]} \to \infty$$

$$R_{NM}^{heat} \approx \frac{\pi/2}{\cos^{-1}\left[\left(\dfrac{2r_{fp}}{r_i+r_{fp}}\right)^{1/2\left(e^{\beta}+1\right)}\right]} \to \mathit{finite}$$



$$R_M \geq R_{NM}$$

Optimal control works better in the Markovian limit

Damped Jaynes–Cummings model

$$H = H_S + H_B + H_I$$

$$H_S = \omega_0 \sigma^+ \sigma^-$$

$$H_B = \sum_k \omega_k b_k^+ b_k$$

$$H_I = \sigma^+ \otimes B + \sigma^- \otimes B^+$$

$$B = \sum_k g_k b_k$$

Markovian / non-Markovian dynamics determined by form of g_k

$$\tilde{g}(\omega) = \frac{1}{2\pi} \frac{\gamma_0 \lambda^2}{(\omega_0 - \omega)^2 + \lambda^2}$$

H. P. Breuer and F. Petruccione 2002 The Theory of Open Quantum Systems (Oxford: Oxford University Press)

Time Local Non-Markovian Master Equation

$$\dot{\rho}(t) = L(t - t_0)\rho(t)$$

$$\rho(t) = \Lambda(t, t_0)\rho(t_0)$$

$$\Lambda(t, t_0) = \exp\left[\int_0^{t-t_0} L(\tau)d\tau\right]$$

$$L(\tau) = \frac{d}{d\tau}\Lambda(\tau).\Lambda^{-1}(\tau)$$

D. Chruscinski and A. Kossakowski, Phys. Rev. Lett. **104**, 070406 (2010).

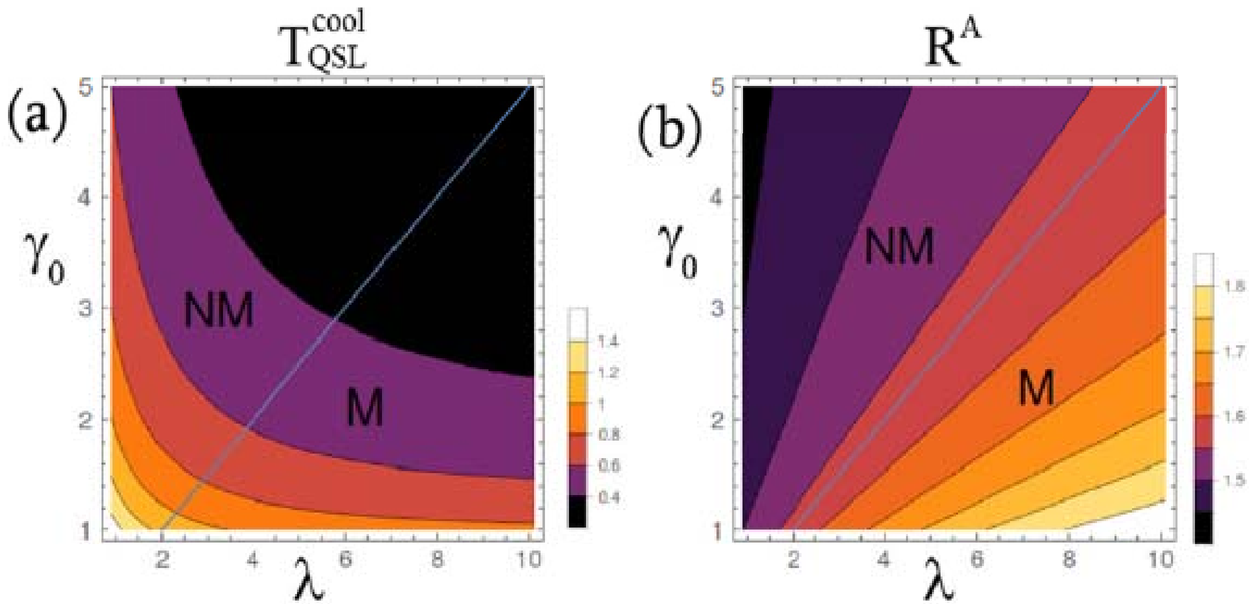
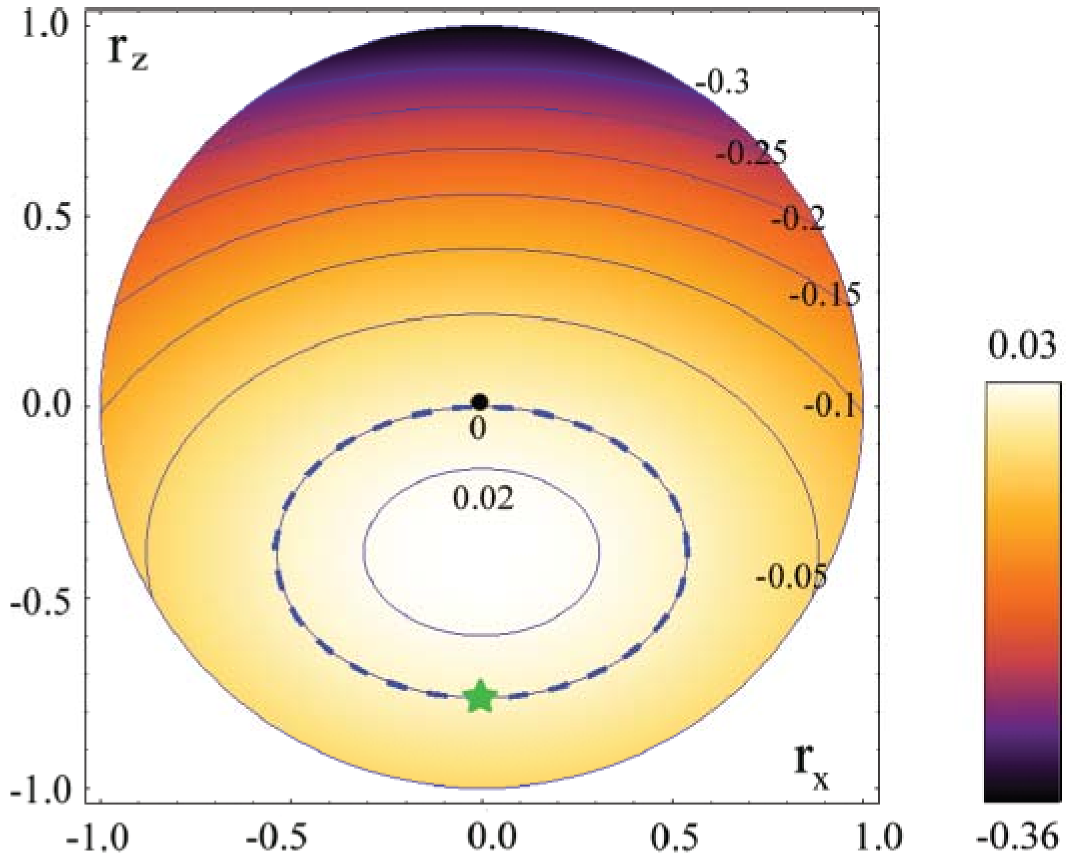


Figure 2. (a) Parametric plot showing variation of time T_{QSL}^{cool} of reaching the fixed point with λ and γ_0 for $\beta = 2, r_i = 0.5$ and $\epsilon = 0.01$. The Markovian (M) and non-Markovian (NM) regions are separated by the blue line on the $\lambda - \gamma_0$ plane. (b) Plot showing variation of quantum speed up ratio R^A with λ and γ_0 for $\beta = 2$, $r_{xi} = 0.3$, $r_{yi} = 0$, $r_{zi} = 0.4$ and $\epsilon = 0.01$. R^A saturates to $R_M^A \approx 2$ ($R_{NM}^A \approx 1$) in the extreme M (NM) limit.

FREEZING OF A QUBIT – STOPPING DECOHERENCE FOR INFINITE TIME



$$\dot{\rho} = -i[H_s, \rho] + \gamma_0 L(\rho) \rightarrow \dot{r} \equiv \partial P / \partial t = 0$$

$$T_{con}^{cool}\left(\vec{r}_i,\varepsilon\right)=\frac{1}{\gamma_0\left(e^{\beta}+1\right)}\ln\left[\frac{\left(r_{fp}-r_i\right)}{\varepsilon}\right]$$

$$T_{con}^{heat}\left(\vec{r}_i,\varepsilon\right)=\frac{1}{\gamma_0\left(e^{\beta}+1\right)}\ln\left[\frac{r_{fp+}r_i}{2r_{fp}+\varepsilon}\right]$$

$$\rho=\frac{1}{2}\left(I+\vec{r}.\vec{\sigma}\right)=\frac{1}{2}\begin{pmatrix}1+r_z & r_x-ir_y\\ r_x+ir_y & 1-r_z\end{pmatrix}$$