Characterizing quantumness of unsteerable correlations

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This talk is based on:

- Debarshi Das, Bihalan Bhattacharya, Chandan Datta, Arup Roy, C. Jebaratnam, A. S. Majumdar, and R. Srikanth, "*Operational characterization of quantumness of unsteerable bipartite states*", Physical Review A **97**, 062335 (2018).

- Debarshi Das, C. Jebaratnam, Bihalan Bhattacharya, Amit Mukherjee, Some Sankar Bhattacharya, and Arup Roy, "*Characterization of the quantumness of unsteerable tripartite correlations*", Annals of Physics **398**, 55 (2018).

Motivation:

- Quantum discord[1] indicates the presence of quantumness even in separable states.

- Certain separable states which have quantumness may improve certain information theoretic protocols if there is a constraint on the possible local hidden variable models[2].

- From an operational perspective, nonlocal or steerable states require augmenting preshared randomness with nonzero communication cost[3].

1) H. Ollivier, and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001).

2) T. K. C Bobby and T. Paterek, New J. Phys. **16**, 093063 (2014).

3) B. F. Toner and D. Bacon, Phys. Rev. Lett. **91**, 187904 (2003); A. B. Sainz, L. Aolita, N. Brunner, R. Gallego, and P. Skrzypczyk, Phys. Rev. A **94**, 012308 (2016).

Motivation continued..

Now the question is

- **How to operationally characterize the quantumness present in local or unsteerable correlations?**

Motivation continued..

- Bowles et. al[1] have shown that the **statistics of all local entangled states can be simulated by using only finite shared randomness**.

- Donohue and Wolfe[2] have demonstrated an interesting feature of certain local boxes which they called **"superlocality": there exist local boxes which can be simulated by certain quantum systems of local dimension lower than that of the shared classical randomness needed to simulate them.** That is, superlocality refers to the dimensional advantage in simulating certain local boxes by using quantum systems.

1) J. Bowles, F. Hirsch, M. T. Quintino, and N. Brunner, Phys. Rev. Lett. **114**, 120401 (2015).
2) J. M. Donohue, and E. Wolfe, Phys. Rev. A **92**, 062120 (2015).

Motivation continued..

- Further it has been demonstrated[1] that superlocality provides an **operational characterization of quantumness of certain local correlations**.

Now the question is:

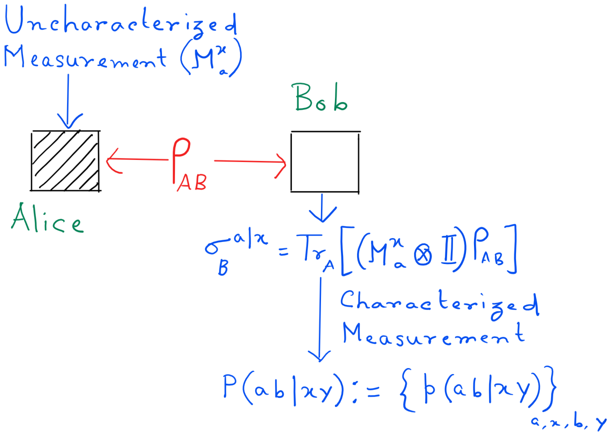
- **Can the notion of 'superlocality' be generalized in steering scenario (i.e. in one-sided device independent scenario) to characterize quantumness of certain unsteerable correlations?**

Answer:

- **Yes.**

1) C. Jebaratnam, S. Aravinda, and R. Srikanth, Phys. Rev. A **95**, 032120 (2017).

Steering Scenario:



Steering Scenario:

The correlation $P(ab|xy)$ is unsteerable from Alice to Bob iff

$$p(ab|xy) = \sum_{\lambda} p(\lambda) p(a|x,\lambda) p(b|y,\rho_\lambda) \quad \forall a,x,b,y, \tag{1}$$

where $\sum_\lambda p(\lambda) = 1$, $p(a|x,\lambda)$ denotes an arbitrary probability distribution arising from local hidden variable (LHV) λ (λ occurs with probability $p(\lambda)$); $p(b|y,\rho_\lambda)$ denotes a quantum probability of outcome b when measurement y is performed on local hidden state (LHS) ρ_λ.

Superunsteerability

Definition: Suppose Alice and Bob share a quantum state in $\mathbb{C}^{d_A} \otimes \mathbb{C}^{d_B}$ and perform measurements which produce an unsteerable bipartite box $P(ab|xy) := \{p(ab|xy)\}_{a,x,b,y}$. Then, superunsteerability holds iff there is no decomposition of the box in the form,

$$p(ab|xy) = \sum_{\lambda=0}^{d_\lambda - 1} p(\lambda) p(a|x,\lambda) p(b|y,\rho_\lambda) \quad \forall a,x,b,y, \qquad (2)$$

with dimension of the shared randomness/hidden variable $d_\lambda \leq d_A$. Here $p(b|y,\rho_\lambda)$ denotes a quantum probability of outcome b when measurement y is performed on LHS ρ_λ in $\mathbb{C}^{d_B}$.

Superunsteerability

• **How to determine whether a given unsteerable correlation is superunsteerable or not:**

We have to consider the LHV-LHS model of the given correlation with **minimum dimension of the shared randomness**.

$\downarrow$

We have to check whether this **minimum dimension** is **greater** than the **local Hilbert space dimension** of the shared quantum system (reproducing the given unsteerable correlation) at Alice's side (untrusted party's side who steers the other party).

• Quantumness in the form of non-zero quantum discord is necessary for demonstrating superunsteerability.

Results: Example 1 of superunsteerability

Consider the white noise-BB84 family given by:

$$P_{BB84}(ab|xy) = \frac{1 + (-1)^{a \oplus b \oplus x.y} \delta_{x,y} V}{4}, \tag{3}$$

where $0 < V \leq 1$;

<u>Quantum simulation:</u> The white noise-BB84 family can be produced when Alice and Bob perform appropriate measurements on the $2 \otimes 2$ dimensional Werner state,

$$\rho_V = V|\psi^-\rangle\langle\psi^-| + \frac{1-V}{4}\mathbb{I}_2 \otimes \mathbb{I}_2, \tag{4}$$

where $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$; $0 < V \leq 1$.

Results: Example 1 of superunsteerability

Classical simulation: The white noise-BB84 family can be simulated by a LHV-LHS model in the range $0 < V \leq \frac{1}{\sqrt{2}}$ and **the minimum dimension of the hidden variable needed to reproduce the correlation by LHV-LHS model is** 4.

• Hence, **the white noise-BB84 family demonstrates superunsteerablity for** $0 < V \leq \frac{1}{\sqrt{2}}$.

Results: Example 1 of superunsteerability

Hence, super-unsteerable white noise-BB84 family ($0 < V \leq \frac{1}{\sqrt{2}}$) certifies quantumness of

i) $2 \otimes 2$ dimensional resource producing it (e.g., two-qubit Werner state), or

ii) $3 \otimes 2$ dimensional resource producing it (e.g., the state $V|\psi^-\rangle\langle\psi^-| + \frac{1-V}{2}|2\rangle\langle 2| \otimes \mathbb{I}_2$)

Results: Example 2 of superunsteerability

Consider the correlation given by:

$$P(ab|xy) = \begin{array}{c|cccc} \text{(x,y)} \backslash \text{(a,b)} & (0,0) & (0,1) & (1,0) & (1,1) \\ \hline (0,0) & \frac{5}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ (0,1) & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & 0 \\ (1,0) & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & 0 \\ (1,1) & \frac{5}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{array} \quad (5)$$

where each row and column corresponds to a fixed measurement setting (xy) and a fixed outcome (ab) respectively.

Results: Example 2 of superunsteerability

Quantum simulation: This correlation (5) can be produced when Alice and Bob perform appropriate measurements on the $2 \otimes 2$ dimensional state,

$$\rho = \frac{1}{2}\Big(|00\rangle\langle 00| + |++\rangle\langle ++|\Big), \tag{6}$$

where, $|0\rangle$ and $|+\rangle$ are the eigenstates of the operators σ_z and σ_x respectively corresponding to the eigenvalue $+1$.

Results: Example 2 of superunsteerability

Classical simulation: The aforementioned correlation (5) can be simulated using a LHV-LHS model and **the minimum dimension of the hidden variable needed to reproduce the correlation by LHV-LHS model is** 3.

• Hence, **the aforementioned correlation (5) demonstrates superunsteerablity**.

• Hence, the aforementioned superunsteerable correlation (5) certifies quantumness of $2 \otimes 2$ dimensional resource producing it (e.g., the state (6)).

Results: Superunsteerability

The present study classifies any bipartite states in the $2-2-2$ experimental scenario (involving 2 parties, 2 measurement settings per party, 2 outcomes per measurement setting) into three types:

(i) States which do not demonstrate superunsteerability. The states having zero discord belong to this class.

(ii) Non-zero discord states which demonstrate superunsteerability with unsteerable boxes having minimum hidden variable dimension 3.

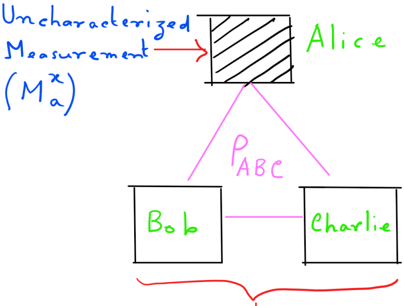
(iii) Non-zero discord states which demonstrate superunsteerability with unsteerable boxes having minimum hidden variable dimension 4.

What is next?

Now the question is

- **How to generalize the notion "superunsteerability" for tripartite unsteerable correlation?**

Bi-unsteerability of tripartite correlation in one-sided device independent scenario



$$\sigma_{BC}^{a|x} = Tr_A\left[\left(M_a^x \otimes \mathbb{I} \otimes \mathbb{I}\right)\rho_{ABC}\right]$$

Characterized Measurement

$$P(abc|xyz) := \{p(abc|xyz)\}_{a,x,b,y,c,z}$$

Bi-unsteerability

The tripartite correlation $P(abc|xyz)$ is called bi-unsteerable across the bipartite cut $A - BC$ (i.e., from Alice to Bob-Charlie) iff

$$p(abc|xyz) = \sum_{\lambda} p(\lambda) p(a|x,\lambda) p(bc|y,z,\rho_{\lambda}^{BC}) \quad \forall a,x,b,y,c,z, \tag{7}$$

with $\sum_{\lambda} p(\lambda) = 1$. Here $P(bc|y,z,\rho_{\lambda}^{BC})$ denotes an arbitrary quantum probability of obtaining the outcomes b and c, when measurements y and z are performed by Bob and Charlie respectively on the shared bipartite LHS ρ_{λ}^{BC}.

Super-bi-unsteerability

Definition: Suppose Alice, Bob and Charlie share a tripartite quantum state ρ_{ABC} in $\mathbb{C}^{d_A} \otimes \mathbb{C}^{d_B} \otimes \mathbb{C}^{d_C}$ producing a correlation box $P(abc|xyz)$ which is bi-unsteerable from Alice to Bob-Charlie. Then super-bi-unsteerability from Alice to Bob-Charlie holds iff there is no decomposition of the form:

$$p(abc|xyz) = \sum_{\lambda=0}^{d_\lambda - 1} p(\lambda) p(a|x,\lambda) p(bc|y,z,\rho_\lambda^{BC}) \quad \forall a,x,b,y,c,z, \tag{8}$$

where $d_\lambda \leq d_A$. Here $P(bc|y,z,\rho_\lambda^{BC})$ denotes a quantum probability of the outcomes b and c, when measurements B_y and C_z are performed by Bob and Charlie respectively on the shared bipartite LHS ρ_λ^{BC} in $\mathbb{C}^{d_B} \otimes \mathbb{C}^{d_C}$.

Genuine Super-bi-unsteerability: Motivation

• It has been demonstrated that bipartite quantum discord is necessary for demonstrating bipartite superunsteerability[1].

• In the tripartite scenario, genuine tripartite quantum discord[2] was defined in order to quantify the genuine quantumness of tripartite quantum states.

• Zhao et. al.[3] has shown that any tripartite state has non-zero genuine tripartite discord iff it has non-zero bipartite discord across all possible bipartitions.

1) D. Das, B. Bhattacharya, C. Datta, A. Roy, C. Jebaratnam, A. S. Majumdar, and R. Srikanth, Phys. Rev. A **97**, 062335 (2018).

2) G. L. Giorgi, B. Bellomo, F. Galve, and R. Zambrini, Phys. Rev. Lett. **107**, 190501 (2011).

3) L. Zhao, X. Hu, R.-H. Yue, and H. Fan, Quantum Inf. Process **12**, 2371 (2013).

Genuine Super-bi-unsteerability

Definition: A tripartite bi-unsteerable correlation is said to be genuinely super-bi-unsteerable iff it is super-bi-unsteerable across all possible bipartitions (i.e., from Alice to Bob-Charlie, from Bob to Alice-Charlie, and from Charlie to Alice-Bob).

• Genuine nonclassicality (in the form of genuine quantum discord) of three-qubit states is necessary for implying genuine super-bi-unsteerability of bi-unsteerable correlations (produced from three-qubit states).

Results: Example of genuine super-bi-unsteerability

Consider the noisy Mermin family given by

$$P_{MF}^{V}(abc|xyz) = \frac{1 + (-1)^{a\oplus b\oplus c\oplus xy\oplus yz\oplus xz}\delta_{x\oplus y\oplus 1,z}V}{8}, \qquad (9)$$

where $0 < V \leq 1$.

Quantum simulation: The noisy Mermin family can be produced when Alice, Bob and Charlie perform appropriate measurements on the $2 \otimes 2 \otimes 2$ noisy GHZ state:

$$\rho_1 = V|GHZ\rangle\langle GHZ| + (1-V)\frac{\mathbb{I}_2}{2} \otimes \frac{\mathbb{I}_2}{2} \otimes \frac{\mathbb{I}_2}{2}, \qquad (10)$$

where $|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$; $0 < V \leq 1$

Results: Example of genuine super-bi-unsteerability

Classical simulation: • The noisy Mermin family can be simulated by a LHV-LHS model (bi-unsteerable from Alice to Bob-Charlie) in one sided device independent scenario for $0 < V \leq \frac{1}{\sqrt{2}}$.

• The above LHV-LHS model cannot be realized with hidden variables having dimension 3 for $V > \frac{1}{\sqrt{5}}$.

• The above LHV-LHS model cannot be realized with hidden variables having dimension 2 or 1 for $V > 0$.

• Hence, the noisy Mermin family demonstrates super-bi-unsteerablity from Alice to Bob-Charlie in one sided device independent scenario for $0 < V \leq \frac{1}{\sqrt{2}}$.

Results: Example of genuine super-bi-unsteerability

<u>Classical simulation:</u> • Since noisy Mermin family is invariant under permutations of parties, the noisy Mermin family demonstrates super-bi-unsteerablity from Bob to Alice-Charlie and from Charlie to Alice-Bob in one sided device independent scenario for $0 < V \leq \frac{1}{\sqrt{2}}$.

• Hence, the noisy Mermin family demonstrates genuine super-bi-unsteerablity in one sided device independent scenario for $0 < V \leq \frac{1}{\sqrt{2}}$.

Results: Example of genuine super-bi-unsteerability

The genuinely super-bi-unsteerable noisy Mermin family certifies the quantumness of

i) the $2 \otimes 2 \otimes 2$ dimensional resource reproducing it in the range $0 < V \leq \frac{1}{\sqrt{2}}$ (e.g. the noisy GHZ state).

ii) the $3 \otimes 2 \otimes 2$ dimensional resource reproducing it in the range $\frac{1}{\sqrt{5}} < V \leq \frac{1}{\sqrt{2}}$ (e.g. the state $\rho_2 = V|GHZ\rangle\langle GHZ| + (1 - V)|2\rangle\langle 2| \otimes \frac{\mathbb{I}_2}{2} \otimes \frac{\mathbb{I}_2}{2}$)

Outlook:

- Whether quantum discord (genuine quantum discord) is *sufficient* for demonstrating superunsteerability (genuine super-bi-unsteerability).
- Generalizing the concept of super-bi-unsteerability in two-sided device independent scenario.
- Investigating information theoretic applications of superunsteerability and super-bi-unsteerability.
- Finding out how to quantify superunsteerability and super-bi-unsteerability.
- Finding out an experimentally testable criteria (like Bell's inequality) to detect superunsteerable and super-bi-unsteerable correlations.

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Thank you.