Identifying and Quantifying Resource for Remote State Preparation using correlations beyond discord

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December 1, 2018

This talk is based on the work:

Som Kanjilal, Aiman Khan, C. Jebaratnam, Dipankar Home, Remote State Preparation using Correlations beyond Discord, arXiv:1809.11123, (To appear in Physical Review A)

Motivation

- Device-independent quantum cryptography requires nonlocal entanglement, entanglement-assisted subchannel discrimination depends on steerable entanglement, while quantum teleportation and dense coding require entanglement not necessarily requiring nonlocality.

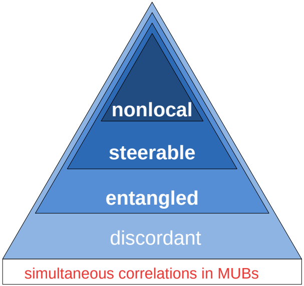

Figure: Hierarchy of correlations in nonproduct states, $\rho_{AB} \neq \rho_A \otimes \rho_B$. Any such state has simultaneous correlations in mutually unbiased bases (MUBs).

Motivation

- The study of quantumness of correlations in *separable states* without entanglement is of much contemporary importance since if separable states can be used as resource for information processing/transfer protocols (ex. quantum state merging, deterministic quantum computational protocol, interferometric power of quantum states, quantum cryptography), such states are much easier to prepare and control even in dissipative environments.
- For states with no entanglement, nonzero quantum discord [Ollivier & Zurek, PRL (2001)] has been argued to be an important resource for quantum communication as well as for computation.

Motivation

- On the other hand, in recent years an essential feature of quantum correlations beyond discord has been argued for any non-product state to be the simultaneous quantum correlations in complementary bases and a quantifier was defined to measure this feature [Guo & Wu, Scientific Reports (2014)].
- In our work, we seek to find implications of this said feature of quantum correlation for the purpose of characterizing efficiency of a quantum information theoretic task like RSP and quantifying quantum effect like quantum steering.

Motivation

Jebarathinam, Khan, Kanjilal and Home, Phys. Rev. A **98**, 042306 (2018)

- We ask the question whether there eixsts an appropriate measure of quantum correlations beyond entanglement that is monotonically related to the degree of steerability of a quantum state in a given steering scenario.
- The above question is addressed for any steerable two-qubit Bell-diagonal state through a novel, analytically derived link, achieved by invoking an appropriate measure of correlations based on the measure of simultaneous correlations in mutually unbiased bases. We have been able to relate such a measure of correlations with a suitable measure of steering which is derived from a linear steering inequality.
- The link thus established is such that for any steerable Bell-diagonal state, a higher value of simultaneous correlations in mutually unbiased bases necessarily implies higher degree of quantum steering.

Overview

Measures of simultaneous correlations in Mutually Unbiased Bases ($C_2, C_3, \cdots$)

- Suppose Alice and Bob share a bipartite quantum state ρ_{ab} and Alice performs a projective measurement in some eigenbasis $\{|i\rangle\}$ of an observable A. Then the measurements on this basis prepares conditional states on Bob's side $\rho_i^b = \text{Tr}_A(|i\rangle\langle i| \otimes \mathbf{1}\rho_{\mathbf{ab}})/\mathbf{p}(\mathbf{i}|\mathbf{A})$, here $p(i|A) = \text{Tr}(|i\rangle\langle i| \otimes \mathbf{1}\rho_{\mathbf{ab}})$, which is the probability of getting the outcome i of the observable A.
- Holevo quantity of conditional states is given by $\chi(\rho_{ab}, |i\rangle\langle i|) = \text{S}(\sum_i \text{p}_i \rho_b^i) - \sum_i \text{p}_i S(\rho_b^i)$, which is interpreted as a measure of correlations for the quantum system in question in the given basis.
- By using the above measure of correlation, we will discuss the definitions for quantification of classical correlation C_1 and simultaneous correlations in Mutually Unbiased Bases ($C_2, C_3, \cdots$).

Classical correlation [Henderson and Vedral (2001)]

For a given nonproduct state, let us consider the Holevo quantity corresponding to the conditional states prepared on Bob's side by varying the measurement basis on Alice's side. Then the particular basis on Alice's side with respect to which the Holevo quantity on Bob's side is maximized is considered to be the optimum basis for quantifying the classical correlation in such a state, and the correlation is defined by taking the maximum value of the Holevo quantity denoted by $C_1(\rho_{ab})$. The optimum basis that quantifies the classical correlation is called C_1-basis.

Quantum discord denoted D [Ollivier & Zurek (2001)]

Quantum discord is defined as $D = I(\rho_{ab}) - C_1(\rho_{ab})$, where $I(\rho_{ab}) = S(\rho_a) + S(\rho_b) - S(\rho_{ab})$ which is called mutual information or total correlations in the given state. $D(\rho) = 0$ iff the given state can be written in the classical-quantum state form, $\rho_{CQ} = \sum_i p_i |i\rangle\langle i| \otimes \rho_i^b$, where $\{|i\rangle\}$ forms an orthonormal basis.

Quantification of simultaneous correlations in two MUBs [Guo & Wu, (2014)] denoted C_2

- Two sets of complete bases, say $\{|a_i^1\rangle\}$ and $\{|a_j^2\rangle\}$ in Hilbert space of dimension d are defined to be mutually unbiased if and only if

$$|\langle a_i^1|a_j^2\rangle| = \frac{1}{\sqrt{d}}.$$

- The measure of simultaneous correlations in any given pair of mutually unbiased bases is defined as follows:

$$\mathcal{C}_2 = \max_{\Pi_1^a, \Pi_2^a \in \Omega} \min[\chi(\rho_{ab}, \Pi_1^a), \chi(\rho_{ab}, \Pi_2^a)], \quad (1)$$

where Π_i^a represents the basis of measurement on Alice's local Hilbert space and

$$\Omega := \{\{\{|a_i^1\rangle\}, \{|a_j^2\rangle\}\} : |\langle a_i^1|a_j^2\rangle| = \frac{1}{\sqrt{d}}\}. \quad (2)$$

Measures of simultaneous correlations in MUBs

- For a bipartite quantum state ρ_{ab} with the local Hilbert space dimension on Alice's side being d_a, one can define quantity $\mathcal{C}_m$ as in Eq. (1) with m mutually unbiased bases, here $3 \leq m \leq d_a + 1$. be more than three mutually unbiased bases. Just like the $\mathcal{C}_2$ quantity, one can define $\mathcal{C}_3$ as the following:

$$\mathcal{C}_3 = \max_{\Pi_1^a, \Pi_2^a, \Pi_3^a \in \Lambda} \min[\chi(\rho_{ab}, \Pi_1^a), \chi(\rho_{ab}, \Pi_2^a), \chi(\rho_{ab}, \Pi_3^a)], \qquad (3)$$

  where the set of all triads of mutually unbiased bases on Alice's Hilbert space is denoted by Λ.

- $\mathcal{C}_m > 0$ if and only if the state is nonproduct. For two-qubit states, $C_1 \geq C_2 \geq C_3$.

Remote state preparation

- Remote state preparation is the variant of quantum state teleportation in which the sender can make the choice of the quantum state to be communicated.
- In contrast to the standard quantum teleportation whose implementation makes use of a shared singlet and two bits of classical communication, remote preparation requires a shared singlet supported by only one classical bit.

An example for RSP Task

- Alice aims to remotely prepare Bobs system in the quantum state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\phi}|1\rangle\right) \tag{4}$$

  which is in the equatorial plane perpendicular to $\hat{z}$.
- Alice applies von Neumann measurement in the basis $\{|\psi_\perp\rangle\langle\psi_\perp|, |\psi\rangle\langle\psi|\}$, where the state $|\psi_\perp\rangle$ is orthogonal to $|\psi\rangle$. Depending on her output Bob's system is in one of the states $|\psi\rangle\langle\psi|$ or $|\psi_\perp\rangle\langle\psi_\perp|$.
- By sending the outcome of her measurement to Bob – which implies sending one classical bit – he either finds his system in the desired state $|\psi\rangle\langle\psi|$, or can correct his state $|\psi_\perp\rangle\langle\psi_\perp|$ by applying the Pauli operator σ_z.

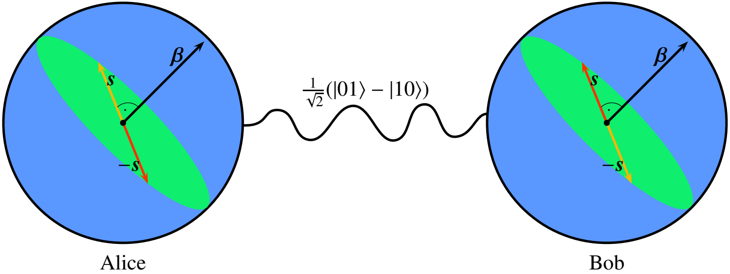

Figure: Deterministic Remote state preparation

- Set of target states that can be remotely prepared deterministically using the aforementioned example of the RSP protocol lies on the equatorial plane. In other words, this task is feasible only for the target states which lie on the equatorial plane.
- The above observation motivates us to operationally characterize a particular instance of the RSP protocol as the set of target states that can be remotely prepared using a fixed shared resource state.
- In other words, the set of all target states for a particular instance of the RSP protocol corresponds to the great circle specified by the unit norm Bloch vector $\vec{\beta}$ perpendicular to the great circle and different instances of RSP protocol correspond to different Bloch vector $\vec{\beta}$.

Remote state preparation using mixed bipartite states

- Dakic *et. al.* considered the implementation of the RSP protocol in which Alice and Bob share a quantum state ρ_{AB}, which can possess various correlations, for example classical correlation, entanglement and quantum discord.

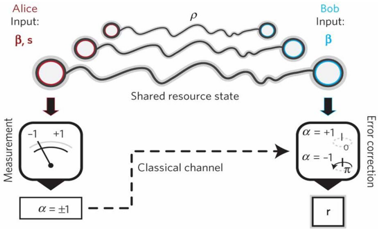

Figure: Dakic *et. al.* Nat. Phys (2012), Remote state preparation in the presence of noise

Remote state preparation using mixed bipartite qubit states

- Alice aims to remotely prepare Bob's system in the quantum state

$$|\psi(\theta,\phi)\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle \tag{5}$$

  having Bloch vector $\hat{s} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ which is in some equatorial plane perpendicular to $\hat{\beta}$. RSP has been implemented if the conditional states prepared on Bob's side by Alice's von Neumann measurement along the direction $\hat{\alpha}$ is of the form

$$\rho = \frac{1 + \vec{r}\cdot\sigma}{2} = r|\psi(\theta,\phi)\rangle\langle\psi(\theta,\phi)| + (1-r)\frac{1}{2} \tag{6}$$

  up to error correction.
- The aim of Alice in this case is to adjust her measurement such that Bob's final Bloch vector $\vec{r}$ becomes as close as possible to the desired vector $\hat{s}$.

Dacik *et. al.* 's way to characterizing the resource for RSP protocol

- As a quantifier of the performance of this procedure Dakic *et. al.* introduced the payoff-function $\mathcal{P} = (\vec{r}.\hat{s})^2$.
- Alice can optimize the payoff for a given $\hat{s}$ and $\hat{\beta}$ by her choice of the local measurement direction $\hat{\alpha}$. Such an optimized payoff is denoted $\mathcal{P}_{\text{max}}$.
- Efficiency of RSP protocol corresponding to the given β is given by the averaged payoff $\langle \mathcal{P}_{\text{max}} \rangle$ which is obtained by averaging the payoff $\mathcal{P}_{\text{max}}$ for all $\hat{s}$ in the given equatorial plane.
- In order to characterize the necessary resource for RSP for any $\hat{\beta}$, Dacik *et. al.* defined the following fidelity formula: $\mathcal{F} = \min_{\hat{\beta}} \langle \mathcal{P}_{\text{max}} \rangle$ which is the above average payoff minimized over all possible choices of the direction β and can be regarded as the quantifier of efficiency for remote state preparation in the worst case.

Relationship between quantum discord and efficiency of RSP in the worst case scenario

- Interestingly, Dacik *et. al.* found that for a given resource state ρ_{AB}, $\min_{\hat{\beta}} \langle \mathcal{P}_{\max}(\rho_{AB}) \rangle$ is nonzero if and only if ρ_{AB} has nonzero discord. This implies that only if quantum discord is non-zero, one can effectively implement the RSP protocol for at least some pure target state corresponding to any great circle in the Bloch sphere.
- Remarkably, for the Bell-diagonal states ρ_{AB}^{BD}, Dacik *et. al.* found that

$$\min_{\hat{\beta}} \langle \mathcal{P}_{\max}(\rho_{AB}^{BD}) \rangle = \mathcal{D}^2(\rho_{AB}^{BD}) \tag{7}$$

where $\mathcal{D}^2(\rho_{AB}^{BD})$ is the geometric discord of the Bell-diagonal state.
- $\mathcal{D}^2(\rho_{AB})$ is defined as as the normalized trace distance to the set of classical states

$$\mathcal{D}^2(\rho_{AB}) = 2 \min_{\rho_{CQ}} ||\rho_{AB} - \rho_{CQ}|| = 2 \min_{\rho_{CQ}} \mathrm{Tr}(\rho_{AB} - \rho_{CQ})^2 \tag{8}$$

Bell-diagonal states

Bell-diagonal states are of the following form:

$$\rho_{AB}^{BD} = \frac{1}{4}\left(1 \otimes 1 + \sum_{i=1}^{3} c_i \sigma_i \otimes \sigma_i\right) = \sum_{ab} \lambda_{ab} |\beta_{ab}\rangle\langle\beta_{ab}|, \qquad (9)$$

where λ_{ab}, with $a, b \in \{0, 1\}$ and $|\beta_{ab}\rangle$ are the corresponding eigenstates given by

$$|\beta_{ab}\rangle = (|0, b\rangle + (-1)^a |1, 1 \oplus b\rangle. \qquad (10)$$

For the Bell-diagonal states with $|c_1| \geq |c_2| \geq |c_3|$, geometric discord, has the following expression:

$$\mathcal{D}^2(\rho_{AB}^{BD}) = \frac{c_2^2 + c_3^2}{2} \qquad (11)$$

Remote state preparation using zero-discord states

- From the above discussion it follows that if a state with vanishing discord is used as resource for RSP, the most one can contend is the existence of at least one great circle for which RSP cannot be implemented for any pure target state on that great circle.
- Nevertheless, this does not rule out using zero discord state as resource for an effective RSP corresponding to at least one great circle.

Efficiency of RSP in the best case scenario for the Bell-diagonal states

- Consider the RSP task in which Alice aims to remotely prepare Bob's system in the quantum state

$$|\psi(\theta = \pi/2, \phi)\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle + e^{i\phi} |1\rangle \right) \tag{12}$$

- For this task, it turns out that

$$\langle \mathcal{P}_{\max}(\rho_{AB}^{BD}) \rangle = \frac{c_1^2 + c_2^2}{2} \tag{13}$$

which is nonzero even for zero-discord states.

- It turns out that the above fidelity captures the efficiency of RSP in the best case since $\max_\beta \langle \mathcal{P}_{\max}(\rho_{AB}^{BD}) \rangle = \frac{c_1^2+c_2^2}{2}$. Note that in case of noisy singlet state, Alice's optimal measurement basis that imply this efficiency correspond to C_2-basis.

Role of $C_2(\rho_{AB}^{BD})$ in the RSP protocol

- For the Bell-diagonal states, the measure of simultaneous correlation in two MUBs is given by

$$C_2(\rho_{AB}^{BD}) = 1 - h\left(\frac{1+\sqrt{(c_1^2+c_2^2)/2}}{2}\right). \quad (14)$$

Here, $h(x) = -xlog_2(x) - (1-x)log_2(1-x)$

- Interestingly, $\langle \mathcal{P}_{\max}(\rho_{AB}^{BD})\rangle$ for remotely preparing $|\psi(\theta=\pi/2,\phi)\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\phi}|1\rangle\right)$ is related to $C_2(\rho_{AB}^{BD}))$ as follows:

$$C_2(\rho_{AB}^{BD})) = 1 - h\left(\frac{1+\sqrt{\langle \mathcal{P}_{\max}(\rho_{AB}^{BD})\rangle}}{2}\right), \quad (15)$$

which implies that the efficiency of RSP in the best case is a monotonic function of the amount of simultaneous correlations in two MUBs.

Averaging RSP Fidelity over all equatorial planes to quantify efficiency of RSP of all pure qubit states

- Effectiveness of the resource state for all $\hat{\beta}$, i.e. efficiency for remotely preparing all pure qubit states can be captured by the following quantity:

$$\mathcal{G} = \left\langle \langle \mathcal{P}_{\mathsf{max}} \rangle_{\phi} \right\rangle_{\theta}. \tag{16}$$

which is the maximal payoff for remotely preparing a pure qubit state with Bloch vector $\hat{s} = (\sin\theta\cos\phi, \sin\theta\cos\phi, \cos\theta)$, and is averaged over ϕ and θ.

Relationship between C_3 and efficiency of RSP for all equatorial planes

- For the Bell-diagonal states, the above defined measure of RSP efficiency is evaluated to be of the form

$$\mathcal{G}(\rho_{AB}^{BD}) = \frac{2}{3}\left(c_1^2 + c_2^2 + c_3^2\right) \tag{17}$$

On the other hand, the measure of simultaneous correlations in three mutually unbiased bases is given by

$$C_3(\tau) = 1 - h\left(\frac{1 + \left(c_1^2 + c_2^2 + c_3^2\right)/\sqrt{3}}{2}\right). \tag{18}$$

- Comparing the above two equations, it follows that

$$C_3(\rho_{AB}^{BD}) = 1 - h\left(\frac{1 + \sqrt{\mathcal{G}(\rho_{AB}^{BD})}}{2\sqrt{2}}\right) \tag{19}$$

Summary

- It is discovered that the quantum communication task of remotely preparing a quantum state (using shared correlated particles as resource assisted by one bit of classical communication) can be efficiently implemented by using any non-product state, including states that have neither entanglement nor quantum discord (the two usually discussed measures of quantum correlation).
- This finding is explained by linking a suitable measure of the efficiency of such a task with a relatively less explored aspect of quantum correlation captured by the existence of the simultaneous correlations in complementary bases whose measure is nonzero for any non-product state.

Thank you for your attention!