Indefinite causal order enables perfect quantum communication with zero capacity channel

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Plan of the talk

- Communication model of Shannon (classical)
- Quantum Shannon theory
- Superposition of quantum channels
 Superposition of path
 Superposition of time order
- Our work: perfect activation of zero capacity channel
- Three main results
- Conclusions

C. E. Shannon: founder of information theory

Claude Shannon's 1948 seminal paper laid the foundations of our current communication technology

The Bell System Technical Journal

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No. 3

A Mathematical Theory of Communication

By C. E. SHANNON

INTRODUCTION

THE recent development of various methods of modulation such as PCM and PPM which exchange bandwidth for signal-to-noise ratio has in-

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C. E. Shannon: founder of information theory

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- Shannon modeled the devices used to store and transfer information as classical systems:
 - state can in principle be determined without errors
 - arrangement in space and time is always welldefined





- Physical systems, at the fundamental level, obey the laws of quantum mechanics.
- Rules of quantum theory have been exploited to achieve communication tasks that are impossible in classical physics





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- Rules of quantum theory have been exploited to achieve communication tasks that are impossible in classical physics
 - Quantum cryptography [BB84; Ekert PRL 67, 661(1991)]
 - Quantum teleportation [PRL 69, 2881 (1992)]
 - Superadditivity of channel capacities [Science 321, 1812 (2008); Nat. Phys. 5, 255 (2009); PRL103, 120501 (2009)]





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- Rules of quantum theory have been exploited to achieve communication tasks that are impossible in classical physics
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 - Quantum teleportation [PRL 69, 2881 (1992)]
 - Superadditivity of channel capacities [Science 321, 1812 (2008); Nat. Phys. 5, 255 (2009); PRL103, 120501 (2009)]
 - Theory of communication with quantum systems is a rich and thoroughly developed discipline, known as Quantum Shannon theory
 - Nevertheless, quantum Shannon theory is still conservative, in that it assumes that the communication channels are combined in a well-defined configuration



Quantum Channel: completely positive and trace preserving (CPTP) map

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□ Classical (HSW) Capacity:

Private Capacity (Devetak):

$$P^{(1)}(\mathcal{N}) \leq P(\mathcal{N}) = \lim_{n \to \infty} \frac{1}{n} P^{(1)}(\mathcal{N}^{\otimes n}) \qquad P^{(1)}(\mathcal{N}) = \max_{\{p_i, \rho_i\}} (\chi_{\{p_i, \rho_i\}}(\mathcal{N}) - \chi_{\{p_i, \rho_i\}}(\tilde{\mathcal{N}}))$$

Quantum Channel: completely positive and trace preserving (CPTP) map



$$\mathcal{E}(\rho) = \sum_i E_i \rho E_i^{\dagger}$$

 $Q^{(1)}(\mathcal{N}) = \max_{\rho} I_c(\rho, \mathcal{N})$

 $I_{c}(\rho, \mathcal{N}) = H(\mathcal{N}(\rho)) - H(\tilde{\mathcal{N}}(\rho))$

□ Classical (HSW) Capacity:

Private Capacity (Devetak):

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□ Classical (LSD) Capacity:

$$Q^{(1)}(\mathcal{N}) \leq Q(\mathcal{N}) = \lim_{n \to \infty} \frac{1}{n} Q^{(1)}(\mathcal{N}^{\otimes n})$$

Quantum mechanics allows for scenarios where the configuration of the communication channels is in a quantum superposition [Aharonov et al. PRL 64, 2965 (1990); K. L. O. Daniel, PRL 91, 067902 (2003)]

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Figure from PRL 91, 067902 (2003)

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For example, a photon could travel through a superposition of different paths



Figure from PRL 91, 067902 (2003)

 Such a superposition offers an opportunity to filter out some of the noise affecting the transmission [N. Gisin et al; Phys. Rev. A 72, 012338 (2005)]



Definite causal order



Definite causal order



Indefinite causal order







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G. Chiribella et al, Phys. Rev. A 88, 022318 (2013)



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- M.Araújo et al, New Journal of Physics 17, 102001 (2015)
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Causal nonseparability



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- Experiment(s)
- L. M. Procopio et al, Nat. Commun. 6, 7913 (2015)
- G. Rubino et al, Science Advances 3, e1602589 (2017)
- K. Goswami et al, Phys. Rev. Lett. 121, 090503 (2018)

Causal nonseparability





$$S_{\omega}(\mathcal{E}_1, \mathcal{E}_2)(\rho) = \sum_{i,j} K_{ij}(\rho \otimes \omega) K_{ij}^{\dagger}$$



Perfect Activation

Qubit Pauli channel:

$\begin{aligned} \mathcal{E}_{\vec{p}}(\rho) &= \sum_{i=0}^{3} p_i \sigma_i \rho \sigma_i^{\dagger} \\ \vec{p} &\equiv (p_0, p_1, p_2, p_3) \end{aligned}$



$\mathcal{E}_{\vec{p}}(\rho) = \sum_{i=0}^{3} p_i \sigma_i \rho \sigma_i^{\dagger}$ **Qubit Pauli channel: Q-SWITCH** channel: $\mathcal{S}_{\omega}(\mathcal{E}_{\vec{p}}, \mathcal{E}_{\vec{p}})(\rho) = q_{+} \mathcal{C}_{+}(\rho) \otimes \omega_{+} + q_{-} \mathcal{C}_{-}(\rho) \otimes \omega_{-}$

Perfect Activation

$$q_{-} = 2(p_{1}p_{2} + p_{2}p_{3} + p_{3}p_{1}), \quad q_{+} = 1 - q_{-}$$

$$\omega_{+} = \omega, \quad \omega_{-} = Z\omega Z$$

$$C_{+}(\rho) = \frac{(p_{0}^{2} + p_{1}^{2} + p_{2}^{2} + p_{3}^{2})\rho + 2p_{0}(p_{1}X\rho X + p_{2}Y\rho Y + p_{3}Z\rho Z)}{q_{+}}$$

 $\vec{p} \equiv (p_0, p_1, p_2, p_3)$

$$\mathcal{C}_{-}(\rho) = \frac{2p_1p_2 Z\rho Z + 2p_2p_3 X\rho X + 2p_1p_3 Y\rho Y}{q_{-}}$$



Perfect Activation

- Qubit Pauli channel: $\mathcal{E}_{\vec{p}}(\rho) = \sum_{i=0}^{3} p_i \sigma_i \rho \sigma_i^{\dagger}$ $\vec{p} \equiv (p_0, p_1, p_2, p_3)$
 - Q-SWITCH channel:

 $\mathcal{S}_{\omega}(\mathcal{E}_{\vec{p}},\mathcal{E}_{\vec{p}})(\rho) = q_{+}\mathcal{C}_{+}(\rho)\otimes\omega_{+} + q_{-}\mathcal{C}_{-}(\rho)\otimes\omega_{-}$

$$q_{-} = 2(p_1p_2 + p_2p_3 + p_3p_1), \quad q_{+} = 1 - q_{-}$$

 $\omega_+ = \omega, \qquad \omega_- = Z \omega Z$

 $\mathcal{C}_{+}(\rho) = \frac{\left(p_{0}^{2} + p_{1}^{2} + p_{2}^{2} + p_{3}^{2}\right)\rho + 2p_{0}\left(p_{1}X\rho X + p_{2}Y\rho Y + p_{3}Z\rho Z\right)}{q_{+}}$

$$\mathcal{C}_{-}(\rho) = \frac{2p_1p_2 Z\rho Z + 2p_2p_3 X\rho X + 2p_1p_3 Y\rho Y}{q_{-}}$$

$$\mathcal{E}_{XY}(\rho) = X\rho X/2 + Y\rho Y/2$$

Related results

Enhanced Communication with the Assistance of Indefinite Causal Order

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¹Department of Computer Science, The University of Hong Kong, Pokfulam Road, Pokfulam 999077, Hong Kong ²Department of Computer Science, University of Oxford, Wolfson Building, Parks Road, Oxford OX1 3QD, United Kingdom ³Canadian Institute for Advanced Research, CIFAR Program in Quantum Information Science, Toronto, Ontario M5G 1Z8, Canada ⁴HKU Shenzhen Institute of Research and Innovation, Kejizhong 2nd Road, Shenzhen 518057, China

(Received 28 November 2017; published 22 March 2018)

In quantum Shannon theory, the way information is encoded and decoded takes advantage of the laws of quantum mechanics, while the way communication channels are interlinked is assumed to be classical. In this Letter, we relax the assumption that quantum channels are combined classically, showing that a quantum communication network where quantum channels are combined in a superposition of different orders can achieve tasks that are impossible in conventional quantum Shannon theory. In particular, we show that two identical copies of a completely depolarizing channel become able to transmit information when they are combined in a quantum superposition of two alternative orders. This finding runs counter to the intuition that if two communication channels are identical, using them in different orders should not make any difference. The failure of such intuition stems from the fact that a single noisy channel can be a random mixture of elementary, noncommuting processes, whose order (or lack thereof) can affect the ability to transmit information.

DOI: 10.1103/PhysRevLett.120.120502



Related results

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DOI: 10.

Communication through coherent control of quantum channels

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arXiv:1810.09826v1

A completely depolarising quantum channel always outputs a fully mixed state and thus cannot transmit any information. In a recent Letter, a surprising result was shown [D. Ebler et al., Phys. Rev. Lett. 120, 120502 (2018)]: if a quantum state passes through two such channels in a quantum superposition of different orders—a setup known as the "quantum switch"—then information can nevertheless be transmitted through the channels. It is perhaps tempting to attribute this result to the indefinite causal order between the channels. Here, however, we show that a similar effect can be obtained when one coherently controls between applying one of two identical depolarising channels to a target system. Such a situation involves no indefinite causal order; we argue that this result should therefore rather be understood as resulting from coherent control of channels. Additionally, we see that when quantum channels are controlled coherently, information about their specific implementation is accessible in the output state of the joint control-target system.



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Theorem 1. Suppose that a message, encoded in a ddimensional quantum system ($d < \infty$) is sent through a superposition of $N < \infty$ independent channels. If all N channels are noisy, then the message cannot be perfectly decoded.

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- Above theorem establishes the maximal activation of the quantum capacity as a fundamental difference between superposition in space and superposition in time.
- □ The origin of the difference is in the fact that the superposition of orders in time involves correlations between all the sub-processes happening within channel E (F) in the configuration EF and all the sub-processes happening within channel E (F) in the alternative configuration F E.

Main Results in our paper:

Theorem 2. Let \mathcal{E} be a qubit channel such that $Q(\mathcal{E}) = 0$ and $Q(\mathcal{S}_{\omega}(\mathcal{E}, \mathcal{E})) = 1$ for some state ω . Then, there exists a basis in which the channel acts as $\mathcal{E}(\rho) = 1/2(X\rho X + Y\rho Y)$.

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Theorem 3. For every pair of complete erasure channels \mathcal{E}_0 and \mathcal{F}_0 and for every state $\omega \in D(\mathcal{H}_C)$, the switched channel $\mathcal{S}_{\omega}(\mathcal{E}_0, \mathcal{F}_0)$ is entanglement-breaking.

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□ Theorem 2 characterizes the qubit channel that exhibit the perfect activation phenomena.

□ Theorem 3 establishes a complementarity between the activation of the quantum capacity and the activation of the classical capacity: every channel that exhibits activation of the classical capacity (from zero to non-zero) does not exhibit activation of the quantum capacity.

Thanks to my collaborators

arXiv.org > quant-ph > arXiv:1810.10457

Search or A

Quantum Physics

Indefinite causal order enables perfect quantum communication with zero capacity channel

Giulio Chiribella, Manik Banik, Some Sankar Bhattacharya, Tamal Guha, Mir Alimuddin, Arup Roy, Sutapa Saha, Sristy Agrawal, Guruprasad Kar

(Submitted on 24 Oct 2018 (v1), last revised 25 Oct 2018 (this version, v2))

Quantum mechanics is compatible with scenarios where the relative order between two events is indefinite. Here we show that two instances of a noisy process, used in a superposition of two alternative orders, can behave as a perfect quantum communication channel. This phenomenon occurs even if the original processes have zero capacity to transmit quantum information. In contrast, perfect quantum communication does not occur when the message is sent along a superposition of paths, with independent noise processes acting on each path. The possibility of perfect quantum communication through noisy channels highlights a fundamental difference between the superposition of orders in time and the superposition of paths in space.

 Comments:
 5+9 pages (minor modifications)

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□ We show that two instances of a completely noisy process (i.e. zero quantum capacity) used in a superposition of two alternative orders, can behave as a perfect quantum communication channel.

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- □ In contrast, perfect quantum communication does not occur when the message is sent along a superposition of paths, with independent noise processes acting on each path.
- □ The possibility of perfect quantum communication through noisy channels depicts a fundamental difference between the superposition of orders in time and the superposition of paths in space.
- □ The phenomenon of causal activation is subject to a general no-go theorem: no quantum channel with zero classical capacity can gain a non-zero quantum capacity with the assistance of the quantum SWITCH. This result establishes a complementarity between the activation of the quantum capacity and the activation of the classical capacity.

When at last the thought came to him that 'time itself was suspect', Einstein had found a new insight into the nature of the physical universe.

And in quantum world if you can make temporal order quantum you can get something perfect out of nothing.

!!Questions / Comments!!

