# Momentum Kicks in Interference Experiments

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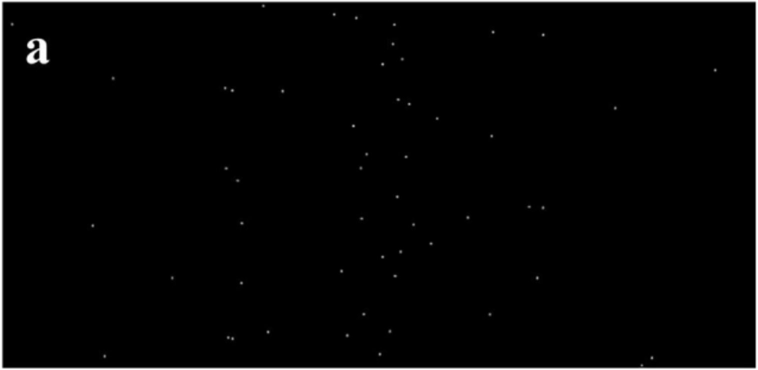
# Outline


1. Two-Slit Experiment and Complementarity
2. Complementarity and Entanglement
3. Momentum kicks

# Two-slit experiment with electrons

Ken Harada et.al., *Scientific Reports* (2018).



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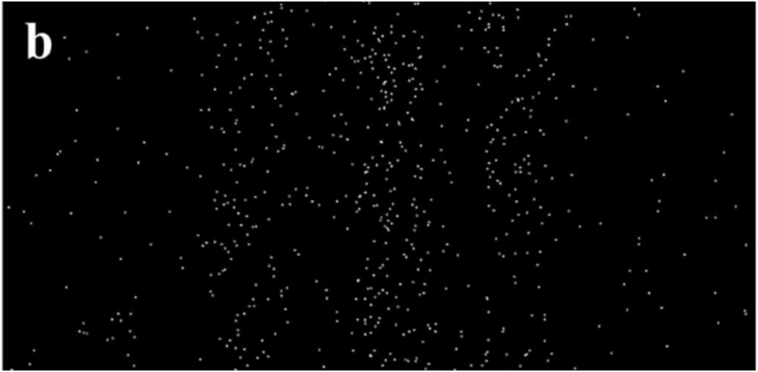


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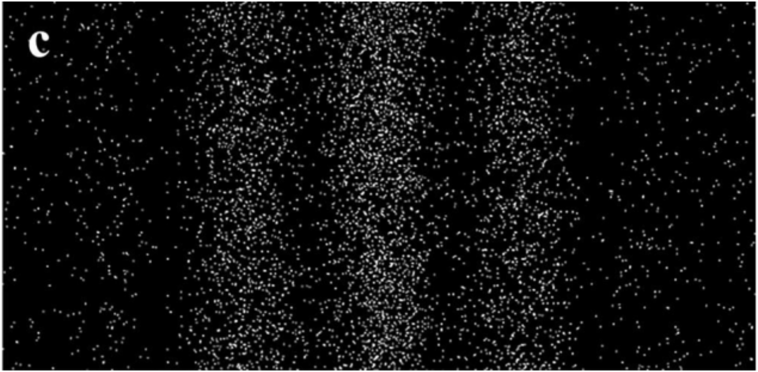


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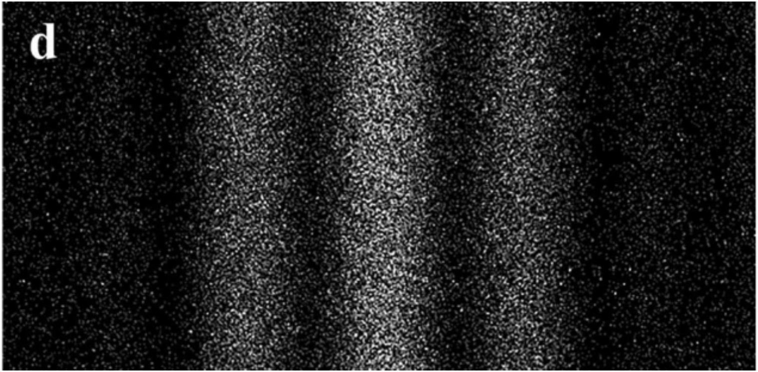
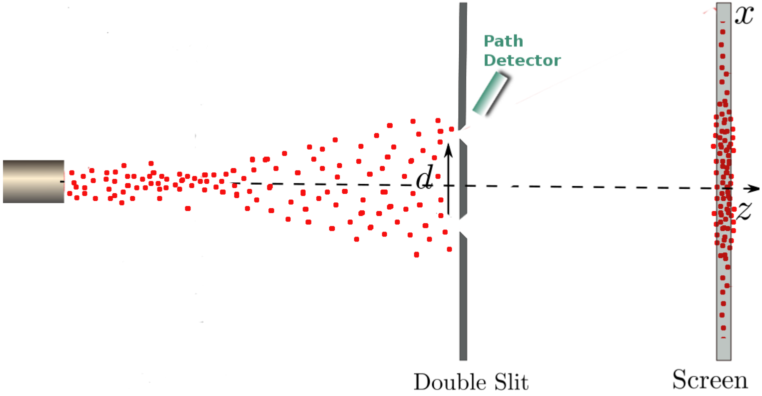



First 2-slit experiment with electrons:

Tonomura, Endo, Matsuda, Kawasaki, Ezawa, *Am. J. Phys.* **57**(2) (1989).

# Which slit did the electron pass through?
## Getting the “*Welcher-Weg*" (which-way) information



# Bohr's Complementarity Principle.



Niels Bohr in 1928

Certain physical concepts are complementary. If two concepts are complementary, an experiment that clearly illustrates one concept will obscure the other complementary one. . . .

- An experiment that illustrates the particle properties of light will not show any of the wave properties of light.
- an experiment that illustrates the wave properties of light will not show any of the particle nature of light.

In the two-slit experiment, the **"which-way"** information and the existence of **interference** pattern are mutually exclusive.

They can NEVER be observed at the same time, in the same experiment.

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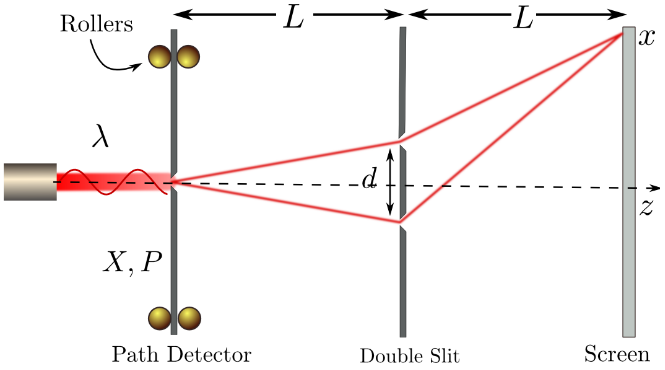
# Einstein's Recoiling-Slit Experiment

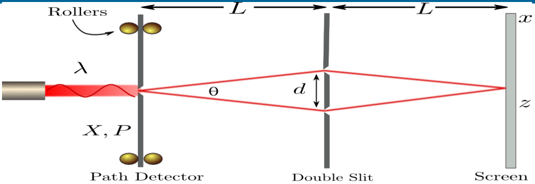
## A thought experiment proposed by Einstein




- Particle going through upper (lower) slit has momentum $p_0$ ($-p_0$)
- Slit experiences momentum recoil $\pm p_0$ (momentum conservation)
- Momentum of slit $\longrightarrow$ which-way information

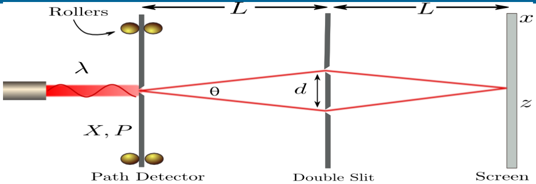
# Bohr's reply






- Difference in the momenta of particles in the two slits
  $p_1 - p_2 = 2p\sin(\theta/2) \approx p\theta = \frac{h}{\lambda}\theta = \frac{h}{\lambda}\frac{d}{L}$
- Recoil momentum of the slit should be measured at least as accurately as $\Delta p_x = \frac{h}{\lambda}\frac{d}{L}$.
- Position of the single-slit is uncertain at least by an amount
  $\Delta x = \frac{\hbar}{2\Delta p_x}$ or $\Delta x = \frac{h}{4\pi}\frac{\lambda}{h}\frac{L}{d} = \frac{\lambda L}{4\pi d}$
- Consequently, position of a fringe is uncertain by an amount $\frac{\lambda L}{4\pi d}$
- Separation between two neighbouring fringes = $\frac{\lambda L}{d}$
- Interference pattern will be washed out.

# Bohr's reply

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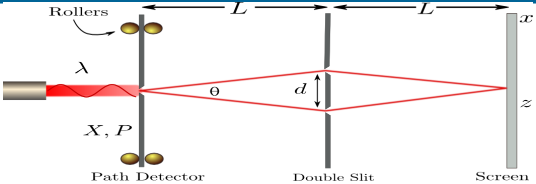


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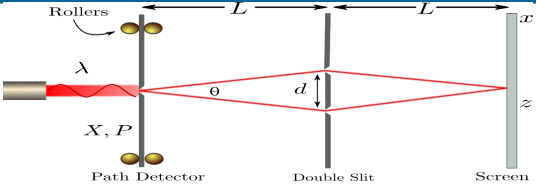


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# Implication of Bohr's resolution



- Uncertainty principle seems to be enforcing complementarity.
- Many people have come to believe:
  Complementarity is a tacit restatement of the uncertainty principle.

Origin of complementarity:

- Getting which-way information will necessarily disturb the state of the particle.
- Disturbance will be enough to wash out interference.

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# Realization of Recoiling-Slit Experiment



PHYSICAL REVIEW A **75**, 062105 (2007)

**Trapped-ion realization of Einstein's recoiling-slit experiment**

Robert S. Utter and James M. Feagin*
*Department of Physics, California State University-Fullerton, Fullerton, California 92834, USA*
(Received 10 July 2006; revised manuscript received 9 October 2006; published 13 June 2007)

Letters to Nature > Abstract



## Letters to Nature

*Nature* **411**, 166-170 (10 May 2001) | doi:10.1038/35075517; Received 22 December 2000; Accepted 7 March 2001

## A complementarity experiment with an interferometer at the quantum–classical boundary

P. Bertet, S. Osnaghi, A. Rauschenbeutel, G. Nogues, A. Auffeves, M. Brune, J. M. Raimond & S. Haroche ← Physics Nobel 2012

1. Laboratoire Kastler Brossel, Département de Physique, Ecole Normale Supérieure, 24 rue Lhomond, F-75231, Paris Cedex 05, France

# Realization of Recoiling-Slit Experiment



PRL 111, 103201 (2013) PHYSICAL REVIEW LETTERS week ending 6 SEPTEMBER 2013

**Momentum Transfer to a Free Floating Double Slit: Realization of a Thought Experiment from the Einstein-Bohr Debates**

L. Ph. H. Schmidt,[1,*] J. Lower,[1] T. Jahnke,[1] S. Schößler,[1] M. S. Schöffler,[1] A. Menssen,[1] C. Lévêque,[2] N. Sisourat,[2] R. Taïeb,[2] H. Schmidt-Böcking,[1] and R. Dörner[1]
[1]*Institut für Kernphysik, Goethe-Universität, Max-von-Laue-Straße 1, 60438 Frankfurt am Main, Germany*
[2]*Laboratoire de Chimie Physique-Matière et Rayonnement, Université Pierre et Marie Curie, 11 Rue Pierre et Marie Curie, 75231 Paris 05, France*
(Received 20 March 2013; revised manuscript received 4 June 2013; published 5 September 2013)

nature photonics ARTICLES
PUBLISHED ONLINE: 1 DECEMBER 2014 | DOI: 10.1038/NPHOTON.2014.289

## Einstein–Bohr recoiling double-slit gedanken experiment performed at the molecular level

Xiao-Jing Liu[1], Quan Miao[2,3], Faris Gel'mukhanov[1,2], Minna Patanen[1], Oksana Travnikova[1], Christophe Nicolas[1], Hans Ågren[2], Kiyoshi Ueda[4] and Catalin Miron[1,5*]

**Double-slit experiments illustrate the quintessential proof for wave-particle complementarity. If information is missing about which slit the particle has traversed, the particle, behaving as a wave, passes simultaneously through both slits. This wave-like behaviour and corresponding interference is absent if 'which-slit' information exists. The essence of Einstein-Bohr's debate about wave-particle duality was whether the momentum transfer between a particle and a recoiling slit**

# Complementarity without uncertainty?



NATURE · VOL 351 · 9 MAY 1991 REVIEW ARTICLE

## Quantum optical tests of complementarity

**Marlan O. Scully, Berthold-Georg Englert & Herbert Walther**

Simultaneous observation of wave and particle behaviour is prohibited, usually by the position–momentum uncertainty relation. New detectors, constructed with the aid of modern quantum optics, provide a way around this obstacle in atom interferometers, and allow the investigation of other mechanisms that enforce complementarity.

$$\Psi(\mathbf{r}) = \frac{1}{\sqrt{2}}[\psi_1(\mathbf{r})|1_1 0_2\rangle + \psi_2(\mathbf{r})|0_1 1_2\rangle]|b\rangle$$

...m ...re ...ct ...to ...o[1]

In a speech delivered in honour of Count Alessandro Volta (1745–1827), quantum theory as we know it today was still new, and all examples used to illustrate complementarity referred to the position (particle-like) and momentum (wave-like) attributes of a quantum mechanical object, be it a photon or a massive particle. This is the historical reason why complementarity is often superficially identified with the 'wave-particle duality of matter'.

Richard Feynman, discussing the two-slit experiment in his admirable introduction to quantum mechanics[2], notes that this wave-particle dual behaviour contains the basic mystery of quantum mechanics. In fact, he goes so far as to say: "In reality it contains the only mystery."

Complementarity, however, is a more general concept. We say that two observables are 'complementary' if precise knowledge of one of them implies that all possible outcomes of measuring the other one are equally probable. We may illustrate this by two extreme examples. (A more general discussion is given in ref. 3.) The first example consists of the position and momentum (along one direction) of a particle: if, say, the position is predetermined then the result of a momentum measurement cannot be predicted and all momentum values are

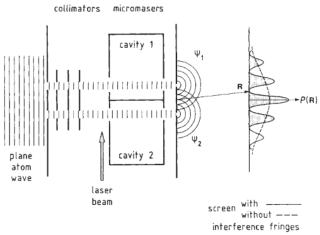


FIG. 3 Two-slit experiment with atoms. A set of wider slits collimates two atom beams which illuminate the narrow slits where the interference pattern originates. The collimation of the atomic beams would actually be done using atomic optics. One could, for instance, employ six-pole fields operating either on the magnetic dipole moment, or in the case of Rydberg atoms on the field-induced electric dipole moment. This set-up is supplemented by two high-quality micromaser cavities and a laser beam to to provide which-path information.

# Momentum kicks



LETTERS TO NATURE

## Path detection and the uncertainty principle

**Pippa Storey, Sze Tan, Matthew Collett & Daniel Walls**

NATURE · VOL 367 · 17 FEBRUARY 1994

Department of Physics, University of Auckland, Private Bag 92019, Auckland, New Zealand

**QUANTUM mechanics predicts that any detector capable of determining the path taken by a particle through a double slit will destroy the interference. This follows from the principle of complementarity formulated by Niels Bohr: simultaneous observation of wave and particle behaviour is prohibited. But such a description makes no reference to the physical mechanism by which the interference is lost. In the best studied *welcher Weg* ('which path') detection schemes[1,2], interference is lost by the transfer of momentum to the particle whose path is being determined, the extent of momentum transfer satisfying the position–momentum uncertainty relation. This has prompted the question as to whether complementarity is always enforced in *welcher Weg* schemes by momentum transfer. Scully *et al.*[3] have recently responded in the negative, suggesting that complementarity must be accepted as an independent component of quantum mechanics, rather than as simply a consequence of the uncertainty principle. But we show here that, in any path detection scheme involving a fixed double slit, the amount of momentum transferred to the particle by a perfectly efficient detector (one capable of resolving the path unambiguously) is related to the slit separation in accordance with the uncertainty principle. If less momentum than this is transferred, interference is not completely destroyed and the path detector cannot be perfectly efficient.**

The best known *welcher Weg* detectors in a double slit

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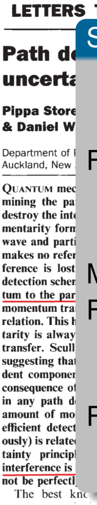
Storey et.al. proved:
Minimum momentum transferred in which-way detection:

$$p_m \geq \hbar/d$$

where $d$ = separation between the two slits.

# Momentum kicks




## Storey et.al. Nature **367**, 626 (1994)

$$|\Psi\rangle = \int dx\,\psi_i(x)|D_x\rangle$$
$$|D_x\rangle = \sum_\xi O_\xi(x)|\xi\rangle$$

Final wavefunction

$$\psi_f(x) = C\psi_i(x)O_\xi(x)$$
$$\tilde{\psi}_f(p) = C\int \tilde{\psi}_i(p')\tilde{O}_\xi(p-p')dp'$$

Momentum distribution of $\psi_i$ gets modified by $\tilde{O}_\xi(p-p')$

Fringe visibility $\mathcal{V} = |\langle D_{x_1}|D_{x_2}\rangle|$ ($x_1, x_2$ position of the slits)

$$1 \geq \mathcal{V} \geq 1 - \frac{p_{max}d}{\hbar}$$

For perfect which-way detection ($\mathcal{V} = 0$)

$$p_{max} \geq \hbar/d$$

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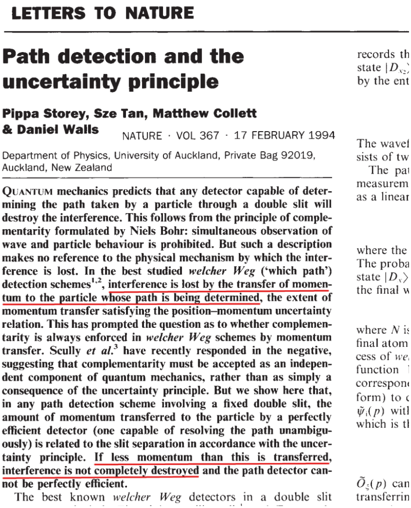
# Momentum kicks



LETTERS TO NATURE

**Path detection and the uncertainty principle**

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The best known *welcher Weg* detectors in a double slit

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Two opposite viewpoints:

- Quantum correlations
  - B-G. Englert, *Phys. Rev. Lett.* **77**, 2154 (1996),
    “Fringe visibility and which-way information: an inequality"
  - M.O. Scully, B.G. Englert, H. Walther, *Nature* **375**, 367 (1995),
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- Uncertainty principle
  - S.M. Tan, D.F. Walls,*Phys. Rev. A* **47**, 4663-4676 (1993),
    “Loss of coherence in interferometry".
  - E.P. Storey, S.M. Tan, M.J. Collett, D.F. Walls, *Nature* **367**, 626 (1994).
  - H. Wiseman, F. Harrison, *Nature* **377**, 584 (1995),
    “Uncertainty over complementarity?"
  - H.M. Wiseman,*Phys. Rev. A* **58** 1740 (1998).
    “Bohmian analysis of momentum transfer in welcher Weg measurements,"
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    “Directly observing momentum transfer in twin-slit which-way experiments"
  - R. Mir, J.S. Lundeen, M.W. Mitchell, A.M. Steinberg, J.L. Garretson, H.M. Wiseman, *New J. Phys.* **9**, 287 (2007).
    “A double-slit ’which-way’ experiment on the complementarity-uncertainty debate"

# “Momentum kicks"

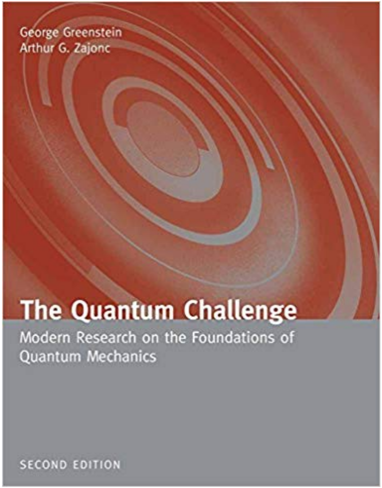
## Controversy on whether particle receives “momentum kick"




- S. Durr, T. Nonn, G. Rempe, *Nature* **395**, 33 (1998), “Origin of quantum-mechanical complementarity probed by a which-way experiment in an atom interferometer."
- Ma X.-S., Kofler J. and Zeilinger A., Rev. Mod. Phys., 88 (2016) 015005.
- Luis A. and Sánchez-Soto L. L., Phys. Rev. Lett., 81 (1998) 4031
- C.S. Unnikrishnan, *Phys. Rev. A* **62**, 015601 (2000).
- Mir R., Lundeen J. S., Mitchell M. W., Steinberg A. M., Garretson JL and Wiseman HM, New J. Phys., 9 (2007) 287, “A double-slit which-way experiment on the complementarity-uncertainty debate"
- Xiao Y., Wiseman H. M., Xu J.-S., Kedem Y., Li C.-F. and Guo G.-C., Observing momentum disturbance in double-slit which-way measurements, arXiv:1805.02059 [quant-ph].

# Controversy on momentum kicks...






having acted on the electron. It is this element that we focus on in thinking about momentum kicks in the Rempe experiment.

***Momentum Kicks in Interference Experiments*** Before doing so, we should note that even a normal interference experiment, for example, the one illustrated in Figure 1–2, delivers a momentum kick. The incoming particles depicted on that figure were moving purely horizontally. Had their momenta not been altered, they would have ended up on the detecting screen directly across from the slits. The fact that they did not do so demonstrates they were given a momentum kick (along the vertical direction on the page) by the slits.

Because quantum mechanics is a probabilistic theory, we must think in terms of a *probability distribution function* describing these kicks. This is a function that tells us the probability of the particle receiving a given momentum kick. This function can be simply read off the interference pattern. For example, the existence of the large interference peak directly across from the two slits (see Figure 1–11) tells us that the most likely situation is to have a zero or very small momentum kick. Similarly, interference minima

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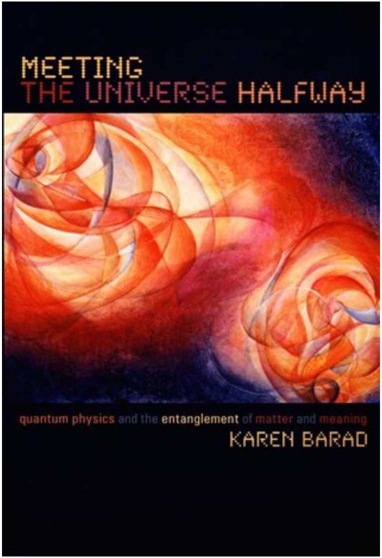
What produces the loss of interference seen when which-path information is obtained? Our personal view is that mechanisms such as that proposed by Einstein, in which a real momentum kick is given to the particle, are too firmly grounded in naive classical concepts. We also regard the principle of complementarity in and of itself as being too abstract to yield much physical insight. Currently, most workers in the field hold to the view that information is an important key to answering this question. There is at present no consensus, however, as to whether the concept of a momentum kick, whether of the classical or quantum-mechanical variety, is also part of the answer.

Differing views can be held as to the significance of complementarity. For example, we can argue that it implies that quantum mechanics is incapable of providing us with a full description of reality. This, of course, was

# Controversy on momentum kicks...






APPENDIX C

CONTROVERSY CONCERNING THE RELATIONSHIP BETWEEN BOHR'S PRINCIPLE OF COMPLEMENTARITY AND HEISENBERG'S UNCERTAINTY PRINCIPLE

The nature of the relationship between Heisenberg's uncertainty principle and Bohr's complementarity, sparked by the work of Scully et al. (1991), has been a matter of some controversy. The claim of Scully et al. that their experiment offers definitive evidence of the loss of interference without any disturbance caused by the detector has been contested by Storey et al. (1994). Storey et al. argue that complementarity is always enforced by the uncertainty relations, that is, by an uncontrollable momentum transfer (disturbance), thereby arguing that it is the more fundamental principle than complementarity, in contradiction to Scully et al. and the point of view that I espouse here. Wiseman and Harrison (1995) argue that the kind of random momentum kick that Storey et al. enlist to explain the destruction of the interference pattern is in general not the same as the classical notion but rather a strange nonlocal beast involving the "more subtle idea of momentum-kick amplitudes" (*within* an entangled state!) (for more details, see Wiseman et al. 1997). Furthermore, Wiseman and Harrison argue that while the Einstein recoiling-slit gedanken experiment *may* be—but *need* not be—understood in terms of uncontrolled classical momentum kicks, this is not the case for the experiment suggested by Scully et al. (and confirmed by Eichmann et al. [1993]). However, as Wiseman and Harrison point out, such a classical analysis of the recoiling-slit experiment is based on a naive-realist interpretation of the uncertainty principle, which, needless to say, Bohr definitively

# Is uncertainty a requirement for Complementarity?



Could Bohr have replied to Einstein without invoking the uncertainty principle?

# Quantum measurement

## According to von Neumann

A quantum measurement consists of two processes.

Process 1: Unitary → establishes correlation between system & detector.

System initial state = $\sum_{i=1}^{n} c_i|\psi_i\rangle$ Detector initial state = $|d_0\rangle$

$$|d_0\rangle \sum_{i=1}^{n} c_i|\psi_i\rangle \xrightarrow[\text{Process 1}]{\text{Unitary evolution}} \sum_{i=1}^{n} c_i|d_i\rangle|\psi_i\rangle$$

Process 2: A non-unitary one which picks out a single result

$$\sum_{i=1}^{n} c_i|d_i\rangle|\psi_i\rangle \xrightarrow[\text{Process 2}]{} |d_k\rangle|\psi_k\rangle$$

with probability $|c_k|^2$.

Process 2 constitutes "The Measurement Problem"

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with probability $|c_k|^2$.

Process 2 constitutes "The Measurement Problem"

# Quantum measurement
## According to von Neumann



A quantum measurement consists of two processes.

Process 1: Unitary $\rightarrow$ establishes correlation between system & detector.

System initial state = $\sum_{i=1}^{n} c_i|\psi_i\rangle$ Detector initial state = $|d_0\rangle$

$$|d_0\rangle \sum_{i=1}^{n} c_i|\psi_i\rangle \xrightarrow[Process\ \ 1]{Unitary\ \ evolution} \sum_{i=1}^{n} c_i|d_i\rangle|\psi_i\rangle$$

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# Which-way detection in Einstein's experiment

## Using von Neumann's process 1



Two orthogonal states of the particle: $|\psi_1\rangle$ and $|\psi_2\rangle$

$|\psi_1\rangle$ $\longrightarrow$ amplitude to go through slit 1.

$|\psi_2\rangle$ $\longrightarrow$ amplitude to go through slit 2.

Two momentum states of the recoiling slit: $|p_1\rangle$ and $|p_2\rangle$.

Points to be noted:

(a) Two different momentum states of the recoiling slit will necessarily get entangled with the states of the particle passing through the two slits: $\Psi(x) = \psi_1(x)|p_1\rangle + \psi_2(x)|p_2\rangle$

(b) In principle it is possible to find an interaction which will not affect the states of the particle $|\psi_1\rangle$ and $|\psi_2\rangle$, but only result in the detector states getting correlated with them.

Point (a) was not part of Bohr's reply.

Point (a) is enough to rule out interference!

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# Which-way information and interference



**Without which-way information**

$$\psi(x) = \psi_1(x) + \psi_2(x)$$

Probability of finding the particle at a point x on the screen,

$$|\psi(x)|^2 = |\psi_1(x)|^2 + |\psi_2(x)|^2 + \psi_1^*(x)\psi_2(x) + \psi_2^*(x)\psi_1(x)$$

The last two terms represent interference.

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Recoiling slit storing which-way information destroys interference!

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Can this argument be made more quantitative?

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# Quantitative Wave-Particle Duality



$$\psi(x) = \psi_1(x)|d_1\rangle + \psi_2(x)|d_2\rangle$$

If detector distinguishes the two paths inaccurately: $\langle d_1|d_2\rangle \neq 0$.

- Define distinguishability of the two paths:

  $\mathcal{D}_Q$ = max. probability with which $|d_1\rangle, |d_1\rangle$ can be unambiguously distinguished
- Visibility of interference

$$\mathcal{V} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

**Symmetric beams**

$$\mathcal{D}^2 + \mathcal{V}^2 \leq 1$$

B.G. Englert, *PRL* **77**, 2154 (1996).

- It has been shown that [1]

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[1] Wave-particle duality in asymmetric beam interference
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# Are there momentum kicks?

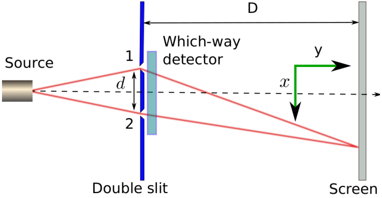


Particle passing through double-slit, with a which-way detector:

$$\Psi(x) = \frac{1}{\sqrt{2}}(\psi_1(x)|d_1\rangle + \psi_2(x)|d_2\rangle)$$

Gives no interference for $\langle d_1|d_2\rangle = 0$.

New basis for path-detector states:
$|d_\pm\rangle = \frac{1}{\sqrt{2}}(|d_1\rangle \pm |d_2\rangle)$




The state

$$\Psi(x) = \frac{1}{2}[\psi_1(x) + \psi_2(x)]|d_+\rangle + \frac{1}{2}[\psi_1(x) - \psi_2(x)]|d_-\rangle$$

Our claim: The state can be written as

$$\Psi(x) = \frac{1}{2}[\psi_1(x) + \psi_2(x)]|d_+\rangle + \frac{1}{2}e^{ip_0x/\hbar}[\psi_1(x) + \psi_2(x)]|d_-\rangle$$

where $p_0 = \hbar\pi/d$ is a momentum-kick.

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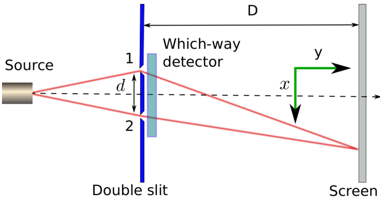


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\end{aligned}
$$

Particle receives a momentum kick of magnitude $p_0 = \hbar\pi/d = h/2d$ whenever the path detector is found in state $|d_-\rangle$ (randomly, 50% of the time).

Detector basis

$|d_1\rangle, |d_2\rangle \quad \longrightarrow$ No momentum kick
$|d_+\rangle, |d_-\rangle \quad \longrightarrow$ Momentum kick of size $h/2d$

# Are there momentum kicks?



The quantum state

$\Psi(x$

## Momentum kick

$$e^{\frac{ip_0 x}{\hbar}}\psi(x) = e^{\frac{ip_0 x}{\hbar}}\int_{-\infty}^{\infty}\tilde{\psi}(p)e^{\frac{ipx}{\hbar}}dp$$

$$= \int_{-\infty}^{\infty}\tilde{\psi}(p)e^{\frac{i(p+p_0)x}{\hbar}}dp$$

$$= \int_{-\infty}^{\infty}\tilde{\psi}(p-p_0)e^{\frac{ipx}{\hbar}}dp$$

Momentum distribution of $\psi$ gets shifted by $p_0$.

$'_-\rangle$

$'_-\rangle$

Particle r

whenever

(randomly

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# Three-slit which-way experiment



$$\Psi(x) = \frac{1}{\sqrt{3}}\left[\psi_1(x)|d_1\rangle + \psi_2(x)|d_2\rangle + \psi_3(x)|d_3\rangle\right]$$

New basis for path-detector states:

$$|d_\alpha\rangle = \frac{1}{\sqrt{3}}(|d_1\rangle + |d_2\rangle + |d_3\rangle)$$
$$|d_\beta\rangle = \frac{1}{\sqrt{3}}(e^{-i2\pi/3}|d_1\rangle + |d_2\rangle + e^{i2\pi/3}|d_3\rangle)$$
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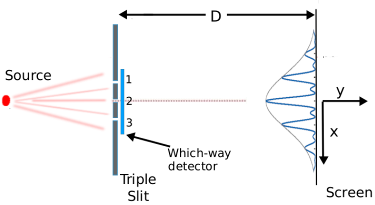


State in the new basis:

$$\Psi(x) = \tfrac{1}{3}\left[\psi_1 + \psi_2 + \psi_3\right]|d_\alpha\rangle + \tfrac{1}{3}\left[e^{\frac{-i2\pi}{3}}\psi_1 + \psi_2 + e^{\frac{i2\pi}{3}}\psi_3\right]|d_\beta\rangle + \tfrac{1}{3}\left[e^{\frac{i2\pi}{3}}\psi_1 + \psi_2 + e^{\frac{-i2\pi}{3}}\psi_3\right]|d_\gamma\rangle$$

Can also be written as

$$\Psi(x) = \tfrac{1}{3}\left[\psi_1 + \psi_2 + \psi_3\right]|d_\alpha\rangle + \tfrac{1}{3}e^{\frac{ip_0x}{\hbar}}\left[\psi_1 + \psi_2 + \psi_3\right]|d_\beta\rangle + \tfrac{1}{3}e^{\frac{-ip_0x}{\hbar}}\left[\psi_1 + \psi_2 + \psi_3\right]|d_\gamma\rangle$$

where $p_0 = h/3d$.

$\psi_1, \psi_2, \psi_3$ are localized at $x = -d, 0, d$

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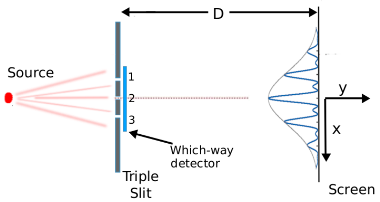


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# Three-slit which-way experiment



Particle recieves a momentum kick of magnitude

$p_0 = h/3d$ when detector state is $|d_\beta\rangle$ (one-third of the time)

$p_0 = -h/3d$ when detector state is $|d_\gamma\rangle$ (one-third of the time)

No kick when detector state is $|d_\alpha\rangle$

# n-slit which-way experiment



Particle going through a multi-slit, with which-way detector:

$$\Psi(x) = \frac{1}{\sqrt{n}} \sum_{k=1}^{n} \psi_k(x)|d_k\rangle$$

Interference

$$\Psi^*(x)\Psi(x) = \frac{1}{n} \sum_{k=1}^{n} |\psi_k|^2 + \frac{1}{n} \sum_{j,k} \psi_j^* \psi_k \langle d_j|d_k\rangle + \psi_k^* \psi_j \langle d_k|d_j\rangle$$

is destroyed by the orthogonality of $\{|d_i\rangle\}$.

# n-slit which-way experiment



A new basis for path-detector states

$$
\begin{aligned}
|d_1\rangle &= \frac{1}{\sqrt{n}}\left(|\alpha_1\rangle + |\alpha_2\rangle + |\alpha_3\rangle + |\alpha_4\rangle + \cdots + |\alpha_n\rangle\right) \\
|d_2\rangle &= \frac{1}{\sqrt{n}}\Big(|\alpha_1\rangle + e^{\frac{i2\pi}{n}}|\alpha_2\rangle + e^{\frac{i4\pi}{n}}|\alpha_3\rangle + e^{\frac{i6\pi}{n}}|\alpha_4\rangle + \\
&\qquad \cdots + e^{\frac{i2(n-1)\pi}{n}}|\alpha_n\rangle\Big) \\
|d_3\rangle &= \frac{1}{\sqrt{n}}\Big(|\alpha_1\rangle + e^{\frac{i4\pi}{n}}|\alpha_2\rangle + e^{\frac{i8\pi}{n}}|\alpha_3\rangle + e^{\frac{i12\pi}{n}}|\alpha_4\rangle + \\
&\qquad \cdots + e^{\frac{i4(n-1)\pi}{n}}|\alpha_n\rangle\Big).
\end{aligned}
$$

$e^{\frac{i2k\pi}{n}} \longrightarrow$ nth root of unity

# n-slit which-way experiment



$$
\begin{aligned}
\Psi(x) \quad = \quad & \tfrac{1}{n}\left(\psi_1 + \psi_2 + \psi_3 + \cdots + \psi_n\right)|\alpha_1\rangle \\
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& + \tfrac{1}{n} e^{\frac{ip_2 x}{\hbar}} \left(\psi_1 + \psi_2 + \psi_3 + \cdots + \psi_n\right)|\alpha_3\rangle \\
& + \ldots \\
& + \tfrac{1}{n} e^{\frac{ip_{n-1} x}{\hbar}} \left(\psi_1 + \psi_2 + \psi_3 + \cdots + \psi_n\right)|\alpha_n\rangle,
\end{aligned}
$$

where $p_j = jh/nd$.

Interpretation:

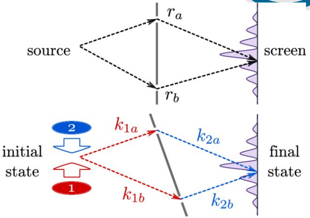
Particle either receives no momentum kick,
or randomly receives a kick of one of the n-1 magnitudes

# "Double-slit experiment in momentum space"

Ivanov et.al, *EPL* **115**, 41001 (2016).

Particle in a superposition of two distinct momentum states

$$\Psi(p) = \frac{1}{\sqrt{2}}\left[\psi_1(p)|d_1\rangle + \psi_2(p)|d_2\rangle\right]$$




State in another basis:

$$\Psi(p) = \tfrac{1}{2}[\psi_1(p) + \psi_2(p)]|d_+\rangle + \tfrac{1}{2}[\psi_1(p) - \psi_2(p)]|d_-\rangle.$$

May also be written as:

$$\Psi(p) = \tfrac{1}{2}[\psi_1(p) + \psi_2(p)]|d_+\rangle + e^{\frac{-ip_1x_0}{\hbar}} e^{\frac{ipx_0}{\hbar}} \tfrac{1}{2}[\psi_1(p) + \psi_2(p)]|d_-\rangle,$$

where $x_0 = \frac{h}{2(p_2 - p_1)}$.

Particle recieves position kicks of magnitude $\frac{h}{2(p_2 - p_1)}$

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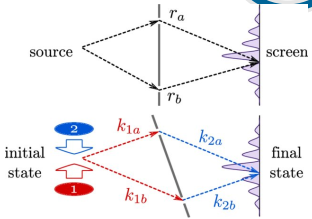


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# Conclusions



- Complementarity is enforced by the ubiquitous entanglement between the particle and the which-way detector - always!
- The loss of interference can be interpreted
  either
  as arising from the entanglement of particle paths with orthogonal state of the which-way detector $|d_1\rangle, |d_2\rangle$ basis
  or
  due to the random momentum kicks the particle appears to experience. $|d_+\rangle, |d_-\rangle$ basis
- The momentum kicks are NOT due to any momentum transfer from the which-way detector.

"Which-way measurement and momentum kicks"
T. Qureshi, *EPL* **123**, 30007 (2018).